INTERNATIONAL ATOMIC ENERGY AGENCY

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

INFORMAL MEETING ON RENORMALIZATION THEORY

25 - 29 August 1969

(SUMMARIES)

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MIRAMARE - TRIESTE

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REFERENCES

I. A. Visconti, Phys. Rev. Letters 21, 652 (1968); Centre de Physique Théorique, Marseille, preprint 68/P. 244.

II. A. Visconti and Y. Le Gaillard, Nuovo Cimento 34, 914 (1964);
Y. Le Gaillard and A. Visconti, J. Math. Phys. 6, 1774 (1965);
J. Soffer and A. Visconti, Nuovo Cimento 38, 917 (1965);
J. Soffer and A. Visconti, Phys. Rev. 162, 1386 (1967);
J. Calmet, J. Soffer, R. Seneor and A. Visconti, Phys. Rev. 162, 1390 (1967).

III. J. Calmet and M. Perrottet, Centre de Physique Théorique, Marseille, preprint 69/P. 284.

The reader will find a complete bibliography of the subject in the above-mentioned papers.

hand, one needs a formulation which algebrized completely all the renormalization processes; this has been achieved by using the functional formalism of field theory (J. Calmet, Y. Le Gaillard, J. Soffer, A. Visconti). In this formalism, one introduces fictitious external sources, and one obtains a generating functional of the propagators defined as vacuum expectation values of time-ordered products.

Then, it may be shown that the renormalization of the generating functional leads to renormalized propagators and that it also leads to an algebraic scheme which is well adapted for programming.

III. The problems which have to be solved for a numerical evaluation of radiative corrections are of three kinds:

a) construction of Feynman diagrams at a given order and their related counter-terms,

b) integration on internal variables and simple renormalization,

c) numerical evaluation of the integrals over Feynman parameters.

This programme (J. Calmet, M. Perrottet), which is complex enough, is in full development: it has been completed for the $g\varphi^3$ theory (which has no physical content but bears some analogy with quantum electrodynamics as far as the form of the final integrations are concerned) and several important programmes for quantum electrodynamics are now running.

The languages which have been used are the LISP language for all formal calculations and FORTRAN language for all numerical evaluations. Two computers, a Univac 1108 and an IBM 360-65, were at our disposal.

CONTENTS

RENORMALIZATION GROUP AND NUMERICAL EVALUATION OF RADIATIVE CORRECTIONS

A. VISCONTI

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The work on which I am going to report may be divided into three parts; its ultimate goal is the evaluation of radiative corrections of any renormalizable field theory using computers.

I. The first part (A. Visconti) deals with generalized renormalization transformations and with the study of the geometry of such transformations defined, roughly speaking, as linear combinations of radiative corrections. The motivation of this work arose partly from considerations on renormalization theory (where in the last analysis one replaces divergent radiative corrections by linear combinations of lower order which are chosen such that the final result remains finite) and partly from considerations on the renormalization group which has been used with success for the study of the asymptotic behaviour of propagators and for the summation of certain classes of graphs.

Rather than elaborating on the applications just mentioned, this work is intended to characterize some fundamental aspects of the former methods. As a first step, we attempt to give a definition of the renormalization transformations general enough to include all the different cases in which such transformations enter into play. Then, one looks for conditions under which such transformations build up a group and one studies its linear representations. Finally, one investigates the connections between such transformations and the renormalization of a renormalizable field theory.

II. As far as the numerical evaluation of radiative corrections is concerned, on one hand, we need to feed in the former framework more information than that already contained. This may be done by considering a definite theory defined by a Lagrangian as quantum electrodynamics or a scalar field interacting with itself ($g\varphi^3$ or $g\varphi^4$ theory). On the other

\- - -

9) K.E. Eriksson, Nuovo Cimento 19, 1010 (1961).

10) K.T. Mahanthappa, Phys. Rev. 126, 329 (1961).

11) N.N. Bogolubov and D.V. Shirkov, "Introduction to the Theory of Quantized Fields" (London 1959), §43.3, 44.2.

12) A. Peres, Nuovo Cimento 35, 322 (1965).

13) J. Schwinger, Phys. Rev. 91, 728 (1953);
R.J. Glauber, Phys. Rev. 130, 2529 (1963).

14) R.J. Glauber, Phys. Rev. 131, 2766 (1963).

15) V. Chung, Phys. Rev. 140, B1110 (1965).

16) T.W.B. Kibble, J. Math. Phys. 9, 315 (1968).

17) P. Planchard, unpublished thesis.

18) J.R. Klauder and J. McKenna, J. Math. Phys. 6, 68 (1965);
J.R. Klauder, J. McKenna and E.J. Woods, J. Math. Phys. 7, 822 (1966).

19) J.K. Storrow, Nuovo Cimento 54A, 15 (1968).

20) T.W.B. Kibble, Phys. Rev. 173, 1527 (1968); 174, 1882 (1968); 175, 1624 (1968).

21) S. Weinberg, Phys. Rev. 140, B516 (1965).

F O R E W O R D

Considerable progress has recently been made in understanding renormalization in field theory. There are perhaps four major areas where deeper understanding has been achieved:

1) Appearance of infinities and necessity of renormalization for exact solutions of field equations in two and three dimensions.

2) Equivalence of different methods of renormalization - the Dyson-Salam method, the Bogolubov-Parasiuk method and the method of analytic renormalization.

3) Renormalization of theories with non-polynomial Lagrangians - with major progress coming from the Dubna school.

4) Renormalization of axial-vector and other currents and of theories with broken symmetries.

The topical conference held by the International Centre for Theoretical Physics brought together about one hundred specialists. No formal Proceedings will be published but abstracts of the talks, with as complete references as possible, have been rapidly issued to mark the range of the topics covered. For copies of the papers, inquiries may be addressed direct to the authors.

ABDUS SALAM

for the distortion phase $\varphi(t)$ is the replacement of $|\underline{v}_i - \underline{v}_j|$ by the relativistic relative velocity u_{ij} .

An S-matrix formalism may be developed for transitions taking place between the finite but large times T ($\lambda T \text{ Min}(1 - v_i, u_{ij}) \gg 1$)

$$S_T = U(\lambda, -T)\, \bar{S}_\lambda\, U(\lambda, T)^\dagger$$

where $\bar{S}_\lambda$ is the S-matrix[21)] of the short-range interaction Hamiltonian $H_I - H_{IR}$.

REFERENCES

1) W. Gordon, Z. Physik 48, 180 (1928);
N.F. Mott and H.S.W. Massey, "The Theory of Atomic Collisions" 2nd ed. (Oxford 1949, 1950), Ch. III.

2) See, for instance, E. Merzbacher, "Quantum Mechanics" (New York 1961).

3) F. Bloch and A. Nordsieck, Phys. Rev. 52, 54 (1937).

4) R.J. Glauber, Phys. Rev. 84, 395 (1951).

5) J. Schwinger, Phys. Rev. 75, 898 (1949).

6) J.M. Jauch and F. Rohrlich, Helv. Phys. Acta 27, 613 (1954).

7) E.L. Lomon, Nucl. Phys. 1, 101 (1956);
N. Nakanishi, Progr. Theoret. Phys. (Kyoto) 19, 159 (1958);
S. Okubo, Nuovo Cimento 18, 70 (1960).

8) D. Yennie, S. Frautschi and H. Suura, Ann. Phys. (N.Y.) 13, 379 (1961). This article contains many references to the earlier literature.

THE LOGICAL NEED FOR RENORMALIZATION

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Non-relativistic quantum mechanics is a much more satisfactory theory than relativistic quantum mechanics. The former is a complete and general theory with the same power as classical mechanics and forms a natural generalization of classical mechanics. The latter consists of a number of special methods with limited applicability and runs into endless trouble when one tries to extend its domain. It is evident that we do not yet have the correct foundation for relativistic quantum mechanics.

One way of proceeding under these circumstances is to take non-relativistic quantum mechanics and try to apply it to higher and higher energies, making it conform to relativity with greater and greater accuracy. We are then working on a sound basis. We start with quantum electrodynamics with a cut-off, which forms a reliable theory for low-energy physics. The cut-off is necessary to make the equations sensible. It destroys the Lorentz invariance. But it preserves approximate Lorentz invariance for low-energy processes.

Proceeding on these lines, one finds that the vacuum state, defined as the stationary state of lowest energy, is not the same as the no-particle state. They are not even approximately the same. One might think it is necessary to determine the vacuum state, i.e., to calculate the wave function representing it. The calculation would be excessively complicated and, further, the result would depend strongly on the cut-off, so it could not be considered as physically significant, as the position and form of the cut-off are unknown and arbitrary.

The calculation of the vacuum state is not really necessary because it would not yield any result comparable with experiment. It is only departures from the vacuum state that are interesting. These are best treated by studying the excitation operators that have to be applied to the

The Hamiltonian is $H = (H_0 + H_{IR}) + (H_I - H_{IR})$ where the infra-red (or infinite range) part H_{IR} is taken together with the free part H_0. The term $H_I - H_{IR}$ vanishes for infinite space-like separation (after ultra-violet renormalization). The spin does not affect the interaction with the soft radiation field. The Hamiltonian H_{IR} is constructed in the Coulomb gauge and the interaction picture with an infra-red radius λ,

$$H_{IR}(t) = \int d k \omega^{-1} a_0^*(\underline{k}, t)\, s_0(\underline{k}, t) - \frac{1}{i}[(a^\dagger, s(t)) - s^*(t), a)] + O(\lambda) \ ;$$

$$s(\underline{k}, t) = i e (2\pi)^{-3/2}\, \theta(\lambda - \omega) \sum_{i=1}^{N} Q_i\, e^{-i(\omega - \underline{v} \cdot \underline{k})t} \left(1, \underline{v}_i - \frac{\underline{v}_i \cdot \underline{k}}{\underline{k}^2}\, \underline{k}\right) \ .$$

$$dk = \frac{d^3 k}{2\omega} \ ; \ (f^*, g) = \int d k\, \underline{f}^*(\underline{k}) \cdot \underline{g}(\underline{k}) \ ; \ \ \underline{k} \cdot \underline{f}(\underline{k}) = \underline{k} \cdot \underline{g}(\underline{k}) = 0 \ .$$

Where Q_i and $\underline{v}_i$ are the charges and the velocities.

The solution of the Schrödinger equation is simplified by the fact that $[H_{IR}(t) \, , [H_{IR}(t') \, , H_{IR}(t'')]\,] = 0$ and gives the result

$$\begin{cases} |\,\rangle_t = U(\lambda, t)\,|\,\rangle_0 \ ; \\ U(\lambda, t) = e^{i\varphi(t)}\, e^{(a^\dagger, S(t)) - (S^*(t), a)} \ . \end{cases}$$

$$\begin{cases} \varphi(t) = \frac{t}{|t|} \ \frac{\alpha}{2} \sum_{i,j=1}^{N} \left((1 - \delta_{ij}) \frac{Q_i Q_j}{u_{ij}} \ \ln|\lambda t| + b(\underline{v}_i, \underline{v}_j) \right) ; \\ u_{ij} = \frac{1}{1 - \underline{v}_i \cdot \underline{v}_j} \ [(\underline{v}_i - \underline{v}_j)^2 + (\underline{v}_i \cdot \underline{v}_j)^2 - \underline{v}_i^2\, \underline{v}_j^2]^{\frac{1}{2}} \ ; \\ \underline{S}(\underline{k}, t) = - \int_0^t dt'\, \underline{s}(\underline{k}, t) = - e (2\pi)^{-3/2}\, \theta(\lambda - \omega) \sum_{i=1} Q_i\, \frac{1 - e^{-i(\omega - \underline{v} \cdot \underline{k})t}}{\omega - \underline{v}_i \cdot \underline{k}} \left(\underline{v}_i - \frac{\underline{v}_i \cdot \underline{k}}{\underline{k}^2}\, \underline{k}\right) \end{cases}$$

where $b(\underline{v}_i, \underline{v}_j)$ can be given by an analytic though lengthy expression in terms of $\underline{v}_i$ and $\underline{v}_j$. The main change from the non-relativistic result vacuum state to produce the departures. The excitation operators are governed by Heisenberg equations of motion. We are thus led to seek solutions of the Heisenberg equations of motion rather than solutions of the Schrödinger equation applying to individual states.

When we proceed to solve the Heisenberg equations of motion, we find that there are some solutions that are insensitive to the cut-off. These are the solutions in which we take the operator of creation of an electron at a certain time and see how it varies with the time. Such calculations, in which one takes into account a static electric or magnetic field, lead to the Lamb shift and anomalous magnetic moment, by a procedure that is logical throughout.

These calculations show definitely the need for renormalization. If g is the cut-off energy, we get $\delta m/m$ and $\delta e/e$ of the order

$\frac{e^2}{2\pi\hbar c} \log \frac{2g}{mc^2}$. It is necessary, for the calculations to be logical, that $\delta m/m$ and $\delta e/e$ shall be small, so that terms involving them can be treated as perturbations. We can make g go up to 10^9 volts and still keep them small, of the order 0.03 . But we must not make $g \to \infty$.

Many physicists like to make $g \to \infty$ and believe that by so doing they get a relativistic theory. But such calculations cannot be made logical.

If one takes the operator of creation of a photon and sees how it varies with time, one is led to a solution of the Heisenberg equations that depends strongly on the cut-off. It corresponds to the photon having a rest-mass of order g^2/mc^2 . One must compensate this mass by introducing a suitable counter-term into the Hamiltonian. Such modifications in the Hamiltonian are permissible provided they can be carried through consistently. The counter-term involves only the field variables, so it does not affect the one-electron calculations.

With a particular g there is still a great deal of arbitrariness in the form of the cut-off. It is possible to arrange the cut-off so that the theory is accurately gauge invariant. The transformation to the Coulomb

ASYMPTOTIC STATES IN QUANTUM ELECTRODYNAMICS

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The infinite range of electromagnetic forces (r^{-1} potential) and the vanishing photon mass give rise to i) distortions of incoming and outgoing waves from the plane-wave form, well known in the case of non-relativistic treatment of Coulomb interaction [1),2)] and ii) the radiation of an infinite number of soft photons in any scattering process except forward scattering.

Consequences of these peculiar features are infinite forward and total cross-sections, infra-red divergences in calculations of radiative corrections, difficulties in incorporating electrodynamics into an axiomatic field theory formalism or the S-matrix formalism.

The infra-red divergences have been well understood since the work of Bloch and Nordsieck [3)] and have been treated in different theoretical frameworks and by different methods [4)-12)]. Through the introduction of coherent states [13),14)] new possibilities opened for the treatment of the infra-red problem. The current responsible for the soft radiation is essentially classical in nature [6)] and, as shown by Glauber [14)], a classical current gives rise to a coherent field. The difficulties with coherent states based on non-normalizable one-photon wave functions [15)] led Kibble [16)] and Planchard [17)] to the use of extended Hilbert spaces [18)]. Coherent states have been incorporated into S-matrix theory [19)] and Green's function formalism [20)].

Kibble [20)] also solves the problem of distortion due to the Coulomb and magnetic interaction. The present work - to appear shortly as a preprint - deals with the same problem of distortion and soft radiation but in a simpler formalism. The charged particles can be treated in first quantization - since pair creation involves energies above pair production thresholds and thus finite range forces.

gauge, which is necessary to show up the Coulomb force between electrons can then be made accurately.

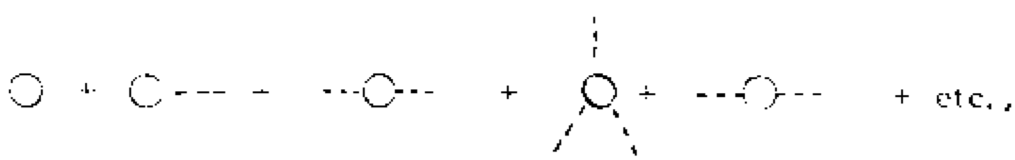
With a gauge-invariant cut-off we find that we must also renormalize c . We get

$$\frac{\delta c}{c} = \frac{1}{6\pi} \frac{e^2}{\hbar c} .$$

The need for this renormalization turns up in the one-electron calculations that lead to the Lamb shift and anomalous magnetic moment, the parameter c that occurs in the formula connecting momentum and energy being slightly different from the original c in the Hamiltonian.

of Feynman functional integral: some terms computed in the late forties with the, then unknown, measure put temporarily equal to one, others discovered in recent years and now identified with the contribution of the measure.

Calculations being the acid test of a theory, it is useful to remark that in gravity theory it is more convenient to work in co-ordinate space than in momentum space. Although working with tempered distributions is in principle completely equivalent to working with their Fourier transforms, even in gravity theory, in practice, momentum space is more convenient in Lorentz-invariant theories and co-ordinate space is more convenient in gravity theory. The reason becomes apparent only when one quantizes gravity (or any other field) relative to an arbitrary background metric. A curved background can produce pairs, both real and virtual (i.e., neutrino pairs, electron-positron pairs, graviton pairs, etc.). Let us, for example, consider the vacuum processes involving only a single closed loop. In momentum space, an infinity of diagrams must be added:



where solid lines denote the flat space propagator of whatever particle is involved in the loop and the dotted line denotes the external (imposed) gravitational field. In co-ordinate space, however, only one single diagram is needed, namely  . The heavy line denotes the propagator in curved space. There are several methods for computing propagators in curved spaces (Refs. 1 and 3). Some have been obtained in closed form (Ref. 3). Radiative corrections have been computed in co-ordinate space; for instance, the contribution of conformal metric fluctuations to the vacuum-to-vacuum amplitude has been summed to all orders (Ref. 1).

REFERENCES

1. B.S. DeWitt, Phys. Rev. 160, 1113 (1967), 162, 1195 and 1239 (1967).
2. C. DeWitt, "L'intégrale fonctionnelle de Feynman", to appear in Annales de l'Institut Henri Poincaré.
3. B.S. DeWitt and C. DeWitt, Physics 1, 3 (1964).

RENORMALIZED HAMILTONIANS IN QUANTUM FIELD THEORY

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In constructive quantum field theory the observables as solutions of field equations and the physical states are both closely related to a proper definition of the Hamiltonian. We shall describe recent progress in obtaining the renormalized Hamiltonian for a finite region in space-time.

Let $V = V(g) = \int d\underline{x}\, g(\underline{x})\, V_0(\underline{x})$ be a local interaction ($V_0(x) = V_0(x)^*$ a Wick polynomial in massive local free fields; $g(\underline{x}) = \overline{g(\underline{x})} \in \mathcal{D}(\mathbb{R}^s)$; $s + 1$ the dimension of space-time). Let φ_0 be the Fock vacuum and D the subset of states in Fock space $\mathfrak{F}$ with finitely many particles and wave functions of compact support. Let W be a Wick polynomial with kernel $w(k)$ and $(\varphi_0, W\varphi_0) = 0$. $\Gamma_\pm(W)$ is obtained from W by replacing $w(k)$ by

$$w(k)\,[E_c(k) - E_a(k) \pm i0]^{-1} \tag{1}$$

with E_c, E_a the sums of the energies of the particles created and annihilated in W. Let V^c be the pure creation part of V and $V^a = V - V^c$. The following classification gives rise to interesting conjectures [1].

