MAGNETIC WILSON LOOP IN THE CLASSICAL FIELD OF HIGH-ENERGY HEAVY-ION COLLISIONS

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This talk gives a brief review of the calculation of the expectation value of a magnetic Wilson loop in the transverse plane of ultra-relativistic heavy-ion collisions. The average of the Wilson loop is determined in the first moments after the impact.

1 Introduction

A complete description of high-energy collisions requires a precise understanding of the initial conditions. The initial conditions are given by the nuclear structure of the participants in the collision, as well as by the structure of the matter formed in the first moments after the impact. How the system evolves at later times depends on the fluctuations in these first two stages. These conditions are the point of interest of many recent works attempting to explain the transition of the collision system from a quantum state far from equilibrium, before the interaction, to the Quark Gluon Plasma that is believe to be formed in experiments at the Large Hadron Collider and the Relativistic Heavy Ion Collider.

In this talk we present the calculation of the expectation value of a spatial Wilson loop in the initial time of heavy-ion collisions. The computations are based on the Color Glass Condensate (CGC) theory for nuclear structure at high energies. The starting concept in the CGC theory is the classical description of the gauge fields, i.e. the McLerran-Venugopalan (MV) model for a large nucleus [1]. The form of the classical field of the individual nuclei before the collision and of the resulting field [2], as well as the form of the chromo-electric and chromo-magnetic fields formed at early times [3], have been calculated previously. We place a non-Abelian magnetic Wilson loop in the transverse plane normal to the longitudinal component of the chromo-magnetic field and compute its average. The numerical result gives an area law behaviour of the loop and indicates a presence of positive and negative domains of chromo-magnetic flux in the plane of the loop [4]. The fluctuations of the chromo-magnetic flux may be viewed as uncorrelated vortices with a typical radius $\sim 0.8/Q_s$. Q_s denotes the saturation momentum which is the scale where the gluon field exhibits non-linear dynamics [5]. A perturbation theory without screening gives a leading non-trivial term proportional to the square of the area of the loop [6].

In the MV model one assumes a momentum scale that separates the partons in the wave function of the nucleus: the partons with large longitudinal momentum act as static sources for the degrees of freedom with a small longitudinal momentum fraction x. The color charge squared per unit transverse area, μ^2, scales as $\mu^2 \sim \Lambda_{QCD}^2 A^{1/3}$, where A is the nucleon number. The momentum μ drives the running of the strong coupling constant, $\alpha_s(\mu^2)$, so for a nucleus with large A the coupling is small and parton distribution functions can be calculated perturbatively. The transverse gluon density in a large, Lorentz contracted, ultra-relativistic nucleus is high and the gauge fields are approximated as classical fields. Their form is obtainable from the classical Yang-Mills equations of motion. Quantum corrections are implemented by including a non-linear small-x evolution of the wave function of the nuclei.

In the picture of classical fields, the solutions of the Yang-Mills equations for a collision of two nuclei are the following. Before the impact, both, the gauge fields of the target and the projectile are a (different) gauge transformation of the vacuum. They are the non-Abelian analogue of the Weizsäcker-Williams field and in light-cone gauge their form is:

$$\alpha_m^i = \frac{i}{g} U_m \, \partial^i U_m^\dagger \quad , \quad \partial^i \alpha_m^i = g \rho_m \ . \tag{1}$$

The subscript m, with values 1 and 2, denotes the projectile and the target respectively. Introducing the gauge potential as

$$\Phi_m = -\frac{g}{\nabla_\perp^2} \rho_m \ , \tag{2}$$

the solution to (1) can be written as [2]:

$$\alpha_m^i = \frac{i}{g} e^{-ig\Phi_m} \partial^i e^{ig\Phi_m} \ . \tag{3}$$

In the forward light cone one needs to solve the equations of motion with two color currents representing the sources in the two nuclei, and with boundary conditions on the light cone. Analytical solution has been found only at proper time $\tau = \sqrt{t^2 - z^2} = 0$ [2]. The resulting transverse field is a sum of two pure gauge fields: $A^i = \alpha_1^i + \alpha_2^i$.

