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Jet azimuthal anisotropy in ep collisions

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Abstract: In this work, we study the azimuthal anisotropy for the azimuthal angle difference ϕ between two azimuthal angles defined in the back-to-back lepton-jet production in lepton-proton collisions – the azimuthal angle of the transverse momentum imbalance q_T of the lepton and jet, and the azimuthal angle of the jet transverse momentum itself. In particular, we provide the theoretical origins for these azimuthal dependence from a factorization formalism derived within the SCET framework. In addition, we find that the directed flow component related to $\cos(\phi)$ azimuthal asymmetry is dominant. The numerical results of such azimuthal anisotropy for EIC kinematics are presented, showing that these are promising observables for studying lepton-jet correlations in future experiments.

Contents

1 Introduction

In recent years, the azimuthal anisotropy has served as an important observable in high-energy physics. For the studies in heavy-ion collisions, the azimuthal anisotropy contribute to characterizing the properties of the quark–gluon plasma (QGP) and provide experimental information on the equation of state of QGP [1–4]. In *ep* collisions, the back-to-back lepton-jet production from *ep* collisions has been providing insights for transverse momentum dependent (TMD) studies, where the jet observables have been expected for probing the three-dimensional nucleon structure encoded in transverse-momentum-dependent parton-distribution functions (TMD PDFs) [5–9]. As proposed in [10], the terminology "azimuthal anisotropy" is also applied and has been introduced as the measure of azimuthal correlations[1]. With the azimuthal asymmetry in the collisions, non-vanishing anisotropic flow harmonics are expected. More specifically, the radial flow, an azimuthal-angle-independent term, corresponds to the isotropic Fourier coefficient [11, 12]. As for the higher order Fourier coefficients that encoding the azimuthal anisotropy, the first term v_1 is the directed flow component, and likewise v_2 is elliptic flow, v_3 is triangular flow, v_4 is quadrapole flow, etc. For instance, in $e + p \to e + \text{jet} + X$ process, the differential cross section of back-to-back lepton-jet production can be decomposed as follows

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^2\boldsymbol{p}_T \mathrm{d}y_J \mathrm{d}\phi_J \mathrm{d}^2\boldsymbol{q}_T} = \frac{\mathrm{d}\sigma}{2\pi \mathrm{d}^2\boldsymbol{p}_T \mathrm{d}y_J q_T \mathrm{d}q_T}\left[1 + 2\sum_{n=1}^{\infty} v_n(p_T, y_J)\cos(n(\phi_q - \phi_J))\right], \quad (1.1)$$

where $\boldsymbol{p}_T$ and y_J are the transverse momentum and rapidity of the produced jet, $\boldsymbol{q}_T$ is the transverse momentum imbalance between the produced electron and jet, while ϕ_J and ϕ_q are the azimuthal angles of jet direction and $\boldsymbol{q}_T$, respectively.

In Eq. (1.1), v_n describes the azimuthal anisotropy of particle spectrum in the momentum space and can be measured in experiments as the expectation of the n-th order harmonics $v_n = \langle\cos(n(\phi_q - \phi_J))\rangle$, the averaged value with respect to the particle spectrum.

[1] Although the term "azimuthal anisotropy" has been adopted in the studies of both heavy-ion collisions and *ep* collisions, it describes different observables. In particular, the azimuthal angle of the heavy-ion collisions is defined with respect to the reaction plane of two colliding nuclei, while for *ep* collisions, the azimuthal angle is related to the transverse momentum imbalance of the produced lepton and jet.

2 Jet azimuthal anisotropy

In this section, we present the factorization theorem for ep to electron plus single jet process with azimuthal anisotropy by projecting the collective flow to different Fourier coefficients and provide the elliptic flow, triangular flow and higher flow harmonics in this collision.

2.1 Factorization and resummation formula

In the back-to-back electron-jet production from ep collision,

$$e(l) + p(P) \to e(l') + J_q(p_J) + X\,, \tag{2.1}$$

we define the transverse momentum of the produced electron $\boldsymbol{l}'_T$, produced jet $\boldsymbol{p}_T$, and the transverse momentum imbalance of the electron and the jet is $\boldsymbol{q}_T = \boldsymbol{l}'_T + \boldsymbol{p}_T$. In the back-to-back limit where $|\boldsymbol{q}_T|$ is small, i.e. $|\boldsymbol{q}_T| \ll |\boldsymbol{p}_T|$, the cross section can be factorized into a convolution of hard, TMDPDFs, and soft functions. For unpolarized proton, TMD factorization is given by [13]

