Gravitons, Inflatons, Twisted Bits:
A Noncommutative Bestiary

John Pearson

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Abstract

In this work, we examine ideas connected with the noncommutativity of spacetime and its realizations in string theory. Motivated by Matrix Theory and the AdS-CFT correspondence, we propose a survey of selected noncommutative objects, assessing their implications for inflation, gauge theory duals, and solvable backgrounds. Our initial pair of examples, related to the Myers effect, incorporate elements of so-called "giant graviton" behavior. In the first, the formation of an extended, supersymmetry-restoring domain wall from point-brane sources in a flux background is related to a nonperturbative process of brane-flux annihilation. In the second, we reexamine these phenomena from a cosmological vantage, investigating the prospect of slow-roll inflation in the noncommutative configuration space of multiple d-branes. For our third and final example, we turn to the solvable pp-wave background, outlining a combinatorial, permutation-based approach to string physics which interpolates between gauge theory and worldsheet methods. This "string bit" language will allow us to find exact agreement between Yang-Mills theory in the large R-charge sector and string field theory on the light cone, resolving some previous discrepancies in the literature.

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My debt to my parents is immeasurable.

For my parents...

Contents

List of Figures

Chapter 1

Introduction: The Noncommutative Bestiary

What are the fundamental degrees of freedom of string theory? In the earliest days of the first string revolution, the answer seemed simple enough: strings, with their two-dimensional worldsheets, were the only renormalizable choices. Yet as our understanding of the theory's dual and nonperturbative structures grew over the following decade, a satisfactory explanation proved more and more elusive. The cast of characters in Planck-scale physics grew beyond the bounds of the formalism to contain it. An explosion of solutions grew up around the problem–matrix models, holography, M-theory–each laying claim to its own regime of validity. Yet which of these descriptions, if any, underlies the rest is no more a closed issue now than it was before the second string revolution of the mid 1990s. Paradoxically, as string theory enters now its third decade as a framework for quantum gravity, it has become nearly impossible for us to say just what exactly the schema called string theory is a physical theory *of*.

Not that the notion of strings has been abandoned entirely. Despite revolutions in our understanding of the theory's nonperturbative structure, it remains true that "fundamental," perturbative strings are the correct description of most phenomena at weak coupling. Yet the idea that strings and strings alone were the key players in Planck-scale physics met its death in the early 1990s, when Polchinski and others unleashed the phenomenon of what came to be known as D-branes. These nonperturbative, solitionic objects, it was found, carried fundamental charge under the mysterious Ramond-Ramond, fields, and their characterization as planes on which strings could end made manifest deep connections between open and closed string theories. These connections, part of a new web of nonperturbative dualities which emerged in the mid-1990s, then implicated the several independent perturbative string theories in existence as part of a connected whole [2]. These newfound dualities, along with the realization that low-energy limits of D-branes could produce a geometric reformulation of gauge theory, set the stage for developments well into the next decade.

Still, these newfound connections left the question of fundamental constituents relatively untouched. While the duality web connected various perturbed limits of the overarching theory (tentatively called "M"), it had also served to blur the distinction between strings and branes. In the context of S-duality, it was soon found that the fundamental string was interchanged with the much heavier D-string under strong-weak coupling duality, while their bound states filled out a complete $SL(2, \mathbf{Z})$ structure of dyonic objects [3]. In this way, the supposedly-elementary perturbative strings found themselves inextricably linked with the solitionic nonperturbative D-branes, and the notion of which was truly fundamental began to seem more than ever a matter of description.

Unfortunately, no formalism has yet been found for treating strings and branes on a completely equivalent footing, though important strides have been made. Most notably, the Matrix Theory proposal of Banks, Fischler, Shenker, and Susskind [4], along with similar proposals in Type IIB, has managed to relate branes of different dimensions to one another through a common language. In such an approach, the fundamental degrees of freedom are taken to be the pointlike D0-branes of Type IIA, and their various bound states can be shown to generate the nonperturbative spectrum of the theory. The conjecture, then, is that in a certain limit, the D0-branes are described by a supersymmetric matrix quantum mechanics [2]

$$S = \int dt\, \mathrm{Tr} \left[\frac{1}{2g_s\sqrt{\alpha'}} D_t X^i D_t X^i + \frac{1}{4g_s\sqrt{\alpha'}(2\pi\alpha')^2}[X^i, X^j]^2 - \frac{i}{2}\psi D_t \psi + \frac{1}{4\pi\alpha'}\psi \Gamma^t \Gamma^i [X^i, \psi] \right], \tag{1.1}$$

the (quantum) solution set of which gives the full nonperturbative spectrum of the theory.

From the vantage of string theory, this is a revelation: the operative language is no longer the extended objects but the familiar point particle, and we have access once again to the techniques of standard quantum field theory. However, the significance of this more conventional theory runs deep. In the interpretation of such a theory, we must remind ourselves that the X^i are not merely quantum fields but coordinates on a target spacetime, and the recognition that these gauge theory coordinates imply a *matrix-valued* geometry constitutes a profound conceptual shift. In the vocabulary of matrix coordinates, diagonal matrices represent the N eigenvalue/coordinates of N distinct particles, a reduction to classical geometry, whereas non-diagonal matrices represent new objects entirely, a departure from the realm of distinct, well-separated objects. Indeed, solutions with $[X^i, X^j] \neq 0$ play a fundamental role, generating extended matrix objects to be identified with D-branes.

This obviously represents a severe departure from traditional notions of space and time. At the string scale, we are told, the coordinates of our familiar geometry are no longer c-numbers but matrix-valued noncommuting objects. Only for simultaneously-diagonalizable matrices can we speak of N distinct point-like objects. All other configurations, subsumed in the language of "collective coordinates," must be matched with known objects in the large-N and long-wavelength limits. Certainly, the idea of the X^i as coordinates on a smooth manifold seems all but lost.

However, this was not the first time an underlying structure for string theory had been suggested. Witten, in his early work on string field theory [5], has argued for a noncommutative product on the algebra of string fields, and he, along with others, had championed Connes's work on gauge theories in noncommutative geometry [6]. What's more, developments in compactification geometry in the early 90s had yielded Landau-Ginzburg descriptions of Calabi-Yau manifolds away from the large-volume, geometric limit [7], and subsequent results in mirror symmetry were driven by increasingly non-geometric, field theory computation [8].

Yet it was not until the late 90s that simple, calculable noncommutative theories were discovered in the context of strings. By examining the worldvolume theories of D-branes in NS B field backgrounds, it was Seiberg and Witten [9] who found the first decoupling limit in which the low-energy theory on D-branes became noncommutative. In the resulting theories, the product between quantum fields was no longer simple multiplication, but an all-orders derivative expansion based on a point-splitting limit. As a consequence, these models illustrated principles which were suspected to be generic to a full theory of quantum gravity, among them the existence of stable extended objects, the presence of quasi-nonlocal interactions, mixing between effects in the deep infrared and far ultraviolet, and an absence of local observables [10].

However, these noncommutative theories are but one example of a class of string theory limits which include the noncommutative open string (NCOS) [11] and the BMN [70] limits[1] —theories derived from considerations of strings in the presence of background gauge fields. In most cases, it is the deforming presence of NS-NS or RR background fields that provides the couplings which modify geometry in the low-energy description. Tied intimately to puzzles regarding the treatment of RR fields in sigma-model actions, these novel limits of conventional string theory have provided controlled laboratories for exploring what are hoped to be generic features of the full theory.

Yet among the many effects stemming from considerations of noncommutativity, the most prevalent and useful from the vantage of D-brane physics has been the "dielectric" or Myers effect [12]. Beginning from multiple coincident D9-branes in Type IIB, it was Myers who generalized the long-wavelength Born-Infeld action by demanding consistency under T-duality. The result, a nonabelian action for branes in curved backgrounds, given in [12], takes the form

$$S_{BI} = -T_p \int d^{p+1}\sigma \ \mathrm{Tr}\left(e^{\phi} \det(\mathcal{P}[E_{ab} + E_{ai}(Q^{-1} - \delta)^{ij} E_{jb}] + 2\pi\alpha' F_{ab}) \det(Q^i_j)\right) \qquad (1.2)$$

[1]The latter to be discussed below.

with $\mathcal{P}$ the pullback, $E_{\mu\nu} = G_{\mu\nu} + B_{\mu\nu}$, and

$$Q^i_j \equiv \delta^i_j + 2\pi i\alpha'[\Phi^i, \Phi^k]E_{kj}. \tag{1.3}$$

Here Φ^i is the nonabelian target space coordinate, and Q^i_j is the term whose expansion in the weak-field limit gives the double-commutator term in (1.1) above. Even more intriguingly, the standard Chern-Simons term is rewritten in the form

$$S_{CS} = \mu_p \int \mathrm{Tr}\left(\mathcal{P}\left[e^{2\pi i\alpha' \mathrm{i}_\Phi \mathrm{i}_\Phi}(\sum C^{(n)} e^B)\right] e^{2\pi\alpha' F}\right), \tag{1.4}$$

with i_Φ the *interior* product which maps p-forms to $p-1$-forms:

$$\mathrm{i}_\Phi F_{\mu_1\mu_2\cdots\mu_p}\, dx^{\mu_1} \wedge dx^{\mu_2} \wedge \ldots dx^{\mu_p} = \Phi^\nu F_{\nu\mu_2\cdots\mu_p}\, dx^{\mu_2} \wedge \ldots dx^{\mu_p}. \tag{1.5}$$

Thus there is now a term in the Chern-Simons action where a pair of i_ϕ operators contract a pair of $F^{(4)}$ indices with a commutator of transverse scalars, and the residual two-form field strength couples as a one-form potential to the collective D0 worldline:

$$\mathcal{L} \sim \frac{i\mu_0}{3}(2\pi\alpha')^2 \int dt\, \mathrm{Tr}(\Phi^i\Phi^j\Phi^k)F^{(4)}_{tijk}. \tag{1.6}$$

As a result, the four-form field strength now couples to the collection of D0-branes as it would to a D2 dipole. Thus not only can branes couple to lower-dimensional field strengths through induced brane charges of the form $C \wedge B$ or $C \wedge F$, but expanded configurations of branes can now couple to higher-dimensional forms through their multipole moments. Conversely, the existence of a background flux of high degree can serve as a source for multipole fluctuations in concentric branes, inducing effective higher-dimensional charges. This is a collective effect altogether new, a reflection of physics entirely dependent upon the nonabelian nature of D-brane geometry. Its inducement of multipole charges further undermines the distinction among branes of differing dimension, and its translation to the long-wavelength limit below will provide a link between the regimes of strings and branes.