<u>Definition</u>: V is of

type A, iff $\| V^c \varphi_0 \| < \infty$

type B, iff $\| V^c \varphi_0 \| = \infty$, $\| \Gamma_\pm(V^c)\, \varphi_0 \| < \infty$

type C, iff $\| \Gamma_\pm(V^c)\, \varphi_0 \| = \infty$, $\| \Gamma_\pm(V^a)\, \varphi \| < \infty \; \forall \varphi \in D$

type D, otherwise.

Jaffe [2] has shown that for a type A interaction $H_0 + V$ is a real symmetric operator on the dense domain D. Perturbation theory suggests (see Refs. 3 and 4) that type B and C models can be properly of the subspace characterized by a gauge condition. In order to obtain results which are independent of the gauge chosen, we shall construct a non-singular operator[*)] F related to S_2 as follows:

$$F \equiv S_2 + \gamma R \hat{\gamma}^{-1} R^{\sim} \gamma$$

where γ and $\hat{\gamma}$ are arbitrary non-singular bitensors (delta functions and their derivatives). Choosing γ and $\hat{\gamma}$ is choosing a gauge. If results are shown to be γ-invariant, they are shown to hold in any gauge. The defining equation for propagators,

$$S_2 G = -1$$

is replaced by

$$F G = -1 \ .$$

Following Faddeev, one integrates on a subspace of Ω, the measure changes correspondingly by a multiplicative term equal to unity for abelian gauge groups and different from unity for non-abelian gauge groups. This term yields the contribution of the fictitious vector particles. First discovered empirically by Feynman to ensure a unitary, manifestly covariant formalism, they have also been discovered by B. S. DeWitt in his formalism which meets the following criteria:

- Manifest covariance under the transformation group; in particular, manifest covariance of expressions involving the classical background field.
- Generalization of Feynman criteria.
- γ-invariance.

Mandelstam, by a different approach, found identical results.

In summary, terms hereto introduced more or less empirically (non-causal chains, fictitious particles) are the expression of the measure in Feynman functional integral. The physical reality is the full expression

*) Similarly, another operator $\hat{F} = R^{\sim} \gamma R$ such that $\hat{F} \hat{G} = -1$ is introduced.

defined by dressing transformations. Let $\kappa \leq \infty$ and V_κ be obtained by restricting the creators and annihilators to $\{|\underline{k}| \leq \kappa\}$.

Conjecture B: Let V be of type B . Then there exist symmetric operators R_κ $(0 \leq \kappa < \infty)$ of order ≥ 2 in g and a family of invertible mappings $T_{\rho\kappa} : D \to D(V_\kappa) \cap D(R_\kappa) \cap D(H_0)$ $(0 \leq \kappa \leq \infty\ ,\ \rho \in \mathbb{Z}_+)$ such that for all $\varphi \in D$

$$\begin{aligned} &\underset{\kappa\to\infty}{\text{s-lim}} \quad T_{\rho\kappa}\varphi = T_{\rho\infty}\varphi && (\rho \in \mathbb{Z}_+) \\ &\underset{\rho\to\infty}{\text{s-lim}} \quad T_{\rho\kappa}\varphi = \varphi && (0 \leq \kappa \leq \infty) \\ &\underset{\kappa\to\infty}{\text{s-lim}} \quad (H_0 + V_\kappa + R_\kappa)\, T_{\rho\kappa}\, \varphi = H_\infty T_{\rho\infty}\varphi \end{aligned} \tag{2}$$

H_∞ is real and symmetric on the dense domain $\bigcup_\rho T_{\rho\infty} D \subset \mathcal{F}$.

Conjecture C: Let V be of type C . Then there exist symmetric operators R_κ $(0 \leq \kappa < \infty)$ of order ≥ 2 in g , a family of invertible mappings $T_\kappa : D \to D(H_0) \cap D(V_\kappa) \cap D(R_\kappa)$ $(0 \leq \kappa < \infty)$ and an invertible mapping $T_\infty : D \to \mathcal{H}$ into a "renormalized" Hilbert space $\mathcal{H} = \overline{T_\infty D}$ with scalar product $\langle \cdot\ ,\ \cdot \rangle$, such that for all $\varphi, \psi \in D$

$$\begin{aligned} &\lim_{\kappa\to\infty} \quad (T_\kappa\varphi, T_\kappa\psi) = \langle T_\infty\varphi, T_\infty\psi\rangle \\ &\lim_{\kappa\to\infty} \quad (T_\kappa\varphi, \psi) = 0 \\ &\lim_{\kappa\to\infty} \quad (T_\kappa\varphi, (H_0 + V_\kappa + R_\kappa)\, T_\kappa\psi) = \langle T_\infty\varphi, H_\infty T_\infty\psi\rangle \ . \end{aligned} \tag{3}$$

H_∞ is real and symmetric on the dense domain $T_\infty D \subset \mathcal{H}$.

In perturbation theory B-interactions (like $(\bar{\psi}\psi\phi^{\mu})_2$ or ϕ_3^3 , lower index: s + 1) only need additive renormalizations, as, for example, the mass renormalization

$$m_\kappa \int d\underline{x}\; g(\underline{x})^2 : \phi_\kappa(\underline{x})^2 : \tag{4}$$

for $(\bar{\psi}\psi\phi)_2$. C interactions also require an infinite wave function renormalization, which leads to weak but not to strong convergence in (3).

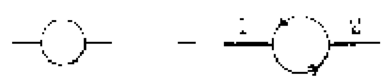
operator equation is not unique: a change[*)] in the field variable $\varphi + \Phi \to \varphi' + \Phi$ leads to a possibly different equation because of the non-commutativity of the Φ's . Comparison between the Feynman field equation and the hermitian equations yields

$$\mathcal{M} = (\det \; G^{adv} + \ldots)^{-\frac{1}{2}}$$

and

$$\langle \mathrm{vac}_{out} | \mathrm{vac}_{in} \rangle = \left(\frac{\det(-S_2^0 \, G)}{\det(-S_2^0 \, G^{adv})} \right) + \cdots .$$

The terms introduced by $\mathcal{M}$ remove the non-causal chains. Typically, when $\mathcal{M}$ is included, the diagram

 is replaced by 

where a line without an arrow represents the propagator G and a line with an arrow represents G^{adv} . Since points 1 and 2 coincide, the chain is non-causal; it contributes only to renormalization. Feynman's prescription to preserve unitarity is equivalent to the removal of the non-causal chain and can be obtained from the expression above by writing $G = G^{adv} - G^{(+)}$. Feynman baskets and the tree theorem are obtained from the set of diagrams in which the non-causal chains have been removed. It would be very pleasing to derive these results completely from the study of the measure in the Feynman functional integral without patch-up work.

If the field φ is invariant under a gauge transformation, S_2 is singular[**)] and the Feynman functional integral is redundant and the above expression for $\langle \mathrm{vac}_{out} | \mathrm{vac}_{in} \rangle$ becomes meaningless. However, although S_2 does not have an inverse in the full space Ω , it has an inverse in any

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*) Invariance under a change in the description of the field has been partially exploited in the study of non-causal chains. As non-causal chains are related to the measure, a more complete study of invariance under a change in the field variables would be valuable in determining the measure.

**) So δS , $S_1 \bar{R} = 0$, $0 = S_2 \bar{R} + S_1 \bar{R}_1 - S_2 \bar{R}$, c.q.f.d.

It is a beautiful confirmation of old physical ideas that conjectures B and C seem to be true outside perturbation theory. One has now (see Refs. 4 to 7) :

Theorem: For all local B-type interactions conjecture B holds. For the ϕ_3^4 interaction and all C-type quadratic interactions conjecture C holds. The infinite renormalizations R_κ can be chosen in the form predicted by perturbation theory.

Therefore the ultraviolet divergences of local interactions are real! The nature of the physical states is changed drastically for C-models: the states in $T_\infty \varphi$ have infinitely many bare particles with probability one[8), 9)].

The most important problem is to deal with the type D interactions. Here the super-renormalizable Yukawa interaction $(\phi\bar{\psi}\psi)_3$ is a most interesting candidate. Do the ultraviolet divergences lead to ghosts, as for the type D Lee models[4)]? Even in formal perturbation theory we do not know what equations of motion could replace the local time translations in the canonical formalism of the ABC-models.

REFERENCES

1) J. Glimm, Varenna Lectures, 1968 (to be published by Academic Press).

2) A. Jaffe, J. Math. Phys. 7,1250 (1966).

3) K.O. Friedricks, "Perturbation of spectra in Hilbert space", Amer. Math. Soc. Transl., Providence.

4) K. Hepp, Paris Lectures, 1968/69 (to be published by Springer-Verlag).

5) J. Glimm, Commun. Math. Phys. 5, 343 (1967); 6, 61 (1967); 10, 1 (1968).

6) J.P. Eckmann, University of Geneva preprint , 1969.

7) K. Osterwalder, Thesis, E.T.H., Zürich (in preparation).

8) J. Fabrey, MIT preprint, 1969.

9) K. Hepp, Inst. des Hautes Etudes Scientifiques, Bures-sur-Yvette, preprint, 1969.

The advantages of working with an arbitrary classical background are the following:

- Gross topology can be determined by background field.
- General covariance can be better displayed, i.e., we are not tied to asymptotic flatness, with inevitable emphasis on the Poincaré group, nor to any other asymptotic symmetry.
- In quasistatic regions, approximate vacua can be defined, relative to the background field, and particle production by the background can be studied.
- Virtual processes can be probed by varying the background subject to the constraint $S_1 = 0$. This accomplishes everything that is accomplished by source techniques but avoids non-abelian gauge difficulties.
- All quantities of interest can be obtained from the transition amplitude from the vacuum at $t = -\infty$ to the vacuum at $t = +\infty$ by taking its suitable functional derivatives.

Thus, let us consider:

$$\langle vac_{out} | vac_{in} \rangle = \int_{\Omega} \exp\left\{ \frac{i}{\hbar} S[\varphi + \phi] \right\} \mathcal{U}[\varphi + \phi] \, d\phi \quad .$$

In particle physics, $\mathcal{U}[\varphi]$ has been determined by imposing unitarity on the theory; in field theory we determine $\mathcal{U}$ by imposing the hermiticity condition - i.e., a condition closely related to unitarity - on the following field equation obtained uniquely from the Feynman functional integral (assuming that, on the boundary of the Ω space, S oscillates rapidly enough to ensure destructive interference):

$$T\left\{ \frac{i}{\hbar} S_1[\varphi + \phi] + (\ln \mathcal{U}[\varphi + \phi])_1 \right\} = 0 \quad .$$

On the other hand, by the correspondence principle, the field equations must be $S_1[\varphi + \phi] = 0$, where the factors are ordered in such a way that the left-hand side is hermitian. Contrary to the classical case, this

ON THE EQUIVALENCES OF RENORMALIZATION SCHEMES

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Among the many more or less clearly defined renormalization schemes for the Green functions in perturbation theory there are three methods which are distinguished by mathematical rigour, generality and simplicity. Since they all work for an arbitrary polynomial Lagrangian, each with certain advantages for definite problems, it is useful to know the precise relation between these schemes.

Consider the formal contribution $\prod_\ell \tilde{\Delta}^F_\ell (x_{i(\ell)} - x_{f(\ell)})$ of any graph G in the Gell-Mann-Low series. G has vertices $\mathcal{V} = \{V_1, \ldots, V_n\}$ and lines $\mathcal{L} = \{\ell_1, \ldots, \ell_L\}$. For $\ell \in \mathcal{L}$ the propagator

$$\Delta^F_\ell(p) = Z_\ell(p)\,(p^2 - m^2_\ell + i0)^{-1} \qquad (1)$$

has a polynomial Z_ℓ of degree r_ℓ and $m_\ell > 0$.

A subgraph $H = H(\mathcal{M})$ of G is determined by any subset $\mathcal{M} \subset \mathcal{L}$. H is called complete if no line $\ell \in \mathcal{L} - \mathcal{M}$ connects two vertices, which are end-points of lines in $\mathcal{M}$. Then H is in one-to-one correspondence to a subset $\mathcal{W} \subset \mathcal{V}$, $H = H(\mathcal{W})$. A subgraph H is 1-particle irreducible (1PI) if it does not become disconnected by cutting any of its lines. The superficial divergence d(H) of a 1PI-subgraph with n(H) vertices is

$$d(H) = \sum_{\ell \in \mathcal{M}(H)} (r_\ell + 2) - 4(n(H)-1) \qquad (2)$$

A 1PI-subgraph H with $d(H) \geq 0$ is called a renormalization part (R part).

Renormalization in <u>momentum space</u> (see Ref.1) introduces counter-terms for all R parts with the freedom of adding as finite renormalization polynomials P(H) of degree d(H) . P(H) should only depend on the

The hope already formulated by Pauli that gravity is the universal regulator which renders all field theories finite, although partially substantiated (Ref. 1, p. 1248), is still not fulfilled. The smearing out of the light cone due to quantum fluctuations of the gravitational field is expressed, with our current methods, as an infinite series of terms so divergent that gravity is classified as unrenormalizable. In some simple classes of diagrams the sum of such a series has turned out to be finite, owing to certain remarkable cancellations. Thus some of the progress made in the full assault of quantum gravidynamics may be relevant to this meeting. The technique used in the work reported here is Feynman functional integration; some of these results can also be obtained by other techniques.

Formally, the Feynman functional integral of particle physics

$$\langle b | a \rangle = \int_{\Omega} \exp\left\{ \frac{i}{\hbar} S[q,b,a] \right\} \mathcal{M}[q]\, dq = \int_{\Omega} \exp\left\{ \frac{i}{\hbar} S[q] \right\} \mathcal{D} q$$

can readily be extended to field theory. An analysis of the measure shows that the measure is not $\mathcal{M}[q]$ but $\exp\left\{ \frac{i}{\hbar} S_{free}[q,b,a] \right\} \mathcal{M}[q]$

(Ref. 2). It follows that the natural origin in the path space Ω is the classical path $\bar{q}$ such that $S_1[\bar{q}] = 0$; consequently, it is advantageous to express an arbitrary path q with reference to the classical path $\bar{q}$. In field theory, the path $q : \mathbb{R} \to M$ (M a manifold which, in some very interesting cases, is curved and non-homotopically equivalent to zero) becomes the field which maps V^4 into a manifold whose properties depend on the field considered. The arbitrary field which enters the Feynman functional integral is expressed with reference to the classical field φ, namely as $\varphi + \phi$, and the variable of integration is ϕ. Correspondingly, the quantum field Ψ is split into the classical background field φ and a quantum remainder Φ

$$\Psi = \varphi + \Phi \ .$$

internal structure of H and (without restriction by theorem 1) $P(H) = 0$ if H is not complete. After having thus replaced the unrenormalized integrand

$$I_G^{\varepsilon}(k,p) = \prod_{\ell \in \mathcal{L}} \frac{Z_\ell(q_\ell)}{q_\ell^2 - m_\ell^2 + i\varepsilon(q_\ell^2 + m_\ell^2)} \tag{3}$$

by $R_{G\{P\}}^{\varepsilon}(k,p)$, one has[1]:

Theorem 1: For every G and every choice of finite renormalizations $\{P\}$

$$R_{G\{P\}} = \lim_{\varepsilon \downarrow 0} \int dk\, R_{G\{P\}}^{\varepsilon}(k,\cdot) \in \varphi'(\mathbb{R}^{4m}) \tag{4}$$

is a tempered distribution in the $1 \leq m \leq n$ external four-momenta. R is a renormalization, which respects the formal structure of perturbation theory.

Renormalization in the α-space of the Feynman parameters introduces counter-terms only for all complete R parts, again with a polynomial $P(H)$ of degree $\leq d(H)$ as finite renormalization. After having completed the unrenormalized α-integrand

$$\mathcal{J}_G^{\varepsilon}(\alpha, p) = \int dk \prod_{\ell \in \mathcal{L}} \frac{1}{i} Z_\ell(q_\ell) \exp i\alpha_\ell(q_\ell^2 - m_\ell^2 + i\varepsilon) \tag{5}$$

to $\mathcal{R}_{G\{P\}}^{\varepsilon}(\alpha, p)$ one obtains[2]:

Theorem 2: For every G and $\{P\}$

$$\mathcal{R}_{G\{P\}} = \lim_{\varepsilon \downarrow 0} \int d\alpha\, \mathcal{R}_{G\{P\}}^{\varepsilon}(\alpha,\cdot) \in \varphi'(\mathbb{R}^{4m}) \tag{6}$$

is a renormalization.

$R_{G\{P\}}$ and $\mathcal{R}_{G\{P\}}$ are both additive renormalizations and their equivalence is not too surprising[3]:

Theorem 3: For every G and $\{P\}$

$$R_{G\{P\}} = \mathcal{R}_{G\{P\}} \ . \tag{7}$$

SOME PROGRESS IN THE COVARIANT FORMALISM, IN PARTICULAR FOR FIELDS INVARIANT UNDER A NON-ABELIAN GAUGE GROUP

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NOTATION:

Field: Classical background field: φ , all discrete and continuous indices have been suppressed; for instance, in gravity theory φ stands for $g_{\mu\nu}(x)$. Thus, implicit summation and integration are assumed. Quantum field operator: Ψ , Φ .

Action: S ; in particle physics $S[q, b, a]$ is the action, functional of the path q and function of the end points of the paths a and b .

n-th functional derivative of S : S_n

$S_1 = 0$: field equation; S_2^0 : S_2 evaluated at flat empty space-time.

Propagators: Feynman propagator G ; advanced propagator G^{adv} ; propagator of positive frequencies on the mass shell $G^{(+)}$; fictitious vector particle propagator G .

Time-ordered product: T

When a gauge group is present: infinitesimal displacement on the group manifold near the origin $\delta\xi$; generator of infinitesimal transformations: R . We shall consider only the cases in which R is a linear functional of φ , namely $R_2 = 0$.

Analytic renormalization[4] starts from the regularized propagators $(\varepsilon, r > 0, \lambda_\ell \in \mathbb{C})$

$$\Delta_\ell^{\varepsilon, r}(\lambda_\ell) = Z_\ell(p)\, \Gamma(\lambda_\ell)^{-1} \int_r^\infty d\alpha_\ell\, \alpha_\ell^{\lambda-1} \exp i\alpha_\ell(p^2 - m_\ell^2 + i\varepsilon) \tag{8}$$

and a finite renormalization $P(H)$ for every full R part $H = H(\mathfrak{V})$, $\mathfrak{V} = \{V_1', \ldots, V_m'\}$, where

$$\mathfrak{X}_{P(H)} = \delta\left(\sum_{j=1}^{m} p_j'\right) P(H; p_1', \ldots, p_m') \tag{9}$$

and Fourier transforms $\tilde{\Delta}_\ell^{\varepsilon, r}(\lambda_\ell)$, $\tilde{\mathfrak{X}}_{P(H)}$.