$$\frac{\mathrm{d}\sigma^{e+p\to e+\mathrm{jet}+X}}{\mathrm{d}^2\boldsymbol{p}_T \mathrm{d}y_J \mathrm{d}^2\boldsymbol{q}_T} = \hat{\sigma}_0 \sum_q e_q^2 H(Q,\mu) \int \frac{\mathrm{d}^2\boldsymbol{b}}{(2\pi)^2}\, \exp(-i\boldsymbol{b}\cdot\boldsymbol{q}_T)\, x\tilde{f}_{q/p}^{\mathrm{TMD}}(x,\boldsymbol{b},\mu,\zeta) J_q(p_T R,\mu) \\ \times S_{\mathrm{global}}(\boldsymbol{b},\mu)\, S_{cs}(\boldsymbol{b},R,\mu)\,, \tag{2.2}$$

where $H(Q,\mu)$ is the hard function that describes the partonic hard scattering and $J_q(p_T R,\mu)$ is the quark jet function, characterizing the production of the outgoing jet from a hard interaction. Here the global soft function $S_{\mathrm{global}}(\boldsymbol{b},\mu)$ at the NLO has the following expression [14]

$$S_{\mathrm{global}}(\boldsymbol{b},\mu) = 1 + \frac{\alpha_s}{2\pi} C_F \Bigg\{ 2\Big[y_J + \ln(-2i\cos(\phi_{bJ}))\Big]\left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{\mu_b^2}\right) \\ + \frac{1}{\epsilon^2} + \frac{1}{\epsilon}\ln\frac{\mu^2}{\mu_b^2} + \frac{1}{2}\ln^2\frac{\mu^2}{\mu_b^2} - \frac{\pi^2}{12}\Bigg\}\,, \tag{2.3}$$

which is universal and contributes in the lepton-jet correlations. Here $\phi_{bJ} \equiv \phi_b - \phi_J$ with ϕ_b and ϕ_J are the azimuthal angles of the vector $\boldsymbol{b}$ and jet transverse momentum $\boldsymbol{p}_T$, respectively. We also provide the expression of the collinear-soft function S_{cs} as below,

$$S_{cs}(\boldsymbol{b},R,\mu) = 1 - \frac{\alpha_s}{2\pi} C_F \left[\frac{1}{\epsilon^2} + \frac{2}{\epsilon}\ln\left(\frac{-2i\cos(\phi_{bJ})\mu}{\mu_b R}\right) + 2\ln^2\left(\frac{-2i\cos(\phi_{bJ})\mu}{\mu_b R}\right) + \frac{\pi^2}{4}\right]. \tag{2.4}$$

And the rapidity-regulator-independent standard TMDPDFs [15] $\tilde{f}_{q/p}^{\mathrm{TMD}}(x,b,\mu,\zeta)$ at scale $\mu^2 = \zeta = Q^2$ in $\boldsymbol{b}$-space are defined by the Fourier transform of the corresponding TMDPDFs in momentum space $f_{q/p}^{\mathrm{TMD}}\left(x,k_T^2,\mu,\zeta\right)$,

$$\tilde{f}_{q/p}^{\mathrm{TMD}}(x,\boldsymbol{b},\mu,\zeta) = 2\pi \int \mathrm{d}k_T\, k_T J_0(k_T b)\, f_{q/p}^{\mathrm{TMD}}\left(x,k_T^2,\mu,\zeta\right)\,, \tag{2.5}$$

We noticed that the Fourier transformation in Eq. (2.2) can be written as an expansion of the Bessel functions using the identity

$$\begin{aligned}\frac{\mathrm{d}^2\boldsymbol{b}}{(2\pi)^2}\exp(-i\boldsymbol{b}\cdot\boldsymbol{q}_T) =&\frac{b\mathrm{d}b\,\mathrm{d}(\phi_{bJ}-\phi_{qJ})}{(2\pi)^2}\left\{J_0(bq_T)+2\sum_{n=1}^{\infty}(-i)^n J_n(bq_T)\cos(n(\phi_{bJ}-\phi_{qJ}))\right\}\\ =&\frac{b\mathrm{d}b}{2\pi}\Bigg\{J_0(bq_T)\frac{d\phi_{bJ}}{2\pi}+2\sum_{n=1}^{\infty}(-i)^n J_n(bq_T)\\ &\times\left[\cos(n\phi_{qJ})\cos(n\phi_{bJ})\frac{d\phi_{bJ}}{2\pi}+\sin(n\phi_{qJ})\sin(n\phi_{bJ})\frac{d\phi_{bJ}}{2\pi}\right]\Bigg\}, \end{aligned}\tag{2.6}$$