But what is the consequence of this coupling? The result is surprising. Specializing once again to the case of D0-branes, we expand (1.2) around flat space with non-vanishing

$F^{(4)}$ field strength,[2] obtaining the potential

$$V(\Phi) = \frac{T_0}{4}\mathrm{Tr}([\Phi^i, \Phi^j]^2) - \frac{i\mu_0}{3}\mathrm{Tr}(\Phi^i\Phi^j\Phi^k)F^{(4)}_{tijk}. \tag{1.7}$$

In the simplest background, in which $F^{(4)}_{t123} = -2f$, this yields static equations of motion which are solved by the ansatz

$$\Phi^i = \frac{f}{2}J^i \tag{1.8}$$

with J^i an $N \times N$ sum of representations of $SU(2)$. In the case of the irreducible representation, this gives an energy $V_N \sim -f^4N^3/g_s$, lower than the commuting configuration with $V_0 = 0$. In fact, the solution (1.8) corresponds to the so-called "noncommutative S^2," or "fuzzy two-sphere," a noncommutative generalization of the smooth topological manifold.

More generally, any sum of $SU(2)$ representations will give a solution to the equations of motion, with the largest irrep giving the lowest energy. These intermediate solutions represent collections of concentric fuzzy spheres, each with its own radius determined by the dimension of the representation. Thus there are a large number of energy levels between 0 and V_N, equal to the number of partitions of N. It will be a question for us in this work whether or not these concentric configurations are in fact unstable to decay into the single graviton irrep.

As proof of our identification of the solution (1.8) with a two-sphere, let us adduce some corroborative evidence. Following [12], we can estimate the radius of these noncommutative spheres by

$$R = \left(\sum_{i=1}^{3}\mathrm{Tr}[(\Phi^i)^2]/N\right)^{1/2} \sim fN\sqrt{1-\frac{1}{N}}. \tag{1.9}$$

Thus the sum of squares of the coordinates is a constant for each representation, and this number grows with both f and N. Even more encouraging, we can reproduce this result in supergravity. Beginning with the Born-Infeld action for a D2-brane and including both an $F^{(4)}$ background flux and N units of worldvolume flux associated with the $U(1)$ charge of

[2] We might well worry about the need to take into account backreaction of the flux on the metric. This problem was considered from the point of supergravity in [13], where it was argued that the instability is removed while the minimum remains.

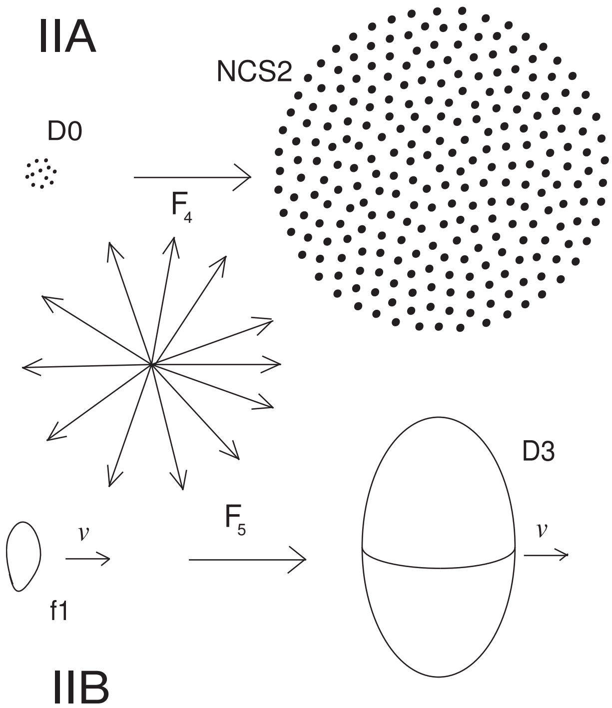

Figure 1.1: The Myers effect for D0 branes and strings. In the presence of background flux, both moving strings and branes have instabilities to expand into "giant graviton" brane configurations.

the dissolved D0 charge, we can write the potential of a spherical D2-brane of radius R as

$$V(R) = 4\pi T_2 \left(\sqrt{R^4 + \frac{(2\pi\alpha')^2 N^2}{4}} - \frac{2f}{3} R^3 \right) \tag{1.10}$$

$$= NT_0 + \frac{2T_0}{(2\pi\alpha')^2 N} R^4 + \ldots - \frac{4T_0}{3(2\pi\alpha')} f R^3, \tag{1.11}$$

where the first term is Born-Infeld, the second Chern-Simons, and we have expanded by assuming $R^2/(2\pi\alpha' N) \ll 1$. We have also substituted $T_0 = 4\pi^2\alpha' T_2$. Minimizing this potential then produces two minima, the first a spurious solution with $R = 0$ and the second an expanded brane with energy $V_N \sim -f^4 N^3/g_s$, just as before. Thus the D0-branes, placed in the background field, expand into a fuzzy sphere configuration with D2 dipole charge, a configuration that increasingly approximates a D2 brane as it expands to macroscopic size.

In fact, it is in precisely this form—the expansion of lower into higher-dimensional branes in background fields—which has proven ubiquitous in string setups with branes. As a matrix description of stringy physics, it has provided a link between UV (matrix theory) and IR (expanded brane) phenomena. As an effect treatable in supergravity, it has demonstrated that uniquely stringy effects can even become relevant in smooth, low-energy contexts. As a result, whenever flux is present as a component of brane backgrounds, noncommutative collective effects are expected to play a role.

Of course, the most fruitful field of application for this technique has been holography, particularly the correspondence between anti-de Sitter space and $\mathcal{N}=4$ super-Yang Mills. In the limit of large 't Hooft coupling $g_s N = g_{YM}^2 N \gg 1$ and small string length $\alpha' \ll 1$, it is known that the geometry is smooth and supergravity accurately approximates string theory, and this is the regime in which most work has been done. Just as important for our purposes, N units of RR five form flux (with $N = 4\pi g_s(\alpha')^2 R^4$) thread both the AdS_5 and S^5 factors of the geometry. Thus probe branes placed in this background are immersed in a nontrivial constant flux, and we find ourselves in a situation strikingly similar to that encountered above.

As a result, Myers-type effects have been discovered in a variety of holographic contexts: from the description of Higgs and confining vacua of the $\mathcal{N}=1^*$ Polchinski-Strassler solution [32]; to the singularity-resolving effects of the enhancon mechanism [14]; to the explanation of the so-called stringy exclusion principle via expanded brane "giant gravitons" [15]. Each of these effects was originally motivated by some apparent paradox in Yang-Mills, and each in turn modified supergravity in some essentially stringy way in order to reconcile gauge and gravity descriptions.

In fact, it is the last of these, the giant graviton mechanism, which will concern us in §2. There, we will examine the work of Kachru, Pearson, and Verlinde [17], in which anti-D3 branes were placed in the conifold background of Klebanov and Strassler. These branes, which break the ambient supersymmetry, are drawn to the IR tip of the conifold by a combination of flux and gravitational forces, and there, via their noncommutative coordinates, they probe a geometry dual to an $SU(N+M) \times SU(N)$ $\mathcal{N}=1$ gauge theory.

This model features a cascade of Seiberg dualities down the AdS throat and chiral symmetry breaking at the deformed tip. Localized to this tip, the anti-branes will interact with the flux background, undergoing nonabelianization to an NS5-brane domain wall. This wall, which separates broken and restored supersymmetry phases of transverse spacetime, then expands, traversing the three-sphere of the tip and initiating a reverse Seiberg duality. In this manner, the anti-D3s reorganize the underlying process of brane-anti-brane annihilation into the nonlinear, dualized mechanism of brane-flux annihilation, connecting unstable and true vacua of the dual gauge theory. In §2, we will examine these processes in detail, taking them as further evidence of the ubiquity of noncommutative objects in string theory.

Afterward, in §3, we will extend this line of reasoning to the mechanism of inflation, evaluating its prospects in the setup of §2. At the same conifold tip, which possesses an almost exact $SO(3)$ invariance, we will describe a scenario in which a single inflaton is selected from among the $\mathcal{O}(N^2)$ scalars describing the positions of the N anti-branes. Following this, we will outline a uniquely stringy exit mechanism for the inflationary phase in which the relevant scalars find their minimum by performing a Myers-like expansion into a noncommutative five-brane. Lastly, following de Wolfe, Kachru, and Verlinde [39], we will assess the difficulty of engineering such a situation, noting both systemic and incidental obstacles to slow-roll inflation in string theory compactifications.

More generally, as we have already noted, the fundamental similarity in the noncommutative scenarios is the presence of nontrivial background flux. Both in the case of noncommutative field theory ($B_{\mu\nu} \neq 0$) and of branes in AdS backgrounds ($F^{(5)} \neq 0$), the crucial ingredient in the emergence of non-geometric behavior is a nonzero flux which couples to the worldvolume of either strings or branes. However, the most vexing corner in the space of such deformations has long been that of fundamental strings in Ramond-Ramond backgrounds. Apart from the case of AdS, in which $\mathcal{N} = 4$ Yang-Mills appears to be an exact dual[3] , few examples of even approximately-tractable RR backgrounds exist.

