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Locality in the Schrödinger Picture of Quantum Mechanics

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Abstract: This paper explains how the so-called Einstein locality is to be understood in the Schrödinger picture of quantum mechanics. This notion is fully compatible with the Bell non-locality exhibited by entangled states. Contrary to the belief that quantum mechanics is incomplete, it is, As a matter of fact, its overcompleteness, as exemplified by the different pictures of quantum physics, that points to the same underlying reality.

Keywords: quantum physics; Schrödinger picture; Heisenberg picture; entanglement; locality; causality

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1. Introduction

The concept of the field was invented in physics in order to preserve the idea of locality. Fields came as an answer to Newton's (and many others') worry that gravity seems to act at a distance, since massive objects attract one another across the vastness of space without any (apparent) mediation in between. In order to remedy this, Faraday came up with the idea of the field, wherein an object only affects the field in its immediate vicinity, and this disturbance then propagates through the field much as a wave would when a stone is thrown into a pond. This wave of disturbance in the field ultimately reaches another object, and the field disturbance then affects it locally at the point where the other object is (as a lotus leaf in the pond would be affected after the wave caused by the stone reaches it). That way, any action at a distance is avoided.

Exactly the same is true for quantum fields [1,2], the only difference being that, in the quantum case, the q-numbers (the "q" stands for quantum) pertaining to the field do the waving [3–5]. The q-numbers are actually just objects that are defined algebraically, and, in quantum physics, they are represented by matrices, or operators. In the case of the electromagnetic field, these are the operators characterising the vector potential, the electric and the magnetic fields [6] (with subtleties involving gauges [7] which are not relevant for the present study). Here, the speed of propagation, certainly is the speed of light. But the speed itself is irrelevant for the concept of locality. Instead, it is just required that acting on one subsystem (whatever this is) cannot instantaneously affect another subsystem. Below, it is exactly spelled out what the latter means, but, suffice to note, this principle—known as Einstein's locality—is also true in non-relativistic quantum mechanics. If there are two qubits located at two different places, then acting on one of the qubits one cannot change any property of the other qubit.

It is sometimes said that quantum physics violates Einstein's locality. But, as explained above, it does not. Quantum physics violates an altogether different idea of locality called Bell locality. In Bell locality, things are described by c-numbers (where "c" stands for classical), or real numbers (which are different from q-numbers), locally. And, as certainly it is known, violation of Bell's inequalities is in direct conflict with the existence of the c-number-based local reality. Some confusingly conclude that the violation of Bell's inequalities means that quantum physics is Einstein non-local (instead of just Bell non-local), but this is only true if one assumes that reality is described by the c-numbers locally. However, given that quantum physics does satisfy Einstein's locality, it is more appropriate to note that quantum physics violates the c-number-based reality (this, too, alas is frequently overstated by stressing that quantum physics proves that reality doesn't exist). The latter actually only means that reality is underpinned by locally given q-numbers, and this is a radically different form of reality from the classical one, though it is a reality nevertheless. Nothing in quantum physics points that there is no reality whatsoever. That statement makes no sense even to a philosopher.

It is necessary to discriminate between different notions of locality since, sometimes, one can hear incorrect conclusions wherein quantum physics is said to be incompatible with general relativity because the former is non-local while the latter is local. This, however, confuses the two notions of locality. The two theories, actually, well agree since both are Einstein local. The fact that quantum physics is Bell non-local and general relativity is not does not represent an obstacle to their unification (no more than it is an obstacle to quantum electrodynamics). When the fundamental entities in general relativity (the metric tensor and the Christoffel symbols) are quantised, the resulting theory of Quantum Gravity also becomes Bell non-local. Any classical theory that is quantised is automatically—by its very construction—Bell non-local.