At $\tau = 0$ the transverse components of the chromo-magnetic and chromo-electric fields are zero. The longitudinal components are [3]:

$$E_z = ig[\alpha_1^i, \alpha_2^i] \ , \quad B_z = ig\epsilon^{ij}[\alpha_1^i, \alpha_2^j] \ , \quad (i,j = 1,2) \ . \tag{4}$$

where ϵ^{ij} is the antisymmetric tensor.

2 Magnetic Wilson loop in the classical field of heavy-ion collisions

The non-Abelian Wilson loop is gauge invariant and is defined as an exponential of an integration of the gauge field matrices, A^μ, ordered along the path. The magnetic Wilson loop is given by:

$$M(R) = \mathcal{P} \exp\left(ig \oint dx^i A^i\right) = \mathcal{P} \exp\left[ig \oint dx^i \left(\alpha_1^i + \alpha_2^i\right)\right] \ , \tag{5}$$

with R the radius of the loop. Note that $M(R) \equiv \mathbb{1}$ if evaluated in the field of a single nucleus (α_1^i or α_2^i) as those are pure gauges.

In [4] it was shown that the expectation value of the magnetic Wilson loop in the field A^i produced in a collision of two nuclei is proportional to the exponent of the area A of the loop:

$$W_M(R) = \frac{1}{N_c} \langle \mathrm{tr}\, M(R) \rangle \sim \exp\left(-\sigma_M A\right) \ . \tag{6}$$

Here, σ_M is the magnetic string tension. For the SU(2) gauge group its value was estimated to be $\sigma_M \simeq 0.12 Q_s^2$ from the fit to the lattice data. The result (6) was obtained for areas $A \gtrsim 2/Q_s^2$. It indicates that the structure of the chromo-magnetic flux at such scales corresponds to uncorrelated vortex fluctuations. This is shown in fig. 1 for proper time $\tau = 0$. For comparison,

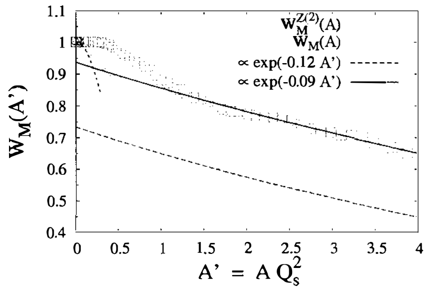

Figure 1 – Expectation value of the magnetic flux loop right after a collision of two nuclei (time $\tau = +0$) as a function of its area $A' \equiv A\,Q_s^2$. We define $Q_s^2 = (C_F/2\pi)\,g^4\mu^2$. Symbols show numerical results for $SU(2)$ Yang-Mills on a 4096^2 lattice; the lattice spacing is set by $g^2\mu_L = 0.0661$. The solid and dashed lines represent fits over the range $4 \geq A' \geq 2$. The short dotted line shows $\cos 2A'$ for $A' < 0.3$.

in the same figure we plot he expectation value of the $Z\,(N_c)$ part of the loop. The two fits have a similar behaviour, confirming the independent domain structure of the flux.

The expectation value in (6) refers to averaging over the color charge distributions in each nucleus. For large nuclei the color sources are treated as random variables with Gaussian probability distribution. Physical observables are then averaged with a Gaussian (McLerran-Venugopalan) action:

$$S_{\text{eff}}[\rho^a] = \frac{1}{2}\int d^2\mathbf{x}\left[\frac{\rho_1^a(\mathbf{x})\rho_1^a(\mathbf{x})}{\mu_1^2} + \frac{\rho_2^a(\mathbf{x})\rho_2^a(\mathbf{x})}{\mu_2^2}\right] , \tag{7}$$

where μ^2 is the color charge squared per unit area, related to the saturation scale via $Q_s^2 \sim g^4\mu^2$.

