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基于手征双重模型研究a_0(980)介子对非对称物质的影响

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Editors' Suggestion

A Study of Effects from $a_0(980)$ Meson to Asymmetric Matter Based on a Parity Doublet Model

KONG Yukkei[1], HARADA Masayasu[1,2,3]

(*1. Department of Physics, Nagoya University, Nagoya 464-8602, Japan*;
2. Kobayashi-Maskawa Institute for the Origin of Particles and the Universe, Nagoya University, Nagoya, 464-8602, Japan;
3. Advanced Science Research Center, Japan Atomic Energy Agency, Tokai 319-1195, Japan)

Abstract: In the present work, we study the effect of the isovector scalar meson $a_0(980)$ (also called δ) to the asymmetric matter properties by constructing a parity doublet model with including the $a_0(980)$ meson based on the approximate chiral $U(2)_L \times U(2)_R$ symmetry. The effect of $a_0(980)$ to the properties of symmetric matter such as symmetry energy $S(n_B)$, symmetry incompressibility K_{sym}, symmetry skewness Q_{sym} is investigated. We find that, $a_0(980)$ meson has significant effect to these nuclear matter properties especially when the chiral invariant mass of nucleon m_0 is small. We also study the effect of the $a_0(980)$ meson to the neutron star (NS) properties and find that it increase the radius of NSs, especially those having intermediate mass. Finally, we further constrain the value of the chiral invariant mass of the nucleon to $640\ \text{MeV} \lesssim m_0 \lesssim 860\ \text{MeV}$ for $L_0 = 57.7$ MeV by comparing with the recent accepted value of K_{sym} and neutron star (NS) observational data.

Key words: parity doublet model; neutron star; asymmetric matter; isovector scalar meson $a_0(980)$

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0 Introduction

Chiral symmetry and its spontaneous breaking is one of the most important ingredient in the low-energy hadron physics. The spontaneous chiral symmetry breaking is expected to generate a part of hadron masses, and also causes mass difference between chiral partners. Here we would like to raise a question: How much mass of nucleon is from the spontaneous chiral symmetry breaking? This question can be addressed using, so called parity doublet models for nucleons[1].

In the parity doublet model (PDM)[1−2] an excited nucleon $N(1535)$ is regarded as the chiral partner to the ordinary nucleon, and both of them have a chiral invariant mass. In this model, the mass of nucleon is schematically written as a sum of chiral invariant mass m_0 and the part coming from the spontaneous chiral symmetry breaking. Since the chiral symmetry is expected to be restored in the high density matter while the chiral invariant mass will be intact, we can study the origin of nucleon mass by applying the parity doublet model to study high-density matter.

There are many works which studied high-density matter based on PDMs. Here, we just list several works closely related to the present work: In Ref. [3], a PDM with a six-point interaction of sigma meson was proposed, the result of which shows that the incompressibility $K_0 = 240$ MeV is reproduced for m_0=500~900 MeV. In Ref. [4], a constraint to the chiral invariant mass in the PDM was obtained from the neutron star (NS) properties. In Ref. [5], a unified equation of state (EoS) for NS was constructed by connecting the PDM and an NJL-type quark model, based on the crossover picture in Ref. [6], to obtain a constraint to m_0 from the NS properties. A novel way to obtain the density dependence of the quark condensate was proposed in Ref. [7]. The effect of $U(1)_A$ axial anomaly was studied in Ref. [8]. These analyses based on the PMD-NJL crossover model are summarized in the review published in Ref. [9]. Ref. [10] studies properties of nuclei such as the binding energy and charge radius using the PDM and shows that $m_0 = 700$ MeV is preferred. Recently in Ref. [11], the effect of the isovector scalar meson $a_0(980)$ meson to the nuclear matter and neutron star properties was studied. The PDM was also used to study the hadron-quark phase transition in Ref. [12].

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Biography: KONG Yukkei(1996−), male, Hong Kong, Doctor of Science/Student, working on theoretical hadron physics;
E-mail: yukkekong2-c@hken.phys.nagoya-u.ac.jp

It is an important topic in nuclear and hadron physics to understand the EoS of the dense hadronic matter. Study of the EoS will provide some clues to understand the properties of the high density nuclear matter as well as the neutron star. Symmetry properties of the nuclear matter such as the symmetry energy are very important for the study of asymmetric matter. In this work, we also study the symmetry properties such as the symmetry energy S, the symmetry incompressibility $K_{\rm sym}$, and the symmetry skewness coefficient $Q_{\rm sym}$, focusing on the effects from the isovector scalar meson $a_0(980)$ which mediates the attractive force in the isovector channel. We find that, the $a_0(980)$ meson has a significant effect to the symmetry properties and the asymmetric matter when m_0 is small. Comparing our predictions with the constraints from the NS observation and the terrestrial experiments, we are able to constrain the size of the chiral invariant mass m_0.