Here, let us target the following related issue (and show that it is not an issue). If quantum physics is local and q-number realistic, then why does the wavefunction (or, more generally, the density matrix) give us the impression of non-locality [8]? Namely, if the state of two qubits is maximally entangled, for instance, $|00\rangle + |11\rangle$ (which is one of the so-called Bell states), then introducing a phase shift on, for instance, the first qubit ($|1\rangle \rightarrow e^{i\phi}|1\rangle$ leads to the state $|00\rangle + e^{i\phi}|11\rangle$ (with the phase ϕ). But the latter state is indistinguishable from the state where the phase shift is introduced in the second qubit. It looks as though the phase "travelled" instantaneously from the first qubit, where it was generated, to the second qubit, which could be arbitrarily far away. Note, however, that there is no way to extract the phase information from the second qubit alone (or the first one for that matter—the key feature for the protocols known as quantum data hiding). In that sense, there is no possibility of using this local phase kick to signal to the second qubit. Formally, the reduced density matrices of either of the qubits are independent of the phase (they are just each in a maximally mixed state). But there is more to this. No property of the second qubit, i.e., no q-number locally pertaining to it, is affected by the phase kick in the first qubit.

Indeed, when one is tempted thinking that something non-local may be happening, one is being deceived by the view that only the wavefunction is relevant to the factual situation of the two systems. The reason is that the state of the system only tells us half of the story in quantum physics. One needs the observables (including the state) as well as the dynamics of the system. Locality is all about the dynamics, and this is true in quantum physics as much as in the wave propagation in the water of the pond. There is also a way of expressing the state in the Schrödinger picture that retains the local knowledge of the phase by keeping track of all the operations executed on the system. But, surely, any given state does not have enough "capacity" of its own to contain all of the dynamics. The current paper is about this point and related issues.

Let us now proceed to review the two main pictures of quantum mechanics, the Heisenberg and the Schrödinger pictures, in order to show how locality is understood in each of them (it is approximately the same since the dynamics is the same in both). The rest of the article then analyses the basic quantum interference as well as entangled states from the local quantum perspective.

2. The Heisenberg and Schrödinger Pictures

It is used to calculating in the Schrödinger picture just because it is frequently more straightforward to do so. However, the Heisenberg picture is much closer to the intuition, which is based on learning Newton's classical mechanics in high school. The Heisenberg picture arises naturally in quantum field theory because one arrives at it by (second) quantising the wavefunction, which therefore becomes an operator in quantum field theory. In this manner, when the Schrödinger picture is used in the first quantisation, it becomes by default the Heisenberg picture upon performing the second quantisation. The two pictures are nevertheless equivalent in much the same way as the Hamilton approach to classical mechanics is equivalent to Newton's. They both lead to exactly the same experimental predictions. But, moreover, both pictures tell the same about what is "really" happening. More formally, in the Schrödinger picture, quantum states evolve in time, while observables are stationary. In the Heisenberg picture, it is the other way round. It is because the observables evolve in the Heisenberg picture that this is closer to the spirit of classical physics, where the position or momentum of a particle changes in time. It is, however, irrelevant what evolves and what does not (this is not real; one cannot tell whether the states or the observables evolve). The only empirically accessible quantity is of the form $\langle\psi|O|\psi\rangle$, where O represents any observable, and this quantity changes in time by either evolving the states $|\psi(t_0)\rangle \rightarrow U(t,t_0)|\psi(t_0)\rangle$ (Schrödinger) or the observables $U^\dagger(t,t_0)O(t_0)U(t,t_0)$, where U represents the unitary transformation and zero subscript denotes the initial state. The expected value is the same since $(\langle\psi|U^\dagger)O(U|\psi\rangle) = \langle\psi|(U^\dagger OU)|\psi\rangle$.

Geometrically, this can be seen as follows. States and operators are vectors in quantum mechanics. The physical meaning lies in their inner product, which, in Euclidean geometry, represent (cosine of) the angle between the vectors. Now, one can evolve one of the vectors in, for instance, the clockwise direction in the two-dimensional plane defined by the vectors while keeping the other vector fixed. This leads to an increased angle between the two and therefore a changed inner product. But the same could be achieved by evolving the other vector by the same amount but in the counter-clockwise direction so that the angle between the two ends up the same as in the former case. This is precisely why, in quantum mechanics, if the density matrix evolves as $\rho \rightarrow U\rho U^\dagger$ (Schrödinger), the operator must evolve ("backwards in time") according to $O \rightarrow U^\dagger\rho U$ (Heisenberg).

