INFLUENCE OF A VACUUM GAP ON A BUNCH WAKEFIELD IN A CIRCULAR WAVEGUIDE FILLED UP WITH DIELECTRIC*

Tatiana Yu. Alekhina#, Andrey V. Tyukhtin, and Victor V. Vorobev,
St. Petersburg State University, St. Petersburg, Russia

Abstract

The electromagnetic field of a bunch moving in a dielectric circular waveguide and crossing a vacuum cavity is under investigation. The main attention is given to the case when Cherenkov radiation (CR) is generated in a dielectric. The behavior of the total field depending on distance and time is explored numerically. Analytical estimations are made as well. Influence of the vacuum gap on the wakefield is considered for different lengths of the gap. It is clarified conditions when the vacuum gap does not practically influence on the wakefield. It is shown that the quasi monochromatic wave ("Cherenkov transition radiation") generated in the vacuum region can be used for the restoration of the field in the area after the gap. This effect can be achieved for some optimal parameters of the problem.

INTRODUCTION

Analysis of the influence of the vacuum gap on the electromagnetic field of a particle bunch in a waveguide loaded with a dielectric is important for the wakefield acceleration technique and other problems of accelerator physics. In reality, it is critical to clarify conditions when the vacuum gap does not practically affect the wakefield (Cherenkov radiation).

In our previous works we considered the case of a single boundary in a waveguide [1-2]. In particular, the effect of "Cherenkov-transition radiation" (CTR) has been investigated for a dielectric - vacuum boundary [2]. It has been shown that the CTR can be dominant in the vacuum area under the conditions

$$\beta_{\mathrm{C}} < \beta < \beta_{\mathrm{CT}}, \quad (1)$$

where the lower threshold $\beta_{\mathrm{C}} = \varepsilon^{-1/2}$ is connected with the condition of the CR generation and the upper threshold $\beta_{\mathrm{CT}} = (\varepsilon - 1)^{-1/2}$ is explained by total internal reflection of the Cherenkov waves from the boundary. The CTR propagates in the dielectric area with a group velocity [1]

$$\mathrm{v}_{g1} = c(\beta\varepsilon)^{-1} \quad (2)$$

and in the vacuum area with a group velocity [2]

$$\mathrm{v}_{g2} = c\beta^{-1}\sqrt{1 - \beta^2(\varepsilon - 1)}. \quad (3)$$

Thus, it is interesting to consider the situation when the CTR penetrates the next vacuum - dielectric boundary.

It should be noted that although the case of a dielectric plate situated in a waveguide (a vacuum – dielectric – vacuum structure) has been analyzed before [3-4, 5], it differs dramatically from the case considered here.

This work is devoted to a dielectric – vacuum – dielectric structure placed in a circular waveguide of radius a. A bunch of charged particles moves uniformly with a velocity $\vec{V} = c\beta\vec{e}_z$ along the axis of the waveguide and intersects a vacuum cavity of length d. The permittivity of a dielectric is ε. The bunch thickness is assumed to be negligible and the longitudinal charge distribution is Gaussian with the density $\rho = q\delta(x)\delta(y)\exp\left(-\zeta^2/\left(2\sigma^2\right)\right)$, where $\zeta = z - c\beta t$ and σ is the half-length of the bunch. The middle of the bunch intersects the first boundary $z = 0$ at the moment $t = 0$. The media are nonmagnetic and have no dispersion.

The analytical solution to this problem is traditionally found as decomposition in an infinite series of normal modes [3]. We investigate the exact solution for n^{th} mode, which presented as integrals with respect to frequency ω, both analytically and numerically. Analytical research is carried out with the methods of complex variable function theory, when singularities of the integrands are studied in a complex plane of (ω) just as it was made in our previous papers [1-2, 5-6]. Numerical approach is based on an algorithm that uses separation between the integration path and the singularities of the integrands by taking into account a small absorption in the media.

The next study reveals two different situations: the case $\beta < \beta_{\mathrm{CT}}$, when the CTR effect takes place in the vacuum cavity, and, the case $\beta > \beta_{\mathrm{CT}}$, when the CTR is absent in the vacuum gap. For ultra-relativistic bunches ($\beta \to 1$), these situations correspond to the dielectric permittivities $\varepsilon < 2$ and $\varepsilon > 2$ accordingly. Here we give only some results of the analytical and numerical investigations for these two cases. Moreover, the field in the area after the vacuum gap is mainly of interest.