Theorem 4: Let $\mathcal{T}_{G\{P\}}^{\varepsilon, r}(\lambda)$ be the Fourier transform of

$$\tilde{\mathcal{T}}_{G\{P\}}^{\varepsilon, r}(\lambda) = \sum_{H_1, \ldots, H_m} \prod_{i=1}^{m} \tilde{\mathfrak{X}}_{P(H_i)} \prod_{\text{comm}} \tilde{\Delta}_\ell^{\varepsilon, r}(\lambda_\ell) \tag{10}$$

where the sum extends over all disjoint full R parts $H_1, \ldots, H_m$ and the product over all lines $\ell \in \mathcal{L} - \cup H_i$. There exists $M = M(G) < \infty$ such that

$$\mathcal{T}_{G\{P\}}(\lambda) = \lim_{\varepsilon \downarrow 0} \lim_{r \downarrow 0} \mathcal{T}_{G\{P\}}^{\varepsilon, r}(\lambda) \in \varphi'(\mathbb{R}^{4m}) \tag{11}$$

exists and is holomorphic for $\mathrm{Re}\,\lambda_\ell > M$, $1 \leq \ell \leq L$.

$$\mathcal{T}_{G\{P\}}(\lambda) \prod_{\mathcal{M} \subset \mathcal{L}} \Gamma\left(\sum_{i \in \mathcal{M}} (\lambda_\ell - \mathcal{A})\right)^{-1} \tag{12}$$

is entire in $\mathbb{C}^L$ with values in $\varphi'(\mathbb{R}^{4m})$. There exists $0 < R_1 < \ldots < R_L$ with $R_i > \sum_{j=1}^{i-1} R_j$, $1 \leq i \leq L$ and $C_i = \{z \in \mathbb{C}^1, |z - 1| = R_i\}$ with positive orientation, such that the following symmetrized integral lies in the domain of holomorph of $\mathcal{T}_{G\{P\}}(\lambda)$

REFERENCES

1) H. P. Dürr and E. Rudolph, Nuovo Cimento 62A, 411 (1969).

2) H. P. Dürr and E. Rudolph, to appear in Nuovo Cimento; Max-Planck Institut, München, preprint June 1969.

3) H. P. Dürr, Proceedings of the International Symposium on Non-Local Quantum Field Theory, Dubna, 1967, p. 69.

4) E. C. G. Sudarshan, Phys. Rev. 123, 2183 (1961).

5) J. Schnitzer and E. C. G. Sudarshan, Phys. Rev. 123, 2193 (1961).

6) K. L. Nagy, "State Vector Spaces with Indefinite Metric in Quantum Field Theory", (Akadémiai Kiadó, Budapest 1965). This book contains a rather complete bibliography of the earlier work on indefinite metric.

7) S. N. Gupta, Proc. Phys. Soc. (London) A63, 681 (1950).

8) K. Bleuler, Helv. Phys. Acta 23, 567 (1950).

9) G. Källén and W. Pauli, Mat. Fys. Medd. Dan. Vid. Selsk. 30, No. 7 (1955).

10) W. Heisenberg, Nucl. Phys. 4, 532 (1957).

11) P. Dennery, N. M. Kroll, Nucl. Phys. 21, 276 (1960).

12) M. Karowski, Hamburg preprint, Dec. 1968.

$$\mathcal{F}_{G\{P\}} = \frac{\cdots}{L!} \sum_{Q} \frac{\cdots}{(2\pi i)^{L}} \int_{C_{Q(1)}} d\lambda_{\ell} \cdots \int_{C_{Q(L)}} d\lambda_{\ell} \frac{\cdots}{\prod_{\ell} (\lambda_{\ell} - 1)} \quad . \qquad (13)$$

Now comes the beauty [4), 5)]:

Theorem 5: $\mathcal{F}_{G\{P\}}$ is a renormalization. There exists a one-to-one mapping $\mathcal{A}$ of the space of finite renormalizations onto itself such that

$$\mathcal{R}_{G\{P\}} = \mathcal{F}_{G\{\mathcal{A}P\}} \quad . \qquad (14)$$

Problem: Let $\sum a_n z^n$ be an entire function such that for $\mathcal{L}_0(y) = \sum a_n : \phi_0(y)^n :$ and all k, ℓ

$$\langle \phi_0(x_1) \ldots \phi_0(x_k) \; \mathcal{L}_0(y_1) \ldots \mathcal{L}_0(y_\ell) \rangle_0 \;\; \in \;\; \mathcal{C}(\mathbb{R}^{4(k+\ell)}) \; . \qquad (15)$$

Is it possible to define

$$\langle T(\phi_0(x_2) \ldots \phi_0(x_k) \; \mathcal{L}_0(y_1) \ldots \mathcal{L}_0(y_\ell)) \rangle_0 \qquad (16)$$

as the limit in the Jaffe class $\mathcal{C}$ [6)] of the sum of all renormalized graphs for a suitable choice of finite renormalizations?

REFERENCES

1) F. J. Dyson, Phys. Rev. 75, 486, 1736 (1949);
Abdus Salam, Phys. Rev. 82, 217 (1951); 84, 426 (1951);
W. Zimmermann, Commun. Math. Phys. 6, 161 (1967); Inst. des Hautes Etudes Scientifiques, Bures-sur-Yvette, preprint, 1969.

2) N. N. Bogolubov and O. S. Parasiuk, Acta Math. 97, 227 (1957);
K. Hepp, Commun. Math. Phys. 2, 301 (1966).

3) W. Zimmermann and K. Hepp, unpublished.

4) C. G. Bollini, J. J. Giambiagi and A. Gonzáles Domingues, Nuovo Cimento 31, 555 (1964).
E. Speer, J. Math. Phys. 9, 1404 (1968).

5) K. Hepp, to appear in Commun. Math. Phys.

6) A. Jaffe, this conference.

The bad-ghost-free states span an invariant subspace with a positive semi-definite metric. Due to their zero norm the good ghosts do not have physical consequences. The normalizable states represent the physical states and are connected by a unitary S-matrix.

More specifically, the Gupta-Bleuler formulation of massless theories with integer spin $j > 0$ necessitates the introduction of a HLG-reducible field consisting of a $(j+1)^2$ component field of the irreducible HLG-representation $D(j/2 \mid j/2)$ and a $(j-1)^2$ component subsidiary field of HLG-representation $D\left(\frac{j-2}{2} \middle| \frac{j-2}{2}\right)$. These components arrange themselves into the two physical field operators and j^2 ghost couple operators connected with the creation and annihilation of ghost couples in the above sense. The theory is restricted by j^2 Lorentz-type conditions which are applied in the Gupta sense and lead to the elimination of the j^2 bad ghosts. In the Heisenberg-Lee model the indefinite metric enters in the form of a single v-particle ghost couple which contains as bad ghost the Heisenberg dipole ghost. The bad ghost is not an eigenstate of energy and hence can be suppressed by the energy eigenvalue condition, at least in the lowest sectors. Investigations of Dennery and Kroll[11] indicate, however, that this condition is not sufficient in higher sectors.

Quantum electrodynamics also indicates that the introduction of an indefinite metric may only be of a technical, although rather convenient, nature. The projection on the physical subspace leads to the Dirac-Schwinger formulation which lacks manifest Lorentz invariance and causality. Similarly, the Heisenberg-Lee model can be reformulated in the physical state space and leads (at least with regard to the lowest sectors) to a non-local Lagrangian with separable potential[12].

RENORMALIZATION OF HAMILTONIANS

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The present programme of constructive field theory [1)-5)] is to find examples of Hamiltonian dynamics in relativistic, local quantum mechanics. Thus we take seriously the requirement that the Hamiltonian H is realized as a self-adjoint operator on a Hilbert space $\mathcal{H}$. The local field $\varphi(x,t)$ should satisfy

$$\varphi(x,t) = e^{iHt}\,\varphi(x,0)\,e^{-iHt} \quad .$$

In addition we want the Hamiltonian to be positive

$$H \geq 0$$

and to have a vacuum vector Ω in $\mathcal{H}$ satisfying

$$H\Omega = 0 \quad .$$

Such a programme is bound to lead to divergences and infinite renormalizations [6)]. The reason for this is that we write the Hamiltonian as a function of the time zero canonical fields on Fock space. Since only the free field Hamiltonian can be a positive operator on Fock space, some renormalization counter-terms are necessary.

The lesson we have learned from the models we have studied is that every renormalization that occurs in perturbation theory is present in the exact solutions of super-renormalizable models. We therefore use perturbation theory as a guide for what to expect in the actual theory, and we must understand all that perturbation theory tells us. While, in principle, the exact solutions may be more singular than perturbation theory indicates, no new divergences have been discovered in the exact solutions of the models studied so far.

INDEFINITE METRIC IN QUANTUM ELECTRODYNAMICS AND MASSLESS THEORIES OF HIGHER SPIN AND THE LEE MODEL

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The introduction of an indefinite metric in the quantum mechanical state space offers an interesting possibility to avoid the divergence difficulties in local quantum field theories with the important and serious drawback that the probability interpretation can no longer hold in the conventional form. This difficulty, however, can be resolved if an invariant subspace exists of the large state space which only contains states with positive norm. These states, then, should be identified with the physical in- and out-states and will be connected by a unitary S-matrix. Hence the main goal of a theory with indefinite metric is to demonstrate that a unitary S-matrix exists. One possible general prescription for the construction of such a unitary S-matrix has been given some years ago by Sudarshan [4), 5)]. However, it also became apparent that by different restriction on the in-states different constructions are possible.

As a consequence of the non-uniqueness of this prescription, it appears more adequate at the moment to study particular theories and models with indefinite metric where the construction of the S-matrix is unambiguous. For this purpose the Gupta-Bleuler quantum electrodynamics [7), 8), 1)] is restudied and its generalization to massless theories of higher spin [2)] (at least in the free field case) as well as the Lee model in the Heisenberg case [10)]. This investigation shows that in all these cases the construction of the S-matrix follows the same formal pattern: the indefinite metric occurs only in the form of one or more ghost couples, each consisting of two non-orthogonal zero-norm states, a good ghost and a bad ghost. The theory provides an invariant subsidiary condition which leads to the suppression of all in-states containing bad ghosts and, as a consequence of the invariance, also to their supppression in the out-states.

The programme of constructive field theory has reached various stages in various models. Since the problem of constructing a theory of electrodynamics or weak interactions is extremely difficult, we have decided to study first models with fewer divergences. Most is known about models in two-dimensional space-time and I shall describe these results. Our meagre knowledge of higher-dimensional models will be reviewed by Hepp. This leaves challenging problems for the future. The main steps of the construction are

1) Formulation of the problem.
2) Construction of a local field.
3) Construction of the Hamiltonian H, the physical Hilbert space $\mathcal{H}$ and the vacuum Ω.
4) Verification of properties of the theory, such as Lorentz covariance, locality, or the particle spectrum.
5) Development of a non-perturbative calculational scheme.

In approaching such a programme we encounter three types of divergences. They are:

a) Volume divergences,

b) Ultraviolet divergences,

c) Particle number divergences.

The infinite volume problem affects global objects, such as the Hamiltonian or the vacuum. It does not affect localized objects such as the energy density or the Heisenberg picture field $\varphi(x,t)$. In order to avoid the infinite volume problem we introduce a spatial cut-off into the interaction Hamiltonian. We write

$$H_V = H_0 + \int H_I(x)\, g(x/V)\, dx \quad ,$$

where

$$g(x) \geq 0$$

is a C_0^∞ function equal to 1 for $|x| < 1$. In a local theory, influence propagates at the speed of light, so the Hamiltonian H_V is formally correct in the space-time region

$$|x| + |t| < V \; .$$

A physical amplitude defined along the real axis will satisfy unitarity but will be split into two different analytic functions as M_1 tends to M_2 and the two branch points coalesce[3),4)]. The consequent lack of an "iε prescription" is unpleasant from a theoretical point of view, but the associated lack of causality is again so small as to be unmeasurable.

More complicated singularities have been studied[4)] and their unpleasant features are again compatible with unitarity and Lorentz invariance.

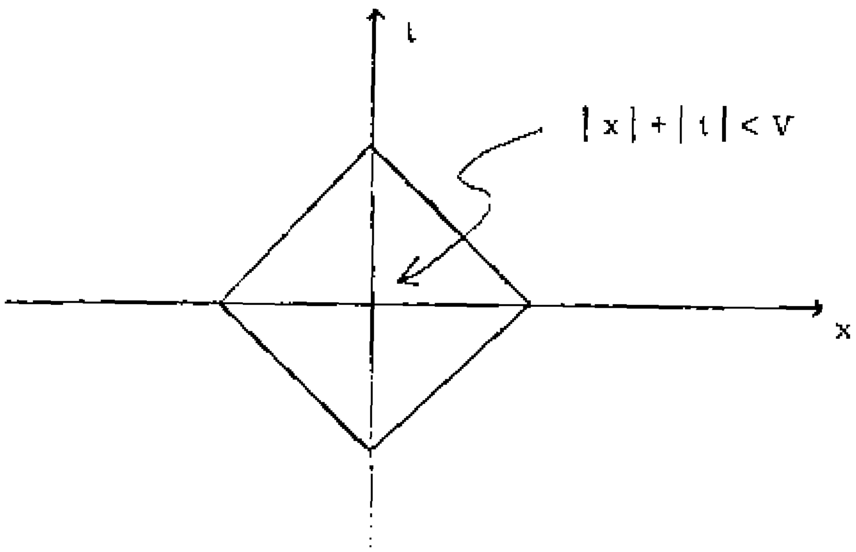
REFERENCES

1) For references see T.D. Lee, "A possible way to remove divergence difficulties in physics", CERN preprint, 1969. (Talk given at the Topical Conference on Weak Interactions, CERN, Jan. 1969.)

2) T.D. Lee and G.C. Wick, Nucl. Phys. B9, 209 (1969).

3) T.D. Lee, "A relativistic complex pole model with indefinite metric", Columbia Univ., N.Y., preprint, 1969.

4) R.E. Cutkosky, P.V. Landshoff, D.I. Olive and J.C. Polkinghorne, to appear in Nucl. Phys.



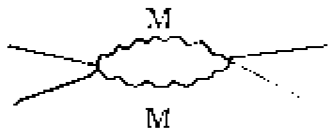
Thus if H_V is a self-adjoint operator, we expect that the field

$$\varphi(x,t) = e^{iH_V t}\, \varphi(x,0)\, e^{-iH_V t}$$

is independent of V for V sufficiently large, and is the correct local field. This is the case for the $(\varphi^4)_2$ theory of a self-interacting boson in space-time of two dimensions[7)-9)]. In order to remove the spatial cut-off in the Hamiltonian, we encounter the vacuum self-energy divergence in every order of perturbation theory:

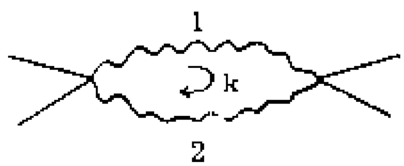
Because of these divergences, each proportional to V , the renormalized Hamiltonian can exist only after changing Hilbert spaces. We must leave Fock space and work on the Hilbert space of physical states. Before giving more details I shall mention the two other types of divergences.

The second type of divergence, the ultraviolet divergence, does not occur in $(\phi^4)_2$ but does occur, for instance, in the $(\text{Yukawa})_2$ theory. For this interaction two ultraviolet divergent Feynman diagrams occur, Corresponding to the diagram

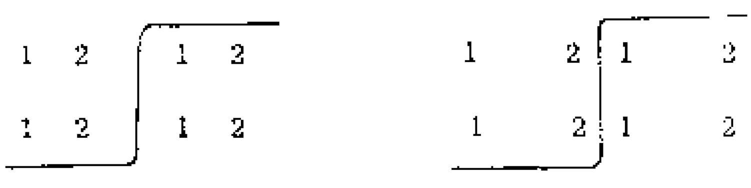


we expect singularities at $(2M)^2$, $(2M^*)^2$ and $(M+M^*)^2$ possibly on the physical sheet. $(M+M^*)^2$ is very unpleasant since it occurs at a real point where there is no stable particle threshold. If this singularity has an "$i\epsilon$ prescription" it disagrees with unitarity, since there is no corresponding new contribution to the imaginary part.

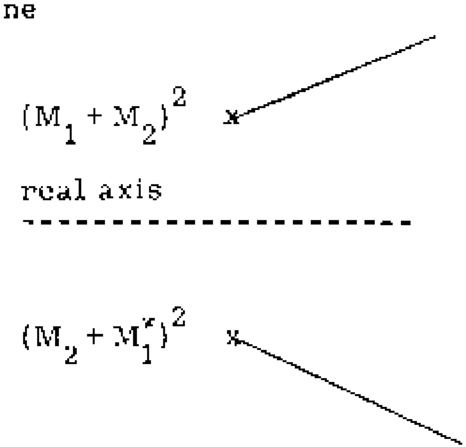
In order to maintain reality for sufficiently low energies the k^0 contour of the Feynman integral



must be as shown below, with pole singularities due to propagators 1 and 2 as indicated.



Near $(M+M^*)^2$ two pinches of the contour occur simultaneously. To analyse this, suppose for the time being that $\text{Im } M_1 > \text{Im } M_2$. Then no pinch occurs on the real axis and we have a complex conjugate pair of branch branch points in the s plane



 and  ,

and each of these diagrams requires a counter-term with a coefficient that diverges logarithmically in the ultraviolet cut-off. Thus we must start with a doubly cut-off Hamiltonian $H_{\kappa, V}$ for this model. Since the ultraviolet cut-off destroys locality, even if we can prove that $\lim_{\kappa \to \infty} H_{\kappa, V}$ is a self-adjoint operator, we still need to verify that the resulting theory is local.

The third type of divergence I mentioned is the particle number divergence. This divergence is caused by the fact that the interaction creates too many particles and hence too many intermediate states. While this divergence does not affect each order of perturbation theory, it becomes evident when we try to sum all orders of perturbation theory. The large number of n^{th} order diagrams yield a divergent sum for Green's functions in $(\phi^4)_2$, see Ref. 10. Likewise, the vacuum self-energy for H_V of $(\phi^4)_2$ is finite in each order of perturbation theory but diverges when summed to all orders. Hence we cannot conclude from perturbation theory that the vacuum energy of H_V is proportional to V . The particle number divergences are the least understood divergences and hence they pose a serious problem for the future.

I shall now describe some results. At the present stage, the $(\varphi^4)_2$ theory is in fairly good shape. We know that H_V is bounded from below [11), 12)] and is self adjoint [8), 9)]. A local field exists for the $(\varphi^4)_2$ theory [13)] and this field is self-adjoint [13)]. The physical vacuum vector Ω exists, as does the Hamiltonian H and the physical Hilbert space . (see Ref. 14). The theory is space-time covariant [14)] and Lorentz covariant [15)].