Let us now give a related account of the thinking around the Heisenberg picture that helps understanding the main argument. The idea is that, as the state is rotated, the basis is also rotated so that the state ultimately stands still with respect to the rotated basis. This is the "active way" of looking at the Heisenberg picture; the state evolves as in the Schrödinger picture but then this evolution is compensated by the evolution of the basis so that the state is stationary (always with the same coordinates—amplitudes in quantum physics—with respect to the current reference frame). For any basis vector $|a\rangle$, let us define $|a_t\rangle = U^\dagger(t,t_0)|a\rangle$. Now, the time evolution of this state is $|a_t(t)\rangle = U(t,t_0)|U^\dagger(t,t_0)|a\rangle = |a\rangle$. If one looks at the spectral decomposition of an operator, $A = \sum_n a_n|a_n\rangle\langle a_n|$, then the evolution of the basis leads to $A(t) = \sum_n a_n|a_{n,t}\rangle\langle a_{n,t}|$ (the observables, if the coordinate system moves with the state, must appear to be moving backwards). When the Hamiltonian is time independent, one has: $U(t,t_0) = \exp\{-iH(t-t_0)\}$.

Incidentally, all the infinitely many pictures in between Schrödinger and Heisenberg are also equally good. The states can be evolved for half the time and the operators for the other half (backwards), and the expected resulting value (the vector inner product) will still be the same. The so-called interaction picture is like that and so is, speaking somewhat loosely, the two-state formalism of quantum mechanics. However, here, the focus is just on the Schrödinger and Heisenberg pictures as they are antipodal to one another, and it would superficially seem that they are as different as it is possible to be.

Let us return to the example of kicking in the phase to one qubit, which is part of a two-qubit maximally entangled state. It looks as though the Schrödinger picture misleads us into believing that the maximally entangled state is non-local, namely, that the two qubits are behaving non-locally in some way, because the phase looks like it could also pertain to the other qubit. The notion of "phase kick-back" in quantum computation arose exactly to convey the feature that only one of the two qubits acquires a phase; then, because the qubits are entangled, the phase is jointly acquired by the other qubit too. In the Heisenberg picture, no such thing happens. Namely, if one considers the X q-numbers of the qubits as $X_1 = X \otimes I$ and $X_2 = I \otimes X$, where I is the unit matrix, X is the Pauli X matrix, then introducing the phase (suppose, for simplicity, that $\phi = \pi$) in the first qubit leads to $X_1 \rightarrow -X_1$, while X_2 stays the same (a similar conclusion is reached for Y, while neither of the Zs changes). The phase kick in the first qubit therefore quite clearly only affects the observables pertaining to the first qubit (X and Y) and not the second one.

Furthermore, the state in the Heisenberg picture can always be taken to be a product state between the two qubits, and, because it never evolves in this picture, there are never any issues with entanglement. The state is also fully local. This last point is not directly relevant as the locality under discussion here, replacing. of the Einstein kind, is all about the dynamics. As one can see in Secton 3 below, any entangled initial state can anyway be thought of as starting as a product state, and the operations needed to create entanglement can then be absorbed into the evolution of the operators. The state is irrelevant in the Heisenberg picture because it does not change. One could choose for it to be entangled, and it does not affect the Einstein locality because this locality is about what happens to the other system when one dynamically changes the first system. Altogether, this is what is meant by Einstein's locality. Both states and operators reflect local operations locally, and this is true in all pictures of quantum physics.

So why does the Heisenberg picture give the impression that it conforms to Einstein's locality fully, while Schrödinger's does not? Because, when working in the Schrödinger picture, one frequently (and mistakenly) forgets to take into account the dynamics itself when the evolution of states is discussed. One only presents the final state of the system and "forgets about" the dynamics through which that state is reached (i.e., one writes the phase down, but does not keep the information regarding how it was induced). This is sufficient as a starting point for all future calculations, but it does not contain the answer to the question of how things happened in the past (like where the phase was inserted). When the dynamics is fully included, not only do the two pictures give the same experimental predictions (since they lead to the same expected values of all quantities), but they also tell one and the same story, in a fully local manner. This topic is addressed next in Section 3.