This state of affairs changed drastically, however, with the work of Berenstein, Malada-

[3]Though a dual in a frustrating regime: the gauge theory in this case is strongly coupled, and until recently [16], little was understood of the structure of the sigma model actions in such a background

cena, and Nastase [70], who identified a limit of both AdS space and $\mathcal{N} = 4$ Yang-Mills which led to tractable theories on both the string worldsheet and gauge theory sides. While the former becomes an exactly-solvable string theory background with nontrivial Ramond-Ramond interactions, the latter reduces to a subsector of Yang-Mills in which large R-charge operators interact in the planar, large-N limit. Thus, instead of the matrix fields characteristic of the fundamental degrees of freedom of Yang-Mills, the new basis of operators is spanned by single and multi-trace "words" of basic matrix fields. Of course, the elements of these words may be permuted by cyclicity of the trace, but their relative order is otherwise fixed. Here, the situation is analogous to one in which a priori noncommuting matrices are reduced to their eigenvalues, but we do not return to a trivial limit. Rather, the existence of a cyclic symmetry, along with the ability to permute indistinguishable eigenvalues, implies an S_N permutation group structure which complicates the problem. In this case, as we will find, the "letters" of the single-trace operators are bits of a discretized string worldsheet, and the S_N symmetry will play a key role in the structuring of a multi-trace/multi-string Hilbert space. In this way, even the "bit-strings" of BMN will find their place in our noncommutative discourse, representatives of an odd regime between the standard geometric and fully noncommutative limits.

Our final chapter, then, will take up this issue of the BMN limit, focusing on the "string bit" approach of Verlinde and collaborators. This formalism, intermediate between those of planar gauge theory and light cone strings, makes fullest and most explicit use of the underlying permutation symmetry, and as such represents an intriguing extension to our earlier noncommutative considerations. In §4, after a review of both the original work of BMN and subsequent papers by Verlinde and collaborators, [71, 72, 73], we will end with a string bit calculation which reconciles several disparate approaches. This result will serve as both a tantalizing comparison of string and graviton methodologies and the final exhibit in our noncommutative bestiary.

Chapter 2

Gravitons

2.1 Introduction

In this chapter, we examine an early application of the first of our noncommutative effects, the giant graviton, to questions in the context of the AdS-CFT correspondence [18]. There, as in the $\mathcal{N}=1^*$ example of Polchinski and Strassler [32], its appearance will provide the resolution of an apparent paradox present in the gauge theory dual. As we shall see, the presence of both non-trivial fluxes and anti-D3 branes in the holographic compactification geometry will lead to the formation of a noncommutative, Myers-like instability.

Our discussion will take place in the context of the supergravity solution given by Klebanov and Strassler [19]. It initially appeared, along with the solutions of Maldacena and Nunez [20], Vafa [21], and others, as an answer to the challenge of finding a geometry dual to pure $\mathcal{N}=1$ Yang Mills Theory. In this solution, we begin with the warped conifold geometry of Klebanov and Witten [65] generated by N D3 branes placed on an orbifold of S^5. To this we add M "fractional" D5 branes which wrap the S^2 of $T^{1,1}$. In the end, all of these branes disappear, leaving a smooth supergravity solution in which the fractional branes have been replaced by M units of RR three-form flux and N units of self-dual F_5 charge. The gauge theory dual to this is then a $d=4$ $SU(N+M)\times SU(N)$ theory with a residual $\mathcal{N}=1$ supersymmetry which undergoes an RG cascade of successive Seiberg dualities relating weakly-coupled descriptions with reduced gauge groups. Furthermore, if $N = KM$ for some integer K, the endpoint of the cascade is pure $SU(M)$ Yang-Mills.

We will begin with this low-energy scenario, embedded as the geometry near a conifold point of a full-blown compact manifold which represents its Planck-scale UV completion [22]. This embedding then implies an exponential hierarchy between the Planck and IR[1] scales, in a mechanism parallel to Randall-Sundrum brane world scenarios [23]. This done, we further introduce a small number $p \ll M$ of anti-D3 branes into the geometry, completely breaking supersymmetry. In the background of the D3-generated manifold, these are no longer BPS objects, and the effects of gravity and 5-form add (rather than cancel as in the D3 case) to produce a force driving these impurities to the tip of the conifold. They then remain there, metastable, unable to directly annihilate with the BPS branes which have been dualized to flux. No analogue of brane-antibrane annihilation is apparent.

In the paper [17], we proposed a mechanism for the decay of this system and the subsequent restoration of supersymmetry, positing the formation of an NS5-brane via the giant-graviton effect which traverses the geometry and relates SUSY and non-SUSY vacua. This process, which may be either exponentially- suppressed or classically allowed, depending on p, then corresponds to an inverse Seiberg duality, and relates a metastable "baryonic" branch of the $SU(2M-p) \times SU(M-p)$ gauge theory to the standard supersymmetric "mesonic" branch. Such a transition thus provides yet another example of holography beyond mere supergravity, as well as further evidence that a proper understanding of string geometry must involve noncommutative elements.

The organization of this chapter is as follows: In §2.2, we review preliminaries and introduce vacua of the KS dual field theory. In §2.3, we give an argument that antibranes in the KS geometry will expand into as NS5-brane via an analogue of the Myers effect. In §2.4, we will then describe this expanded brane in terms of a Born-Infled fivebrane action with worldvolume flux. This will present us with a picture in which the initial brane clump expands to wrap an S^2 in the deformed S^3 at the conifold tip and creates a domain wall

[1] given in this case by the scale of chiral symmetry breaking

in four-dimensional spacetime. For p/M small, this is a metastable false vacuum. Details of the euclidean domain wall instanton will be presented in §2.4.3, along with a description of the final supersymmetric state with $M-p$ D3-branes. Finally, in §2.5, we discuss the relevant gauge theory configurations on either side of the transition and give an explanation in terms of Type IIA theory. We conclude with remarks on warped compactifications in more generality and some novel features of our effective supergravity.

2.2 Preliminaries

Here, we review essentials of the Klebanov and Strassler geometry and its gauge theory dual. Our supersymmetry-breaking scenario is detailed in §2.2.2 and the structure of KS moduli spaces in §2.2.3.

2.2.1 The Klebanov-Strassler geometry from F-theory

Though most of our discussion will only concern the non-compact limit of KS geometry, for purposes of describing our model in the context of warped compactifications, we will begin with the perspective of F-theory. That scenario, containing both H_3 and F_3 three-form fluxes, has been previously examined in both M and F-theory in [26, 25, 27, 28, 22, 29] and others. We will work in the limit [22] that the F-theory fourfold compactification can be treated as the orientifold of Type IIB theory on a CY threefold Y, and we will allow the addition of both D3 and anti-D3 branes to the geometry. With numbers N_3 and $\overline{N}_3$, respectively, the net charge $Q_3 = N_3 - \overline{N}_3$ is then fixed by the global tadpole condition:

$$\frac{\chi(X)}{24} \;=\; Q_3 + \frac{1}{2\kappa_{10}^2 T_3}\int_Y H_3 \wedge F_3 \;. \tag{2.1}$$

Here $\chi(X)$ is the Euler characteristic of the CY fourfold that specifies the F-theory compactification, and T_3 is the D3-brane tension.

We then proceed to engineer a KS geometry by focusing on a single conifold singularity in Y, threading M units of F_3 RR flux[2] through the deformed S^3 A-cycle. Complementing

[2]corresponding to the M fractional branes of [19]

this, there will be a companion H_3 flux through the dual noncompact B-cycle:

$$\begin{aligned} \frac{1}{4\pi^2}\int_A F_3 &= M \ , \\ \frac{1}{4\pi^2}\int_B H_3 &= -K \ . \end{aligned} \tag{2.2}$$

In the case that no other crossed fluxes are present, we can then choose M and K such that $MK = \frac{\chi}{24}$ and we have no need of additional D3-branes to cancel the tadpole. This results in a smooth geometry with nontrivial superpotential for the complex structure modulus z of the A-cycle, stabilizing it at an exponentially-suppressed value

$$z \sim \exp(-2\pi K/M g_{\mathrm{s}}). \tag{2.3}$$

The manifold thus consists of a long warped throat, terminating at its infrared end in the geometry of a deformed confold

$$\sum_{i=1}^{4} z_i^2 = \varepsilon^2 \ , \tag{2.4}$$

with $\varepsilon \sim z$ exponentially small.

From the dual perspective, this exponential hierarchy (2.3) has a natural explanation [19, 22]: The geometry (2.4) is dual to an $SU(N+M)\times SU(N)$ gauge theory with nontrivial β-function proportional to $g_s M$. Thus its RG flow toward the infrared consists of a cascade of strong coupling transitions at scales μ_n with $\log(\mu_n/\mu_{n+1}) \sim (g_s M)^{-1}$. Each transition then involves a Seiberg duality which switches the theory from a strongly to weakly-coupled description and lowers the rank of the larger gauge group by $2M$. In the case $N = KM$, there are K such dualities, leaving an IR theory which is pure $SU(M)$ super-Yang-Mills. The theory thus traverses a range of scales $\log(\mu_0/\mu_K) \sim K/g_s M$ over the course of its cascade, a value consonant with the supergravity estimate (2.3).