The first step in constructing the physical representation is to prove that H_V has a vacuum vector Ω_V in Fock space,

$$H_V \geq E_V \ , \quad H_V \, \Omega_V = E_V \, \Omega_V \ .$$

The vacuum vector Ω_V does exist and, in fact, it is unique [13)]. We pass to the infinite volume limit with the vacuum expectation values ω_V determined

THE NON-ANALYTIC S-MATRIX

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One old way of making divergences finite[1] is to replace propagators

$$\frac{1}{q^2 - m^2} \rightarrow \frac{1}{q^2 - m^2} - \frac{1}{q^2 - M^2} = \frac{m^2 - M^2}{(q^2 - m^2)(q^2 - M^2)} \sim O(1/q^4) .$$

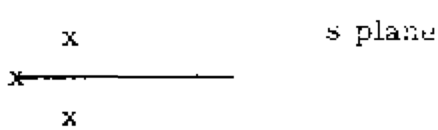
Thus one introduces a new particle of mass M with similar couplings to m but with a negative metric. This negative metric is an undesirable feature which could possibly be removed by making M unstable so that it does not contribute to unitarity. Suppose these particles couple to a two-particle system, then unitarity of the s-wave amplitude A states that

$$\mathrm{Im}\, \frac{1}{A} = 1 \quad .$$

Since now the M pole $A \sim G/s-M^2$ with G approximately a negative real number, we have

$$\mathrm{Im}\, M^2 > 0 \quad .$$

Thus the unstable particle pole M^2 is on the physical sheet rather than the unphysical sheet, which is usual for normal particles. In fact we have a complex conjugate pair of poles violating dispersion relations



This phenomenon has been verified in the Lee model[2] where it leads to a violation of causality so small as to be unmeasurable.

by Ω_V,

$$\omega_V(A) = (\Omega_V, A\Omega_V) .$$

The limit

$$\lim_{V_j \to \infty} \omega_V(A) = \omega(A)$$

exists as V_j passing through a subsequence of volumes tending to infinity[14]. On the other hand, the vacuum vectors Ω_V converge weakly in Fock space to zero, the Van Hove phenomenon.

We gain control over the limit of infinite volume by proving an estimate on the vacuum energy

$$E_V \geq -MV .$$

While this estimate is true in each order of perturbation theory, the exact result requires non-perturbative methods. We prove this using the Feynman-Kac history integral formula[14]. As a result of this estimate we find that the bare particle energy per unit volume is finite in the physical vacuum. This yields a local unitary equivalence between the Fock space and the physical representation[14]. For a bounded space-time region B and for $(x, t) \in B$, there is a unitary operator U_B mapping Fock space into $\mathcal{H}$, such that

$$\varphi_{Ren}(x, t) = U_B \, \varphi_{Fock}(x, t) \, U_B^*$$

$$= U_B \, e^{iH_V t} \varphi_{Fock}(x, 0) \, e^{-iH_V t} U_B^* .$$

The problem of ultraviolet divergences in the $(Yukawa)_2$ model needs to be treated differently. Since the Hamiltonian for a finite volume can be defined on Fock space only as a limit of cut-off Hamiltonians $H_{\kappa, V}$, we need a precise mathematical definition of such a limit. A useful concept is the graph limit[9]. Suppressing the fixed subscript V, we consider all sequences of vectors ψ_κ in the domain of H_κ such that

$$\{\psi_\kappa\} \to \psi$$

and

$$\{H_\kappa \psi_\kappa\} \to \chi .$$

In comparison with the Hilbert space approach of Glimm and Jaffe[4], Euclidean quantum field theory has the disadvantage of yielding, at best, necessary conditions for the existence of a theory with non-negative energies to a given Lagrangian. Its advantage is its manifest covariance, and that it bypasses the (in the other approach difficult) problems of the vacuum and of the representation of the canonical commutation relations, and it appears to be the most direct route to investigate the values of the renormalization constants. The mathematical tool of this approach is probability theory, in particular the theory of stochastic processes and of their transformations, and it appears to offer beautiful mathematical problems in this area.

REFERENCES

1) K. Symanzik, J. Math. Phys. 7, 510 (1966); in "Mathematical Theory of Elementary Particles", ed. R. Goodman, I. Segal (MIT Press, Cambridge, Mass. 1966) p.125.

2) J. Schwinger, Proc. Nat. Acad. Sci. U.S. 44, 956 (1968); Phys. Rev. 21, 241 (1959);
T. Nakano, Progr. Theoret. Phys. (Kyoto) 21, 241 (1959).

3) K. Symanzik, Proceedings of the 1968 Varenna Summer School "Enrico Fermi", to be published by Academic Press, Bologna.

4) A. Jaffe, these Proceedings.

If the set of such ψ is dense then there is a symmetric operator H such that[9)]

$$H\psi = \chi \quad .$$

We say that H is the graph limit of $\{H_\kappa\}$. The graph limit of the cut-off $(\text{Yukawa})_2$ Hamiltonians was proved to exist by Glimm[16)] and Hepp[4)]. To show that an operator H exists, the dressing transformation is used, and this is described by Hepp in these Proceedings.

The next question is whether this graph limit H is self-adjoint. The self-adjointness follows if in addition we have resolvent convergence,

$$(H_\kappa + b)^{-1} \rightarrow (H + b)^{-1} \quad .$$

We believe it is possible to prove resolvent convergence[17)]. Once self-adjointness is established, we can ask whether H yields a local dynamics. This also appears to be true[17), 18)]. Thus we hope that soon the $(\text{Yukawa})_2$ theory will be at the level of $(\phi^4)_2$ one year ago.

We can look forward to future progress and perhaps some day a new scheme will emerge for non-perturbative calculations, stage 5 of the programme above.

REFERENCES

1) A. M. Jaffe, "Constructive quantum field theory", Zürich Lectures E.T.H. polycopy.

2) A. M. Jaffe, Varenna Lectures, 1968.

3) J. Glimm, Varenna Lectures, 1968.

4) K. Hepp, Paris Lectures, Ecole Polytechnique, 1968-69.

5) A. M. Jaffe, "Whither quantum field theory?", to appear in Rev. Mod. Phys.

6) J. Glimm and A. M. Jaffe, "An infinite renormalization of the Hamiltonian is necessary", to appear in J. Math. Phys.

7) A. M. Jaffe, J. Math. Phys. 7, 1250 (1966).

8) J. Glimm and A. M. Jaffe, Phys. Rev. 176, 1945 (1968).

EUCLIDEAN QUANTUM FIELD THEORY

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Euclidean quantum field theory [1] is the analysis of a field theory through the study of its Euclidean Green functions [2], or Schwinger functions, which are the analytic continuations of ordinary Green functions from real to imaginary time, or, in terms of Fourier transforms, from real to imaginary energy components. These functions possess, compared with the real-time Green functions, relatively simple properties. In particular, if a Lagrangian is given, they satisfy an infinite set of coupled elliptic rather than of coupled hyperbolic differential equations. The existence of functions with these properties is a necessary condition for the existence of the real-time Green functions.

For scalar theories with $g A^4$ or $g(B^+B)^2$ interaction Hamiltonian density, the Schwinger functions can be represented as multiple Wiener integrals, the integrands of which are functions on the Wiener spaces of continuous trajectories. These functions of trajectories obey an infinite set of coupled Wiener integral equations, which are the Kirkwood-Salsburg equations of classical statistical mechanics of grand canonical ensembles of occupation time distributions of Wiener trajectories. The convergence of the iteration solution of these equations, yielding the analogue of the fugacity expansions in classical statistical mechanics, has been proven for one-space-"time" dimension (anharmonic oscillator).

For two and three space-time dimensions, only partial results concerning the elimination of ultraviolet divergences have been obtained [3]. Hereby, in three dimensions, a "self-improvement" of the theory is found: divergences tend to inhibit themselves. Physically, this stems from the repulsive character of the interaction if $g > 0$, and thus this effect could not be obtained in an expansion in powers of g.

9) J. Glimm and A. M. Jaffe, "Singular perturbations of self-adjoint operators", to appear in Commun. Pure Appl. Math.

10) A. M. Jaffe, Commun. Math. Phys. 1, 127 (1965).

11) E. Nelson, "A quartic interaction in two dimensions" in "Mathematical Theory of Elementary Particles", ed. R. Goodman and I. Segal (MIT Press).

12) J. Glimm, Commun. Math. Phys. 5, 343 (1967).

13) J. Glimm and A. M. Jaffe, "The $\lambda(\varphi^4)_2$ quantum field theory without cut-offs - II: The field operators and the approximate vacuum", to appear in Ann. Math.

14) J. Glimm and A. M. Jaffe, "The $\lambda(\varphi^4)_2$ quantum field theory without cut-offs - III: The physical vacuum", preprint in preparation.

15) J. Cannon and A. M. Jaffe, "Lorentz covariance of the $\lambda(\varphi^4)_2$ quantum field theory", preprint in preparation.

16) J. Glimm, Commun. Math. Phys. 5, 343 (1967); 6, 61 (1967).

17) J. Glimm and A. M. Jaffe, Commun. Math. Phys. 11, 9 (1968).

18) J. Glimm and A. M. Jaffe, work in progress.

cribed by a Lagrangian without the need of a limiting process. One can also discuss the conditions for the occurrence of the Goldstone phenomenon in terms of properties of the ground state energy density as a function of the strength of the symmetry-breaking term or of those vacuum expectation values of fields which do not vanish. It turns out that familiar stationarity principles [5] here become extremal principles for that energy density.

In the case of symmetry breaking by a quadratic term, however, the Goldstone limit cannot be taken in the renormalized perturbation expansion since a condition for reaching that limit is the occurrence of a "composite" Goldstone particle. This cannot be dealt with in perturbation theory.

REFERENCES

1) For the case of linear symmetry breaking see K. Symanzik, Lettere al Nuovo Cimento 2, 10 (1969);
B.W. Lee, Nucl. Phys. B9, 649 (1969) for an earlier different treatment.

2) K. Hepp, Commun. Math. Phys. 1, 95 (1965) and references given there.

3) E. Kazes, Nuovo Cimento 13, 1226 (1959).

4) R.J. Rivers, J. Math. Phys. 7, 385 (1966).

5) G. Jona-Lasinio, Nuovo Cimento 34, 1790 (1964).

THE ROLE OF LOCALITY IN PERTURBATIVE RENORMALIZATION

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We present a new method of obtaining the usual perturbation renormalization theory which simultaneously proves that each order is finite and that it satisfies the requirements of locality. We treat the case of a scalar neutral self-interacting boson field $\hat{A}(x)$; the Lagrangian density of interaction $g\mathcal{L}(x)$ is expressed in terms of the free field $A(x)$ by

$$g\mathcal{L}(x) = :A(x)^{\nu}: \qquad (\nu \geq 3) \quad .$$

The starting point is the Gell-Mann-Low formula for generalized retarded products

$$(\Omega , \hat{A}(x)\updownarrow \ldots \updownarrow \hat{A}(x_p)\,\Omega) =$$

$$= \sum_{n=0}^{\infty} \frac{i^n g^n}{n!} \int (\Omega , \mathcal{L}(y)\updownarrow \ldots \updownarrow \mathcal{L}(y_n)\updownarrow A(x_1)\updownarrow \ldots \updownarrow A(x_p)\Omega)\, dy_1 \ldots dy_n \quad .$$

$\updownarrow = \uparrow$ or $\downarrow$ = Steinmann arrows. (1)

To give meaning to the r.h.s. one must first define generalized retarded products (g.r.p.) of Wick polynomials of the free field A . Once this is done the integration offers no difficulty because of analyticity properties of the (correctly defined) generalized retarded functions (of any set of local fields) in momentum space; it automatically yields a correct definition of the l.h.s.

The construction of the g.r.p. of Wick powers is done by induction on the number of such fields.

Induction hypothesis

For any $p \leq n$ we assume we have already constructed

$$\mathcal{L}^{(r_1)}(y_1)\updownarrow \ldots \updownarrow \mathcal{L}^{(r_p)}(y_p)$$

RENORMALIZATION OF RENORMALIZABLE MODELS WITH SIMPLE SYMMETRY BREAKING

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From a Lagrangian density invariant under an N-parameter group of linear transformations can be derived N conserved currents. If to that Lagrangian density a term linear in Bose fields or quadratic in Bose or Fermi fields is added and the invariance thereby reduced, the currents of the original functional form obey the same algebra as before but only less than N of them will be conserved, the divergences of those remaining being linear and quadratic in the fields, respectively, in the two cases. If the theory to the original symmetric Lagrangian is renormalizable, so are the theories to the modified Lagrangians [1].

In the framework of Bogolubov-Parasiuk-Hepp renormalization theory [2], the values of all superficially divergent vertex functions at suitable subtraction points must be given. These are, in general, more in number than the unrenormalized Lagrangians have parameters. The Ward-Takahashi-Kazes [3] - Rivers [4] identities for vertex functions, evaluated for all momenta zero, provide linear relations between the subtraction values and involve in addition only known higher-order terms from vertex functions that do not require final subtractions. These relations just suffice to express all constants by as many of them as the unrenormalized Lagrangians have parameters and exhaust all the extra information the identities contain beyond that contained already in the usual non-linear relations between vertex functions.

In the case of symmetry breaking by terms linear in fields (PCAC), the Goldstone limit can be performed continuously within the framework of renormalized perturbation theory provided one chooses subtraction points at non-zero momenta (except for the Goldstone particle propagators). The resulting theory with "elementary" Goldstone particles can even be des-

(where $\mathcal{L}^{(r)}(y) = \nu(\nu-1)\ldots(\nu-r+1) : A(y)^{\nu-r} :$) with correct algebraic and geometric properties, and verifying

$$\mathcal{L}^{(r_1)}(y_1)\updownarrow \ldots \updownarrow \mathcal{L}^{(r_p)}(y_p) =$$

$$= \sum_{\beta_1,\ldots,\beta_p} : A(y_1)^{\beta_1} \ldots A(y_p)^{\beta_p} : (\Omega, \mathcal{L}^{(r_1+\beta_1)}(y_1)\updownarrow \ldots \updownarrow \mathcal{L}^{(r_p+\beta_p)}(y_p)\Omega) \tag{2}$$

and

$$A(x)\uparrow \mathcal{L}^{(r_1)}(y_1)\updownarrow \ldots \updownarrow \mathcal{L}^{(r_{p-1})}(y_{p-1}) =$$

$$= \sum_{j=1}^{p-1} \frac{1}{i} \Delta_R(x-y_j)\, \mathcal{L}^{(r_1)}(y_1)\updownarrow \ldots \updownarrow \mathcal{L}^{(r_j+1)}(y_j)\updownarrow \ldots \updownarrow \mathcal{L}^{(r_{p-1})}(y_{p-1}) \quad . \tag{3}$$

<u>Going from n to $n+1$</u>

To construct g.r.p. of $n+1$ Wick powers satisfying the induction hypothesis, in particular (2) and (3), it is necessary and sufficient to construct $(\Omega, \mathcal{L}^{(r_1)}(y_1)\updownarrow \ldots \updownarrow \mathcal{L}^{(r_{n+1})}(y_{n+1})\Omega)$ for $r_j \leq \nu - 2$, with correct properties. To do this, we note that if the problem is soluble the difference

$$\mathcal{L}^{(r_1)}(y_1)\uparrow \ldots \uparrow \mathcal{L}^{(r_{n+1})}(y_{n+1}) - \mathcal{L}^{(r_1)}(y_1)\downarrow \ldots \downarrow \mathcal{L}^{(r_{n+1})}(y_{n+1})$$

can be expressed as a sum of commutators $D(1,2,\ldots,n;n+1) = \sum_j [O_j, O_j']$ where the O_j, O_j' are already constructed g.r.p. We verify that this expression has the correct algebraic and geometric properties. In particular, its support is the union of two opposite closed convex cones $\Gamma^{\pm} = \pm \left\{ y : y_j - y_{n+1} \in \overline{V}^+ ,\ 1 \leq j \leq n \right\}$, having only the origin in common. The problem then is reduced to that of splitting tempered distributions $(\Omega, D\Omega)$, having support in $\Gamma^+ \cup \Gamma^-$, into a difference of two tempered distributions having support in Γ^+ and Γ^-, respectively. This is done by multiplying first $C(y) = (\Omega, D\Omega)$ by $y^\beta \sqcup\!\sqcup (y)$; here $\perp (y)$ is a

ANALYTIC REGULARIZATION, ELECTROMAGNETIC MASS DIFFERENCES AND $\pi^0 \to 2\gamma$ DECAY [1)]

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The simplest graphs contributing to the electromagnetic mass differences of the baryon octet and the decay rate for the process $\pi^0 \to 2\gamma$ are calculated including anomalous magnetic moments at the vertices. To obtain finite results the gauge-invariant formulation of analytic regularization[2)] is applied. In order to get only a reduced amount of arbitrariness compared with the whole class of Bogoliubov-Parasiuk-Hepp regularizations, this arbitrariness is introduced into the modified propagators by a function $f(\lambda) = \Lambda + \sum_{i=1}^{\infty} C_i \lambda^i$, the C_i being arbitrary constants, λ the variable in which the analytic continuation is performed. This corresponds to a certain class of generalized evaluators in the sense of Speer[3)].

In the regularized result there appears the arbitrary constant C_1 of the expansion of $f(\lambda)$. If this is supposed to be equal in all cases and fitted to the p-n mass difference taking the anomalous magnetic moments from SU(3) predictions, rather good agreement with experimental results is obtained.

Calculating a higher-order graph with n loops, the regularized result of the above-mentioned subclass will contain at most the arbitrary constants $C_1 \ldots C_n$ from the expansion of $f(\lambda)$.[4)] This is valid independent of the character of the particles involved (scalar, spinor or vector). Perhaps this may offer certain possibilities for renormalizing higher-order perturbation theory of non-renormalizable theories.

REFERENCES

1) W. Becker, H. Cameron and H. Mitter, preprint (to be published).

2) See the contribution of H. Mitter in these Proceedings, and P. Breitenlohner and H. Mitter, Nucl. Phys. B7, 443 (1968).

3) E. R. Speer, J. Math. Phys. 9, 1404 (1968).

4) W. Becker, Doctoral thesis (in preparation).

function infinitely differentiable everywhere except at 0, equal to 1 (and 0) in a cone larger than Γ^+ (and Γ^-), respectively, with $\sqcup\!\sqcup(\rho y) = \sqcup\!\sqcup(y)$ for all $\rho > 0$, $y \neq 0$; y^β denotes $\prod_{j=1}^{n} \prod_{\mu=0}^{3} (y_j^\mu - y_{n+1}^\mu)^{\beta_{j\mu}}$ and β is a multi-index with $|\beta| \geq M$ (M depends on C). This corresponds in momentum space to the convolution

$$H^\beta(p) = \int \tilde{\sqcup\!\sqcup}(p - p')\, D^\beta\, \tilde{C}(p')\, dp' \quad , \quad D^\beta = \prod_{j,\mu} \left(\frac{\partial}{\partial p_j^\mu}\right)^{\beta_{j\mu}} \quad ,$$

where $\tilde{\sqcup\!\sqcup}$ plays the role of a generalized Cauchy kernel. The functions H^β are the M^{+n} derivatives of the result we need. The latter is given by

$$H(p) = M \int_0^1 dt\, (1-t)^{M-1} \sum_{|\beta| = M} \frac{p^\beta}{\beta!}\, H^\beta(tp)$$

$$+ \text{ arbitrary polynomial of degree } (M-1).$$

Finding at each step a minimal value of M is the object of the power-counting theory. This (in our approach) consists in evaluating the behaviour of $C(p)$ at ∞. It is found that $M - 1$ coincides with the "superficial index of divergence", $\omega = (n+1)(\nu - 4) - E + 4$, where $E = r_1 + \ldots + r_{n+1}$. The above subtraction procedure can also be understood in terms of counter-terms, as usual. It is probably possible to extend this theory to non-polynomial $\mathcal{L}(y)$ in Jaffe's class.