3. Entanglement and Locality in the Schrödinger Picture

This Section presents the Schrödinger picture in a slightly different way that is quite easier to apply to the problem at hand. Let us first think of the familiar free evolution of a single particle. For a free particle, all the operators can be taken to be functions of its position and momentum, $O(p,q)$. This is true for the density matrix too, $\rho(p,q)$.

In the Heisenberg picture, one can assume that $\rho(p_0,q_0)$, where p_0,q_0 are the initial momentum and position of the particle. The dynamics, just as in classical mechanics, is given by $p(t) = p_0$ and $q(t) = q_0 + p(t)/mt = q_0 + p_0/mt$, where m is the particle mass. The momentum operator never changes (since the particle is free), while the position operator becomes a function of the initial position plus vt, with v the velocity, just like in classical physics. Here, the observables change in time and acquire (in general) an explicit time dependence through $O(p(t),q(t)) = O(p_0,q_0 + p_0/mt)$.

To reach the Schrödinger picture, let us assume that observables are stationary, while the states acquire an explicit time dependence. In that sense, $O(p_0,q_0)$, while $\rho(p_0,q(t) - p(t)/mt)$. Note the minus sign in the evolution of the density matrix in the Schrödinger picture with respect to the plus sign in the operator evolution in the Heisenberg picture. "Time flows backwards" between the two pictures. The same is true for quantum bits, but it might be more straightforward to see in the case of continuous variables, which is why I believe the above example is a helpful illustration.

This consideration leads to the calculation in this paper that shows that the Schrödinger picture is as Einstein local as the Heisenberg one. Let us now discuss an entangling protocol between two qubits which start in the state $|+\rangle|+\rangle$. Here, $|\pm\rangle = |0\rangle \pm |1\rangle$ (up to the omitted normalisation). For simplicity, it is assumed that the evolution is a phase kick that occurs only if the state of the qubits is $|1\rangle|1\rangle$. If, again, the phase is assumed to be π, the overall unitary just transforms $|1\rangle|1\rangle$ into $-|1\rangle|1\rangle$ and leaves all the other three basis states unchanged.

Quite straightforwardly in the Schrödinger picture, the final state is the maximally entangled state $|0\rangle|+\rangle + |1\rangle|-\rangle$. Up to the local Hadamard transformation of the second qubit, this state is the same as the Bell state considered in Section 2. It, therefore, looks Einstein non-local in the sense that, if a phase is introduced to only one of the two qubits, from the form of the state, one cannot tell which qubit it is that suffered the phase. The reduced density matrices computed from the state $|0\rangle|+\rangle + e^{i\phi}|1\rangle|-\rangle$ are, again, both maximally mixed and contain no information about where the phase was located.

To remedy this, let us switch to the density matrix notation. Initially, the density matrix is given by

$$\rho = \frac{1}{2}(I + X_1) \times \frac{1}{2}(I + X_2), \tag{1}$$

where, as in Section 2, $X_1 = X \otimes I$ is the X operator pertaining to the first qubit, while $X_2 = I \otimes X$ is the X operator pertaining to the second qubit. The product between the two is just an ordinary product (not the tensorial one) so that $X_1 \times X_2 = X \otimes X$ when one chooses the representation above. The evolution is easily calculated since $X_1(t) = X_1 Z_2$ and $X_2(t) = Z_1 X_2$. The final state of the density matrix is, therefore:

$$\rho(t) = \frac{1}{2}(I + X_1(t)Z_2) \times \frac{1}{2}(I + Z_1 X_2(t)), \tag{2}$$

which is still a product state in this new notation. The time t just indicates the state after the phase gate has been applied.