2.2.2 The SUSY-Breaking Model

We will study the model outlined in §2.2.1 in the case

$$N_3 = 0,\ \overline{N}_3 = p,\ \frac{\chi}{24} \ = \ KM - p \tag{2.5}$$

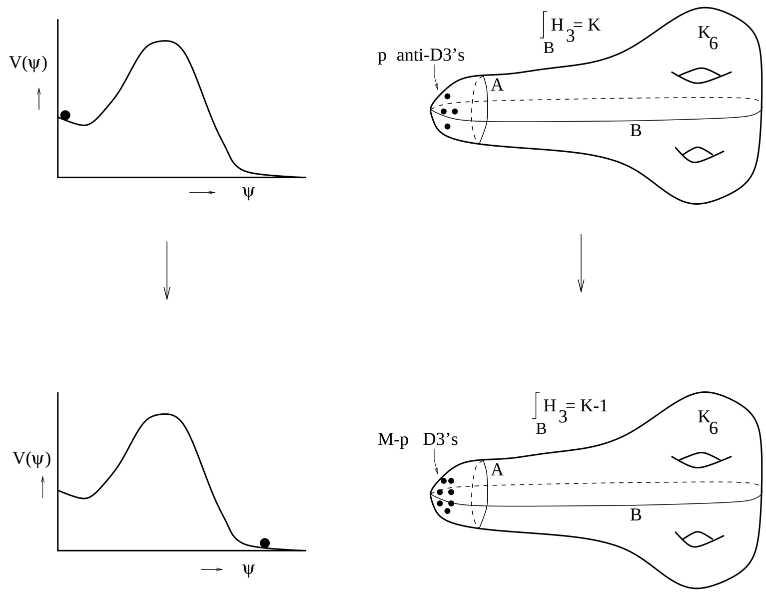

Figure 2.1: The decay, described in §2.2.2 and §2.4.3, takes place between an intitial non-supersymmetric situation with p $\overline{D3}$ branes near the tip of the conifold, to a final supersymmetric situation with $M - p$ D3-branes. The total D3-charge is preserved via the simultaneous jump in the H_3 flux around the B-cycle by one unit, from K to $K - 1$.

with $p \ll K, M$. That is, we will study the theory of p anti-D3 branes probing the Klebanov-Strassler geometry.

Of course, the addition of anti-D3 branes to this geometry breaks the remaining supersymmetry: the branes' own SUSY is opposite that preserved by the imaginary self-dual 3-form flux of the background. As a result, the configuration should be unstable, and in the case that the flux were replaced by its D3 sources, should annihilate the excess negative charge. However, in our configuration, all D3-branes have been dualized to flux, and there is no clear pathway for the decay to proceed.

In the dual field theory, however, the problem assumes a more lucid form. If we identify the total D3 charge of the compactification, $\chi/24$, with the N of the $SU(N+M) \times SU(N)$ gauge theory, we can imagine taking $N = KM + p$ with $p \ll M$ and proceeding by K cascades until we are left with a $SU(M+p) \times SU(p)$ in the infrared [19]. As explained in [19], this theory has a moduli space parametrized by "meson" fields N_{ij} which can be convincingly matched with the moduli space of p D3 branes moving on the deformed conifold geometry.

By analogy, we might reason that our situation corresponds to one in which $\frac{\chi}{24} = KM - p$. In that case, however, only $K-1$ Seiberg dualities would be allowed, ending in an $SU(2M-p) \times SU(M-p)$ theory. Clearly this theory is supersymmetric, and corresponds not to (2.5), but to a geometry with $M-p$ D3 branes and only $K-1$ units of H_{NS} flux through the B-cycle:

$$N = M - p, \ \overline{N} = 0, \ \frac{\chi}{24} \ = \ (K-1)M + (M-p) \ = \ KM - p. \tag{2.6}$$

It is this, we will argue, which must be the supersymmetric endpoint of the decay, the final description of the gauge theory after (2.5) has undergone brane-flux annihilation. Indeed, IIB string theory admits a class of domain walls across which H_3 changes by a single unit across the noncompact B-cycle—NS5-branes partially wrapping the A-sphere. Thus it is natural to expect that the decay between (2.5) and (2.6) proceeds via nucleation of an NS5-brane domain wall surrounding a patch of supersymmetric vacuum. Driven by the differential of vacuum energies, this patch will then expand to encompass the entire space, a process we will describe in §2.4.3. There, we will find that the process is indeed nonperturbative for a wide range of parameters.

However, not even the $\overline{D3}$ description is fully appropriate before the decay. Even that system, as we shall see, demonstrates *perturbative* instabilities, the onset of a Myers-like transition from nonabelianized $\overline{D3}$-branes to an expanded NS5 partially wrapping the A-cycle. Provided the number of anti-branes p is sufficiently smaller than M, this maximal giant graviton fivebrane will prove classically stable, decaying quantum mechanically by an exponentially-suppressed tunneling process.

To simplify our analysis, we will assume that all of the interesting dynamics takes place very close to the tip of the conifold. Indeed, it is not difficult to argue that the anti-D3 branes will in general feel a net radial force $F_r(r)$ proportional to the 5-form flux F_5

$$F_r(r) = -2\mu_3 F_5(r) \tag{2.7}$$

that will attract them to the tip at $r = 0$. This force is a sum of gravitational and 5-form contributions: For a $D3$-brane these two terms cancel; in the case of $\overline{D3}$-branes they add up to a net attractive force (2.7). Therefore, even if we began with a more generic initial distribution, we expect the anti-D3 branes to accumulate quickly near the apex at $r = 0$.

The metric near the apex reads [30] [3]

$$\begin{aligned} ds^2 &= a_0^2\, dx_\mu dx_\mu + g_s M\, b_0^2 \Big(\frac{1}{2} dr^2 + d\Omega_3^2 + r^2 d\tilde{\Omega}_2^2\Big) \\ a_0^2 &\simeq \frac{\varepsilon^{4/3}}{g_s M} \qquad\qquad b_0^2 \approx 0.93266. \end{aligned} \tag{2.8}$$

Since we assume that all physics takes place at $r = 0$, our space-time has the topology $\mathbf{R}^4 \times S^3$. The RR field $F_3 = dC_2$ has a quantized flux around the S^3

$$\int_{S^3} F_3 = 4\pi^2 M, \tag{2.9}$$

while in the supersymmetric background dictates $\star_6 H_3 = -g_s F_3$, so that

$$dB_6 = \frac{1}{g_s^2} \star_{10} H_3 = -\frac{1}{g_s}\, dV_4 \wedge F_3, \tag{2.10}$$

with $dV_4 = a_0^4\, d^4x$. The dilaton field is constant, and the self-dual 5-form field vanishes at the tip.

2.2.3 Branches of Moduli Space in the KS system

Here we review one of the relevant features of the KS low energy field theory, following section 7 of [19]. This will be important in understanding our proposal for the holographic field theory description of the metastable false vacuum, as well as of the tunneling process that describes its quantum decay.

Consider the RG cascade of the KS gauge theory with $N = KM$ in its penultimate step. In this case, the unbroken gauge group is $SU(2M) \times SU(M)$, and the $SU(2M)$ factor has an equal number of flavors and colors. There is also a quartic superpotential whose rough form is

$$W = \lambda\, (A_i B_j A_k B_l)\, \epsilon^{ik} \epsilon^{jl} \tag{2.11}$$

[3] we work in string units, $\alpha' = 1$

where the A_i are in the $(2M, \overline{M})$ representation, the B_i are in the conjugate representation, and $i = 1, 2$. We know from the analysis of [31] that in $\mathcal{N} = 1$ supersymmetric QCD with $N_f = N_c$, the moduli space is quantum mechanically modified. Treating the $SU(M)$ as a global symmetry (i.e. taking its dynamical scale to vanish), this situation reduces to the one studied in [31] (with the added complication of the quartic tree-level superpotential).

Define the "meson" fields $N_{ij,\alpha\beta}$ and the "baryon" fields $\mathcal{B}, \tilde{\mathcal{B}}$

$$N_{ij,\alpha\beta} \;=\; A_{i,\alpha} B_{j,\beta}, \;\; \mathcal{B} \;=\; (A_1)^M (A_2)^M, \;\; \tilde{\mathcal{B}} \;=\; (B_1)^M (B_2)^M \;, \tag{2.12}$$

where α and β are $SU(M)$ "flavor" indices. In order to reproduce the quantum modified moduli space of the $SU(2M)$ theory, we should add a Lagrange multiplier term to (2.11)

$$W = \lambda \left(N_{ij,\alpha\beta} N_{kl}^{\alpha\beta} \right) \epsilon^{ik} \epsilon^{jl} + X(\det(N) \;-\; \mathcal{B}\tilde{\mathcal{B}} - \Lambda^{4M}) \;, \tag{2.13}$$

where the determinant is understood to be that of a $2M \times 2M$ matrix (coming from the i, j and color indices on N).

There are distinct "mesonic" and "baryonic" branches of supersymmetric vacua arising from the superpotential (2.13). On the baryonic branch

$$X \;=\; N \;=\; 0, \;\; \mathcal{B} \;=\; \tilde{\mathcal{B}} \;=\; i\Lambda^{2M}. \tag{2.14}$$

On this branch, the $SU(M)$ factor in the gauge group remains unbroken, and one is left with pure $\mathcal{N} = 1$ gauge theory in the IR. However, there is also a branch where $\mathcal{B} = \tilde{\mathcal{B}} = 0$ and the mesons have non-vanishing VEVs.

$$\det(N) \;=\; \Lambda^{4M}, \quad \mathcal{B} \;=\; \tilde{\mathcal{B}} \;=\; 0 \;. \tag{2.15}$$

Now the gauge group is generically Higgsed, and there is a moduli space of vacua consisting of the theory of M D3 branes probing a deformed conifold geometry.

The fact that the mesonic and baryonic branches of moduli space are disconnected is a result of the tree level superpotential (2.11). In the theory without (2.11), the quantum moduli space is defined by the equation $\det(N) - \mathcal{B}\tilde{\mathcal{B}} = \Lambda^{4M}$ and one can smoothly interpolate between these branches, via a continuous path of supersymmetric vacua. In the present case, on the other hand, field configurations that interpolate between the two branches take the form of localized supersymmetric domain walls with non-zero energy density. These domain walls solve BPS equations of the form

$$\partial_z \Phi^I = g^{IJ} \partial_J W(\Phi) \tag{2.16}$$

with $\Phi^I = \{N_{ij}^{\alpha\beta}, \mathcal{B}, X\}$, and z the coordinate transverse to the domain wall. The wall tension is proportional to

$$|\Delta W| = |W(\Phi_b) - W(\Phi_m)|, \tag{2.17}$$

where Φ_b and Φ_m denote the respective vacuum values (2.14) and (2.15) of the two branches. By considering e.g. the special mesonic vacuum where all meson fields have the same expectation value (i.e. independent of their "flavor" index), we deduce

$$|\Delta W| = 2\lambda M \Lambda^4. \tag{2.18}$$

In §2.4.2 we will consider the dual supergravity description of the domain wall. In this case it will represent the transition from the smooth deformed conifold with only flux (corresponding to the baryonic branch) to the situation with one less unit of NS 3-form flux and M D3-branes. In §2.5 we will use this dual understanding of the supersymmetric domain wall to motivate a similar holographic description of our non-supersymmetric $\overline{D3}$ background and its decay process. Here we will similarly argue that there are two relevant branches: The first, the analogue of the baryonic branch, will be the (now nonsupersymmetric) metastable vacuum; and the second, the analogue of the mesonic branch, will be the theory with $M-p$ D3 branes probing the deformed conifold (2.6).