(A full list of references will be given in a forthcoming paper.)

REFERENCES

1) E. Speer, J. Math. Phys. 9, 1404 (1968);
E. Speer, Annals of Mathematical Studies, No. 62, Princeton Univ. Press 1969.

2) P. Breitenlohner and H. Mitter, Nucl. Phys. B7, 443 (1968).

3) N. Kroll, Nuovo Cimento 45A, 65 (1966).

4) P. Breitenlohner and R. Seiler (unpublished results).

5) D. Großer, Doctoral thesis, Inst. of Tech., München, 1969 (it will be issued as a preprint by Max-Planck-Inst.)

6) W. Heisenberg, Nucl. Phys. 4, 532 (1957).

7) T.D. Lee and G. Wick, Nucl. Phys. B9, 209 (1969).

FORM-INVARIANT RENORMALIZATION

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We study the equations which connect the propagators (i.e., Green functions multiplied by the vacuum-vacuum transition amplitude) of a field theory; they form an infinite set since propagators with increasingly high numbers of external lines occur. This work is done best in configuration space and with propagators rather than Green functions because only thus can one achieve combinatorial expansions of striking symmetry and compactness [1]. The perturbative expansions of the propagators are assumed to be only asymptotic.

Having settled all combinatorial problems, one has [2]:

a) branching equations of I type (containing only propagators)

b) branching equations of II type (containing derivatives of propagators also, with respect to all masses and charges)

c) formal perturbative expansions

(The familiar Lehmann-Symanzik-Zimmermann equations are, for instance, of I type.)

There arise at this point structural and analytic problems. To investigate the first, we took several [3] simple models, consisting of spurion fields interacting among themselves or with ordinary spinor fields, and found that in all such cases the exact solutions have an essential singularity at the origin of the coupling constant, with the perturbative expansions as their asymptotic limits; we also found that the propagators are, for models corresponding to renormalizable theories, functions of Hadamard's class 2 in the coupling constant. Heuristic criteria were also devised to construct the exact solutions by using the knowledge of i) the asymptotic expansions and ii) the branching equations of both types [3].

identity. As can be shown by considering a special example[2], the invariance is not regained after application of the evaluation procedure. One could apply as a remedy a procedure described by Kroll[3] which ensures gauge-invariant results for an arbitrary propagator modification by introducing appropriate vertex modifications (additional one- and multiphoton-vertices). This procedure is, however, very impractical for the modification studied here. A much simpler remedy has been shown to work in Ref. 2. It consists in modifying only photon propagators if there are no closed loops (this does not spoil gauge invariance). For closed loops a procedure is described which is, in effect, a modification of the whole loop (only one λ for the whole loop). The procedure is given in Ref. 2 for a general closed loop with $n + 1$ corners, at one of which there is a γ_μ vertex (the other vertices are arbitrary). The procedure reads complicated, but the calculation turns out to be very simple (much simpler than with Pauli-Villars regulators; this is in general the case if one uses analytic regularization!). Some applications have been discussed in Ref. 2, a study of the anomaly of the axial vector divergence (i.e., the triangle with one $\gamma_5\gamma_\mu$ and two γ_μ vertices) is done at present. An analogous treatment of "axial gauge invariance" seems to be rather difficult[4].

The technique of analytic regularization can be applied, after some minor technical adaptions, to the Lee model, which can then be studied without any cut-off in the lowest sector[5]. The results which one obtains are exactly the same as those obtained with cut-off (in the limit in which the cut-off is removed): there are always ghosts present, the only interpretable solutions are Heisenberg's dipole solution[6] and the one discussed recently by T.D. Lee and G. Wick[7] in which the ghost is unstable. Thus the regularization procedure does no harm to the solution in this model.

The analytic problems come from the obvious fact that branching equations and formal expansions are meaningless as they stand because they require integrations over products of tempered distributions (the causal free propagators). We have proved that it is possible to change all such expressions into meaningful ones by carefully redefining these integrations by some continuation procedure which, starting from a subspace of test functions where integrations are meaningful, extends the result to the whole space of interest. It was found that any such procedure must satisfy a set of axioms [4), 5)] after which one is certain that the said procedure will leave invariant all formal expressions which now make perfect mathematical sense. It is convenient to denote any such procedure formally still with a symbol of integration, which is called "finite part integral" because it is in fact the generalization of Hadamard's "partie finie" integral.

We have used, in particular, the Güttinger and Speer [6)] procedure of analytic continuation (which is termed by some "analytic renormalization"), suitably extended to configuration space [5)]. On one hand, the renormalization programme is completely achieved, on the other, all terms which are subtracted by this technique are grouped together and found to be, in fact, mass and charge renormalizations in the Dyson sense. The ambiguity involved in any such procedure causes no trouble, provided the theory is renormalizable in Dyson's sense; otherwise things are still finite, but an infinite number of counter-terms in the standard formal treatment would be required. Explicit expressions for the renormalization group are easily obtained from the branching equations, upon requiring their form invariance under a change of procedure [7)].

This technique fully solves the proposed problem: it is independent of any approximation method used to solve the branching equations; it yields, in particular, the correctly renormalized asymptotic expansions. It is worth remarking that the combinatories used for this purpose [1), 2)] do <u>not</u> require the specific consideration of graphs but act quite automatically, subtracting in a correct manner all unwanted divergences. A full proof of these statements is given for the $g\varphi^4$ theory in Ref. 4.

SOME COMMENTS ON ANALYTIC RENORMALIZATION

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Analytic renormalization is one of the few methods which can be used to give meaning to a Lagrangian field theory in the framework of perturbation theory. Some problems which arise in dealing with this method in practical cases and some results which have been obtained will be reported in what follows.

A class of renormalization procedures, with which we have worked, arises from the prescription [1] to replace every Feynman denominator of a general Feynman diagram

$$\frac{1}{m^2 - p^2 - i\epsilon} \quad \text{by} \quad \frac{f(\lambda)\, m^{2\lambda}}{(m^2 - p^2 - i\epsilon)^{1+\lambda}} \quad \text{where} \quad f(\lambda) = 1 + \lambda C_1 + \lambda^2 C_2 + \ldots$$

where C_i are arbitrary constants and one has to take a different λ for every internal line (but the same C_i 's for every line belonging to a special sort of particle). The diagram is then evaluated along standard lines keeping λ large enough. After the integrations have been performed a certain evaluator is applied, which essentially consists in symmetrizing with respect to the λ's , continuing analytically towards $\lambda \to 0$ and omitting contributions which are singular in this limit. These contributions are of the type of a polynomial in $\Box$ times a δ-function $\delta(z_i)$ in co-ordinate space where $z_i = 0$ is a point at which the T-product (which forms the diagram) is not defined. In the result obtained the constants C_i occur and the interesting problem is how many of them are involved and how they can be fixed from physics. In renormalizable theories they have to be accommodated in renormalization constants.

In theories in which this is relevant, it turns out that the prescription given above does not lead to gauge-invariant results. As long as λ is finite this is clear, since the modified propagator does not fulfill Ward's

Among the applications already made of this formalism, we mention the study of a truncated model of the $g\varphi^4$ theory [8], which is both relativistically invariant and invariant under the renormalization group.

REFERENCES

1) E.R. Caianiello, Nuovo Cimento 10, 1634 (1953); 11, 692 (1954).

2) E.R. Caianiello, Nuovo Cimento 13, 637 (1959); 14, 185 (1959);
E.R. Caianiello, A. Campolattaro and B. Preziosi, Nuovo Cimento 18, 505 (1960);
E.R. Caianiello, F. Guerra and M. Marinaro, Progr. Theoret. Phys. (Kyoto) Suppl. No. 37-38, 183 (1966).

3) E.R. Caianiello, A. Campolattaro and M. Marinaro, Nuovo Cimento 38, 1777 (1965);
F. Guerra and M. Marinaro, Nuovo Cimento 42A, 285 (1966).

4) E.R. Caianiello, F. Guerra and M. Marinaro, Nuovo Cimento 60A, 713 (1969).

5) F. Guerra and M. Marinaro, Nuovo Cimento 60A, 756 (1969).

6) W. Güttinger, Fortschr. Phys. 14, 489 (1966);
E.R. Speer, J. Math. Phys. 9, 1404 (1968).

7) F. Esposito, U. Esposito and F. Guerra, Nuovo Cimento 60A, 772 (1969).

8) M. Marinaro and G. Scarpetta, to appear in Nuovo Cimento.

REFERENCES

1) C. Bouchiat, J. Iliopoulos and J. Prentki, Nuovo Cimento 56A, 1150 (1968).

2) J. Iliopoulos, Nuovo Cimento 62A, 209 (1969) and references given there.

3) S.L. Glashow and S. Weinberg, Phys. Rev. Letters 20, 224 (1968).

4) M. Gell-Mann, R.J. Oakes and B. Renner, CALTECH, preprint 68-165.

5) R. Gatto, G. Sartori and M. Tonin, Phys. Letters 28B, 128 (1968), and CERN preprint TH 964 (1968).

6) N. Cabibbo and L. Maiani, Phys. Letters 28B, 131 (1968) and Istituto Superiore di Sanità (Roma) preprint ISS 69/18.

7) R.N. Mohapatra and P. Olesen, Phys. Rev. 179, 1417 (1969) and references given there.

STRONG INTERACTIONS AS NATURAL CUT-OFF IN ELECTRODYNAMICS

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It is a long-cherished hope of many physicists that the strong interactions might reduce the divergences of hadron electrodynamics (in particular of the electromagnetic mass shift) by introducing a natural cut-off. In order to investigate this possibility we must consider the electromagnetic interactions as a perturbation not of a theory of free fields but of a theory of fields which are already strongly interacting among themselves. Attempts to develop such a theory lead at once to great mathematical difficulties. We shall therefore approach the problem in the LSZ framework.

We consider a model involving a real scalar "hadron" field $A(X)$ of mass $M > 0$ and a real scalar "photon" field $B(x)$ of mass $m > 0$. (Capital letters will be used to denote hadron variables, small ones for photon variables.) A is interacting strongly with itself, weakly with B. According to LSZ the theory can be characterized by its Green functions $\tau(P_1, \ldots, P_a, p_1, \ldots, p_b)$, i.e., the Fourier transforms of the amputated time-ordered functions. The τ have to satisfy a number of linear properties (symmetry, reality in certain points, invariance) plus a system of quadratic integral equations, the generalized unitarity equations.

We develop τ in a perturbation series with respect to the weak coupling constant and solve the unitarity equations in ascending order, taking the linear properties as subsidiary conditions. In <u>zeroth order</u> no weak interactions are present. This means that the $\tau_0(p_1, \ldots, p_b)$ are those of a free field B_0, the $\tau_0(P_1, \ldots, P_a)$ those of a <u>self-interacting</u> field A_0, and the truncated parts $\tau_0^T(P_i, p_j)$ of the mixed functions vanish.

In <u>first order</u> the unitarity equations become a set of linear homogeneous integral equations for τ_1. The choice of a particular solution

$$\left[\ldots\left[[\partial_\mu J_\mu(x_1), J_0(x_2)]\,,\; J_0(x_3)\right]\ldots, J_0(x_j)\right]\delta(x_1^0 - x_2^0)\ldots\delta(x_{j-1}^0 - x_j^0) \tag{4}$$

where the J's stand for both J and J^+. Using this form and working out the combinatorials, one can obtain the following results:

a) By introducing an infinite number of local counter-terms in the Lagrangian (1), one can get rid of all leading divergences. The explicit form of these counter-terms can be obtained to every order of perturbation theory.

b) Let $\mathcal{L}_0$ be a sum of two terms $\mathcal{L}_0^{SYM.} + \mathcal{L}_0^{BR.}$ where $\mathcal{L}_0^{SYM.}$ is invariant under chiral SU(3) x SU(3) and $\mathcal{L}_0^{BR.}$ is the symmetry breaking. It is clear from (4) that the leading divergences will depend only on $\mathcal{L}_0^{BR.}$.

If $\mathcal{L}_0^{BR.}$ is a member of a $(3\bar{3}) + (\bar{3}3)$ representation of the chiral group its general form will be

$$\mathcal{L}_0^{BR.} = C_0 U_0 + C_8 U_8 + C_3 U_3 \tag{5}$$

where the C's are real constants. Then one can prove the following:

b(i)) Without introducing any counter-term in the original Lagrangian (1), the non-diagonal transitions (i.e., parity and/or strangeness violating) are free from leading divergences.

b(ii)) In order to cancel the leading divergences from the diagonal matrix elements we must introduce subtractions to (1). However, the infinite number of such counter-terms required in the general case now collapses to a finite number:

$$\sim f(G\Lambda^2)\,[b_0 U_0 + b_8 U_8 + b_3 U_3 + b_7 V_7] \tag{6}$$

where f is a certain function of $G\Lambda^2$ and the b_i's are functions of the C_i's and the Cabibbo angle. Under certain prescriptions, this form can be used to determine the parameters of the theory and in particular the Cabibbo angle.

fixes the interaction, i.e., it replaces the specification of L_{int} . We choose τ_1 such that all $\tau_1^T = 0$, except those with exactly one photon variable. This choice corresponds to an L_{int} linear in B . In higher orders we shall restrict the ambiguity in the solution by demanding optimal behaviour for large momenta.

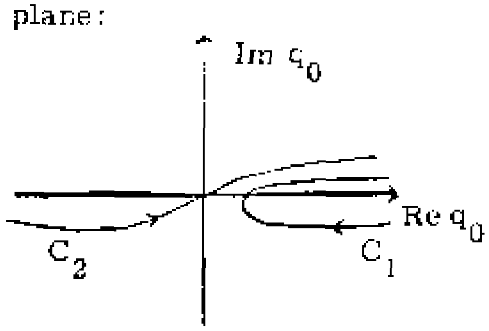
In <u>second order</u> the interesting cases are those with zero or two photon variables. The b = 2 case can easily be solved. The no-photon functions can then be obtained from the relations

$$\tau_2(P_1, \ldots, P_a) = \text{const} \int dq \, \frac{\tau_2(P_1, \ldots, P_a, q, -q)}{q^2 - m^2}$$

known to be valid in perturbation theory with respect to the strong interactions. In our formalism this expression satisfies unitarity in general only if the q_0-integration is taken over a path which differs from the Feynman description by an infinite semicircle in the lower half plane. For the second-order hadron mass shift we obtain

$$(\delta M^2)_2 = \text{const} \int dq \, \frac{M(P, q)}{q^2 - m^2} \quad ,$$

where $M(P, q)$ is the forward scattering amplitude of a hadron with four-momentum P (on the mass shell) and a photon of four-momentum q (off the mass shell). In the rest system of P the integrand is, as a function of q_0 , analytic in a cut plane:



Our prescription is to integrate over C_1 , in contradistinction to the traditional formula derived in a less rigorous way, which demands integration over C_2 . The two expressions coincide among themselves and with a third expression involving integration along the imaginary axis if the

LEADING DIVERGENCES TO ALL ORDERS OF WEAK INTERACTIONS

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I am going to present in this talk some results obtained last year about the leading divergences of weak interactions. Let me consider a Lagrangian of the form

$$\mathcal{L} = \mathcal{L}_0 + \sqrt{G}\ J_\mu(x)\ W^+_\mu(x) + \text{h. c.} \tag{1}$$

where $\mathcal{L}_0$ contains all interactions which do not vanish when the Fermi coupling constant G goes to zero. $J_\mu(x)$ is the hadronic part of the weak current and $W_\mu(x)$ is the field of the intermediate boson. We shall not consider any leptonic or semileptonic interactions in this talk.

The general matrix element for a transition $\alpha \to \beta$, with $|\alpha\rangle$ and $|\beta\rangle$ being eigenstates of $\mathcal{L}_0$, is given by

$$S_{\alpha\to\beta} \approx G^n \int \langle\beta|\, T\big(J_{\mu_1}(x_1), J^+_{\nu_1}(y_1), \ldots, J_{\mu_n}(x_n), J^+_{\nu_n}(y_n)\big)|\alpha\rangle$$

$$\Delta_{\mu_1\nu_1}(x_1-y_1)\ldots\Delta_{\mu_n\nu_n}(x_n-y_n)\, d^4x_1\, d^4y_1 \ldots d^4x_n\, d^4y_n \tag{2}$$

where $\Delta_{\mu\nu}(x-y)$ is the W-meson propagator. The most divergent parts of (2) come presumably from the $q_\mu q_\nu$ parts of the propagators and, under more or less general assumptions, one can show that they are of the form

$$G^n \Lambda^{2n} \int \langle\beta|\, T\big(\Sigma_1(x_1), \ldots, \Sigma_m(x_m)\big)|\alpha\rangle\, d^4x_1 \ldots d^4x_m \tag{3}$$

where Λ is a cut-off and $\Sigma_i(x)$ is an operator given by a multiple commutator at equal times of the form

q_0-integration is carried out before the d^3q integration. Changes of this order, or the introduction of curvilinear co-ordinates, may then lead to wrong results if carried out indiscriminately.

REFERENCES

The details of this work can be found in "Electromagnetic mass shifts in non-Lagrangian field theory", ICTP, Trieste, preprint IC/69/41 (to appear in Commun. Math. Phys.)

For the LSZ formalism and the properties of Green's functions see:

1) H. Lehmann, K. Symanzik and W. Zimmermann, Nuovo Cimento 1, 205 (1955); 6, 319 (1957).

2) V. Glaser, H. Lehmann and W. Zimmermann, Nuovo Cimento 6, 1122 (1957).

3) H. Epstein, in "Axiomatic Field Theory", ed. M. Chrétien and S. Deser (Gordon and Breach, New York 1966).

For the LSZ formulation of ordinary perturbation theory (i.e., with a free zeroth approximation) see:

4) K. Nishijima, Phys. Rev. 119, 485 (1960).

5) M. Muraskin and K. Nishijima, Phys. Rev. 122, 331 (1961).

6) H.M. Fried, J. Math. Phys. 3, 1107 (1962).

7) P. Curtius, Helv. Phys. Acta 36, 389 (1963).

8) O. Steinmann, Ann. Phys. (N.Y.) 29, 76 (1964); 36, 267 (1966).

For earlier derivations of the mass shift formula see:

9) M. Cini, E. Ferrari and R. Gatto, Phys. Rev. Letters 2, 7 (1959).

10) Riazuddin, Phys. Rev. 114, 1184 (1959).

