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Computing the $qg \to qg$ cross section using the BCFW recursion and introduction to jet tomography in heavy ion collisions via MHV techniques

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Abstract. To make predictions for the particle physics processes, one has to compute the cross section of the specific process as this is what one can measure in a modern collider experiment such as the Large Hadron Collider (LHC) at CERN. Theoretically, it has been proven to be extremely difficult to compute scattering amplitudes using conventional methods of Feynman. Calculations with Feynman diagrams are realizations of a perturbative expansion and when doing calculations one has to set up all topologically different diagrams, for a given process up to a given order of coupling in the theory. This quickly makes the calculation of scattering amplitudes a hot mess. Fortunately, one can simplify calculations by considering the helicity amplitude for the *Maximally Helicity Violating* (MHV). This can be extended to the formalism of on-shell recursion, which is able to derive, in a much simpler way the expression of a high order scattering amplitude from lower orders.

1. Introduction

Shortly after the Big Bang, for a few millionths of a second, the universe was filled with an ultra-hot, dense soup (Plasma) of elementary particles mostly composed of quarks and gluons moving at near the speed of light. These conditions have been recreated at powerful accelerators by making head-on collisions between massive ions. The first evidence for jets was seen in 2003 at Brookhaven National Laboratory's Relativistic Heavy Ion Collider (RICH) in the United States and recent experiments at CERN's Large Hadron Collider (LHC) have confirmed the phenomenon of jet quenching in heavy-ion collisions. The study of the radiative energy loss in Quark-Gluon Plasma (QGP) provides insight into the dynamics of the early universe. In heavy ion collisions, the interaction between a high energy quark and a thermal gluon can yield the emission of gluon radiations and the decrease of the energy of the quark [1]. The aim of the project is to compute the cross section for $qg \to qg + ng$ process with $n = 0, 1$ (and more, time permitting). Due to the $n!$ increase in complexity of the usual Feynman approach, novel mathematical structures in gauge theory scattering amplitudes will be used such as Maximally Helicity Violating (MHV) techniques and Britto-Feng-Cachazo-Witten (BCFW) on-shell recursion relations. Indeed, in quantum field theory, one adds up all the Feynman diagrams while each diagram represents a possible collision with an associated probability, and the redundancy due to the gauge invariance leads to a complicated calculation [2]. Recently, significant simplifications in the calculation of scattering amplitudes have been achieved by using new representations for vector polarizations, and the recursion relations. When doing the computation of the $qg \to qg$ scattering amplitude , we will have a look at the case where the conservation of the helicity is violated to the maximum extent possible. Then, the BCFW recursion will be used to construct the 4 degree scattering amplitude from a 3 point amplitudes which we can easily compute using the *spinor helicity formalism.*

2. Spinor helicity formalism and color decomposition

In the usual Feynman approach in quantum field theory, there is too much redundancy among momenta and polarizations in the calculation of scattering amplitudes using Lorentz 4-vectors, especially when one has to deal with particles with spin. That arises the need to find new variables to represent the kinematics. It appears that one can organize efficiently the spin quantum numbers of external states in the *helicity basis.* In high energy process, almost all fermions are ultra-relativistic, behaving as if they were massless. Massless fermions that interact through gauge interactions have a conserved helicity, which we can exploit by computing in the helicity basis [3]. Although vector particles like photons and gluons do not have conserved helicity, it turns out that the most helicity-violating processes one can imagine are zero at tree level. That is, vectors can be written as *bispinors* $p_{a\dot{b}} = p_\mu \sigma^\mu_{a\dot{b}}$ where $\sigma^\mu = (\sigma^0, \sigma^j)_{j=1,2,3}$ are the usual Pauli matrices. For massless particles, $\det(p_{a\dot{b}}) = 0$, so we can write the momentum as a product of *Weyl spinors* [4] $p_{a\dot{b}} = \lambda_a \tilde{\lambda}_{\dot{b}}$. We can define the angle and square product as:

$$\langle ij \rangle = \epsilon_{ab} \lambda_i^a \lambda_j^b, \qquad [ij] = \epsilon_{\dot{a}\dot{b}} \tilde{\lambda}_i^{\dot{a}} \tilde{\lambda}_j^{\dot{b}}. \tag{1}$$

As a result, the usual Lorentz invariant product is written as $s_{ij} = \langle ij \rangle [ij]$. And for vector particles which have polarizations, the polarizations can be expressed as:

$$\epsilon_k^-(p_i) = -\frac{1}{\sqrt{2}} \frac{[k\gamma^\mu i\rangle}{[ki]}, \qquad \epsilon_k^+(p_i) = -\frac{1}{\sqrt{2}} \frac{[i\gamma^\mu k\rangle}{\langle ki \rangle}. \tag{2}$$

These equations satisfy $\epsilon_k^+(p_i)\epsilon_k^-(p_i) = -1$ and $p_i \epsilon_k^\pm(p_i) = 0$, where k is the *reference momentum.* In order to deal with the color factors in QCD, one can do a factorization of the scattering amplitude. This allows us to compute separately the *kinematic parts* and the *color-structures* of the scattering amplitude. For a process involving $(n-2)$ gluons and 2 fermions, one can do the following decomposition:

$$\mathcal{A}_n(1_q, 2_g, ..., n_{\bar{q}}) = g_s^{n-2} \sum_{\sigma \in S_n} \left(T^{\sigma(2)} T^{\sigma(3)} ... T^{\sigma(n-1)} \right) A_4[1, \sigma(2), ..., \sigma(n-1), n], \tag{3}$$

where the sum is over the permutations of the $(n-2)$ external legs. The color-independent amplitude $A_4[1, ..., n]$ is called ***color-ordered amplitude*** which is gauge invariant.