THE CASE OF GENERATION OF THE CTR ($\varepsilon < 2$)

If the condition for the CTR effect (1) is satisfied, the CR generated in the dielectric area $z < 0$ penetrates into the vacuum gap, and through the next boundary into the dielectric area $z > d$. The expression for the CTR can be obtained analytically. As can be seen, the CTR has two summands in the area after the gap. Because the first summand equals the CR (wakefield) in the dielectric taken with the opposite sign, it compensates for the wakefield in some area after the vacuum cavity as in the case of a singular vacuum – dielectric boundary [1].

*This research was supported by Saint Petersburg State University (grant 11.0.61.2010).
#tanya@niirf.spbu.ru

ISBN 978-3-95450-132-8

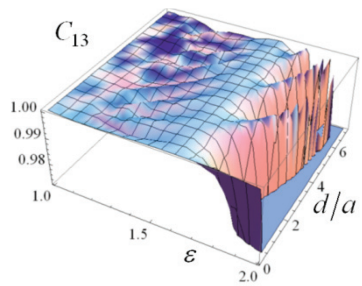


Figure 1: The first mode of the normalized fiel amplitude.

However, the second summand, which is proportional to C_{n3}, can restore the wakefield in the same domain $d < z < Z_3(t)$, where

$$C_{n3} = \left[1 + \left(\varepsilon^2 S^2 - 1\right)^2 (2\varepsilon S)^{-2} \sin^2(\kappa_n d S)\right]^{-1/2}, \quad (4)$$

$\kappa_n = \chi_n a^{-1}\left(\varepsilon\beta^2 - 1\right)^{-1/2}$, $S = \sqrt{1 - \beta^2(\varepsilon - 1)}$, χ_n is the n^{th} zero of the Bessel function ($J_0(\chi_n) = 0$). The magnitude $Z_3(t) = d + (t - d/v_{g2})v_{g1}$ is the front of the CTR in the dielectric after the cavity for $t > d/v_{g2}$. It can be evaluated by taking into account the finite length of the gap and the group velocities of the CTR (2) and (3).

As a result, the amplitude of the total field in the area $d < z < Z_3(t)$ is determined by the coefficient C_{n3} (Fig. 1), and it depends periodically on the cavity lengths with maximums $C_{n3\max} = 1$. These maximums occur at

$$d = d_{nk} = \frac{\pi k}{\kappa_n S} = \frac{a\pi k\sqrt{\beta^2\varepsilon - 1}}{\chi_n\sqrt{1 - \beta^2(\varepsilon - 1)}}, \quad k = 1, 2, \ldots. \quad (5)$$

For this length of the gap, the wave field is strictly equal to the wakefield. So, in the cavity, a standing wave arises at (5) and a resonant phenomenon takes place. For ultra-relativistic bunches ($\beta \to 1$), the above condition (5) is written in the form $d = a\pi k\sqrt{\varepsilon - 1}\left(\chi_n\sqrt{2 - \varepsilon}\right)^{-1}$. Naturally, this condition can be only fulfilled for $\varepsilon < 2$.

It should be noted that there is also a special case when $C_{n3} = 1$ for any cavity length *d*. One can see from (4) that it holds true when $\varepsilon^2 S^2 = 1$, which results in the relation

$$\varepsilon = \varepsilon_0 = \left(1 + \sqrt{1 + 4\beta^2}\right)\Big/2\beta^2. \quad (6)$$

