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SNAKE MATCHING THE EIC'S HADRON STORAGE RING

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Abstract

Pairs of Siberian Snakes allow the avoidance of first-order spin resonances during energy ramping. However, a high density of first-order resonances correlates with the presence of higher-order resonances after the installation of snakes. Thus, one effective tactic of mitigating higher-order resonances is by weakening the surrounding first-order ones, equivalent to minimizing the spin-orbit coupling integrals. Such a proxy helps sidestep a multi-hour polarization transmission simulation for each lattice configuration. In a three-fold super-periodic ring, using 12 snakes is a sufficient condition for completely eliminating the spin-orbit coupling integrals at all energies and tunes. Since the HSR will only have up to 6 snakes, we opt to focus on suppressing the strongest first-order resonances instead of the whole spectrum. By varying the snake reflection axes and the betatron phase advance in two of the arcs, we search in a 7-dimensional lattice space for the weakest resonance structure using a variety of metrics and find the configuration with highest polarization transmission.

INTRODUCTION

Since its inception in 2000, RHIC has been delivering the highest energy polarized proton and heavy ion beams in the world [1]. What begun as just over 30 % polarization has been enhanced and optimized over the years to consistently deliver over 50 % polarization at up to 255 GeV [2, 3]. RHIC offers the only high-energy physics setting to study the spin substructure of nuclear matter, and will be getting an upgrade to 275 GeV in the form of the Hadron Storage Ring of the Electron-Ion Collider [4].

While it's true that most of the magnets of the HSR will be reused from RHIC, the HSR may also be facing new spin polarization challenges. In particular, spatial as well as physics-program constraints such as having 1 or 2 interaction points will impose further breaking of the three-fold symmetry in the lattice [4]. Past wisdom favors symmetric lattices in terms of polarization transmission, and the current incarnation of the HSR with storage optics adds credence to that wisdom. From Fig. 1 it is clear that the HSR's spin resonance spectrum has a large RMS value, even though the peaks are approximately the same as RHIC's. These densely-packed low-lying resonances overlap with the large peaks and interfere strongly, causing severe higher-order resonances.

We propose the solution of snake-matching, invented for the design of HERA-p by one of the authors [5, 6].

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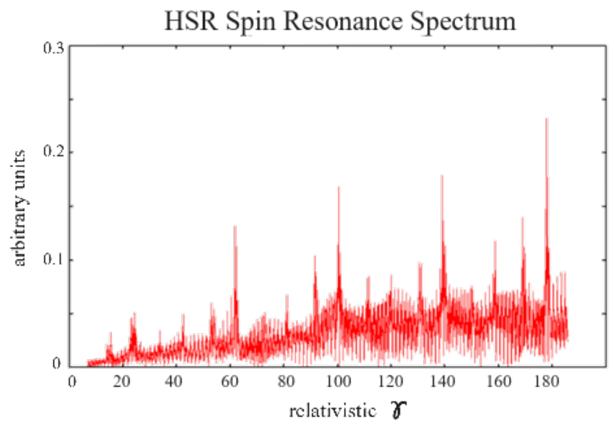

Figure 1: Spin resonance spectrum of Helion in the HSR using storage optics and 1π mm mrad normalized emittance.

SPIN-ORBIT COUPLING INTEGRAL

The general philosophy behind preserving hadron polarization in storage rings is to minimize the linearized spin-orbit coupling integrals $I_j^{\pm}$, since there are much less stochastic effects compared to electrons. The linearized integrals are [5]:

$$I_j^{\pm} \equiv \frac{G\gamma + 1}{i\sqrt{2}} \oint k(s)\sqrt{\beta_j(s)}\, e^{\pm i\phi_j(s)} e^{-i\psi(s)} ds \qquad (1)$$

where $G \equiv \frac{g-2}{2}$ is the anomolous magnetic moment, k is the normalized quadrupole strength, β_j is the twiss beta function for the j-th coordinate, ϕ_j is the betatron phase advance and ψ is the spin phase advance or amount of precession around the stable spin direction $\vec{n}_0$. See [7, 8] for the higher-order analog of Eq. (1).

Such an integral is typically disguised as a well-known quantity called the first-order "spin-orbit resonance strength," which is also the Fourier coefficient of the spin precession vector with respect to the generalized machine azimuth [9]. The resonance strength, however, is only well-defined while satisfying the corresponding resonance condition $\nu \equiv \pm\vec{l} \cdot \vec{Q} \bmod 1$ where ν is the spin-tune, $\vec{l}$ is a triplet of integers and $\vec{Q}$ are the beta-synchrotron tunes. For the remainder of the paper, we will only be concerned with the vertical integrals.