3. Computing the cross section using BCFW recursion

Let us now compute the cross section of the $qg \to qg$ process. Let us consider all the momentum to be outgoing. Thus, instead we compute $qgg\bar{q}$ and apply the crossing symmetry at the end. Denoted by 1 and 4 are the quark and the anti-quark, and 2 and 3 the two gluons with color indices a and b respectively. By virtue of the color decomposition formula, we can write down the full expression of the scattering amplitude:

$$\mathcal{A}_4(1_q, 2_g, 3_g, 4_{\bar{q}}) = g_s^2 T^a T^b A_4[1_q, 2_g, 3_g, 4_{\bar{q}}] + g_s^2 T^b T^a A_4[1_q, 3_g, 2_g, 4_{\bar{q}}]. \tag{4}$$

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Let us make a choice of helicity such that we have a *Maximally Helicity Violating* (MHV)-amplitude, which means that we have exactly two particles with negative helicity, say $(-,-,+,+)$. Since different helicity do not interfere, as shown by the color decomposition of the full amplitude, to get the full answer of the amplitude one only has to compute the squares of all the possible helicity amplitude which can contribute to the process. The idea of *BCFW construction* is based on a linear momentum shift in complexified momentum space [5]. Let us consider the $[2,3\rangle$-shift:

$$|\widehat{2}] = |2] + z|3],\ |\widehat{2}\rangle = |2\rangle \quad \text{and} \quad |\widehat{3}\rangle = |3\rangle - z|2\rangle,\ |\widehat{3}] = |3]. \tag{5}$$

Hence, we have the following **BCFW on-shell diagrams**:

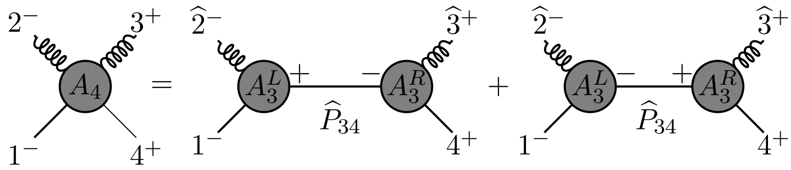

Figure 1. BCFW diagram for the $qgg\bar{q}$ process.

which can be mathematically expressed as:

$$A_4[1_q^-, 2_g^-, 3_g^+, 4_{\bar{q}}^+] = A_L^3[1_q^-, \widehat{2}_g^-, \widehat{P}_{34}^+]\frac{1}{P_{34}^2}A_R^3[-\widehat{P}_{34}^-, \widehat{3}_g^+, 4_{\bar{q}}^+]. \tag{6}$$

Here, the internal momentum $\widehat{P}_{34}$ is on-shell. The second diagram on the right hand-side vanishes and does not contribute to the calculation. Indeed, amplitudes with all the particles having the same helicity vanish $A_L^3[1_q^-, \widehat{2}_g^-, -\widehat{P}_{34}^-] = A_R^3[-\widehat{P}_{34}^+, \widehat{3}_g^+, 4_{\bar{q}}^+] = 0$. Using the momentum conservation to cancel the *momentum term* in the expression of the polarization, we end up with an expression of the *color-ordered subamplitude* in terms of the internal momentum $\widehat{P}_{34}$:

$$A_L^3[1_q^-, \widehat{2}_g^-, \widehat{P}_{34}^+] = \frac{\langle 12\rangle^2}{\langle \widehat{P}_{34}\rangle}, \quad \text{and} \quad A_R^3[-\widehat{P}_{34}^-, \widehat{3}_g^+, 4_{\bar{q}}^+] = \frac{[34]^2}{[4\widehat{P}_{34}]}. \tag{7}$$

Therefore, the total color-ordered amplitude can be written as:

$$A_4[1_q^-, 2_g^-, 3_g^+, 4_{\bar{q}}^+] = \frac{\langle 12\rangle^2}{\langle \widehat{P}_{34}\rangle}\frac{1}{\langle 34\rangle[34]}\frac{[34]^2}{[4\widehat{P}_{34}]}. \tag{8}$$

Since the internal momentum $\widehat{P}_{34}$ is *on-shell*, we have $\langle\hat{3}4\rangle[\hat{3}4] = 0$, and using the *shift equations* in (5), we find that the complex component z is equal to $\langle 34\rangle\langle 24\rangle^{-1}$. Hence,

$$|\hat{2}] = |2] + \frac{\langle 34\rangle}{\langle 24\rangle}|3],\ |\hat{2}\rangle = |2\rangle \quad \text{and} \quad |\hat{3}\rangle = |3\rangle - \frac{\langle 34\rangle}{\langle 24\rangle}|2\rangle,\ |\hat{3}] = |3]. \tag{9}$$