11) W.R. Theis, Nucl. Phys. 21, 170 (1960).

12) V. Barger and E. Kazes, Nuovo Cimento 28, 385 and 394 (1963).

13) Riazuddin and Fayyazuddin, Phys. Rev. 158, 1447 (1967).

It is noted that in all renormalizable theories dilation invariance implies conformal invariance, while for arbitrary theories a condition is exhibited which is necessary and sufficient for dilation symmetry to imply conformal symmetry [1), 2)].

REFERENCES

1) C. Callan, S. Coleman and R. Jackiw, in preparation.

2) The connection between dilation symmetry and conformal symmetry has been previously established in slightly weaker form by G. Mack and Abdus Salam, Ann. Phys. (N.Y.) 53, 174 (1969); and D.J. Gross and J. Wess, in preparation.

NON-LOCAL QUANTUM FIELD THEORY

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The construction of the S-matrix in the non-local quantum field theory of one-component scalar field $\varphi(x)$ with the fixed interaction Lagrangian $gL_I(x)$ can conditionally be represented in the following main steps:

I. Formulation of the general axioms of the S-matrix theory

We introduce in the system of axioms of Bogolubov, Medvedev and Polivanov the following modifications:

a) The S-matrix must be unitary only on the mass shell.

b) The macrocausality condition can be written in the following form:

$$\frac{\delta}{\delta\varphi(x)}\left(\frac{\delta S}{\delta\varphi(y)}\ S^{-1}\right) = C(x,y) \qquad (1)$$

where any matrix elements of the operator $C(x,y)$ decrease rapidly enough in the region $y \tilde{<} x$ when $\|y-x\| \to \infty$.

II. The interaction Lagrangians

We shall consider the interaction Lagrangians of the kind

a) $$g\,\mathcal{L}_I(x) = g\varphi^N(x) \qquad (2)$$

where N is a fixed number.

b) $$g\,\mathcal{L}_I(x) = g\,U(\varphi(x)) = g\sum_{n=3}^{\infty}\frac{u_n}{n!}\,[\varphi(x)]^n \qquad (3)$$

where $U(\varphi)$ is a function of φ.

RENORMALIZING THE ENERGY-MOMENTUM TENSOR

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(Presented by R. Jackiw)

Contrary to common belief, the matrix elements of the energy-momentum tensor are not in general finite in renormalized perturbation theory, regardless whether this tensor is evaluated from the canonical formula

$$\theta^{\mu} = \sum_r \frac{\delta}{\delta\partial_\mu\varphi_r} \partial^\nu \varphi_r - g^{\mu\nu}\mathcal{L}$$

or from the symmetric formula of Belinfante. It is shown that for <u>all</u> renormalizable theories it is possible to add to $\theta^{\mu\nu}$ an additional term, which does not destroy the symmetries and conservation properties of $\theta^{\mu\nu}$, which does not contribute to the Poincaré generators, but which removes the divergences in renormalized perturbation theory, thus yielding a finite energy-momentum tensor. For example, for the scalar $\lambda\varphi^4$ theory, the addition is proportional to $(\partial^\mu\partial^\nu - \Box g^{\mu\nu})\varphi^2$.

The modified tensor $\theta^{\mu\nu}$ has the additional interesting property that in terms of it the dilation and conformal currents D^μ and $K^{\mu\nu}$, respectively, have the simple form

$$D^\mu = X_\nu \theta^{\mu\nu}$$

$$K^{\mu\nu} = X^2 \theta^{\mu\nu} - 2X^\nu X_\alpha \theta^{\alpha\mu} \quad .$$

III. The correspondence principle

The S-matrix has the form

$$S = 1 - ig \int dx \; \mathcal{L}_I(x) \tag{4}$$

for infinitesimally small g .

IV. The definition of T-product of two field operators:

$$\mathcal{D}(x_1 - x_2) = \langle 0 | T(\varphi(x_1)\, \varphi(x_2)) | 0 \rangle \; = \; ? \tag{5}$$

As the theory is non-local we shall consider that the T-product is undefined *not only in the point* $x_1 = x_2$ *but in some small domain near* $x_1 = x_2$.
Thus

$$\mathcal{D}(x_1 - x_2) = \frac{1}{(2\pi)^4 i} \int \frac{d^4p\, e^{ip(x_1 - x_2)}}{m^2 - p^2 - i\varepsilon} + \frac{1}{i} \sum_{n=0}^{\infty} \frac{c_n}{(2n)!} \Box^n \delta^{(4)}(x_1 - x_2) \tag{6}$$

where the second term is a non-local distribution. In momentum space we obtain

$$\tilde{\mathcal{D}}(p^2) = \frac{V(p^2 \ell^2)}{m^2 - p^2 - i\varepsilon} \tag{7}$$

where $V(p^2)$ is an entire function in the complex plane p^2 . The parameter ℓ^2 has the sense of elementary length. Thus we have introduced non-locality into the theory.

V. The construction of the higher orders of perturbation theory

From the mathematical point of view this problem is equivalent to the definition of the product of two distributions of the type (6). We define a matrix element corresponding to any connected Feynman diagram by means of the following improper limiting procedure:

$$F(p_1, \ldots, p_n) = \lim_{\delta \to 0} \int \ldots \int \prod_j d^4\ell_j \prod_{m \in \Gamma} \frac{V(k_m^2)}{m^2 - k_m^2 - i\varepsilon} \; R^\delta(k_m^2) \tag{8}$$

WILSON'S NON-LAGRANGIAN APPROACH TO FIELD THEORY

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A recent work of K. Wilson is discussed which proposes the following modifications of Lagrangian field theory:

I) Equal-time commutators are replaced by operator product expansion. By doing so, one avoids the ambiguous nature of equal-time commutators and preserves Lorentz covariance.

II) To have an estimate of the singular structure of the theory, Wilson proposes to determine the singularities by dimensional arguments, thereby following a suggestion of Kastrup and Mack that scale invariance is the most crucial broken symmetry in field theory.

These proposals were then applied by Brandt to renormalize theories like neutral pseudoscalar theory and quantum electrodynamics. It must be stressed that scale invariance arguments break down for vector meson theories with mass.

A recent application of these proposals to two-dimensional quantum electrodynamics brought an exact result for the equal-time commutator of the electric current and the vacuum polarization tensor.

REFERENCES

1) K. Wilson, Phys. Rev. 179, 1499 (1969);
K. Wilson, Cornell University, preprint, 1969, to appear in Phys. Rev.

2) P. Haberler, CERN preprints 1025 and 1046, to appear in Nuovo Cimento.

3) R. Brandt, Ann. Phys. (N.Y.) 52, 122 (1969); 44, 221 (1967); University of Maryland preprint UMD-910, to appear in Phys. Rev.; Phys. Rev. 166, 1795 (1968).

4) G. Mack, Nucl. Phys. B5, 499 (1968).

5) W. Zimmermann, Nuovo Cimento 10, 597 (1958).

where $p_1, \ldots, p_n$ are the external four-momenta, ℓ_j are the four-momenta over which the integration is performed. The product over m is performed over the lines corresponding to the given diagram Γ ; k_m is the four-momentum corresponding to the m-line. Then

$$R^{\delta}(k^2) = \exp\left\{-\delta(k^2 + i)^{\frac{1}{2}+\nu} e^{-i\pi\sigma}\right\} \tag{9}$$

where $0 < \nu < \sigma < \frac{1}{2}$. The function $R^{\delta}(k^2)$ is analytic in the upper half plane of the complex k^2 and

$$|R^{\delta}(k^2)| \leq \exp\left\{-\delta|k^2|^{\frac{1}{2}+\nu}\right\} \tag{10}$$

when $-\gamma \leq \arg k^2 \leq \pi + \gamma$, $\gamma > 0$ $|k^2| \to \infty$. Let $V(k^2)$ be the entire analytic function of the order $\rho < 1$ and

$$V(k^2) = O\left(\frac{1}{k^2}\right) \tag{11}$$

when $k^2 \to -\infty$. Then it is possible to go over to the Euclidean metric in the amplitude (8) rotating the integration contours $\ell_{j0} \to \ell_{j4}$. Then one can go to the limit $\delta \to 0$ as the integrals corresponding to any connected Feynman diagrams converge in the limit $\delta = 0$.

Thus we obtain the perturbation series which contain no ultraviolet divergences.

VI. The proof of the unitarity of the perturbation theory is grounded on the proof of the Cutkosky theorem for the normal thresholds for any Feynman diagram.

VII. The summation of the perturbation series

We make the following additional suggestions:

A) The S-matrix is a functional of a scalar real function $\varphi(x)$.

B) The S-matrix is written in the Euclidean metric.

so that they do not change the analysis recalled in 1 above but are coupled only to the external fields. Using this method we have determined the anomalies.

Finally, the PCAC equation reads

$$\partial_\mu A^\mu = c\pi + \mathcal{A}$$

where $\mathcal{A}$ contains the contribution of the two types of extra regulators which are only coupled to the vector and axial vector fields. If those last fields are quantized one gets a violation of the Goldberger-Treiman relation already at the level of strong interaction which seems to be hard to accept. If, on the contrary, the vector and axial vector are not associated with fundamental fields but, rather, are dynamical resonances, PCAC is only modified to the order α. Also, we have verified that in this case $\mathcal{A}$ does not modify the computation of the $\pi^+ - \pi^0$ mass difference[8].

REFERENCES

1) B.W. Lee, Nucl. Phys. B9, 649 (1969).

2) J.-L. Gervais and B.W. Lee, to appear in Nucl. Phys.

3) A similar conclusion has been reached, using a different method, by K. Symanzik, Lettere al Nuovo Cimento 2, 10 (1969) and this conference.

4) J.S. Bell and R. Jackiw, to appear in Nuovo Cimento.

5) S.L. Adler, Phys. Rev. 177, 2426 (1969).

6) D. Amati, C. Bouchiat and J.-L. Gervais, to appear in Nuovo Cimento.

7) This disagrees with the result one can get if the currents are defined by a limiting procedure which is too simple-minded a transposition of the method which works in spinor electrodynamics (B. Zumino, unpublished).

8) J. Das, et al., Phys. Rev. Letters 18, 759 (1967).

C) The form factor $V(p^2 \ell^2)$ in (7) satisfies in the Euclidean metric

$$\text{a)} \quad V(-k_E^2\, \ell^2) > 0 \tag{12}$$

$$\text{b)} \quad \mathcal{D}(0) = \frac{1}{(2\pi)^4} \int \frac{d^4 k_E\, V(-k_E^2\, \ell^2)}{m^2 + k_E^2} < \infty \quad , \tag{13}$$

D) We consider the non-linear interaction Lagrangians of the type

$$\mathcal{L}_I(x) = \int_{-\infty}^{\infty} d\alpha\, A(\alpha) : e^{i\alpha\varphi(x)} : \tag{14}$$

where

$$\int_{-\infty}^{\infty} d\alpha |\, A(\alpha) |\, e^{\frac{1}{2}\alpha^2 \mathcal{D}(0)} < \infty \quad . \tag{15}$$

E) The integration over x is performed over the finite four-volume V_4 in the Euclidean x-space.

The S-matrix can be represented in the form

$$S[\varphi] = \sum_{n=0}^{\infty} \frac{g^n}{n!} \int_{V_4} d^4 x_1 \ldots \int_{V_4} d^4 x_n \int_{-\infty}^{\infty} d\alpha_1 \ldots \int_{-\infty}^{\infty} d\alpha_n \quad \times$$

$$\times\, A(\alpha_1) \ldots A(\alpha_n)\, e^{i[\alpha_1 \varphi(x_1) + \ldots + \alpha_n \varphi(x_n)]} \quad \times$$

$$\times \exp\left\{ - \sum_{1 \le i < j \le n} \alpha_i\, \mathcal{D}(x_i - x_j)\, \alpha_j \right\} \tag{16}$$

One can easily show that

$$|\, S[\varphi] \,| \le \exp\left\{ g\, V_4 \int_{-\infty}^{\infty} d\alpha |\, A(\alpha) |\, e^{\frac{1}{2}\alpha^2 \mathcal{D}(0)} \right\} \quad . \tag{17}$$

The divergences arising when $V_4 \to \infty$ are connected with the vacuum-vacuum transition and they are picked out of the perturbation series in the form of the non-essential phase multiplier.

REFERENCES

All these results are published in the articles:

1) G.V. Efimov, Institute for Theoretical Physics, Kiev, preprints 68-52, 68-54, 68-55, 1968.

2) G.V. Efimov, JINR, Dubna, preprint R2-4546, 1969.

ization of the model, namely, one which is achieved by adding chiral-symmetric mass counter-terms and by simple re-definition of the bare constants of the Lagrangian[3].

Turning now to the case where vector fields are introduced in the σ model, one can see that the parity doublets do not regularize loops with an odd number of axial vertices like the one which leads to the $\pi^0 \to 2\gamma$ decay. For those diagrams one must use new regulators, only coupled to the vector and axial vector fields, which have the same chiral properties as the nucleon. One thus gets anomalies, in agreement with the analysis recently carried out by Bell and Jackiw[4] and Adler[5]. In collaboration with Amati and Bouchiat[6], a general study of those anomalies has been made for the case of external vector and axial vector fields with a non-abelian gauge group. Two cases have to be distinguished:

a) Anomalies involving an even number of axial fields

In this case the parity doublets do not play any role and, as the regulators introduced are not coupled to the π, σ or nucleon, the anomalies are the same as if the nucleon was only interacting with the external fields. We have studied the anomalies in this case. In particular, we find that the anomaly involving two axial fields is the third of the anomaly involving two vector fields[7]. Also, it appears, on the basis of gauge invariance, that anomalies involve higher power of external fields when the gauge group is non-abelian. The reason for this is that, in the abelian case, the amplitude has to be transverse with respect to the vector external lines. This reduces the degree of divergence, thus decreasing the number of diagrams which lead to anomalies. In the non-abelian case, on the contrary, there are longitudinal parts which lead to anomalies up to the fourth order in the external fields.

b) Anomalies involving an odd number of axial fields

In this case the situation is more involved as the parity doublets also contribute. In fact, they already regularize the corresponding loops by themselves, so that the new regulators lead to an ill-defined result. A way out is to introduce a third type of regulator which are parity doublets

MOMENTUM SPACE BEHAVIOUR OF INTEGRALS IN NON-POLYNOMIAL LAGRANGIAN THEORIES

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Most Lagrangians of physical interest - e.g., the chiral Lagrangians for strong interactions, the intermediate boson mediated weak Lagrangian and Einstein's Lagrangian for gravity - appear to be of the non-polynomial form in the field variables. By suitable field-transformations they can in general be expressed in the form of rational functions. This class of Lagrangian was examined in an earlier paper [1] (referred to as I) following a method due to Efimov and Fradkin [2], with particular reference to the ultraviolet infinities of physical amplitudes. The discussion was carried out in x-space with the amplitudes defined as Borel sums of divergent series like $\sum a_{nm\ldots} \Delta_F^n(x)\, \Delta_F^m(y)\ \ldots$. The singularity behaviour of these Borel sums was examined in the limits, $x^2 \to 0$, $y^2 \to 0$, ... With Efimov and Fradkin we concluded that if the Dyson index D of these rational Lagrangians was less than or equal to 4, all ultraviolet infinities associated with amplitudes in these theories could be compensated by a finite number of counter-terms (Dyson index D is defined by the limit, $\lim_{\phi \to \infty} L(\phi) = \phi^D$) and in this respect the theories behave like renormalizable theories.

For their physical use we need the renormalized amplitudes not in x-space but in p-space. What we did not examine in I were the momentum-space Fourier transforms, their analyticity properties and their asymptotic behaviour. This talk is devoted to a consideration of these problems, following a method first discussed in this context by Volkov [3] and which in its essentials goes back to a discussion (in the appropriate region of x and n) of the Fourier transform of $[\Delta_F(x)]^n$ by Gel'fand and Shilov [4]. A study of the same Fourier transforms has recently been made by Lee and Zumino [5] using different methods; we reproduce their results for the examples they consider. In particular we show:

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ON THE AXIAL CURRENT WARD-TAKAHASHI IDENTITY

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A general study is presented on the so-called anomalous terms in the Ward-Takahashi identity. It is based on a systematic use of the Pauli-Villars regularization method which automatically ensures the gauge invariance of the result while canonical formalism can be used on the regularized Lagrangian as long as the regulator masses are kept finite. In this method, anomalies show up as regulator contributions in the limit of infinite regulator masses.

Anomalous terms have been exhibited only in the presence of vector fields. In collaboration with B.W. Lee [1], we have tried to see whether anomalies cannot occur also when there are no vector fields by studying the σ model. In this model PCAC holds as an operator equation between bare quantities, and the question of anomalies in PCAC is linked to the question of renormalizing the σ model without spoiling the structure of the bare Lagrangian where the breaking of chiral symmetry comes only from the fact that the σ can go into the vacuum. This question had already been studied by B.W. Lee [2] but without considering the fermion contributions. In Ref. 1 we have shown that no anomaly will be encountered if one makes an appropriate choice of the regularization procedure which has to be such that it can also be applied to the symmetric theory, namely, the one where the σ-tadpoles are absent. As a consequence, we find that one cannot regularize by adding spin 1/2 fields with the same chiral properties as the nucleon. In particular, we exclude the prescription of Bell and Jackiw [3], in which the regulators are given zero mass, as inconsistent. A new regularization procedure is proposed where the regulators appear in parity doublets in such a way that their mass terms can be made chiral symmetric. It is shown to be applicable to the symmetric theory and therefore does not lead to any anomaly in PCAC when applied to the complete σ model. More generally, it leads to a consistent renormal-

a) The Fourier transforms, if properly defined, give correctly the singularity structure consistent with the unitarity requirements.

b) Our method gives immediately the asymptotic behaviour for large and space-like p^2 .

c) The discussion of ultraviolet infinities, previously carried out in x-space (I), is closely paralleled for p-space and the same conclusions are reached.

d) The closed loop integrations in p-space have exactly the same form in polynomial and non-polynomial Lagrangian theories. The methods of analytic renormalization [6] studied recently for polynomial Lagrangians are particularly appropriate to the p-space method discussed in this talk.

REFERENCES

1) R. Delbourgo, Abdus Salam and J. Strathdee, ICTP, Trieste, preprint IC/69/17, to appear in Phys. Rev.

2) G.V. Efimov, Soviet Phys.-JETP 17, 1417 (1963); E.S. Fradkin, Nucl. Phys. 49, 624 (1963).

3) M.K. Volkov, Ann. Phys. (N.Y.) 49, 202 (1968).

4) I.M. Gel'fand and G.E. Shilov, "Generalized Functions", Vol.I (Academic Press, New York and London 1964).

5) B.W. Lee and B. Zumino, CERN preprint TH.1053 (1969).

6) E.R. Speer, J. Math. Phys. 9, 1404 (1968).

Further references will be found in ICTP, Trieste, preprint: Abdus Salam and J. Strathdee, IC/69/120.

REFERENCES

1) J. Schwinger, Phys. Rev. 82, 664 (1951).