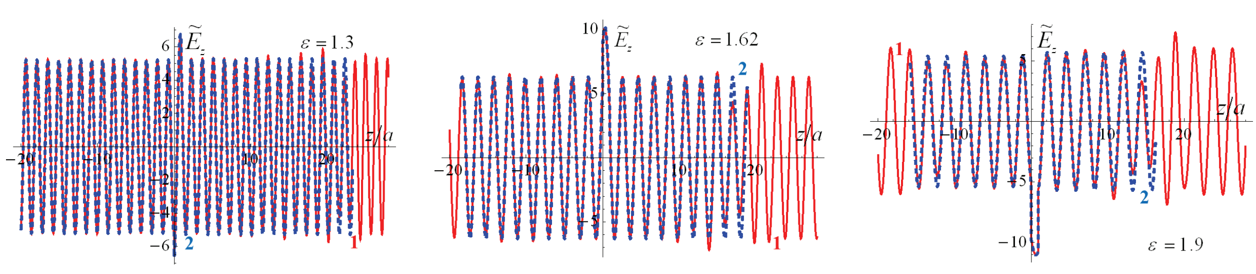
This is explained by the Brewster phenomenon, when a wave having the polarization under consideration does not reflect on the boundary at the incident angle $|\theta_i| = \theta_B = \operatorname{atan}\left(\sqrt{\varepsilon_0}\right)$. For ultra-relativistic bunches, the optimal value of the permittivity equals $\varepsilon_0 = \left(1 + \sqrt{5}\right)/2 \approx 1.62$.

Thus, we have found two cases ((5) and (6)) when all of the radiation is directed forward (because $C_{n3} = 1$). As can be shown, the amplitude of the reflected waves in the dielectric before the vacuum cavity is negligible at these cases. It is interesting that in the case when $\varepsilon = \varepsilon_0$, we can achieve coincidence the CTR with the CR not only in amplitude but also in phase.

Numerical results are presented at Fig. 2 for some parameters of the problem, including the optimal ones. As the condition for the CTR effect (1) is fulfilled, the CR penetrates into the vacuum cavity and through the second boundary, which results in the restoration of the wakefield in the dielectric after the gap (Fig. 2 a,b) for the optimal parameters (which was discussed above). If the optimality is broken (Fig. 2 c), then there is also some backwards radiation, and the restoration of the wakefield does not occur in full. The border of the area of this phenomenon moves with the group velocity of the CTR in the dielectric area (2).

THE CASE IN THE ABSENCE OF THE CTR EFFECT ($\varepsilon > 2$)

Another situation takes place in the case $\beta > \beta_{CT}$, where the CTR effect is absent in the vacuum gap because of the total internal reflection of the Cherenkov waves



(a) (b) (c)

Figure 2: The dependence of normalized longitudinal component $\tilde{E}_z = E_z a^2 q^{-1}$ of the first mode of the total field (continuous line 1) and the CTR together with the wakefield (dashed line 2) on distance z/a for different permittivities ε; $d/a = 0.93$, $\sigma/a = 0.2$, $ct/a = 30$, $\gamma = 100$ ($\gamma = \left(1 - \beta^2\right)^{-1/2}$), and $a = 5\,\text{mm}$.