This work is organized as follows. The model used in this work is presented in Section 1. Then, we discuss the results of nuclear matter such as the symmetry energy S, symmetry incompressibility $K_{\rm sym}$, and symmetry skewness coefficient $Q_{\rm sym}$, as well as the neutron star properties computed from the present model and the effect of $a_0(980)$ meson to these matter properties in Section 2. We then constrain the chiral invariant mass of nucleon m_0 by comparing the results to the recent accepted constraints to the symmetric properties and the neutron star observations. Finally, we summarize this work in Section 3.

1 A parity doublet model with $a_0(980)$ meson

We construct a parity doublet model with $a_0(980)$ meson based on $\mathrm{U}(2)_L\times\mathrm{U}(2)_R$ chiral symmetry. The effective Lagrangian is given by

$$\mathcal{L}=\mathcal{L}_N+\mathcal{L}_M+\mathcal{L}_V\,, \tag{1}$$

where the Lagrangian is separated into three parts: the Lagrangian for baryons including the Yukawa interactions to the scalar and pseudoscalar mesons expressed by $\mathcal{L}_N$, the Lagrangian for the scalar and pseudoscalar mesons expressed by $\mathcal{L}_M$, and the vector meson Lagrangian for the vector mesons including the Yukawa interaction of the baryons to vector mesons expressed by $\mathcal{L}_V$. In $\mathcal{L}_M$, the scalar and pseudoscalar meson field M is introduced as the $(2,2)_{-2}$ representation under the $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\times\mathrm{U}(1)_A$ symmetry which transforms as

$$M\to \mathrm{e}^{-2\mathrm{i}\theta_A}g_L M g_R^{\dagger}\,, \tag{2}$$

where $g_{R,L}\in\mathrm{SU}(2)_{R,L}$ and $\mathrm{e}^{-2\mathrm{i}\theta_A}\in\mathrm{U}(1)_A$. We parameterize M as

$$M=[\sigma+\mathrm{i}\boldsymbol{\pi}\cdot\boldsymbol{\tau}]-[\boldsymbol{a_0}\cdot\boldsymbol{\tau}+\mathrm{i}\eta], \tag{3}$$

where $\sigma,\boldsymbol{\pi},\boldsymbol{a_0},\eta$ are scalar and pseudoscalar meson fields, and $\boldsymbol{\tau}$ are the Pauli matrices. The vacuum expectation value (VEV) of M is given by

$$\langle 0|M|0\rangle=\begin{pmatrix}\sigma_0 & 0\\ 0 & \sigma_0\end{pmatrix}, \tag{4}$$

where $\sigma_0=\langle 0|\sigma|0\rangle$ is the VEV of the σ field. The explicit form of the Lagrangian $\mathcal{L}_M$ is given by

$$\mathcal{L}_M=\frac{1}{4}\mathrm{tr}\left[\partial_\mu M\partial^\mu M^{\dagger}\right]-V_M\,, \tag{5}$$

where V_M is the potential for M. In the present model, V_M is taken as

$$\begin{aligned}V_M=&-\frac{\bar{\mu}^2}{4}\mathrm{tr}[M^{\dagger}M]+\frac{\lambda_{41}}{8}\mathrm{tr}\left[(M^{\dagger}M)^2\right]-\\&\frac{\lambda_{42}}{16}\left\{\mathrm{tr}[M^{\dagger}M]\right\}^2-\frac{\lambda_6}{12}\mathrm{tr}\left[(M^{\dagger}M)^3\right]-\\&\frac{m_\pi^2 f_\pi}{4}\mathrm{tr}[M+M^{\dagger}]-\frac{K}{8}\left\{detM+detM^{\dagger}\right\}.\end{aligned} \tag{6}$$

For the vector meson, the iso-triplet ρ meson and iso-singlet ω meson are included based on the hidden local symmetry[13−14] to account for the repulsive force in the matter. In this work, we just show the Lagrangian after taking the mean-field approximation later. The complete Lagrangian for the vector mesons is seen in Ref. [15].