2.3 The $\overline{D3}$ brane Perspective

We are interested in the dynamics of p $\overline{D3}$ branes sitting at the end of the KS throat, under the influence of the F_3 and H_3 fluxes (2.2). We will do our analysis within the probe approximation, taking the KS background as fixed, while ignoring the backreaction due to the $\overline{D3}$ branes. The characteristic size of the geometry is set by $R \simeq \sqrt{g_s M}$, while we can estimate that the backreaction due to the p anti-branes extends over a region of order $r^2 \simeq \sqrt{g_s p}$. Hence the distortion of deformed conifold due to presence of the $\overline{D3}$ branes remains small as long as $p \ll M$. We will assume that we are in this regime.

It is now rather well understood how, in a non-trivial flux background, p D3-branes can expand to form a spherical D5-brane (this phenomenon played an important role in the analysis of [32], for example). Here, due to specific form of the background fluxes, we will need to consider the S-dual phenomenon, where the $\overline{D3}$ branes expand into wrapped NS 5 branes. A technical difficulty, however, is that it is not yet known how to consistently couple the background B_6-flux (2.10) to the matrix Born-Infeld action of the $\overline{D3}$ branes. (This problem is directly related to that of finding a Matrix theory description of the transverse NS 5-brane.) A way around this obstacle, is to use the S-dual description of the $\overline{D3}$ world-volume. In §2.4, we will turn things around, and view things from the perspective of a wrapped NS5-brane with $\overline{D3}$ charge p coming from a world-volume magnetic flux.

Before getting started, we need to warn the reader that strictly speaking, since g_s is assumed to be small and thus the S-dual coupling $\tilde{g}_s = 1/g_s$ large, we are far outside of the regime of validity of the S-dual Born-Infeld action. We will nonetheless proceed with using it; in §2.4 will find a posteriori justification of our description when we establish a precise match with the results obtained from the NS 5-brane perspective.

2.3.1 Dielectric $\overline{D3}$ Branes

The worldvolume action of the p anti-D3 branes, placed at the apex of the deformed conifold and in the S-dual frame, is given by the Born-Infeld action

$$S_{BI} = \frac{\mu_3}{g_s} \int \mathrm{Tr} \sqrt{\det(G_{\|} + 2\pi g_s F) \det(Q)} - 2\pi\mu_3 \int \mathrm{Tr}\, \mathbf{i}_\Phi \mathbf{i}_\Phi B_6 \tag{2.19}$$

Here

$$Q^i{}_j = \delta^i{}_j + \frac{2\pi i}{g_s}[\Phi^i, \Phi^k](G_{kj} + g_s C_{kj}) \ . \tag{2.20}$$

Because we are working in the S-dual frame, relative to the usual Born-Infeld action, we replaced B_2 by C_2 and C_6 by B_6. The two-form F here denotes the non-abelian field strength on the $\overline{D3}$ brane worldvolume.

The scalar fields Φ parametrize the transverse location X of the $\overline{D3}$ branes, via the relation $\Phi = 2\pi X$. By making these matrix coordinates non-commutative, the anti D3-branes can collectively represent a 5-dimensional brane which can be identified with the NS 5-brane. The topology of this "fuzzy NS 5 brane" is $\mathbf{R}^4 \times S^2$, where the two-sphere S^2 has an approximate radius R equal to

$$R^2 \simeq \frac{4\pi^2}{p}\mathrm{Tr}((\Phi^i)^2). \tag{2.21}$$

It is instructive to look for the non-commutative solution for $p \ll M$, in which case Φ remains small relative to the radius of curvature of the surrounding space-time and variations in the 3-form field strengths. In this case we may write $C_{kj} \sim \frac{2\pi}{3}F_{kjl}\Phi^l$, and locally we may approximate the metric in the compact space by the flat metric $G_{kj} = \delta_{kj}$. We find that

$$Q^i_j = \delta^i_j + \frac{2\pi i}{g_s}[\Phi^i, \Phi_j] + i\frac{4\pi^2}{3}F_{kjl}[\Phi^i, \Phi^k]\Phi^l \ . \tag{2.22}$$

So we can expand

$$\mathrm{Tr}\sqrt{\det(Q)} \simeq p - i\frac{2\pi^2}{3}F_{kjl}\mathrm{Tr}\Big([\Phi^k, \Phi^j]\Phi^l\Big) - \frac{\pi^2}{g_s^2}\mathrm{Tr}\Big([\Phi^i, \Phi^j]^2\Big) \ . \tag{2.23}$$

Furthermore, we are in an imaginary self-dual flux background where $dB_6 = -\frac{1}{g_s}dV_4 \wedge F_3$. In an imaginary *anti* self-dual flux background, the cubic terms in the full potential for the $\overline{D3}$ worldvolume fields Φ would have to cancel (by a "no-force" condition between anti-branes and IASD fluxes). This logic, or alternatively, direct calculation of the second term in (2.19), tells us that the full potential coming from (2.19) in this ISD flux background will be

$$g_s V_{\text{eff}}(\Phi) \ \simeq \ \sqrt{\det(G_{||})}\left(p - i\frac{4\pi^2}{3}F_{kjl}\mathrm{Tr}\Big([\Phi^k, \Phi^j]\Phi^l\Big) - \frac{\pi^2}{g_s^2}\mathrm{Tr}\Big([\Phi^i, \Phi^j]^2\Big) + \dots\right) \ . \tag{2.24}$$

As in [12], this potential has extrema away from the origin $\Phi = 0$. To get some intuition, let us set $F_{kjl} = f\epsilon_{kjl}$, where we make the approximation that the A-cycle S^3 is large (which is good in the limit of large $g_s M$). The magnitude of f can be read off from the normalization of the integrated RR flux as in (2.2), which requires

$$f \simeq \frac{2}{b_0^3\sqrt{g_s^3 M}} \ . \tag{2.25}$$

Demanding that $\frac{\partial V(\Phi)}{\partial \Phi^i} = 0$ yields the equation of motion

$$[[\Phi^i, \Phi^j], \Phi^j] - i g_s^2 f \epsilon_{ijk} [\Phi^j, \Phi^k] \;=\; 0 \;. \tag{2.26}$$

To solve (2.26), notice that if one takes constant matrices Φ^i satisfying the commutation relations

$$[\Phi^i, \Phi^j] \;=\; - i g_s^2 f \, \epsilon_{ijk} \Phi^k, \tag{2.27}$$

then (2.26) is automatically satisfied. But, up to rescaling the Φ, (2.27) are just the commutation relations which are satisfied by a $p \times p$ dimensional matrix representation of the $SU(2)$ generators

$$[J^i, J^j] = 2 i \epsilon_{ijk} J^k \;. \tag{2.28}$$

So by setting $\Phi^i = -\frac{1}{2} g_s^2 f \, J^i$, with J^i the generators of any p-dimensional $SU(2)$ representation, we find solutions of (2.27).

Which solution is energetically preferred? Using the known value of the quadratic Casimir $c_2 = \mathrm{Tr}((J^i)^2)$ in each p-dimensional $SU(2)$ representation, one can see that the energetically preferred solution is to take the p-dimensional irreducible representation of $SU(2)$, for which one finds

$$V_{\rm eff} \;\simeq\; \frac{\mu_3}{g_s} \Big(p - \frac{\pi^2}{6} g_s^8 f^4 \, p \, (p^2 - 1) \Big) \tag{2.29}$$

$$\simeq\; \frac{\mu_3 p}{g_s} \left(1 \,-\, \frac{8 \pi^2 \, (p^2 - 1)}{3 \, b_0^{12} M^2} \right) \;. \tag{2.30}$$

The negative term in (2.29) comes about through a competition between the (positive) quartic term and the (negative) cubic term in (2.24). The other p-dimensional reducible representations occur as metastable vacua of (2.24), where the $\overline{D3}$ branes have blown up to a number of "less giant" NS 5 branes but can still satisfy $V_{\rm eff} < p/g_s$. These separate 5-branes all want to cluster together to form the "most giant" NS 5-brane, with minimal energy equal to (2.29).

It is interesting to compare the energy in the giant graviton vacuum (2.29) to the final energy in the supersymmetric ground state, $V = 0$. We see that $V_{\rm eff} > 0$ implies that

$p \ll M$, which is the condition we have chosen to impose. In this regime, we find a self-consistent picture: the $\overline{D3}$ branes are driven by a perturbative instability to expand into an NS5-brane wrapped on an S^2 in the A-cycle, and must await a non-perturbative effect to decay to the supersymmetric vacuum. As we will discuss in §2.4, the complementary analysis in terms of the NS5-brane worldvolume action indicates that, for sufficiently large $\frac{p}{M}$, the decay to a final supersymmetric state can occur without the intermediate metastable false vacuum. Based on the above story, we can obtain a reasonable estimate for the onset of this classical instability by considering the radius R (given in eqn (2.21)) of the fuzzy NS5-brane in comparison to the radius $R_0 = b_0\sqrt{g_s M}$ of the S^3

$$R^2 \simeq \frac{4\pi^2(p^2-1)}{b_0^8 M^2} R_0^2 \tag{2.31}$$

We see that there is a classical minimum only if p/M is sufficiently smaller than $b_0^4/2\pi$; otherwise the radius of the NS 5-brane will get too close to R_0 and the configuration will become classically unstable.