So called "Siberian Snakes" were conceived precisely for this reason: where by flipping a propagating spinor around an axis in the horizontal plane at one point in the ring and then again at another point exactly half-way around the ring and around an (horizontal) axis normal to the former axis of rotation, they achieve an exact cancellation of anomalous magnetic precessions around the ring (encapsulated by

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$\psi \to -\psi$). What is left is the $2\pi\nu_0 = \pi$ precession due to orthogonal snake axes of rotation [10].

With the use of snakes it becomes rare to satisfy the spin resonance conditions and hard to diagnose. This means two things: One, there will be less resonances to worry about. Two, any remaining resonances must be higher-order ones, observed by particles whose amplitude-dependent spin-tune satisfies higher-order resonance conditions only for large-amplitude particles (by deviating from $\nu_0 = 1/2$).

SIBERIAN SNAKES SAVE SPIN

Minimizing the spin-orbit coupling integral involves carefully cancelling contributions from each section with a different section [10]. With snakes, the natural way to do this is by varying the horizontal snake axes of rotation denoted ψ_i within the range required for maintaining a closed-orbit spin-tune of 1/2.

$$\psi_1 - \psi_2 + \psi_3 - \psi_4 + ... = \frac{\pi}{2} \tag{2}$$

ψ_i is the angle (rad) measured radially outward from the longitudinal direction in the horizontal plane.

Of course, it is also possible to vary the betatron phase advances $\phi(s)$ as well to cause these cancellations, however we explore this in the next section.

To determine a greatest lower bound for the number of snakes necessary for completely cancelling the integrals, the quantities of interest are the super-periodicity of the machine and its energy range. For a given choice of these parameters, one can sometimes arrange the snake rotation axes ψ_i in such a way to perfectly cancel contributions between arcs in a ring, independent of orbital tune. This was planned for HERA-p which had an approximate 4-fold symmetry, and required 8 snakes for a perfect match [5, 6]. In the case of the EIC's HSR, the approximate three-fold symmetry is a good starting point before any fine-tuning. Based alone on the super-periodicity, it is clear that the greatest lower bound must be a multiple of 3, leaving us with either at least 6 or 12 snakes as sufficient.

To be more precise, the spin-orbit coupling integrals come in $\pm$ pairs which must be simultaneously cancelled. The 6 or 12 snakes will split each of these integrals further down into smaller terms, which we know are of approximately equal magnitude due to the super-periodicity. For each integral terms are then arranged into two families $\{I_i^\pm, I_o^\pm\}$ representing the inner and outer arcs of a super-period, respectively. In the case of 6 snakes with axes of rotation $\psi_i = \alpha_i/2$, and $\alpha_{ij} \equiv \alpha_i - \alpha_j$, we have:

$$\begin{aligned} I^\pm = I_i^\pm &\left[1 + e^{i(\pm 2\pi Q_y/3 - \alpha_{21})} + e^{i(\pm 4\pi Q_y/3 - \alpha_{43} - \alpha_{21})}\right] \\ + I_o^\pm &\left[1 + e^{i(\pm 2\pi Q_y/3 - \alpha_{32})} + e^{i(\pm 4\pi Q_y/3 - \alpha_5 - \alpha_{32})}\right]. \end{aligned} \tag{3}$$

Note that in a three-fold symmetric ring, I_i and I_o are independent quantities, meaning that there are much fewer combinations of terms that can coherently cancel each other out in $I^\pm$. Upon inspection of Eq. (3), it appears that the sufficient number for a perfect snake match is to six snake only if the ring has a precise three-fold symmetry. Otherwise 12 snakes would be needed unless an alternate method was used.

Nevertheless, in the case of HERA-p, the design was developed further to maintain good polarization using only 4 snakes, instead of 8. This was done by carefully varying both the snake axes and phase advances between arcs [5, 6]. The same can be done in the HSR, using a similar technique.

SNAKE AXIS OPTIMIZATION

We first investigate the dependence of polarization transmission on the choice of snake axes. Based on Eq. (2) we see that using 6 snakes results in a 5-dimensional parameter space of axes. The efficiency of scanning such a parameter space really depends on what is being calculated: it would be easy to calculate the spin-orbit coupling integral at a given energy, but challenging to calculate polarization transmission since that requires tracking the full ramp in real-time. We compare the benefits of using two snakes to using six snakes, while taking two representative pairs of locations around the ring in the 2 snake case. We note that different locations of snakes result in different equilibrium polarization.

Two Snakes

Beginning with two snakes, 2 locations are considered for the first snake: CCW of IR-12 or CCW of IR-10. The corresponding locations for the second snake would be CCW of IR-6 and CCW of IR-4, respectively. For each placement, the choice of rotation axes for the snakes was swept over 100 different values and a single particle was ramped through each choice of rotation axes and snake placement. Our findings demonstrated that the IR 12-6 placement is insensitive to choice of snake axis and preserved about 75 % polarization on average, whereas the IR 10-4 placement is highly sensitive to choice of snake axes and averaged 40 % transmission.