where $\phi_{bJ}=\phi_b-\phi_J$ and $\phi_{qJ}=\phi_q-\phi_J$. In the factorization formalism, we found that only the global soft function and collinear-soft function contain ϕ_{bJ} dependence. As a consequence, we let $S_q(\boldsymbol{b},R,\mu)=S_{\text{global}}(\boldsymbol{b},\mu)S_{cs}(\boldsymbol{b},R,\mu)$ and define the azimuthal-angle average of $S_q(\boldsymbol{b},R,\mu)$ as $\bar{S}_q(b,R,\mu)$ and $S_q^{\langle\mathcal{F}(\phi_{bJ})\rangle}(b,R,\mu)$ when weighted by a function $\mathcal{F}(\phi_{bJ})$ of angle ϕ_{bJ}. More specifically,

$$\bar{S}_q(b,R,\mu)=\int\frac{\mathrm{d}\phi_{bJ}}{2\pi}S_{\text{global}}(\boldsymbol{b},\mu)S_{cs}(\boldsymbol{b},R,\mu)\,,\tag{2.7}$$

$$S_q^{\langle\mathcal{F}(\phi_{bJ})\rangle}(b,R,\mu)=\int\frac{\mathrm{d}\phi_{bJ}}{2\pi}S_{\text{global}}(\boldsymbol{b},\mu)S_{cs}(\boldsymbol{b},R,\mu)\mathcal{F}(\phi_{bJ})\,,\tag{2.8}$$

where $\mathcal{F}(\phi_{bJ})$ can be ϕ_{bJ} or $\cos(n\phi_{bJ})$, depending on the context. Specifically, when $\mathcal{F}(\phi_{bJ})=1$, one has $S_q^{\langle 1\rangle}(b,R,\mu)\bar{S}_q(b,R,\mu)$. To that end, Eq. (2.2) is further expanded as

$$\begin{aligned}\frac{d\sigma^{e+p\to e+\text{jet}+X}}{d^2p_T dy_J d\boldsymbol{q}_T} =&\hat{\sigma}_0\sum_q e_q^2 H(Q,\mu)\int\frac{bdb}{2\pi}x\tilde{f}_1^{\text{TMD}}(x,b,\mu,\zeta)J_q(p_TR,\mu)\Bigg\{J_0(bq_T)\bar{S}_q(b,R,\mu)\\ &+2\sum_{n=1}(-i)^n J_n(bq_T)\left[\cos(n\phi_{qJ})S_q^{\langle\cos(n\phi_{bJ})\rangle}(b,R,\mu)\right]\Bigg\}\,.\end{aligned}\tag{2.9}$$

Compared to Eq. (2.6), the azimuthal anisotropy expansion of the differential cross section shown in Eq.(2.9) has no $\sin(n\phi_{qJ})$-related terms. Since the soft function $S_q(\boldsymbol{b},R,\mu)$ is an even function of ϕ_{bJ}, with $\mathcal{F}(\phi_{bJ})=\sin(n\phi_{bJ})$ for all n, the weighted soft function will vanish, namely $S_q^{\langle\sin(n\phi_{bJ})\rangle}(b,R,\mu)=0$.

2.2 Soft functions

As given in Eq. (2.3), the global soft function $S_{\text{global}}(\boldsymbol{b},\mu)$ is free of rapidity divergence but depends on the magnitude b and azimuthal angle ϕ_b of the vector $\boldsymbol{b}$. Similarly for the collinear soft function $S_{cs}(\boldsymbol{b},R,\mu)$ shown in Eq. (2.4). In this section, we present the trigonometric function-averaged $S_{\text{global}}(\boldsymbol{b},\mu)S_{cs}(\boldsymbol{b},R,\mu)$ at NLO ($\sim\mathcal{O}(\frac{\alpha_s C_F}{\pi})$) up to the order of quadrapole flow $4\phi_{bJ}$. Then the product of the global soft function and collinear soft function up to NLO is

$$S_q(\boldsymbol{b},R,\mu)=S_{\text{global}}(\boldsymbol{b},\mu)S_{cs}(\boldsymbol{b},R,\mu)$$

$$=1+\frac{\alpha_s}{2\pi}C_F\left[2\left(\ln R^2\ln(-2i\cos(\phi_{bJ}))-\ln^2(-2i\cos(\phi_{bJ}))\right)-\frac{\pi^2}{3}\right.$$
$$\left.+2y_J\left(\frac{1}{\epsilon}+\ln\frac{\mu^2}{\mu_b^2}\right)+\ln R^2\left(\frac{1}{\epsilon}+\ln\frac{\mu^2}{\mu_b^2}\right)-\frac{1}{2}\ln^2 R^2\right]. \qquad (2.10)$$