Finally, the baryonic Lagrangian $\mathcal{L}_N$ based on the parity doubling structure [1−2] is given by

$$\begin{aligned}\mathcal{L}_N=&\bar{N}_1 i\partial\!\!\!/ N_1+\bar{N}_2 i\partial\!\!\!/ N_2-m_0[\bar{N}_1\gamma_5 N_2-\bar{N}_2\gamma_5 N_1]-\\&g_1[\bar{N}_{1l}MN_{1r}+\bar{N}_{1r}M^{\dagger}N_{1l}]-\\&g_2[\bar{N}_{2r}MN_{2l}+\bar{N}_{2l}M^{\dagger}N_{2r}]\,,\end{aligned} \tag{7}$$

where N_{ir} (N_{il}) ($i=1,2$) is the right-handed (left-handed) component of the nucleon fields N_i. By diagonalizing $\mathcal{L}_N$, we obtain two baryon fields N_+ and N_- corresponding to the positive parity and negative parity nucleon fields, respectively. Their masses at vacuum are obtained as[1−2]

$$m_{\pm}^{(\rm vac)}=\frac{1}{2}\left[\sqrt{(g_1+g_2)^2\sigma_0^2+4m_0^2}\pm(g_1-g_2)\sigma_0\right]. \tag{8}$$

In the present work, N_+ and N_- are identified as $N(939)$ and $N(1535)$ respectively, with m_+ (m_-) to be the mass of $N(939)$ ($N(1535)$).

To construct the finite matter, we adopt the mean-field approximation following[5, 9], by taking

$$\sigma(x)\to\sigma\,,\qquad \pi(x)\to 0\,,\qquad \eta(x)\to 0\,. \tag{9}$$

The $a_0(980)$ mean field is assumed to have non-zero value only in the third axis of iso-spin as

$$a_0^i(x) \to a\delta_{i3}\ . \tag{10}$$

We note that the mean field of $a_0(980)$ vanishes ($a=0$) in the symmetric nuclear matter as well as vacuum, due to the iso-spin invariance. The mean fields for vector mesons are taken as

$$\omega_\mu(x) \to \omega\delta_{\mu 0}, \qquad \rho_\mu^i(x) \to \rho\delta_{\mu 0}\delta_{i3}\ . \tag{11}$$

Redefining the parameters as

$$\begin{aligned}
\bar{\mu}_\sigma^2 &\equiv \bar{\mu}^2 + \frac{1}{2}K,\\
\bar{\mu}_a^2 &\equiv \bar{\mu}^2 - \frac{1}{2}K = \bar{\mu}_\sigma^2 - K,\\
\lambda_4 &\equiv \lambda_{41} - \lambda_{42},\\
\gamma_4 &\equiv 3\lambda_{41} - \lambda_{42},
\end{aligned} \tag{12}$$

we write V_M in terms of the meson mean fields as

$$\begin{aligned}
V_M =& -\frac{\bar{\mu}_\sigma^2}{2}\sigma^2 - \frac{\bar{\mu}_a^2}{2}a^2 + \frac{\lambda_4}{4}(\sigma^4 + a^4) + \frac{\gamma_4}{2}\sigma^2 a^2 - \\
&\frac{\lambda_6}{6}(\sigma^6 + 15\sigma^2 a^4 + 15\sigma^4 a^2 + a^6) - m_\pi^2 f_\pi \sigma.
\end{aligned} \tag{13}$$

The Lagrangian $\mathcal{L}_V$ is written in terms of the mean fields of the vector mesons as

$$\begin{aligned}
\mathcal{L}_V =& -g_\omega \sum_{\alpha j} \bar{N}_{\alpha j}\gamma^0 \omega N_{\alpha j} - g_\rho \sum_{\alpha j} \bar{N}_{\alpha j}\gamma^0 \frac{\tau_3}{2}\rho N_{\alpha j} + \\
&\frac{1}{2}m_\omega^2\omega^2 + \frac{1}{2}m_\rho^2\rho^2 + \lambda_{\omega\rho} g_\omega^2 g_\rho^2 \omega^2\rho^2.
\end{aligned} \tag{14}$$

We note that the ω-ρ mixing is included in our model to reduce the slope parameter following Ref. [8].

Now, the thermodynamic potential for the nucleons is written as

$$\Omega_N = -2\sum_{\alpha=\pm, j=p,n} \int^{k_f} \frac{\mathrm{d}^3 p}{(2\pi)^3}\left[\mu_j^* - E_{\alpha j}\right], \tag{15}$$

where $\alpha = \pm$ denotes the parity and $j = p, n$ the iso-spin of nucleons. μ_j^* is the effective chemical potential given by

$$\mu_j^* \equiv (\mu_B - g_\omega\omega) + \frac{j}{2}(\mu_I - g_\rho\rho)\ , \tag{16}$$

and $E_{\alpha j}$ is the energy of the nucleon defined as $E_{\alpha j} = \sqrt{(\boldsymbol{p})^2 + (m_{\alpha j}^*)^2}$ where $\boldsymbol{p}$ and $m_{\alpha j}^*$ are the momentum and the effective mass of the nucleon. The effective mass $m_{\alpha j}^*$ is given by

$$m_{\alpha j}^* = \frac{1}{2}\left[\sqrt{(g_1+g_2)^2(\sigma - ja)^2 + 4m_0^2} + \alpha(g_1 - g_2)(\sigma - ja)\right]. \tag{17}$$

We stress that the masses of proton and neutron become non-degenerate in the asymmetric matter due to the non-zero mean field of $a_0(980)$.