2.4 The NS 5-brane perspective

In this section, we take the perspective of an NS 5-brane moving near the tip of the conifold geometry. The 5-brane is wrapped on a two-sphere S^2 inside the internal S^3 and carries p units of world-volume two-form flux which induce $\overline{D3}$ charge. As noted previously, it is a point of some concern that the NS 5 world-volume description used below has only limited validity for small sizes of the S^2. For sufficiently large S^2 radius, however, the NS 5 world-volume curvature is small compared to the string scale and one may expect that the description as given below becomes reasonably accurate.

2.4.1 Giant Graviton 5-brane

Consider an NS 5-brane of type IIB string theory located at an S^2 inside S^3 with radius specified by a polar angle ψ. The bosonic worldvolume action reads [32]

$$S = \frac{\mu_5}{g_s^2}\int d^6\xi \left[-\det(G_{\|})\det(G_{\perp}+2\pi g_s\mathcal{F})\right]^{1/2} + \mu_5\int B_6 \ , \tag{2.32}$$

$$2\pi\mathcal{F}_2 = 2\pi F_2 - C_2 \ . \tag{2.33}$$

This action has been obtained by S-duality from that of the D5-brane. Here $F_2 = dA$ is the two-form field strength of the world-volume gauge field, $G_\perp$ denotes the induced metric along the internal S^2, and $G_{||}$ encodes the remaining components along the $d\psi$ and $\mathbf{R}^4$ directions. Using the explicit form (2.8) of the metric, we have

$$\begin{aligned} ds^2_{induced} &= b_0^2\, g_s M\Big[\, dx_\mu dx^\mu + d\psi^2 \;\; + \;\; \sin^2\psi\, d\Omega_2^2\Big] \\ &= \qquad\qquad ds^2_{||} \qquad\qquad + \qquad ds^2_\perp \end{aligned} \tag{2.34}$$

where (relative to eqn (2.8)) we have absorbed the factor of a_0/R_0 into x^μ. (This means that from now on, all time and distance scales in the $\mathbf{R}^4$ directions are measured in red-shifted string units, or in holographic dual terminology, in terms of the dimensional transmutation scale Λ of the low energy gauge theory.) We can evaluate the following integrals over S^2

$$\int_{S^2}\sqrt{\det G_\perp} \;=\; 4\pi\, b_0^2\, g_s M \sin^2\psi \ , \tag{2.35}$$

$$\int_{S^2} C_2(\psi) \;=\; 4\pi M\Big(\psi - \frac{1}{2}\sin(2\psi)\Big) \ , \tag{2.36}$$

$$2\pi\int_{S^2} F_2 \;=\; 4\pi^2\, p \ . \tag{2.37}$$

This last equation reflects the fact that the NS 5-brane carries $\overline{D3}$ brane charge p. Combining (2.35)-(2.37) gives:

$$\int_{S^2}\sqrt{\det(G_\perp + 2\pi g_s\mathcal{F})} = 4\pi^2 M g_s\, V_2(\psi) \tag{2.38}$$

$$V_2(\psi) = \frac{1}{\pi}\sqrt{b_0^4\sin^4\psi + \Big(\pi\frac{p}{M} - \psi + \frac{1}{2}\sin(2\psi)\Big)^2}. \tag{2.39}$$

Adding the $\mu_5\int B_6$ term, obtained from eqn (2.10), gives a total NS 5-brane action

$$S = \int d^4x\,\sqrt{-\det G_{||}}\;\mathcal{L}(\psi) \ , \tag{2.40}$$

$$\mathcal{L}(\psi) \;=\; A_0\Big(\, V_2(\psi)\sqrt{1-\dot\psi^2} - \frac{1}{2\pi}\,(2\psi - \sin 2\psi)\Big) \ , \tag{2.41}$$

with

$$A_0 = \frac{4\pi^2\mu_5 M}{g_s} = \frac{\mu_3 M}{g_s}. \tag{2.42}$$

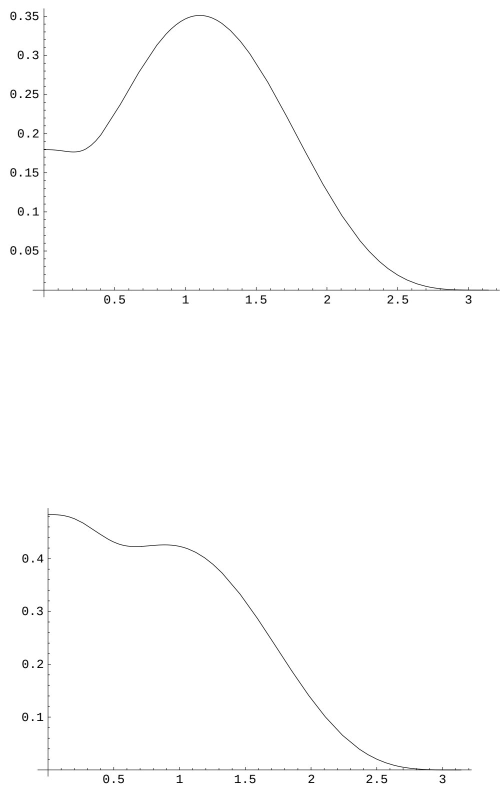

Figure 2.2: The effective potential $V_{\text{eff}}(\psi)$ for $\frac{p}{M} \simeq 3\%$, showing the stable false vacuum, and for $\frac{p}{M} \simeq 8\%$, with only a marginally stable minimum.

We can use this action to introduce a canonical momentum P_ψ conjugate to ψ, and write the resulting Hamiltonian density $\mathcal{H}$ as

$$\mathcal{H}(\psi, P_\psi) \;=\; -\frac{A_0}{2\pi}\Big(2\psi - \sin 2\psi\Big) \;+\; \sqrt{A_0^2\Big(V_2(\psi)\Big)^2 + P_\psi^2} \tag{2.43}$$

which generates the time evolution of $\psi(t)$. For the moment, however, we are just interested in finding whether there exists a static solution corresponding to the "giant graviton" of §2.3.

Some useful intuition is obtained by considering the effective potential

$$
\begin{aligned}
V_{\text{eff}}(\psi) &\equiv \mathcal{H}(\psi, P_\psi = 0) \\
&= A_0 \Big(V_2(\psi) - \frac{1}{2\pi}(2\psi - \sin 2\psi) \Big)
\end{aligned} \tag{2.44}
$$

In Figure 2.2 we have plotted this effective potential $V_{\text{eff}}(\psi)$ for two different values of p relative to M: the top graph corresponds to $p/M \simeq 3\%$, and shows a stable false vacuum, and the lower graph corresponds to the special case $p/M \simeq 8\%$ at which there is only a marginally stable intermediate minimum. For $p/M > 8\%$ the slope of the effective potential is everywhere negative.

In both cases we can draw an interesting conclusion. In the regime $p/M > 8\%$, the nonsupersymmetric configuration of p $\overline{D3}$ branes relaxes to the supersymmetric minimum via a *classical* process: The anti-branes cluster together to form the maximal size "fuzzy" NS 5-brane which rolls down the potential until it reduces to $M-p$ D3-branes located at the north-pole at $\psi = \pi$. In the regime with p/M (sufficiently) smaller than 8%, the branes reach a meta-stable state, the fuzzy NS-5 brane located at the location ψ_{min} for which $\frac{\partial V_{\text{eff}}}{\partial \psi}(\psi_{min}) = 0$. This configuration is classically stable, but will decay via quantum tunneling. We will study this tunneling process in §2.4.3. In both cases, the end result of the process is $M-p$ D3-branes in place of p anti-D3-branes with the H_3 flux around the B-cycle changed from K to $K-1$.

Now let us check the correspondence with the non-abelian description of §2.3. For small values of ψ we can expand

$$
V_{\text{eff}}(\psi) \simeq A_0 \Big(\frac{p}{M} - \frac{4}{3\pi}\psi^3 + \frac{b_0^4 M}{2\pi^2 p}\psi^4 \Big) \tag{2.45}
$$

which has a minimum at $\psi_{min} = \frac{2\pi p}{b_0^4 M}$ equal to

$$
V_{\text{eff}}(\psi_{min}) \simeq \frac{\mu_3 p}{g_s} \Big(1 - \frac{8\pi^2 p^2}{3\, b_0^{12} M^2} \Big), \tag{2.46}
$$

in exact agreement with the value (2.29) found earlier. Moreover, the size $R \simeq \psi_{min} R_0$ (with R_0 the S^3 radius) of the NS 5 brane "giant graviton" exactly matches with our earlier result (2.31).

Another quantitative confirmation of the result (2.44) for the effective potential is that the difference in vacuum energy between the south and north-pole is equal to twice the tension of the anti-D3 branes

$$V_{\text{eff}}(0) - V_{\text{eff}}(\pi) = \frac{2p\mu_3}{g_s}. \tag{2.47}$$

This is the expected result. One way to understand this [33] is to compare our nonsupersymmetric model with a hypothetical situation with all p anti-D3 branes replaced by $-p$ D3-branes, i.e. branes with opposite charge and tension as $D3$-branes. This last situation would preserve supersymmetry and would therefore have zero vacuum energy. To change it back to our physical situation, however, one needs to add back p D3/anti-D3 pairs, with zero charge but with total tension $2p\mu_3/g_s$. Notice, however, that in order to obtain the true total vacuum energy, we need to add to the result (2.44) a term

$$V_{\text{tot}}(\psi) = V_{\text{eff}}(\psi) + \frac{p\mu_3}{g_s}, \tag{2.48}$$

so that the supersymmetric vacuum indeed has $V_{\text{tot}} = 0$. This extra contribution comes from a term $\frac{\mu_3}{g_s}\chi/24 - \int H_3 \wedge G_3$ in the string action which, via the global tadpole condition, adds up to $p\mu_3/g_s$.