Six Snakes

With six snakes, the available locations for placement of snakes are severely restricted due to their 12 m extent and the requirement of 2 additional spin rotators around each IP. Possible variations in position could circumvent at most 2 additional quadrupoles (in contrast there are 24-26 quadrupoles between each snake). Even after fixing the position of each snake, there remains a 5-dimensional parameter space for snake axes of rotation. A direct scan of polarization transmission over the full space would take considerable computational resources, as computing just one point in that space takes roughly 15 hrs with 1 particle/core.

Instead we opt for a more top-down approach which is to avoid crossing resonance conditions in general. Even though the closed-orbit spin-tune is fixed to 1/2 by design, variations from this emerge in the amplitude-dependent spin-tune (ADST) ν away from the closed orbit. Unfortunately,

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the ADST is taxing to numerically compute and we instead use a figure-of-merit known as the perturbed spin tune (PST), which comes from the eigenvalues of a 1-turn matrix as a function of phase-space coordinates [11]. While it is not necessary for the average of PST to converge to the ADST after long times, it does provide a good substitute for short-term behavior.

Thus, one of the authors [12] minimizes the PST variation around the most severe resonances, and finds those snake axes which produce this minimized variation. As a result, it is possible to improve polarization transmission to 95 % without any further changes. Of course, however, the story does not end here.

PHASE ADVANCE OPTIMIZATION

Next, we move on to exploring the space of vertical phase advance per arc with fixed overall vertical tune. This is a similar constraint as Eq. (2) but is not an alternating sum. Nevertheless, some benefits of opting to vary betatron phase advance instead of snake axes for use in optimization are that this circumvents managing the various orbital and optical defects that snakes produce (which strongly depend on the snake axes). Furthermore, it is much more common to measure and control betatron phase advances in rings, so this is also a more accessible approach. Finally, through Eq. (1) we see an equivalence between betatron phase shifts ϕ_y and spin phase advances ψ_i, so there should be no tradeoff in performance in the two approaches.

To enable phase advance control in different sections of the ring, we propose the installation of an extra pair of trim power supplies for the arc quadrupoles, to allow independent control of the phase advances across each arc. We find the best wiring scheme for fine control to be a series connection, with alternating polarity from focusing to defocusing quadrupoles. This offers a greater range of phase advance control over a fixed polarity wiring scheme.

It is clear that we will have again a 5-dimensional parameter space of options that satisfy our constraints, too large for our desire. Luckily, it is straightforward to initially organize them according to the spin-orbit coupling contributions of the respective arc. In this way, we find that the arcs near the IPs of the lattice generate the most coupling, and we only directly vary those 2 out of the 6 arcs. By leaving the remaining 4 arcs unfixed, they use the main power supplies for compensating the changes in the first 2 in order to maintain tune. For each pair of phase advances, we launch a single Helion particle across an energy ramp through the most depolarizing energy range $\gamma \in [170, 190]$, and see that there are drastic effects on polarization transmission in Fig. 2. Specifically there is an island of stability on the right side in terms of polarization transmission, demonstrating the success of the snake-matching procedure.

CONCLUSION

Siberian snakes are by far the most effective tool for maintaining polarization through very high energies, and by all means indispensable. However, these big, bulky magnets take up a lot of real estate, only come in pairs and introduce strong coupling, optical and orbital distortions. Therefore it is important to maintain the efficacy of polarization transmission with a minimal number of snakes. In this report, we address the unique polarization challenges facing the Hadron Storage Ring in the EIC by using two independent but very powerful methods from the framework of linearized spin-orbit dynamics called snake matching. These methods prove very successful at addressing the higher order depolarizing resonances at fault, and the phase advance control method provides experimental knobs for the variation of polarization. Future investigations will be in the direction of applying this method to different species of ions for experimental physics programs.

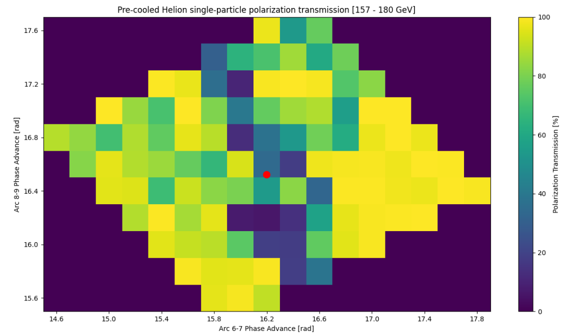

Figure 2: 2D scan of Helion polarization transmission in the 2-IR lattice over phase advances in arcs 6-7 and 8-9, while accelerating from $G\gamma = 700$ to $G\gamma = 800$. The red dot is the working point.

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