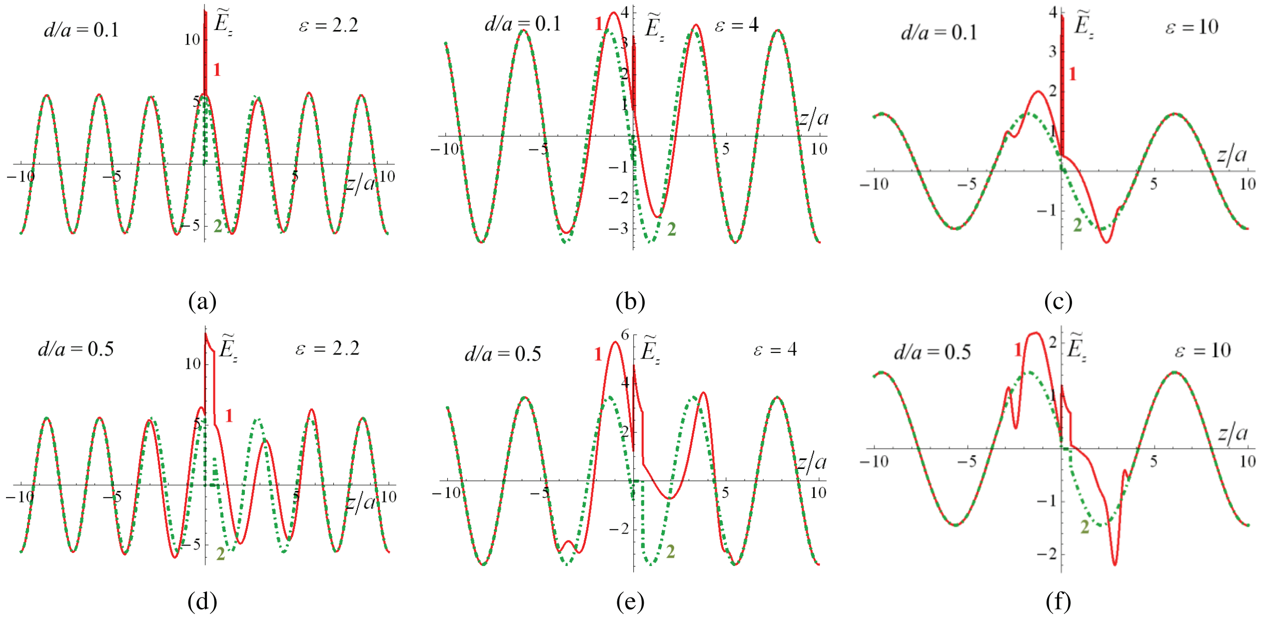


Figure 3: The dependence of the normalized longitudinal component $\tilde{E}_z = E_z a^2 q^{-1}$ of the first mode of the total field (continuous red line 1) and the wakefield (dot-dashed green line 2) on distance z/a for different lengths of the gap *d/a* and permittivities ε; *ct/a* = 10, $\sigma/a = 0.2$, $\gamma = 100$, and $a = 5\,\text{mm}$.

from the first boundary. In this case, a major contribution to the field makes transition radiation (TR), which can be evaluated for small gaps with the steepest descent technique [7]. It can be seen that the TR for small cavities is proportional to $(d/a)^1$. The TR exists only in the area $z < z_{03}(t)$, where $z_{03}(t) = d + (t - d/c)v_{\text{ph1}}$ for $t > d/c$, and $v_{\text{ph1}} = c/\sqrt{\varepsilon}$ is the phase velocity of the TR in a dielectric. Thus, one can obtain that the magnitude of the TR increases with increases in *d*.

Therefore, if $\varepsilon > 2$ and the CTR effect is absent in the vacuum gap, then the restoration of the wakefield for large lengths of the gap is impossible. In this situation, the TR on the cavity becomes significant, and the investigation of the influence of the vacuum gap on the wakefield is important. In Fig. 3, the longitudinal components of the first mode of the total field and of the wakefield are presented for different dielectric permittivities ε and cavity lengths *d*. One can see that relatively small gaps are minimally influential on the wakefield for dielectrics with relatively small permittivity (Fig. 3a). The influence of the cavity increases with increase in ε (Fig. 3 b,c) and *d* (Fig. 3 d,e,f). Furthermore, the domain of this influence is defined by the phase velocity of the TR in dielectric.

CONCLUSION

We have shown that, if the CTR exists in the vacuum cavity, then it influences on the field in the third (dielectric) segment of the waveguide. In this case, one can choose certain optimal parameters that enable to restore the field amplitude in the whole area after the vacuum gap. These parameters can be connected with the resonance effect as well as with the Brewster phenomenon. The TR is negligibly small in this situation, and the CTR and CR are the main parts of the total field. The border of area of the CTR moves with the group velocity in the dielectric area. In certain situations, it is possible to achieve coincidences in the phases of the CTR and CR as well.

If the CTR effect is absent in the vacuum cavity, then the TR is significant. The influence of the vacuum cavity increases with increase in ε and *d*. The domains of this influence are determined by the phase velocity of the TR in the dielectric.

REFERENCES

[1] T.Yu. Alekhina and A.V. Tyukhtin, Journal of Physics: Conference Series 357, 012010 (2012).

[2] T.Yu. Alekhina and A.V. Tyukhtin, Phys. Rev. ST Accel. Beams 15, 091302 (2012).

[3] K.A. Barsukov, Zh. Tekh. Fiz. 30, 9 1337 (1960).

[4] K.A. Barsukov, E.D. Gazazian, and E.M. Laziev, Izvestiya vuzov - Radiofizika 15, 191 (1972).

[5] T.Yu. Alekhina and A.V. Tyukhtin, Journal of Physics: Conference Series 517, 012012 (2014).

[6] T.Yu. Alekhina and A.V. Tyukhtin, Phys. Rev. E 83, 066401 (2011).

[7] B. Felsen and N. Marcuvitz, *Radiation and Scaterring of Waves* (New Jersey: Wiley Interscience, 2003).

ISBN 978-3-95450-132-8