To obtain $W_M(R)$ we need to determine the deviation of A^i from a pure gauge:

$$W_M(R) \simeq \frac{1}{N_c}\left\langle \text{tr}\,\exp\left(-\frac{1}{2}\left[X_1, X_2\right]\right)\right\rangle \simeq 1 - \frac{1}{2N_c}\left\langle g^2h^2\right\rangle , \tag{8}$$

where:

$$g^2h^2 = \frac{1}{8}f^{abc}f^{\bar{a}\bar{b}c}X_1^aX_1^{\bar{a}}X_2^bX_2^{\bar{b}} \quad \text{with :} \quad X_m = ig\oint dx^i\alpha_m^{ai}t^a . \tag{9}$$

f^{abc} are the structure constants of the special unitary group and h^2 corresponds to a four gluon vertex of the fields. In addition, we expand the fields α^i from eq. (3) perturbatively in terms of the coupling constant:

$$\alpha_m^i = -\partial^i\Phi_m + \frac{ig}{2}\left(\delta^{ij} - \partial^i\frac{1}{\nabla_\perp^2}\partial^j\right)\left[\Phi_m, \partial^j\Phi_m\right] + \mathcal{O}\left(\Phi_m^3\right) . \tag{10}$$

A non-trivial result for the Wilson loop gives the term of quadratic order: $\alpha^{i,a} \sim gf^{abc}\Phi^b\partial^i\Phi^c$:

The expectation value $\langle h^2\rangle$ that enters in the expression for the magnetic loop involves the fields of both nuclei. The leading diagram is shown in fig. 2 corresponds to two sources, for both projectile and target, whose field is evaluated at second order in the gauge potential. The final result we obtain for the expectation value of the magnetic Wilson loop for classical fields α^i:

$$W_M(R) \simeq 1 - \frac{\pi^2N_c^6}{64(N_c^2-1)^3}\frac{Q_{s1}^4Q_{s2}^4}{\Lambda^4}A^2 . \tag{11}$$

In this result, A is the area of the loop, and Q_{s1} and Q_{s2} are the saturation scales of the projectile and the target, respectively. We use the relation:

$$Q_s^2 = \frac{g^4C_F}{2\pi}\mu^2 , \qquad \text{where :} \qquad C_F = \frac{N_c^2-1}{2N_c} . \tag{12}$$

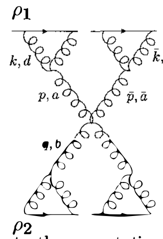

Figure 2 – Classical contribution to the expectation value of the magnetic Wilson loop.

The cut-off Λ regulates the infrared divergence of the integrals over the gluon momentum k shown in diagram 2. It sets the mass scale for the gluon propagator.

The perturbative result for the expectation value of the magnetic Wilson loop gives a first non-trivial contribution that is proportional to the square of the area, and therefore does not reproduce the numerical result. The analytical expansion of the magnetic loop holds only for small areas, and not in the onset of area law behaviour. A term proportional to the area of the loop would involve single powers of thc target's and projectile's saturation scales: $\sim A\ Q_{s1}Q_{s2}$ [4]. However, Gaussian contractions can only give powers of Q_{s1}^2 and Q_{s2}^2:

$$\langle \rho_m^a(\mathbf{x})\, \rho_m^b(\mathbf{y})\rangle = \mu_m^2 \delta^{ab}\delta(\mathbf{x}-\mathbf{y}) \sim Q_{s_m}^2 \ , \tag{13}$$

and thereforc a term $\sim A^2$. Area law scaling of the Wilson loop presumably requires resummation of screening effects [7]. "Naive" perturbation theory cannot capture the presence of screening corrections.

In summary, the magnetic Wilson loop at proper time zero in heavy-ion collisions shows an area law behaviour which indicates a presence of independent domains of magnetic flux. The perturbative result for the average of the loop gives a term proportional to the area squared.

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