By combining the above contributions, the entire thermodynamic potential for hadronic matter is expressed as

$$\begin{aligned}
\Omega_H =& \Omega_N \frac{\bar{\mu}_\sigma^2}{2}\sigma^2 - \frac{\bar{\mu}_a^2}{2}a^2 + \frac{\lambda_4}{4}(\sigma^4 + a^4) + \frac{\gamma_4}{2}\sigma^2 a^2 - \\
&\frac{\lambda_6}{6}(\sigma^6 + 15\sigma^2 a^4 + 15\sigma^4 a^2 + a^6) - \\
&m_\pi^2 f_\pi \sigma - \frac{1}{2}m_\omega^2\omega^2 - \frac{1}{2}m_\rho^2\rho^2 - \lambda_{\omega\rho} g_\omega^2 g_\rho^2 \omega^2\rho^2 - \Omega_0\ ,
\end{aligned} \tag{18}$$

where we subtracted the potential at the vacuum

$$\Omega_0 \equiv -\frac{\bar{\mu}_\sigma^2}{2}f_\pi^2 + \frac{\lambda_4}{4}f_\pi^4 - \frac{\lambda_6}{6}f_\pi^6 - m_\pi^2 f_\pi^2. \tag{19}$$

2 Results

In the present model, there are 10 parameters to be determined for a given value of the chiral invariant mass m_0: $g_1, g_2, \bar{\mu}_\sigma^2, \bar{\mu}_a^2, \lambda_4, \gamma_4, \lambda_6, g_\omega, g_\rho, \lambda_{\omega\rho}$. We determine them from the vacuum properties as well as nuclear saturation properties as follows: The vacuum expectation value of σ is taken to be $\sigma_0 = f_\pi$ with the pion decay constant $f_\pi = 92.4$ MeV. The Yukawa coupling constants g_1 and g_2 are determined to reproduce the nucleon masses at vacuum given by Eq. (8), with $m_+ = m_N = 939$ MeV and $m_- = m_{N^*} = 1\,535$ MeV. The parameters $\bar{\mu}_\sigma^2$, λ_4, λ_6 and g_ω are determined from the saturation density n_0, binding energy B_0 and incompressibility K_0 listed in Table 1 together with the vacuum condition: $-\bar{\mu}^2 f_\pi + \lambda_4 f_\pi^3 - \lambda_6 f_\pi^5 - m_\pi^2 f_\pi = 0$. Then, $\bar{\mu}_a^2 = \bar{\mu}_\sigma^2 - K$ is determined from K given by

$$K = m_\eta^2 - m_\pi^2\ . \tag{20}$$

Similarly, γ_4 is given by

$$\gamma_4 = \frac{m_a^2 + 5\lambda_6 f_\pi^4 + \bar{\mu}_a^2}{f_\pi^2}\ , \tag{21}$$

from the mass of $a_0(980)$. We summarize the values of masses of mesons in Table 2.

The values of the parameters g_1, g_2, $\bar{\mu}_\sigma^2$, λ_4, λ_6, g_ω for given values of the chiral invariant mass m_0 are shown in Table 3.

Table 1 Saturation properties used to determine the model parameters: saturation density n_0, binding energy B_0, incompressibility K_0, symmetry energy S_0, and slope parameter L_0.

n_0/fm^{-3}	B_0/MeV	K_0/MeV	S_0/MeV	L_0/MeV
0.16	16	240	31	57.7

Table 2 Values of meson masses at vacuum in unit of MeV.

π	$a_0(980)$	η	ω	ρ
140	980	550	783	776

Table 3　Values of g_1, g_2, $\bar{\mu}_\sigma^2$, λ_4, λ_6, g_ω for m_0 = 500 ~ 900 MeV.

m_0/MeV	500	600	700	800	900
g_1	9.02	8.48	7.81	6.99	5.96
g_2	15.47	14.93	14.26	13.44	12.41
$\bar{\mu}_\sigma^2/f_\pi^2$	22.70	22.35	19.28	11.93	1.50
λ_4	41.94	40.39	35.46	23.12	4.43
$\lambda_6 f_\pi^2$	16.94	15.75	13.89	8.89	0.64
g_ω	11.34	9.13	7.30	5.66	3.52
$\bar{\mu}_a^2/f_\pi^2$	−10.43	−10.79	−13.86	−21.21	−31.64
γ_4	191.41	185.07	172.70	140.38	88.67

Tables 4 and 5 show the values of the parameters g_ρ and $\lambda_{\omega\rho}$ that are determined from fitting S_0 and L_0. We note that the existence of ω-ρ mixing interaction allows to tune the slope parameter L_0 in the present model. Reference [16] shows that $L_0 = 57.7 \pm 19$ MeV, so we take $L_0 = 40-80$ MeV as the physical input to compare the effect of $a_0(980)$. In this work, we just show the results for $L_0 = 57.7$ MeV. The results for other choices of L_0 are seen in Ref. [11].