2) S. L. Adler, Phys. Rev. 177, 2426 (1969).

3) C.R. Hagen, Phys. Rev. 177, 2622 (1969).

4) R. Jackiw and K. Johnson, to appear in Phys. Rev.

5) R.A. Brandt, Phys. Rev. 180, 1490 (1969).

6) M. Gell-Mann and F.E. Low, Phys. Rev. 95, 1300 (1954).

SOME REMARKS ON THE EFIMOV-FRADKIN METHOD IN NON-LINEAR FIELD THEORIES

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A discussion is given of the method of formal summation of the perturbation series in non-linear field theories proposed by Efimov[1] and Fradkin[2]. Interest in this method has been revived recently[3),4),5)], in view of the suggestion that the method may be used to give a meaning to theories described by chiral Lagrangians as well as certain vector theories.

The proposal is investigated in detail in the context of a simple example, where the complications due to derivative and vector couplings are absent. We consider the problem of constructing the two-point function for a field V(x) which is a function of a free field $\phi(x)$. In particular we take

$$V(x) = :\phi(x) \sum_{\ell=0}^{\infty} [\alpha \phi^2(x)]^\ell : = : \frac{\phi(x)}{1-\alpha\phi^2(x)} .$$

but different functions could be treated in a similar way. The Fourier transform of the two-point function for the field V(x) is calculated by introducing a Pauli-Villars regularization (or other) in the propagator of the field ϕ and carrying out the summation of the perturbation series. The result is given in terms of an integral over a parameter: one finds that the path of integration can be specified in such a way as to produce an amplitude in momentum space having the physically correct analyticity properties in the variable $s = p^2$. As long as the regulator mass is kept finite, this amplitude can be expanded in powers of f^2 generating in this way the regularized perturbation expansion which is shown to be the asymptotic expansion in f^2 of the amplitude. In this way a precise meaning to the formal summation is given.

This apparent paradox is, however, not an isolated instance of the apparent shortcomings of the ϵ technique. In particular, one finds that it implies i) a non-covariant polarization tensor, ii) contradictory results for the j^0, j_5^0 commutator, iii) a breakdown of electric charge conservation in the presence of vector and axial vector fields and iv) a contradiction with the perturbation theory calculation of j_5^μ in the presence of axial vector fields.

The resolution proposed for the above-listed paradoxes consists in realizing that the source-driven spinor field is not consistent as it stands because of the fact that not all the matrix elements of j^μ (i.e., as generated by the external vector field) are finite. When they are made finite (as, for example, by a regularization technique) it is trivial to show that all of the aforementioned difficulties are resolved. Furthermore, the so-called anomalous commutators vanish and certain non-canonical high-energy modifications of the theory, which have been predicted elsewhere, are found not to occur. In addition, the Schwinger-Adler result is found to be true not as an operator equation but as a relation between matrix elements in which the regulator mass term generates the anomalous contribution.

Although this approach can be criticized because of the "unphysical" regulator field, it is to be emphasised that the spirit in which this is done is that although a regulator is required order by order, the regulator dependence should drop out in the end by means of an eigenvalue condition of the Gell-Mann-Low type on the bare coupling constant. Such a condition renders the entire theory finite (and thereby consistent) while at the same time leaving intact the conclusion concerning the absence of anomalous commutators. Should it seem that this approach (i.e., an eigenvalue condition on the fine structure constant) is somewhat far-fetched, one should emphasise that it represents the only hope for a consistent electrodynamics. It can also be argued that the very existence of this conference implies that we are a long way from giving up such hopes for this most successful of all field theories.

The choice of the integration path over the parameter, alluded to above, can also be described as an appropriate average of the two possible analytic continuations for the sum from negative values of f^2 to the physical positive value. Each of the two analytic continuations, taken alone, would give for positive f^2 an amplitude with a non-vanishing imaginary part in the region for s below the threshold, where the amplitude must be real. The average gives a real function there, in agreement with unitarity, and can also be shown to have the right analytic continuation to values of s above the threshold. Let us compare, for s below the threshold, one of the two complex analytic continuations from negative to positive f^2, with their real average. For finite regulator mass, both these functions have the same regularized perturbation expansion as their asymptotic expansion in f^2. This apparent paradox is resolved by the verification that the imaginary part has a vanishing asymptotic expansion in f^2. As the regulator mass tends to infinity, however, this is no longer true and the imaginary part has a non-vanishing asymptotic expansion.

The summation method of Efimov and Fradkin, with its precise interpretation given here, produces a finite function having the correct analyticity properties and the correct imaginary part, as dictated by unitarity requirements. One may wish to compare the result of this method with the function represented by a dispersion integral over the imaginary part. For the non-linear model considered, the imaginary part increases exponentially for large s, corresponding to real multiple particle production. A dispersion integral, therefore, requires an infinite number of subtractions. The ambiguity can be described in terms of an arbitrary real entire function. The summation method of Efimov and Fradkin gives rise to the same degree of ambiguity because the averaging procedure mentioned above is not unique. Different ways of averaging give functions which differ by a real entire function of s. It is interesting to observe that, for finite regulator mass, this entire function has a vanishing asymptotic expansion in f^2, so that in any case agreement with the perturbation expansion is obtained. One could eliminate this ambiguity by requiring that the real part of the amplitude should vanish as $s \to +\infty$, in which case the averaging procedure becomes

NON-EXISTENCE OF ANOMALOUS COMMUTATORS IN SPINOR ELECTRODYNAMICS*

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Shortly after the rediscovery by Adler of Schwinger's expression for the divergence of the axial current in spinor electrodynamics, it was shown that this result could be derived rather elegantly by means of a point-splitting definition of the current operator. Thus, instead of the naive expression $\frac{i}{2}\psi\beta\gamma_5\gamma^\mu\psi$, one writes the axial current as

$$j_5^\mu = \lim_{x',x''\to x} \frac{i}{2}\psi(x')\,\beta\gamma_5\gamma^\mu \exp\left\{ie_0 q\int_{x''}^{x'} dx^\nu A_\nu\right\}\psi(x'')$$

with the limit to be understood as implying

$$x' = x + \epsilon/2$$
$$x'' = x - \epsilon/2$$

with ϵ a purely spatial vector (we employ here a formalism in which ψ is hermitian so that q is the antisymmetrical matrix which acts in the internal charge space). The remarkable aspect of the calculation of the axial current divergence is the fact that with the above definition the result is independent of the way in which the $\epsilon = 0$ limit is taken.

The motivation of the present work was a desire to develop an action principle formalism which allows the introduction of the point-splitting technique and leads to the Schwinger-Adler result for the current divergence. Although the electric current can indeed be handled by this technique with no particular difficulty, it was found that the pseudovector defined by this approach is not independent of the details of the $\epsilon = 0$ limit. Only in the case in which a four-dimensional averaging is performed (an approach which is not admissible in a canonical formalism) is the desired result obtained.

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* This work is based on a University of Rochester preprint UR-875-274 of the same title.

unique; however, there does not appear to be sufficient motivation for this restriction.

The imaginary part of the two-point function increases exponentially in s as $s \to +\infty$. In view of the exponential increase for positive s, the theory under consideration is therefore not "strictly local"[6] and is not even "local"[7),8].

The question of the ambiguity in the two-point function and that of the possible further ambiguities in higher-point functions, seems to us one of the main problems in the method discussed.

REFERENCES

1) G.V. Efimov, Soviet Phys.-JETP 17, 1417 (1963); Phys. Letters 4, 314 (1963); Nuovo Cimento 32, 1046 (1964); Nucl. Phys. 74, 657 (1965).

2) E.S. Fradkin, Nucl. Phys. 49, 624 (1963).

3) M.K. Volkov, Commun. Math. Phys. 7, 289 (1968).

4) H.M. Fried, Suppl. New Physics (Korean Physical Society) 7, No.2, 23 (1968).

5) R. Delbourgo, Abdus Salam and J. Strathdee, ICTP, Trieste, preprint IC/69/17 (to appear in Phys. Rev.).

6) A.M. Jaffe, Phys. Rev. 158, 1454 (1967).

7) G.V. Efimov, Commun. Math. Phys. 5, 42 (1967).

8) F. Constantinescu, Munich University preprint (April 1969), other references can be found there.

two-photon state. If a PCAC assumption is made for the naive divergence operator, the low-energy theorem is found to give a good fit to the decay $\pi^0 \to 2\gamma$ in several interesting models.

REFERENCES

1) For discussions of the anomalous Ward identities for the triangle diagram see: S. L. Adler, Phys. Rev. 177, 2426 (1969);
J. Schwinger, Phys. Rev. 82, 664 (1951);
C. R. Hagen, Phys. Rev. 177, 2622 (1969);
R. Jackiw and K. Johnson, Nuovo Cimento 60, 47 (1969).

2) For discussions of the anomalous Ward identities for other spinor loops see: W. A. Bardeen, to appear in Phys. Rev.;
I. Gerstein and R. Jackiw, to appear in Phys. Rev.;
D. Amati, G. Bouchiat and J.-L. Gervais, Orsay preprint.

3) For discussions of higher-order corrections to the anomalous terms see: S. L. Adler and W. A. Bardeen, to appear in Phys. Rev.;
D. Amati, G. Bouchiat and J.-L. Gervais, Orsay preprint.

4) For discussions of the low-energy theorem for $\pi^0 \to 2\gamma$ decay see:
D. G. Sutherland, Nucl. Phys. B2, 433 (1967);
S. L. Adler, Phys. Rev. 177, 2426 (1969);
R. Jackiw and K. Johnson, Nuovo Cimento 60, 47 (1969);
S. L. Adler and W. A. Bardeen, to appear in Phys. Rev.

THE MEANING OF NON-LINEAR LAGRANGIANS

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It should be pointed out that the action integral of a non-linear chiral Lagrangian need not be considered as the basic Lagrangian of a non-renormalizable theory, but can be taken, instead, as an approximation to the effective action which generates the one-particle irreducible vertices of a renormalizable theory. From this point of view, the problem of the higher-order effects is not that of calculating the contribution of diagrams with one or more loops, but rather that of giving an improved description of the one-particle irreducible vertices. The correct effective action which generates them has the property that, when used in the tree approximation, it reproduces the exact propagators and S-matrix.

Let

$$Z = \left\langle T^{*} e^{i\int(\eta_i \phi_i + v_{\mu a} V^{\mu}_{a})} \right\rangle_0 = e^{iW}$$

be the functional which generates the time-ordered functions (τ-functions) of the renormalized fields Φ_i and of the currents V^{μ}_{a}. The functional $W[\eta_i, v_{\mu a}]$ of the sources η_i and external fields $v_{\mu a}$ generates the connected τ-functions. The effective action $A[\varphi_i, v_{\mu a}]$, defined by the functional Legendre transformation [1), 2)] (φ_i are c-number fields)

$$W[\eta_i, v_{\mu a}] = A[\varphi_i, v_{\mu a}] + \int \eta_i \varphi_i \, d_4x$$

$$\frac{\delta W}{\delta \eta_i} = \varphi_i \qquad\qquad \frac{\delta A}{\delta \varphi_i} = -\eta_i \quad ,$$

generates the one-particle irreducible vertices. Using these vertices and the complete propagator (also obtainable from A), one obtains the exact τ-functions by using, however, only tree diagrams. Clearly A is complex and non-local, but it satisfies unitarity and locality constraints.

WARD IDENTITIES
IN RENORMALIZABLE LAGRANGIAN FIELD THEORIES

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The properties of the currents, their divergences and their commutators, have been studied in the context of perturbation theory for renormalizable Lagrangian field theories. In some cases the results of the perturbation calculations were found to be inconsistent with the naive use of formal manipulations of the field operators. The anomalous terms arise due to ambiguities in the definition of local products of field operators which do not allow the use of formal manipulations.

In a large class of theories it is possible to find the precise form of the anomalous terms in the divergence equations for the currents. These divergence equations may be analysed using a Pauli-Villars regularization of the boson lines which preserves the exact formal Ward identities for currents attached to the boson lines. The remaining divergences and ambiguities are contained in the smaller spinor loops. As the spinor loops are non-overlapping, each loop may be defined independently. A direct calculation shows that the spinor loops may be defined so that the vector currents satisfy the naive Ward identities. Anomalous terms then appear in the Ward identities for the axial vector currents in those smaller loops with only vector and axial vector vertices having an abnormal parity relation. These anomalous terms may be represented as anomalous divergences for the axial vector currents. These results are shown to be valid for essentially all renormalizable theories.

The presence of the anomalous terms in the divergence of the axial vector currents imply a modification of the Sutherland-Veltman low-energy theorem for the naive divergence operator. Hence we may obtain an exact low-energy theorem valid to any finite order in perturbation theory for the matrix element of the naive divergence operator between the vacuum and a

If the basic Lagrangian is invariant under a group G having generators associated with the currents $V_{\mu a}$, the τ-functions satisfy an infinite set of Ward identities, to which correspond Ward identities for the irreducible vertices generated by A. These can be stated most simply as the requirement that A be invariant

$$\delta A = 0$$

under the local (Yang-Mills) transformations associated with the group G

$$\delta\varphi_i = \Lambda_a M_{aij} \varphi_j$$

$$\delta v_{\mu a} = \Lambda_b C_{bac} v_{\mu c} + \partial_\mu \Lambda_a \quad ,$$

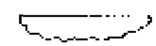
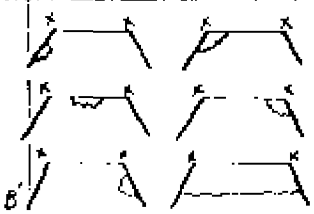
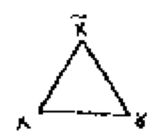
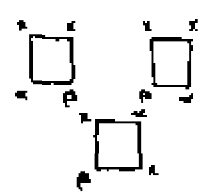
where C_{bac} are the structure constants and M_{aij} the matrix representatives of the generators, while $\Lambda_a(x)$ is a gauge function. If the symmetry is spontaneously broken, in which case a stable solution $\varphi_i = \bar{\varphi}_j$ (a non-vanishing constant) corresponds to $\eta_i = 0$, one can always[3] introduce new fields χ_i, which are local non-linear functions of the φ_i and $\bar{\varphi}_i$ and which transform by irreducible non-linear realizations of the group. The effective action, expressed in terms of the fields χ_i, is invariant under non-linear realizations of the group. The local non-linear Lagrangians which one finds discussed in the literature can be considered as lowest-order approximations to the correct effective action.

REFERENCES

1) J. Schwinger, Proc. Nat. Acad. Sci. 37, 452 and 455 (1951).

2) G. Jona-Lasinio, Nuovo Cimento 34, 1790 (1964).

3) S. Coleman, J. Wess and B. Zumino, Phys. Rev. 177, 2239 (1969).

$\langle 0\vert[\psi(x,t),\bar\psi(y,t)]_+\vert 0\rangle = \gamma^0\delta(x-y)$	[illegible]	$(1-\frac{3g^2}{32\pi^2})\gamma^0\delta(x-y)$		[illegible]	[illegible]
$\langle 0\vert[\psi(x,t),j^0(y,t)]\vert i\rangle=\delta(x-y)$ $\langle 0\vert[\psi(x,t),j^i(y,t)]\vert f\rangle=\gamma^0\gamma^i\delta(x-y)$	$\Gamma^\mu \xrightarrow{k\to\infty} \gamma^\mu$	$\delta(x-y)$ $(1-\frac{g^2}{8\pi^2})\gamma^0\gamma^i\delta(x-y)$		[illegible]	
$\langle\psi\vert[j^i(x,t),j^j_5(y,t)]\vert\psi\rangle = \delta(x-y)\,i\epsilon^{ijk}\langle\psi\vert j^k_5(x)\vert\psi\rangle$ $\int d^3x\,\langle\psi\vert[j^i(x,t),j^j(y,t)]\vert\psi\rangle = (\delta^{ij}p^2-p^ip^j)A+\delta^{ij}B$	[illegible] radiative corrections [illegible]	$\delta(x-y)\,i\epsilon^{ijk}(1-\frac{3g^2}{16\pi^2})\times\langle\psi\vert j^k_5(x)\vert\psi\rangle$ $(\delta^{ij}p^2-p^ip^j)A+\delta^{ij}B+\delta^{ij}\not p B'$		[illegible]	[illegible]
$\langle 0\vert[j^0(x,t),j^i(y,t)]\vert 0\rangle = 0$	See remark 4	$S\partial^i\delta(x-y)-\frac{1}{24\pi^2}\nabla^2\partial^i\delta(x-y)$		[illegible]	[illegible]
$\langle 0\vert[j^0(x,t),j^\mu_5(y,t)]\vert\gamma\rangle = 0$ $\langle 0\vert[j^\mu(x,t),j^\nu_5(y,t)]\vert\gamma\rangle = 0$	[illegible]	$\delta([x-y]\cdot n)\langle 0\vert[j^\mu(x,t),j^\nu_5(y,t)]\vert\gamma\rangle$ $= -Cn^\nu P_{\alpha\beta}\,\partial^\alpha\delta(x-y)\,\partial^\beta\langle 0\vert\tilde F^{\mu\beta}(x)\vert\gamma\rangle$ $+C\left(g^\mu_\sigma[g^\nu_\tau n^\nu+g^{\alpha\nu}n_\tau]+g^\nu_\sigma[g^\mu_\tau n^\mu+g^{\alpha\mu}n_\tau]\right)\times$ $P_{\alpha\beta}\,\partial^\beta\delta(x-y)\langle 0\vert\tilde F^{\sigma\tau}(y)\vert\gamma\rangle$		[illegible]	[illegible]
$\langle\alpha\vert[j^0(x,t),j^i(y,t)]\vert\beta\rangle = 0$	See remark [illegible]	[illegible] if α and β are scalar or pseudoscalar particles $-\langle\alpha\vert\phi^2(y)\vert\beta\rangle\partial^i\delta(x-y)$ if α and β are [illegible] particles		[illegible]	[illegible]

In addition to the entries of this table, there exist the large number of calculations from the triangle graph by Jackiw and Low[1])

TABLE

STRICTLY LOCALIZABLE FIELDS

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The usual Wightman axioms for quantum field theory are based on the assumptions of relativistic quantum mechanics:

1) There is a Hilbert space of states.

2) The theory is Lorentz covariant.

3) The energy operator is positive.

4) The theory is local.

5) A particle interpretation exists.

On the basis of these axioms, posed in a mathematically precise form, several general results about field theories have been proved. These include:

6) The connection between spin and statistics: Lüders, Zumino, Burgoyne.

7) The PCT theorem: Jost.

8) Scattering theory: Haag, Ruelle, Araki, Hepp.

9) Crossing symmetry: Bros, Epstein, Glaser.

10) Dispersion relations: Bogolubov, Medvedev, Polivanov, Bremmermann, Oehme, Taylor, Hepp.