Finally, let us return to the validity of our description. As mentioned, this is a slightly problematic question, since both the S-dual D3-brane variables and the NS 5 world-volume theory are strongly coupled. It seems reasonable, however, that at least our main qualitative conclusions, (i) the anti-D3 branes expand to a "fuzzy" NS 5 brane, (ii) for small enough p/M, the NS 5 brane will stabilize at some S^2-radius ψ_{min} proportional to p/M, and (iii) for large enough p/M the anti-D3/NS 5 configuration will be classically unstable, will remain unchanged in a more complete treatment. One forseeable quantitative difference, for example, is that inclusion of the backreaction of the NS 5-brane on the S^3 geometry might trigger the classical instability for smaller values of p/M than found above.

2.4.2 BPS Domain Wall

As discussed in §2.2.3, one can consider supersymmetric domain walls that interpolate between the supersymmetric "mesonic" and "baryonic" vacuum branches in the pure KS gauge theory with $p = 0$. As we now show, it is possible to write a special BPS solution to the NS 5 brane equations of motion that describes a supersymmetric domain wall between the two phases. Specifically, we assume that the mesonic vacuum is such that all meson fields $(N_{ij})^{\alpha\beta}$ have the same expectation value. In the supergravity, this is described by the configuration of M D3-branes located at the same point on the S^3, which we take to be the north-pole $\psi = \pi$.

Before we describe this domain wall solution, we note that, for general p, the total $\mathcal{F}_2$ flux through the S^2 satisfies

$$\begin{aligned} 2\pi \int_{S_2} \mathcal{F}_2 &= 4\pi\Big(\pi p - M(\psi - \frac{1}{2}\sin(2\psi))\Big) \\ &= -4\pi\Big(\pi(M-p) - M(\tilde{\psi} - \frac{1}{2}\sin(2\tilde{\psi}))\Big) \end{aligned} \tag{2.49}$$

with $\tilde{\psi} = \pi - \psi$. In other words, we can think of the $\mathcal{F}$ background from the "south-pole perspective" as carrying p units of $\overline{D3}$ charge, or from the "north-pole perspective" as carrying $M-p$ units of D3-charge. Notice that this implies that there must be $M-p$ units of F_2 flux placed at the north-pole, that is, $M - p$ D3-branes. Hence, in the special case that $p = 0$, the NS 5-brane at the north-pole represents M D3 branes, while at the south-pole it can simply shrink from view without a trace.

The domain wall solution corresponds to an NS 5-brane configuration described by a spatial trajectory $\psi(z)$ (z the coordinate transverse to the wall) interpolating between $\psi = \pi$ and $\psi = 0$. Following the same steps as above, we find that the z-evolution of $\psi(z)$ is governed by the Hamilton equations $\partial_z\psi = \frac{\partial\mathcal{H}_z}{\partial P_\psi}$ and $\partial_z P_\psi = -\frac{\partial\mathcal{H}_z}{\partial\psi}$ with

$$\mathcal{H}_z(\psi, P_\psi) = -\frac{A_0}{2\pi}\Big(2\psi - \sin 2\psi\Big) + \sqrt{A_0^2\Big(V_2(\psi)\Big)^2 - P_\psi^2} \tag{2.50}$$

We look for a trajectory that, for large negative z, starts at rest at the north-pole, i.e.,

$\psi = \pi$ and $P_\psi = 0$. Therefore, this solution has $\mathcal{H} = 0$. Solving for P_ψ we find

$$P_\psi = b_0^2 \sin^2 \psi. \tag{2.51}$$

We thus obtain the following first order equation for $\psi(z)$

$$\partial_z \psi = -\frac{b_0^2 \sin^2 \psi}{\psi - \frac{1}{2}\sin 2\psi}. \tag{2.52}$$

This can be integrated to (choosing the location of the domain wall around $z \simeq 0$)

$$z = \frac{\psi \cot \psi}{b_0^2} \ . \tag{2.53}$$

The right-hand side covers the half-space from $z = -\infty$ to $z = 1/b_0$, where the NS 5-brane trajectory has reached the south-pole at $\psi = 0$. At this point the brane has disappeared, leaving behind the pure flux KS solution. In the dual gauge theory, this is the baryonic vacuum.

It is useful to think of the domain wall as an NS 5 brane wrapped around the S^3 A-cycle of the conifold, deformed near the north-pole due to the presence of the M D3-branes ending on it. From this perspective, it clearly has the property of inducing a jump by one unit in the H_3 flux around the B-cycle. Indeed, if we consider two such B-cycles $B(z_+)$ and $B(z_-)$ located at opposite sides of the domain wall, their difference $B(z_+) - B(z_-)$ represents an (otherwise contractible) 3-cycle that surrounds the NS 5-brane once. Since the NS 5-brane acts like a magnetic source for H_3 we have

$$\int_{B(z_+)} H_3 - \int_{B(z_-)} H_3 = 4\pi^2. \tag{2.54}$$

The tension of the domain wall is obtained by evaluating the classical action of the above solution per unit time and area. We have

$$S \ = \ \int dt d^2x \sqrt{-\det G_3}\, T_{wall} \tag{2.55}$$

with

$$T_{wall} \ = \ A_0\, b_0 \sqrt{g_s M} \int_0^\pi d\psi \ \frac{b_0^2 \, \sin^2 \psi}{\pi} \ = \ \frac{\mu_5 \, 2\pi^2 b_0^3 \, (g_s M)^{3/2}}{g_s^2} \tag{2.56}$$

As expected, this result for the domain wall tension breaks up as the product of the NS 5-brane tension μ_5/g_s^2 times the volume of the S^3 (with radius $R_0 = b_0\sqrt{g_s M}$) wrapped by it. From a holographic viewpoint, this should be compared with the formula (2.18) obtained from the gauge theory effective superpotential.

We should mention that the probe approximation used here is no longer strictly valid near the north-pole $\psi = \pi$, since the M D3-branes represent an appreciable stress-energy that will have a non-negligible effect on the background geometry. Previous experience with supersymmetric configurations of this kind [32], however, suggests that such backreaction effects do not significantly alter the results for quantities like the domain wall tension.

2.4.3 Vacuum Tunneling

We now turn to a description of the decay of the non-supersymmetric configuration with p non-zero. This takes place via nucleation of a bubble of supersymmetric vacuum (2.6) surrounded by a spherical NS 5 domain wall which expands exponentially as a consequence of the pressure produced by the drop in the vacuum energy. To obtain the nucleation rate, it is standard practice to look for a corresponding Euclidean solution. The relevant solution for us is an NS-5 brane trajectory $\psi(R)$, where R is the radial coordinate in $\mathbf{R}^4$, connecting the "giant graviton" configuration at $\psi = \psi_{\min}$ at large R to an instantonic domain wall located near some appropriate radius $R = R_*$ at which the solution reaches the supersymmetric minimum $\psi = \pi$.

The total action functional for such a trajectory reads

$$S = B_0 \int_{R_*}^{\infty} dR\, R^3 \left(V_2(\psi)\sqrt{1 + (\partial_R \psi)^2} + \frac{1}{\pi}\,(\pi\frac{p}{M} - \psi + \frac{1}{2}\sin 2\psi)\right) \tag{2.57}$$

with

$$B_0 = 2\pi^2 b_0^4 \mu_3 g_s M^3. \tag{2.58}$$

As before, it is convenient to write the classical equations of motion for this action in the form of Hamilton equations $\partial_R \psi = \frac{\partial \mathcal{H}_R}{\partial P_\psi}$ and $\partial_R P_\psi = -\frac{\partial \mathcal{H}_R}{\partial \psi}$ with

$$\mathcal{H}_R(\psi, P_\psi) \;=\; -\frac{B_0 R^3}{2\pi}\left(2\psi - \sin 2\psi\right) \;+\; \sqrt{B_0^2\, R^6 \left(V_2(\psi)\right)^2 - P_\psi^2} \tag{2.59}$$

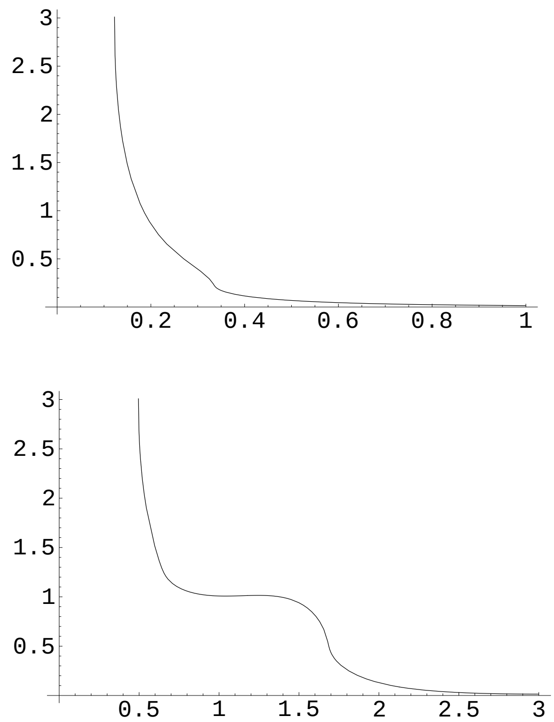

Figure 2.3: The Euclidean NS5-brane trajectory $\psi(R)$ for two values of p/M: the upper trajectory corresponds to $\frac{p}{M} \simeq 3\%$, and the lower one to the near critical value $\frac{p}{M} \simeq 8\%$.

In Figure 2.3 we have drawn the resulting Euclidean NS 5-brane trajectories $\psi(R)$ for two values of p/M.