Table 4　g_ρ of model with a_0 (980) meson.

m_0/MeV	500	600	700	800	900
L_0 = 40 MeV	19.43	15.52	13.89	12.64	11.40
L_0 = 50 MeV	18.75	15.03	13.35	12.00	10.69
L_0 = 57.7 MeV	18.54	14.84	13.08	11.63	10.24
L_0 = 60 MeV	18.14	14.59	12.87	11.45	10.09
L_0 = 70 MeV	17.58	14.18	12.44	10.97	9.59
L_0 = 80 MeV	17.07	13.81	12.05	10.54	9.15

Table 5　$\lambda_{\omega\rho}$ of model with a_0 (980) meson.

m_0/MeV	500	600	700	800	900
L_0 = 40 MeV	0.012 7	0.025 1	0.076 1	0.291 6	2.459 5
L_0 = 50 MeV	0.011 9	0.022 1	0.064 9	0.241 5	1.942 7
L_0 = 57.7 MeV	0.011 3	0.019 9	0.056 3	0.203 4	1.550 7
L_0 = 60 MeV	0.011 0	0.019 2	0.053 7	0.191 4	1.425 9
L_0 = 70 MeV	0.010 1	0.016 2	0.042 5	0.141 3	0.909 1
L_0 = 80 MeV	0.009 2	0.013 2	0.031 3	0.091 1	0.392 3

To compare how the matter properties are affected by $a_0(980)$ meson, we eliminate the $a_0(980)$ meson by taking the mean field $a = 0$ in the potential (18) and the masses in Eq. (17). The parameters are determined by fitting them to nuclear saturation properties and vacuum properties. We note that, in the model without $a_0(980)$, the model parameters g_1, g_2, $\bar{\mu}_\sigma^2$, λ_4, λ_6 are the same as in the $a_0(980)$ model, since these parameters are determined from the properties of symmetric matter. We also note that γ_4-terms do not exist in the model without $a_0(980)$ which takes account of the cross interactions between σ and $a_0(980)$. We summarize the values of the parameters g_ρ and $\lambda_{\omega\rho}$ in the model without a_0 meson in Tables 6 and 7.

Table 6　g_ρ of model without a_0 (980) meson.

m_0/MeV	500	600	700	800	900
L_0 = 40 MeV	12.48	10.99	10.72	10.64	10.61
L_0 = 50 MeV	10.72	10.01	9.91	9.90	9.91
L_0 = 57.7 MeV	9.78	9.40	9.39	9.42	9.46
L_0 = 60 MeV	9.54	9.24	9.25	9.29	9.34
L_0 = 70 MeV	8.68	8.63	8.71	8.78	8.86
L_0 = 80 MeV	8.02	8.13	8.26	8.35	8.44

Table 7　$\lambda_{\omega\rho}$ of model without a_0 (980) meson.

m_0/MeV	500	600	700	800	900
L_0 = 40 MeV	0.055 4	0.085 7	0.169 5	0.415 9	2.518 6
L_0 = 50 MeV	0.045 1	0.067 1	0.130 1	0.315 3	1.890 3
L_0 = 57.7 MeV	0.037 2	0.052 9	0.099 8	0.237 8	1.406 4
L_0 = 60 MeV	0.034 8	0.048 6	0.090 7	0.214 7	1.261 9
L_0 = 70 MeV	0.024 5	0.030 1	0.051 3	0.114 0	0.633 6
L_0 = 80 MeV	0.014 2	0.011 6	0.012 0	0.013 4	0.005 2

Let us first study the effect of $a_0(980)$ to the symmetry energy. The results are shown in Fig. 1. We observe that, in most cases, the symmetry energy is stiffened by the existence of $a_0(980)$ and the difference of the symmetry energy between the model with $a_0(980)$ indicated by solid curves and the models without $a_0(980)$ by dashed curves is larger for smaller m_0. At $n_B = 2n_0$, the symmetry energy $S(2n_0)$ can be enlarged as large as 45% in the a_0 model when $m_0 = 500$ MeV. This increase of symmetry energy can be understood as a result of the competition between the repulsive ρ meson interaction (modified by the ω-ρ mixing interaction) and the attractive $a_0(980)$ interaction, in addition to the kinetic contribution from the nucleons. In the model without $a_0(980)$, on the other hand, only repuls-