Furthermore, considering $\langle\hat{3}4\rangle[\hat{3}4] = 0$, since $[\hat{3}4]$ cannot be equal to zero, we have $|\widehat{3}\rangle$ is collinear to $|4\rangle$. Thus, we can write $|\hat{3}\rangle = k|4\rangle$. Using the new shift equations in (9), it follows that $k = \langle 23\rangle\langle 24\rangle^{-1}$. However, according to our definition, $\widehat{P}_{34} = k|4\rangle[3| + |4\rangle[4|$. We can therefore write the internal on-shell momentum only in terms of the momentum of the external particles. So, we have:

$$A_4[1_q^-, 2_g^-, 3_g^+, 4_{\bar{q}}^+] = \frac{\langle 12\rangle^3\langle 42\rangle}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle} \quad \text{and} \quad A_4[1_q^-, 3_g^+, 2_g^-, 4_{\bar{q}}^+] = \frac{\langle 12\rangle^3\langle 42\rangle}{\langle 13\rangle\langle 32\rangle\langle 24\rangle\langle 41\rangle}. \tag{10}$$

To get the expression of $A_4[1_q^-, 3_g^+, 2_g^-, 4_{\bar{q}}^+]$, we have to swap 2 and 3 in the denominator from the $A_4[1_q^-, 2_g^-, 3_g^+, 4_{\bar{q}}^+]$-amplitude. Therefore, the expression of the full scattering amplitude can be written as:

$$\mathcal{A}_4(1_q, 2_g, 3_g, 4_{\bar{q}}) = g_s^2 \left[T^a T^b \frac{\langle 12\rangle^3 \langle 42\rangle}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle} + T^b T^a \frac{\langle 12\rangle^3 \langle 42\rangle}{\langle 13\rangle\langle 32\rangle\langle 24\rangle\langle 41\rangle} \right]. \tag{11}$$

Squaring this amplitudes, taking into account the factors for averaging over the initial colors and helicities, adding the case with two gluons exchanged, and applying the crossing symmetry rules we get the final expression of the cross section of the process:

$$\frac{d\sigma}{d\cos\theta}(qg \to qg) = \frac{2}{9}\frac{\pi\alpha_s^2}{s}\left[-\left(\frac{s^2+u^2}{su}\right) + \frac{9}{4}\left(\frac{s^2+u^2}{t^2}\right) \right]. \tag{12}$$

4. Summary and Outlook

We have performed to derive correctly and in a much more efficient way the cross section of the $qg \to qg$ process [6]. The fact that the spinor helicity formalism fully exploit the properties of gauge invariance allows us to better organize the diagrammatic expansion. The BCFW on-shell recursion is a powerful tool to compute higher order scattering amplitude from a few number of external legs. The BCFW recursion relations can be used to build a generalization formula for scattering amplitude involving n external legs. For the case of $(n-2)$-gluons and 2-quarks, we have the generalization:

$$A_n[1_q^-, 2_g^+, ..., i_g^-, ..., n_{\bar{q}}^+] = \frac{\langle 1i\rangle^3 \langle ni\rangle}{\langle 12\rangle\langle 23\rangle ... \langle (n-1)n\rangle\langle n1\rangle}. \tag{13}$$

The aim of the project (AIMS essay) is to understand the energy loss of light quark passing through a Quark-Gluon Plasma. When a quark interacts with a thermal gluon, this leads to a emission of radiated gluons. The decrease of the energy of the quark is due to these emitted radiations. For that, we consider the following process:

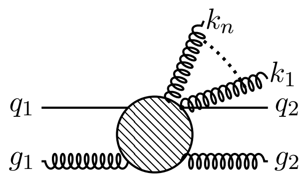

Figure 2. $q_1 g_1 \to q_2 g_2$ process with n bremsstrahlung gluons.

The idea is to start with 1 gluon radiation and then increase accordingly the number of bremsstrahlung gluons and we hope to generalize the concept to order n. We assume that the gluon radiation is *soft* and *collinear* to the outgoing quark. At *soft gluon limit* [7], we can factorize the amplitude into a divergent piece that depends on the energy and the angle of the emitted gluon (say k_s), and a second piece which is the amplitude omitting the soft gluon $\mathcal{A}_n(1, ..., i, k_s^{\pm}, j, ..., n) \to soft(i, k_s^{\pm}, j)\mathcal{A}_{n-1}(1, ..., i, j, ..., n)$. The expression of the *soft-amplitude* [8] is given by the MHV-formula. Similarly, if the gluon radiation is collinear to the outgoing quark, the amplitude factorizes into an amplitude with one fewer external state and a *splitting amplitude* depending on the particle going collinear and the internal state $\mathcal{A}_n(1, 2, \cdots, a, b, \cdots, n) \to \sum_{\lambda=\pm} split(a, b)\mathcal{A}_{n-1}(P^{\lambda}, 1, ..., n)$. These factorization properties of MHV-amplitudes will be used in the project.

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