Therefore, the azimuthal-angle-averaged soft function $S_q(\boldsymbol{b}, R, \mu)$ as defined in Eq. (2.7) is

$$\bar{S}_q(b,R,\mu)=1+\frac{\alpha_s}{2\pi}C_F\left[2y_J\left(\frac{1}{\epsilon}+\ln\frac{\mu^2}{\mu_b^2}\right)+\ln R^2\left(\frac{1}{\epsilon}+\ln\frac{\mu^2}{\mu_b^2}\right)-\frac{\ln^2 R^2}{2}-\frac{\pi^2}{3}\right], \qquad (2.11)$$

and the azimuthal-angle-weighted soft functions defined in (2.8) can also be computed accordingly.

2.3 Numerical results

In this section, we present the numerical results for $\cos\phi$, $\cos 2\phi$, $\cos 3\phi$ and $\cos 4\phi$ azimuthal asymmetries where $\phi \equiv \phi_{qJ}$ defined in the previous section for the EIC kinematics. For jet radius we have $R = 1$ in the left plot and $R = 0.5$ in the right plot. At small q_T range, the directed flow v_1 is dominant. We also find that the elliptic flow v_2 is negative for larger R and positive for small R as expected in theoretical calculations. The other higher order flows triangular flow v_3 and quadruple flow v_4 are negative for both jet radius. And the magnitudes for the four flows are all in a measurable region for the EIC kinematics, so these can be possible measurements for the future experiments.

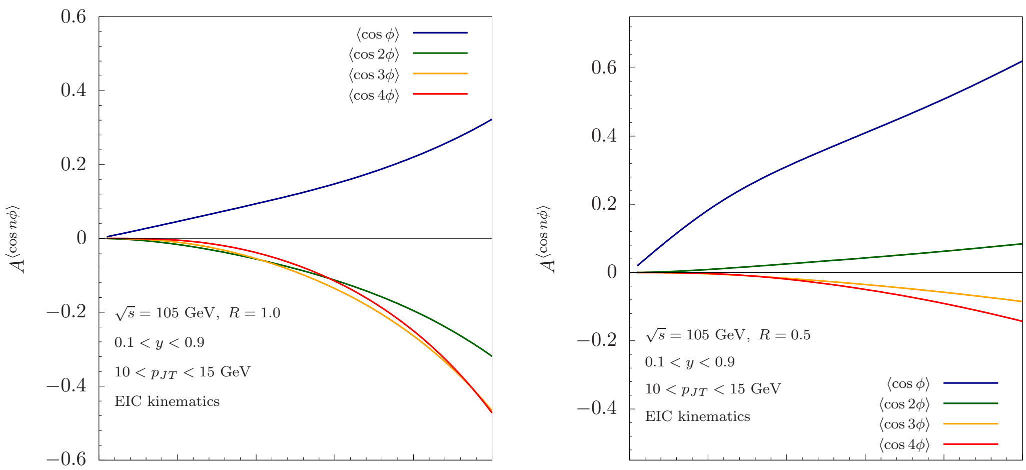

Figure 1. The jet azimuthal anisotropy with EIC kinematics at jet radius $R = 1.0$ (left) and $R = 0.5$ (right) for $10 < p_T < 25$ GeV, $0 < q_T < 5$ GeV.

3 Conclusion

To summarize, we study the azimuthal anisotropy of back-to-back lepton-jet production in ep collisions. This observable is related to difference between the azimuthal angle ϕ_q of the transverse momentum imbalance of the lepton and the jet, and the azimuthal angle ϕ_J

of the jet transverse momentum: $\phi_{qJ} = \phi_q - \phi_J$. In this work, we derive the factorization formalism within the SCET framework and present numerical results for EIC kinematics. We find that the directed flow component related to $\cos(\phi_{qJ})$ azimuthal asymmetry is dominant and show that the anisotropic harmonic flows can be promising observables for studying lepton-jet correlations in future experiments.

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