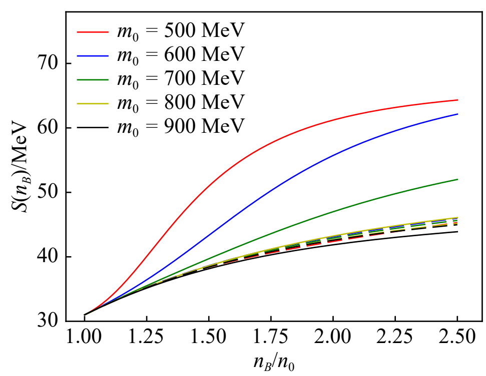

Fig. 1　(color online) Symmetry energy $S(n_B)$ for $m_0 = 500$ ~ 900 MeV and $L_0 = 57.7$ MeV. Solid curves represent the $S(n_B)$ of the model including $a_0(980)$, while dashed curves show the results of the model without $a_0(980)$.

ive contribution from the ρ meson exists. Since the symmetry energy at saturation density is fixed as $S_0 = 31$ MeV in both the models with and without $a_0(980)$ meson, the ρ meson coupling g_ρ is strengthened by the existence of the attractive $a_0(980)$ contribution in the model with a_0 comparing to the model without a_0. Actually, from Tables 4 and 6 it is clear that g_ρ is larger in the a_0 model than in the no-a_0 model for a fixed m_0.

The symmetry incompressibility $K_{\rm sym}$ and the symmetry skewness coefficient $Q_{\rm sym}$ are important properties to study the asymmetric matter. It is highly related to the asymmetric matter EoS and symmetry energy at higher density $n_B > n_0$. $K_{\rm sym}$ and $Q_{\rm sym}$ are defined as the coefficients of the Taylor expansion of symmetry energy $S(n_B)$ at density around n_0 as

$$S(n_B) = S_0 + \left(\frac{n_B - n_0}{n_0}\right)\frac{L_0}{3} + \left(\frac{n_B - n_0}{n_0}\right)^2 \frac{K_{\rm sym}}{18} + \left(\frac{n_B - n_0}{n_0}\right)^3 \frac{Q_{\rm sym}}{162} + O(n_B^4), \tag{22}$$

where

$$K_{\rm sym} = 9n_0^2 \left.\frac{\partial^2 S}{\partial n_B^2}\right|_{n_0}, \quad Q_{\rm sym} = 27n_0^3 \left.\frac{\partial^3 S}{\partial n_B^3}\right|_{n_0}. \tag{23}$$

Figure 2 shows the m_0 dependence of $K_{\rm sym}$ in the present models with and without a_0 meson for different L_0 together with the recent accepted value $K_{\rm sym} = -107 \pm 88$ MeV given in Ref. [16] which is shown by pink region. We observe that the presence of a_0 meson has strong effect on the $K_{\rm sym}$. In particular, the $K_{\rm sym}$ becomes very large when $m_0 \lesssim 600$ MeV in the a_0 model and reach $K_{\rm sym} \approx 1\,200$ MeV when $m_0 = 500$ MeV for $L_0 = 57.7$ MeV. We note that the $K_{\rm sym}$ become positive when m_0 is small. This seems to imply that the pure neutron matter is more stable than symmetric nuclear matter at saturation when m_0 is small. By

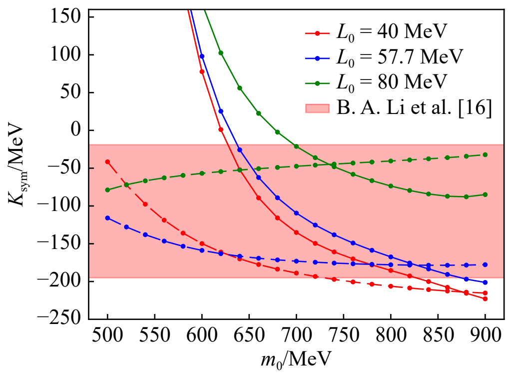

Fig. 2　(color online) m_0 dependence of $K_{\rm sym}$ with different $L_0 = 40, 57.7, 80$ MeV. Solid curves represent the results from the model with a_0 meson and dashed curves are from the model without a_0 meson. The pink region shows the recent accepted value of $K_{\rm sym}$ as summarized in Ref. [16].

comparing with the constraint shown by pink region, the present results seem to give strong constraint to the value of m_0 as $640 \lesssim m_0 \lesssim 860$ MeV for $L_0 = 57.7$ MeV. We also note that larger L_0 results in larger $K_{\rm sym}$ and the effect of L_0 is much smaller than the effect of m_0 when m_0 is small ($m_0 \lesssim 600 \sim 700$ MeV) in the present a_0 model. However, when m_0 is large where the effect of m_0 is small, L_0 effect dominates.