Since these results depend on the precise details of the mathematical framework, we can ask for what fields they apply. Wightman made the assumption that fields are tempered distributions in the sense of Laurent Schwartz. This assumption automatically limits the momentum space growth of field amplitudes to be no faster than a polynomial. On the basis of this assumption, the results 6)-10) were proved using certain mathematical tools for tempered distributions:

REFERENCES

(for the table)

1) H. Lehmann, Nuovo Cimento 11, 342 (1954).

2) R. Jackiw and G. Preparata, to appear in Phys. Rev.

3) G. Preparata and W.I. Weisberger, Phys. Rev. 175, 1965 (1968).

4) C. Callan and D.J. Gross, Phys. Rev. Letters 22, 156 (1969).

5) R. Jackiw and G. Preparata, Phys. Rev. Letters 22, 975 (1969).

6) S.L. Adler and Wu-Ki Tung, Phys. Rev. Letters 22, 978 (1969).

7) J. Schwinger, Phys. Rev. Letters 3, 296 (1959).

8) T. Goto and I. Imamura, Progr. Theoret. Phys. (Kyoto) 14, 396 (1955).

9) D.G. Boulware and R. Jackiw, preprint 1969.

10) D.G. Sutherland, Nucl. Phys. B2, 433 (1967).

11) K. Johnson and F.E. Low, Progr. Theoret. Phys. (Kyoto), Suppl. No. 37-38, 74 (1966).

12) R. Jackiw and K. Johnson, to appear in Phys. Rev.

13) S.L. Adler and D.G. Boulware, Inst. of Advanced Studies, Princeton, preprint, 1969.

14) S. Weinberg, Phys. Rev. Letters 18, 507 (1967).

15) T. Nagylaki, Phys. Rev. 158, 1534 (1967).

a) Characterization of the Laplace transforms of distributions with support in a cone.

b) Edge of the wedge theorem.

c) Jost-Lehmann-Dyson representation.

However, it is clear that non-renormalizable theories, such as those described in this conference by Lehmann and Zamino, do not fall into this class. Many authors had suggestions on this problem, and papers were written by Güttinger, Meiman, Schroer, van Hieu and Efimov, to mention a few. However, none of these authors posed a precise mathematical framework for non-renormalizable but local fields.

Such a framework is given in Refs. 1-3. A large bibliography of earlier work can be found in Ref. 2 and also in Ref. 4. In order to make a mathematically precise framework, three steps are involved:

i) Formulation of locality.

ii) Development of the mathematical tools such as a)-c).

iii) Applications such as 6)-10).

We study a field φ defined on test functions f,

$$\varphi(f) = \int \varphi(x)\, f(x)\, dx = \int \tilde{\varphi}(p)\, \tilde{f}(-p).dp \ .$$

In order to allow $\tilde{\varphi}$ to grow faster than a polynomial in momentum space we take test functions that decay in momentum space faster than any polynomial. We take momentum space test functions $\mathcal{M}$ such that

$$\mathcal{M} = \left\{ f : \sup_{p} \left| (1 + \|p\|^2)^n\, g(A\|p\|^2)\, D^m f(p) \right| < \infty \right.$$

$$\left. \text{for all } n,\ m,\ A \right\} .$$

Here $\|p\| = (p_0^2 + \vec{p}^{\,2})^{\frac{1}{2}}$ is the Euclidean norm of p and $g(t^2)$ is an entire function

$$g(t^2) = \sum_r c_{2r}\, t^{2r} \ , \quad c_{2r} \geq 0\ , \quad c_0 > 0 \ .$$

REMARKS
(for the table)

a) The solid line is a Fermion.

b) The wavy line is a vector boson in the Landau gauge (so that $Z_1 = Z_2$ is finite) coupled with strength g to $\overline{\psi}\gamma^{\mu}\psi$.

c) G(p) is the unrenormalized Fermion propagator.

d) $\Gamma^{\mu}(p, q)$ is the unrenormalized vertex function.

e) x in the diagram represents the vector current.

f) The state $|\psi\rangle$ is a Fermion state with momentum p normalized so that $\langle 0|\psi|\psi\rangle = 1$.

g) Crossed diagrams must also be included.

h) Schwinger[7] showed that positivity and Lorentz covariance force the commutator to be non-zero. Previously, Goto and Imamura[8] derived a representation for this object which involved one derivative of the delta function. Their result is not verified by calculation.

i) $\tilde{x}$ in the diagram is the axial vector current.

j) $|\gamma\rangle$ is a one-photon state.

k) $\tilde{F}^{\mu\nu}$ is the dual electromagnetic tensor.

l) c is a constant.

m) The commutator has been written in explicitly covariant notation, with the help of a unit time-like vector n , and $P_{\alpha\beta} \equiv g_{\alpha\beta} - n_{\alpha} n_{\beta}$. Only the anomalous portion of the commutator is explicitly indicated.

n) α , β are bosons.

o) φ is a scalar or pseudoscalar field.

p) Some derivations of Weinberg's first sum rule assume a c-number Schwinger term[14]. In spite of the presence of q-number Schwinger terms this theorem remains true.

Since $g(t^2)$ indicates up to a polynomial the allowed momentum space growth of the fields, we call g the <u>indicator function.</u> The configuration space test functions $\mathscr{C}$ are obtained by Fourier transforming $\mathscr{A}$.

We say that the field φ is strictly localizable if φ has configuration space test functions with compact support. Hence $\varphi(f)$ can be localized in a bounded space-time region and locality can be defined in the usual sense. This imposes a requirement on the momentum space growth of the indicator function. In fact, φ is strictly localizable if and only if

$$\int_1^{\infty} \frac{\log g(t^2)}{t^2} \, dt < \infty \quad .$$

Hence growth such as

$$g(s) \leq \exp(s^{\alpha}) \quad , \qquad \alpha < 1/2$$

or

$$g(s) \leq \exp(s^{\frac{1}{2}} / (\log s)^2)$$

is allowed, but

$$g(s) \geq \exp(\epsilon \, s^{\frac{1}{2}}) \quad , \quad \epsilon > 0 \; ,$$

is not allowed.

Within the framework of strictly localizable field theory, we can prove a)-c). For instance, we have the following characterization of the local singularities of the vacuum expectation values of strictly localizable fields.

Let

$$G(t^2) = \frac{1}{t} \int_0^{\infty} g(\mu^2) \, e^{-u/t} \, du$$

$$= \sum_{r=0}^{\infty} c_{2r} (2r)! \; r^{2r}$$

be the Borel transform of g. In particular, if $g(s)$ is an entire function of order $\alpha < 1/2$, then $G(s)$ is an entire function of order $\alpha(1-2\alpha)^{-1}$, which may be arbitrarily large.

NON-CANONICAL BEHAVIOUR IN CANONICAL THEORIES

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A summary of recent work concerning the nature of commutators in model field theories is presented. It is shown that commutators defined via the Bjorken-Johnson-Low theorem [1] in perturbation theory will differ in the general case from their canonical value. These modified commutators are relevant, by construction to high-energy theorems such as those of Preparata and Weisberger [2] or Callan and Gross [3]. These high-energy theorems are accordingly modified. Modification of low-energy theorems, such as the one of Sutherland and Veltman for $\pi^0 \to 2\gamma$ decay [4], are shown to follow from the fact that the non-canonical commutators can be of a form which makes it impossible for Feynman's conjecture to hold, i.e., Schwinger terms do not cancel against divergences of seagulls [5)-7)]. The solution of the general problem of constructing Lorentz-covariant and gauge-invariant T^* products from a knowledge of the T product and of the commutators is indicated [7]. The dependence of the commutators on the dynamics is exhibited. A summary of results is presented in the Table.

REFERENCES

1) J.D. Bjorken, Phys. Rev. 148, 1467 (1966);
K. Johnson and F.E. Low, Progr. Theoret. Phys. (Kyoto) Suppl. No. 37-38, 74 (1966).

2) G. Preparata and W.I. Weisberger, Phys. Rev. 175, 1965 (1968).

3) C. Callan and D.J. Gross, Phys. Rev. Letters 22, 156 (1969).

4) D.G. Sutherland, Nucl. Phys. B2, 433 (1967).

5) R. Jackiw and K. Johnson, to appear in Phys. Rev.

6) S.L. Adler and D.G. Boulware, to appear in Phys. Rev.

7) D.J. Gross and R. Jackiw, CERN preprint, 1969.

Let $F(x-iy)$ be analytic for $x \in R^4$, and y in the forward light cone V^+. This is the expected analyticity for the two-point vacuum expectation value. For vacuum expectation values in $\mathcal{C}'$ it is ensured by the properties of elements of $\mathcal{M}'$ with support in V^+.

<u>Theorem.</u> $F(x-iy)$ has a boundary value in $\mathcal{C}'(R^4)$ if and only if for each compact $\kappa \subset V^+$ there exists a polynomial $P_\kappa(x)$ and constants A, N such that for $0 < t \leq 1$ and $y \in \kappa$,

$$|F(x-ity)| \leq P_\kappa(x) \frac{1}{t^N} G(At^{-2}) .$$

A similar theorem also holds for the n-point vacuum expectation value.

This bound shows that local singularities of the type

$$e^{\left(\frac{1}{x}\right)^N} ,$$

for any integer N, may occur in strictly localizable theories. In particular, an example of a strictly localizable field is

$$A(x) = :\exp f\,\varphi(x):$$

where f is an arbitrary complex number and $\varphi(x)$ is a free field of any spin, in space time of any number of dimensions.

Using the framework of strictly localizable fields, we can verify whether the results 6)-10) above are valid. The results 6)-9) are immediate consequences of the material in Refs. 1-3. The proof of dispersion relations has been given by Epstein, Glaser and Martin [5].

To finish, I shall remark that a time-ordered propagator can be defined in strictly localizable field theory. Given a strictly localizable field $A(x)$,

$$\langle 0 | A(x) A(y) | 0 \rangle = \int \rho(a) \frac{1}{i} \Delta^{(+)}(x-y; a)\, da ,$$

where

$$\int \frac{\rho(a)}{g(a)}\, da < \infty$$

propagator for time-like momentum vanishes asymptotically, a condition which is not clearly motivated.

For the definition of general Green functions, it is necessary to consider products of propagators. Volkov[2] has given a recipe for this and it appears possible to view this construction as an analytic regularization related to the work of Speer[4]. The question whether the Green functions so defined satisfy the conditions of unitarity and locality requires, in our opinion, further investigation. Also, as emphasised by Zumino in his talk, the non-uniqueness problem is not at all understood.

REFERENCES

1) S. Okubo, Progr. Theoret. Phys. (Kyoto) 11, 80 (1954).

2) M.K. Volkov, Commun. Math. Phys. 7, 289 (1968). This paper gives further references.

3) A.M. Jaffe, Phys. Rev. 158, 1454 (1967) and unpublished manuscript.

4) E.R. Speer, J. Math. Phys. 9, 1404 (1968).

for a suitable indicator function $g(a)$. Thus we may define

$$\langle 0| TA(x)\, A(y) |0\rangle_g = g(-\Box) \int \frac{\rho(a)/g(a)}{p^2 - m^2 + i\epsilon}\, da\, e^{-ipx}\, dp ,$$

where $g(-\Box)$ maps $\mathcal{C}'$ into $\mathcal{C}'$. The ambiguity in this definition is given by

$$\langle 0| TA(x)\, A(y)| 0\rangle_{g_1} - \langle 0| TA(x)\, A(y)| 0\rangle_{g_2}$$

and has the form

$$g_3(-\Box)\, \delta(x-y) = \kappa(x-y)$$

where $\kappa(x)$ is a generalized function in $\mathcal{C}'$ with support at the origin. In momentum space, $\tilde{\kappa}(p)$ is an entire function of p satisfying

$$|\tilde{\kappa}(p)| \leq g_4(p^2)$$

for a suitable indicator function g_4. This ambiguity does not affect the value of the residue of the pole in the propagator.

One can ask whether the analytic renormalization procedure of Lehmann and Pohlmeyer (in these Proceedings) or the renormalization procedure of Epstein and Glaser (in these Proceedings) can be applied to non-polynomial Lagrangians in the strictly localizable class.

REMARKS ON THE GREEN FUNCTIONS OF EXPONENTIALS OF FREE FIELDS

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Let

$$B(x) = :e^{ifA(x)}: - 1$$

where A is a free scalar or pseudoscalar field. The problem of defining time-ordered Green functions of the B-field has been treated repeatedly in the literature [1], in recent years in particular by Volkov [2]. We have re-examined some aspects of this problem and, since Volkov is not present at this meeting, give a brief summary of results on the two-point function. We add some vague remarks on n-point functions which, in our opinion, are little understood at present.

Simple examples of Lagrangian models where exponentials of free fields occur are the derivative coupling of the scalar field to a spinor field (which has a trivial scattering matrix) and ps(pv) coupling of a spinor field to a neutral pseudoscalar field [1), 2)].

A mathematical basis for treating a certain class of entire functions of free fields which includes the operator B has been given by Jaffe [3]. The unordered products

$$\langle 0 | B(x)\, B(y) | 0 \rangle \qquad \text{and} \qquad \langle 0 | B(x)\, B^{+}(y) | 0 \rangle$$

are well-defined generalized functions. The corresponding time-ordered products require discussion along lines indicated by Jaffe [3] and contain arbitrary parameters. In the case of the BB product uniqueness is achieved by demanding that the propagator in momentum space vanishes for large space-like momenta. The BB^{+} product is more singular and uniqueness follows only from the requirement that the real part of the

REFERENCES

1) A. M. Jaffe, "Local Quantum Fields as Operator-Valued Generalized Functions", Proceedings of the MIT Conference on Scattering Theory, March 1966, Ed. I.E. Segal.

2) A. M. Jaffe, "High-energy behaviour in quantum field theory - I: Strictly localizable fields", Phys. Rev.

3) A. M. Jaffe, "High-energy behaviour in quantum field theory - II: Mathematical tools", unpublished.

4) G. V. Efimov, "Non-local quantum theory of the scalar field", Kiev preprint and these Proceedings.

5) H. Epstein, V. Glaser and A. Martin, "Polynomial behaviour of scattering amplitudes at fixed momentum transfer in theories with local observables", to appear in Commun. Math. Phys.

The vacuum expectation values in a local theory are boundary values of analytic functions in $\mathcal{T}_{n-1}$ where the convergence is in the sense of generalized functions in $(S^1)'$. The Wightman functions are polynomially bounded in $\mathcal{T}_{n-1}$ but can grow arbitrarily fast near the real points. At the same time the Wightman functions have a unique analytic continuation in the extended tube $\mathcal{T}'_{n-1}$. We remark that our convergence in the sense of $(S^1)'$ is weaker than the corresponding convergence for tempered and strict localizable theories.

An "edge of the wedge" type theorem is still valid. A limit theorem in the sense of Jaffe is also valid [4].

Our definition of local commutativity is based on the notion of the value of a generalized function at a point [5]. Let us take a generalized function $T(x)$ and let Δ_T be the set of points where $T(x)$ has the value zero. We define supp $T = \overline{C\Delta_T}$ where C denotes the complement. We say that the scalar field $A(x)$ satisfies local commutativity if the functional $(\Psi, [A(f), A(g)], \Phi)$; $\Psi, \Phi \in D$ has the support in the difference of variables $x_f - x_g$ in or on the light cone.

Now we can state that with this definition of local commutativity the TCP theorem, the cluster properties and the spin and statistics theorem are valid. At the same time, it is possible to define asymptotic states and develop a corresponding Haag-Ruelle theory.

REFERENCES

1) R. F. Streater and A. S. Wightman, "PCT, Spin, Statistics and All That" (Benjamin, New York 1964).

2) A. Jaffe, Phys. Rev. Letters 17, 661 (1966); Phys. Rev. 158, 1454 (1967); unpublished.

3) I. M. Gel'fand and G. E. Shilov, "Generalized Functions", Vol. 2 (Academic Press, New York 1964).

4) F. Constantinescu, Munich preprints.

5) F. Constantinescu, Commun. Math. Phys. 7, 225 (1968); 8, 345 (1968).

SOME REMARKS ABOUT NON-STRICTLY LOCALIZABLE FIELDS*

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In the usual Wightman framework, great attention has been paid to the investigations of the consequences of some basic requirements such as Hilbert space, covariance of the fields under the inhomogeneous Lorentz group, positive energy, local commutativity and particle interpretation.

On the basis of the first two requirements it follows that a field $A(x)$ will be an operator-valued generalized function. At this point we have to answer the following question: how to take the test functions $f(x)$? It was pointed out that some basic properties of relativistic fields can be obtained if we choose tempered functions [1]. Some years ago, A. Jaffe [2] showed that the above requirements can be incorporated in a theory of strictly localizable fields with test functions which allows, in momentum space, a non-tempered increase of fields. Jaffe has also shown that the main properties of tempered fields are also obtainable for strictly localizable fields.

From the mathematical point of view the property of a field being strictly localizable is closely related to the existence of sufficiently many test functions with compact support in configuration space. On the other hand, the existence of test functions with compact support in configuration space requires test functions in momentum space which have a decrease at infinity like $\exp(-\|p\|^{\mu})$, $\mu < 1$. Let us denote with $g(\|p\|^2)$ a positive function which behaves asymptotically like $\exp(\|p\|^{\mu})$. The test functions for strict localizable theories will satisfy the following inequality.

$$\sup_{p} g(\|p\|^2) | D^m f(p) | < \infty . \qquad (1)$$

The exponential type of $g(\|p\|^2)$ is not relativistic invariant. To make the space of test functions in momentum space relativistic invariant we have two possibilities:

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i) To take functions $f(p)$ such that

$$\sup_p \; g(N\|p\|^2)|D^m f(p)| < \infty \quad \text{for } N = 1, 2, \ldots \tag{2}$$

ii) To take a function $g(\|p\|^2)$ with the type zero.

These two possibilities correspond the first to a projective limit and the second to an inductive limit of spaces. In the first category enter the spaces which were used by Jaffe to formulate strict localizability and in the second category enter some spaces of Gel'fand and Shilov[3] which can also be used to formulate a strict localizable theory.

It seems to be difficult to formulate a Wightman theory for the case $\mu > 1$. Excepting the Haag-Ruelle theory, it seems that, physically, results such as TCP and spin and statistics are no longer obtainable, at least in the usual form. We develop here a Wightman theory only for the case $\mu = 1$. We try to show that in this case, with an adequate definition of local commutativity, the main results of a Wightman theory still remain valid. At the same time, in this case, some new problems spring up because of the non-existence of test functions with compact support in co-ordinate space. We divide our results into two categories.

A) Results which do not depend on the definition of local commutativity.

B) Results which depend on the definition of local commutativity.

First we take test functions which belong to some spaces of Gel'fand and Shilov. We prefer these spaces because for $\mu = 1$ the test functions in co-ordinate space have a definite domain of analyticity. In this way we allow only an increase of fields in momentum space like $g(\|p\|^2)$ where g has the order of growth one and the type zero. At first it appears that the extension of μ from strictly localizable fields $(\mu < 1)$ to local fields $(\mu \leq 1)$ is of little significance. Nevertheless, the class of local fields $(\mu \leq 1)$ is much larger than the class of strictly localizable ones $(\mu < 1)$.[4]