More to the point, one can see that the state of the first qubit has now acquired a dependence on the initial Z component of the second qubit, and the state of the second qubit has, likewise (since the unitary is symmetric with respect to the qubits), acquired the dependence of the initial Z of the first qubit. It is through this mutual dependence that one can see that the qubits have become entangled. The qubits have exchanged information through the phase gate. Indeed, it is fully appropriate to note that each qubit measures the other one.

Note that this does not mean that all local operations are detectable locally. Quite the contrary. For any given initial state, one has many operations that lead to the same final state of affairs as far as local observables are concerned. This feature is intrinsic and cannot be changed by changing the notation—and it is true both in the Schrödinger and the Heisenberg picture. However, the product notation just introduced for the Schrödinger picture allows us to formally keep track of the dynamics, just like in the Heisenberg picture. Thus, one can also keep track of where local gates have been applied by checking which factor in the product has been affected. Once more, this only means that, in both pictures, states alone do not contain all the relevant information, and one needs to keep track of the dynamics (which, in the Heisenberg picture, is achieved by default by transforming all the algebra of the relevant operators).

Now, as soon as the state in Equation (2) is multiplied out, one obtains (in the tensor product notation)

$$\rho(t) = \frac{1}{4}(I + X \otimes Z + Z \otimes X + Y \otimes Y), \tag{3}$$

which is certainly the maximally entangled state $|0\rangle|+\rangle + |1\rangle|-\rangle$. It is this form that tricks us into believing that something non-local is occurring in quantum physics. Performing the phase operation on the first qubit here leads to the state $\rho(t) = \frac{1}{4}(I - X \otimes Z + Z \otimes X - Y \otimes Y)$, and this form just does not tell us which of the two qubits was affected (since one we could have obtained the same state by a suitable phase kick on the second qubit). In the product notation, on the other hand, the state would become

$$\rho(t) = \frac{1}{2}(I - X_1(t)Z_2) \times \frac{1}{2}(I + Z_1 X_2(t)), \tag{4}$$

which exhibits the minus sign in the state pertaining to the first qubit. So, if one looks for a fuller account of what is happening, the product notation is possibly better than the tensor product. Even here, naturally, there are operations on the second qubit that would lead to the same state since, ultimately, the results and the information contained in the two pictures are the same.

4. Discussion and Conclusions

Two points are worth summarising. One is that the state in the product notation is as local in the Schrödinger picture as it is in the Heisenberg picture, meaning that the evolution of the whole can be specified by the "local factors" as defined here. This is due ultimately to unitarity. As was said in Section 3, this does allow for different unitary dynamics to lead to the same quantum state (the same statistics), like when the phase gate is applied to one or the other qubit in a maximally entangled state. The local and global statistics of the final state alone does not tell which of the unitaries was applied. A separate point is, therefore, that, for completeness, one needs to keep track of all operations. The key insight is that, when Schrödinger's and Heisenberg's quantum mechanics are compared, one must compare them on an equal footing. For such a comparison, it is just wrong to consider only one state in the Schrödinger picture while keeping the information about the full algebra of observables in the Heisenberg case (the latter clearly contains much more information). Certainly, one could track the full basis of states in the Schrödinger picture, but this is not in the spirit of this version of quantum mechanics as the initial state is assumed to be given as a "boundary condition".

Two formulations of locality are relevant for this paper: Einstein and Bell locality. It is worth pointing out that Einstein almost certainly expected quantum theory to obey Bell locality—that seems to be the whole point of the Einstein–Podolsky–Rosen argument. Classically, there is no difference between the two since, in classical physics, all quantities are represented by real numbers only, pertaining locally to the system (this is taken to be so transparent that it is not even worth discussing). However, as things stand, quantum physics satisfies the Einstein locality but violates the Bell locality. Namely, if an action takes place in one subsystem in quantum physics, this cannot instantaneously affect any property of other subsystems (the state or the observables). Here, I have argued that this is true in all representations of quantum physics and, in particular, in the Schrödinger picture, which is frequently (and wrongly, as shown here) seen as "non-local".