Figure 3 shows the m_0 dependence of $Q_{\rm sym}$ of the present models with and without a_0 meson for different L_0 together with the range of $Q_{\rm sym}$ calculated in the models with Skyrme parameterization[17] which is shown by blue region. Similar to the $K_{\rm sym}$, the isovector scalar a_0 have huge effect to the $Q_{\rm sym}$. While $Q_{\rm sym}$ can be negative for small m_0 in the model without a_0 meson, $Q_{\rm sym}$ becomes very large in the a_0 model when m_0 becomes small. When $m_0 = 500$ MeV, $Q_{\rm sym} \approx 12\,000$ MeV in the present model which is very large comparing to other models with Skyrme parameterization. The m_0 dependence of $K_{\rm sym}$ and $Q_{\rm sym}$ also agree with the fact that the matter becomes softer when m_0 is smaller. When $m_0 \gtrsim 700$ MeV, the effect of m_0 reduces rapidly and L_0 effect dominates. We also note that increasing L_0 reduces $Q_{\rm sym}$ in the a_0 model. We see that the results from the present model with $L_0 = 57.7$ MeV agree with those from other models shown by blue region when $m_0 \gtrsim 700$ MeV. While $Q_{\rm sym}$ is very difficult to be measured in the experiment, we believe that further experimental constraint on the $Q_{\rm sym}$ will bring valuable insight to the understanding of asymmetric matter as well as the chiral invariant mass of nucleon.

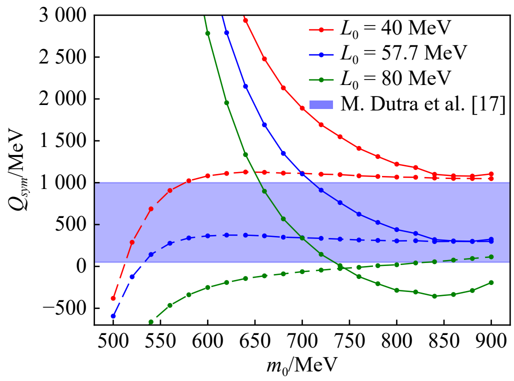

Fig. 3　(color online) m_0 dependence of $Q_{\rm sym}$ with different L_0. Solid curve represents the model with a_0 meson and dashed curve represents the model without a_0. The blue region is the recent accepted value of $Q_{\rm sym}$ as summarized in Ref. [17].

We also study the effect of $a_0(980)$ meson to the neutron star properties by constructing a unified EoS using the interpolation method as introduced in Ref. [6]. We smoothly connect the hadronic matter EoS to the quark matter EoS constructed from an NJL-type quark model, assuming a crossover phase transition. Then, we compute the

M-R relation by solving the Tolman-Oppenheimer-Volkoff (TOV) equation[18−19], and compare the results with the observational data. We first examine how the existence of $a_0(980)$ affects to the M-R relation. In Fig. 4, we show some typical examples of the M-R relations computed with and without the existence of $a_0(980)$ for $m_0 = 500 \sim 900$ MeV where $L_0 = 57.7$ MeV is taken. This clearly shows that inclusion of the $a_0(980)$ meson increases the radius for the neutron stars with the mass of $0.5 \lesssim M/M_\odot \lesssim 2$, by the amount of $\lesssim 1$ km depending on the parameters. We see that the difference becomes smaller for larger m_0 similarly to the one for the symmetry energy. This is because the $a_0(980)$ meson couples to the matter weaker for larger m_0. In addition, we observe that the $a_0(980)$ meson has a little effect on the maximum mass, since the core of such heavy neutron star includes the quark matter and the maximum mass is mainly determined by the parameters of the NJL-type quark model.

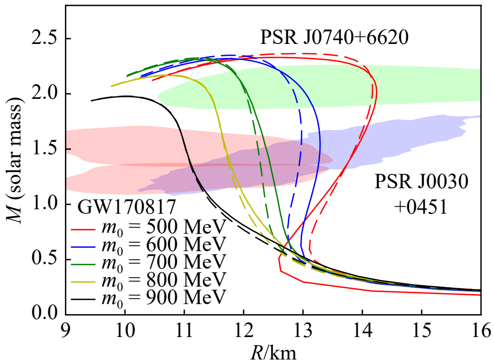

Fig. 4 (color online) M-R relations computed from models with and without the existence of $a_0(980)$ for different m_0 with $L_0 = 57.7$ MeV. Solid curves represent the M-R relations from the model with $a_0(980)$ meson and dashed curves the ones of the model without $a_0(980)$.

We compare the results with the observational constraints on neutron star mass and radius obtained from PSR J0030+0451[20], PSR J0740+6620[21], and GW170817[22], which restricts the value of the chiral invariant mass as

$$580 \text{ MeV} \lesssim m_0 \lesssim 860 \text{ MeV}, \tag{24}$$

for $L_0 = 57.7$ MeV. This should be compared with the constraint obtained in the model without $a_0(980)$: $540 \text{ MeV} \lesssim m_0 \lesssim 870 \text{ MeV}$. We see that the constraint is slightly changed by the inclusion of $a_0(980)$ meson.