This study has been emphasizing the point that the Schrödinger wavefunction is, by itself, not enough. However, even here, one could sidestep this problem by assuming that the given wavefunction is not just that of the system under consideration, but includes all other relevant systems. In that sense, all the actions on the system itself are recorded in the states of the other systems participating in them. So, in this example of the maximally entangled state of two qubits undergoing a local phase kick, the knowledge about which of the qubits was kicked would reside somewhere in the wavefunction but not the two qubits. Instead, it could, for instance, be recorded in the quantum state of the memory of the experimentalist who implemented the transformation. This picture, akin to what is known as the Page–Wootters formulation of dynamics without dynamics [9,10], assumes that the wavefunction also contains all the relevant information regarding the dynamics of observables.

The closest classical physics comes to the Schrödinger picture is when using the phase space representation of states and dynamics [11]. Imagine two particles elastically colliding in one dimension but each starting with a distribution of positions and momenta $f(p_1, q_1, p_2, q_2, t_0) = f(p_1, q_1, t_0) f(p_2, q_2, t_0)$. There are no initial correlations between particles. From energy and momentum conservation, it follows that

$$\begin{aligned} p_1(t) &= g_1(p_1(t_0), p_2(t_0)), \quad p_2(t) = g_2(p_1(t_0), p_2(t_0)), \\ q_1(t) &= q_1(t_0) + \frac{p_1(t)}{m} t, \quad q_2(t) = q_2(t_0) + \frac{p_2(t)}{m} t, \end{aligned}$$

where the exact forms of the functions g_1 and g_2 are not relevant (they can easily be calculated), and $p_1(t), p_2(t), q_1(t), q_2(t)$ are the momenta and positions of the two particles after the collision has taken place. One can see how each particle has now obtained the information about the initial momentum of the other particle. Positions of the particles thus also become correlated, and so do their densities in phase space. The only difference with quantum mechanics is that all these quantities become q-numbers, resulting in the equal time commutation relations of the form $[q_i(t), p_i(t)] = i\hbar$, where $\hbar$ is the reduced Planck constant. The q-numbers for different particles always commute with one another. The quantum correlations arising from this could be entanglement, something that does not have any classical analogue. But this in itself is not relevant as far as Einstein's locality is concerned. The classical and quantum accounts are here identical as long as the c-numbers are replaced with the q-numbers [11].

There are other notions of locality in quantum physics, such as the frequently emphasised micro-causality in quantum field theory. Field operators at different space-like separated points must commute with one another (for fermions, it is the quadratic Hamiltonians that commute with quadratic observables). This is a sufficient (but not a necessary) condition to ensure that no causes can propagate faster than the speed of light [12]. The Einstein locality condition used here is more general. It does not refer to any specific speed of propagation or any particular relationship between space and time. Micro-causality is, therefore, a stronger condition, and it implies Einstein's locality. The former not only prohibits an instantaneous action at a distance, but also puts a specific speed limit (the speed of light) on any propagation.

There is one last "non-locality" that is frequently noted in quantum physics that deserves a mention. It is the Aharanov–Bohm effect [13]. Here, the argument has frequently been made that the magnetic field acts non-locally on an interfering electron to generate a phase that affects the electron interference at a distance (i.e., where the field is zero). But, as we have shown [14], in the fully quantum mechanical analysis, the Aharanov–Bohm phase is also acquired locally. All transformations in quantum mechanics, just as described in this paper, always take place locally. The specific formalism used may or may not directly reflect this finding, but that is completely irrelevant since all accounts lead ultimately to one and the same set of outcomes.

The interesting question for a physicist, therefore, is whether there are instances where locality is truly violated, and not just in some accounting procedures executed on a piece of paper while calculating. Can an action at a distance really take place instantaneously under some circumstances? Non-local extensions of quantum field theory have been considered, motivated frequently by trying to avoid various infinities, but—to the best of my knowledge—there is no evidence whatsoever that any of them is more successful than the Standard Model. More importantly, perhaps, it is questionable whether one could handle such notions as "information" or "observables" in a theory that is Einstein non-local. A theory that violates the Einstein locality may therefore not even be testable.

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