Since the K_{sym} is determined from the nuclear matter at saturation density while the NS observational constraints are at high density, K_{sym} and NSs provide independent constraints at high and low density to the m_0. Figure 5 shows the constraints to m_0 from K_{sym} and the neutron star observations in the present model with a_0 meson, as a function of L_0. When we require both constraints are satisfied, m_0 is constrained as

$$640 \text{ MeV} \lesssim m_0 \lesssim 860 \text{ MeV}, \tag{25}$$

for $L_0 = 57.7$ MeV. We note that due to the difficulty of determining Q_{sym} experimentally, we do not obtain any constraints to m_0 from Q_{sym} at this moment. We expect that future experimental study on Q_{sym} will provide further constraint to m_0.

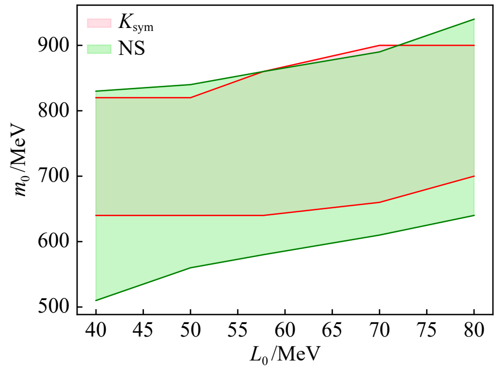

Fig. 5 (color online) Summary of the constraints on m_0 from K_{sym} and NSs with different L_0 in the a_0 model. Pink region is the constraint from the recent accepted value of K_{sym} as summarized in Ref. [16]. Green region is the constraint from the NS observations PSR J0030+0451[20], PSR J0740+6620[21], and GW170817[22].

3 Summary

In this work, we studied the effect of isovector scalar meson $a_0(980)$ to the nuclear matter and neutron star matter properties. In particular, we focused on studying the asymmetric matter properties such as symmetry energy, symmetry incompressibility K_{sym}, symmetry skewness coefficient Q_{sym}, and the neutron star M-R relation. We find that, $a_0(980)$ has significant impact on the asymmetric matter: The symmetry energy in the high density as well as K_{sym} and Q_{sym} is largely enhanced when m_0 is small. The EoS of the neutron star matter is stiffened, which results in the increase of the radius of the neutron star. Finally, we constrain the value of the chiral invariant mass to $580 \text{ MeV} \lesssim m_0 \lesssim 860 \text{ MeV}$ for $L_0 = 57.7$ MeV by comparing with the recent observational data on neutron stars. The constraint is further restricted to $640 \text{ MeV} \lesssim m_0 \lesssim 860 \text{ MeV}$ for $L_0 = 57.7$ MeV by considering the constraint from K_{sym}. We believe that further experimental constraint on the asymmetric matter properties such as K_{sym}, Q_{sym}, and $S(n_B > n_0)$ will bring valuable insight to the understanding of asymmetric matter as well as the constraint of chiral invariant mass of nucleon.

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基于手征双重模型研究 $a_0(980)$ 介子对非对称物质的影响

江育基[1,1)], 原田正康[1,2,3]

(1. 名古屋大学物理系，日本 名古屋 464-8602；
2. 名古屋大学小林-益川基本粒子与宇宙起源研究所，日本 名古屋 464-8602；
3. 日本原子能研究开发机构先端基础研究中心，日本 茨城 319-1195)

摘要: 在本工作中，通过基于近似 $U(2)_L \times U(2)_R$ 对称性构建包含 $a_0(980)$ 介子的手征双重模型，研究了同位旋矢量标量介子 $a_0(980)$（即 δ）对不对称物质性质的影响，并研究了 $a_0(980)$ 对对称物质性质的影响，如对称能 $S(n_B)$、对称能不可压缩系数 K_{sym}、对称能偏斜系数 Q_{sym}。发现 $a_0(980)$ 介子对这些核物质性质有显著影响，特别是在核子手征不变质量 m_0 较小的情况下。还研究了 $a_0(980)$ 介子对中子星 (NS) 性质的影响，发现它增加了 NS 的半径，特别是那些具有中等质量的中子星。最后，通过与最近约束的 K_{sym} 和 NS 观测数据比较，进一步将 L=57.7 MeV 时核子的手征不变质量值限制在 640 MeV<m_0<860 MeV。

关键词: 手征双重模型；中子星；非对称核物质；同位旋矢量标量介子 $a_0(980)$

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1) E-mail: yukkekong2-c@hken.phys.nagoya-u.ac.jp