In principle, we could extract the nucleation rate from the above formulas for general p/M by evaluating the total action of the (numerically obtained) classical solutions, though this is a laborous prospect. Instead, we can quite easily obtain the leading order decay rate in the limit of small p/M as follows: From Figure 2.3, the classical trajectory for small p naturally divides up into two separate regions: it stays flat near the non-supersymmetric minimum until coming very close to the domain wall radius R_* where it quickly moves toward the north-pole value $\psi = \pi$. On these grounds, we divide the total action into a contribution coming from the non-zero vacuum energy (given in (2.48) and (2.46)) of the non-supersymmetric "giant graviton" at $R > R_*$ and a second contribution coming from the tension of the domain wall. Using the "thin wall approximation" [34] we may write

$$S \;=\; T_{wall}\,\mathrm{Vol}_3(R_*) \;-\; V_{\rm tot}(\psi_{min})\,\mathrm{Vol}_4(R_*) \tag{2.60}$$

where

$$\mathrm{Vol}_3(R_*) \;=\; 2\pi^2 b_0^3 (g_s M)^{3/2} R_*^3 \qquad\qquad \mathrm{Vol}_4(R_*) \;=\; \frac{1}{2}\pi^2 b_0^4 (g_s M)^2\, R_*^4. \tag{2.61}$$

denote, respectively, the 3-volume of and the 4-volume inside a 3-sphere of radius R_* as measured by the metric (2.35).

It now seems reasonable to assume that, when $p \ll M$, the profile of the domain wall around $R = R_*$ approaches the supersymmetric configuration described in §2.4.2. Based on this intuition, we will set the domain wall tension T_{wall} equal to its the supersymmetric value (2.56). Taking the leading order value $V_{\rm tot}(\psi_{min}) \simeq 2p\,\mu_3/g_s$ for the energy of the false vacuum, we thus get the following result for the classical action of the domain wall solution with radius R_*

$$S(R_*) \;=\; \pi^2 \mu_3 b_0^4 g_s M^2 \Big(M\, b_0^2 R_*^3 \;-\; p\, R_*^4 \Big) \tag{2.62}$$

The two terms represent the two competing forces on the NS 5-brane domain wall bubble: the tension pulling it inward and the outward pressure caused by the lower energy of the supersymmetric vacuum inside the bubble. At the critical radius

$$R_* = \frac{3Mb_0^2}{4p} \tag{2.63}$$

the two forces are balanced. Notice that, as expected, the domain wall becomes flat in the limit that p/M approaches zero.

Plugging the critical value for R_* back into the action, gives the final leading estimate of the nucleation rate

$$\text{Decay rate} \;\simeq\; \exp\left(-\frac{27\, b_0^{12} g_s M^6}{512\pi\, p^3} \right) \tag{2.64}$$

where we set $\mu_3 = 1/2\pi^3$. We see that the rate is very highly suppressed in the $p \ll M$ regime we have been considering here.

In general, the above expression gives a negligible rate in the supergravity limit of large M with $g_s M$ fixed. Note, however, that in our derivation of (2.64) we have assumed that p/M is far below the critical value (about 8% in our probe approximation) where the false vacuum becomes classically unstable. We conclude that, by tuning p/M, we can make the non-supersymmetric vacuum arbitrarily long or short-lived.

2.5 Two Dual Perspectives

In this section we consider two dual perspectives on our model. In particular, we propose a dual holographic description in terms of a nonsupersymmetric, metastable vacuum in the KS field theory. Secondly, as an additional motivation for this proposal, we summarize how our model may be described from the type IIA point of view.

2.5.1 Holographic Dual Field Theory

For simplicity, we first consider the case that $p = 1$, which we expect to be related to the $SU(2M-1) \times SU(M-1)$ KS field theory. In addition to the $\mathcal{N} = 1$ gauge multiplets, this theory has bifundamental fields, two in the $(2M-1, \overline{M-1})$ representation and two in the $(\overline{2M-1}, M-1)$ representation. So, from the perspective of the $SU(2M-1)$, there are $N_f = 2(M-1)$ flavors, one less than the number of colors. Hence it is no longer possible to write the color neutral baryonic combinations $\mathcal{B}$ and $\tilde{\mathcal{B}}$, and, as a result, the only supersymmetric vacuum of the system is the mesonic branch (2.15). This is the holographic dual of the stable vacuum with $M-1$ D3-branes, the situation we expect to land on after the decay process in the case $p = 1$. The question now becomes, "Where do we find the other, nonsupersymmetric metastable vacuum corresponding to a single anti-D3 brane probing the conifold?"

Though qualitatively important, for large M, the presence of the single anti-D3 brane acts only as a small perturbation of the situation with $p = 0$. (For example, the effective

NS 5-brane potential V_{eff} given in (2.44)-(2.39) has a perfectly smooth limit for $\frac{p}{M} \to 0$.) It seems reasonable to assume, then, that the nonsupersymmetric minimum for $p = 1$ can be thought of as closely related to the baryonic vacuum of the supersymmetric theory with $p = 0$. Indeed, as we have argued in §2.4.2, the NS 5 domain wall that separates it from the theory with $M-1$ D3-branes in the supersymmetric case represents a transition between the baryonic and mesonic branch.

We will now try to use this intuition to obtain a description of the nonsupersymmetric minimum. We introduce, in spite of the fact that $N_f = N_c - 1$ in our case, the two "baryonic" superfields

$$\mathcal{B}^a \;=\; (A_1)^{M-1}(A_2)^{M-1}, \;\; \tilde{\mathcal{B}}^a \;=\; (B_1)^{M-1}(B_2)^{M-1} \;, \tag{2.65}$$

which are no longer colorless but carry a color index a transforming in the fundamental representation of $SU(2M-1)$. The idea is that the nonsupersymmetric theory corresponds to a false vacuum of the $p = 1$ KS gauge theory characterized by a non-zero expectation value of these color charged "baryon" fields. Naturally, this will cost energy, but it seems a reasonable assumption that (for p/M very small) this nonsupersymmetric vacuum may nonetheless be classically stable because it is separated from the supersymmetric minimum via a potential barrier.

To make this proposal more concrete, let us derive the form of the superpotential of our model with $\mathcal{B}^a$ and $\tilde{\mathcal{B}}^a$ included. To this end, we start by adding to the $SU(2M–1)\times SU(M–1)$ gauge theory a single pair of scalar multiplets A^a and B^a (with a denoting the $2M-1$ color index), which we will then make very massive. The motivation for introducing these extra fields is that, before we decouple them, they augment the system to have $N_f = N_c$ so that we *can* introduce color neutral baryonic fields. Define the combination $\phi_{ab} = A_a B_a$. Now write the superpotential

$$W = \lambda \left(N_{ij,\alpha\beta} N^{\alpha\beta}_{kl} \right) \epsilon^{ik} \epsilon^{jl} + X(\det N' - \phi_{ab}\mathcal{B}^a\tilde{\mathcal{B}}^b - \Lambda^{4M-2}) + m\mathrm{tr}\phi \tag{2.66}$$

with $\det N'$ the determinant of the $(2M-1)\times(2M-1)$ meson matrix obtained by including A_a and B_a. Here the last term gives rise to a mass m for the extra fields A_a and B_a. We can decompose

$$detN' = \left(\mathrm{tr}\phi + \phi_{ab}A_i^{\alpha,a}B_j^{\beta,b}(N^{-1})^{ij}_{\alpha\beta}\right)\det N. \tag{2.67}$$

The lagrange multiplier term in (2.66) is the standard one for an $\mathcal{N}=1$ gauge theory with $N_f = N_c$. Notice that the extra field ϕ_{ab} does not appear in the first term in eqn (2.66); we omit it here to avoid a symmetry breaking expectation value for ϕ_{ab}. Instead, we would like to keep N_c equal to $2M-1$ after integrating out ϕ_{ab}. The supersymmetric ϕ_{ab} equations of motion now read

$$m + X(\det N - \mathrm{tr}\mathcal{B}\tilde{\mathcal{B}}) = 0 \tag{2.68}$$

from which we can solve for X, and

$$A_i^{\alpha,a}B_j^{\beta,b}(N^{-1})^{ij}_{\alpha\beta}\det N = \mathcal{B}^a\tilde{\mathcal{B}}^b. \tag{2.69}$$

Inserting the solution for X back into W gives the superpotential (with $p=1$)

$$W = \lambda(N_{ij})^{\alpha}_{\beta}(N_{k\ell})^{\beta}_{\alpha}\epsilon^{ik}\epsilon^{j\ell} + p\left(\frac{\Lambda_1^{4M-p}}{\det_{ij,\alpha\beta}N - \mathrm{tr}\,\mathcal{B}\tilde{\mathcal{B}}}\right)^{\frac{1}{p}} \tag{2.70}$$

with $\Lambda_1^{4M-1} = m\Lambda^{4M-2}$. This is our proposed superpotential of the theory with the "baryonic" superfields present.

In case of general p, we can similarly write color charged "baryon" fields, which transform in the p-th anti-symmetric product of the fundamental. Although we have not done the explicit analysis in this general case, a natural guess is that the superpotential will take the form (2.70). As a primitive reasonability check, we note that the supersymmetric equation of motion for the "baryon" field, $\partial_{\mathcal{B}}W = 0$, yields the condition that $\mathcal{B}^a = \tilde{\mathcal{B}}^a = 0$, so the mesonic vacuum remains present as the only supersymmetric vacuum, according with our expectations.

In general, without more detailed control over the dynamics, the superpotential on its own provides at most inconclusive evidence of the possible existence of other, nonsupersymmetric vacua of the theory. Still, if our proposal is right, it should at least give some hint. A general comment: Formally, the equation $\partial_{\mathcal{B}}W = 0$ also admits one other solution, namely $\mathrm{tr}\,\mathcal{B}\tilde{\mathcal{B}} \to \infty$. While it is of course dangerous to suggest that this implies the existence of another supersymmetric vacuum, it does indicate that, as a function of the baryon condensate, the full potential $V = |dW|^2$ of the theory will have a maximum at some intermediate scale (which one would expect to be near $\mathrm{tr}\,\mathcal{B}\tilde{\mathcal{B}} \sim \Lambda_1^{4M-4p}$). It is conceivable, therefore, that there exists another minimum at large $\mathrm{tr}\,\mathcal{B}\tilde{\mathcal{B}}$.

The strongest evidence for the existence of the nonsupersymmetric "baryonic" vacuum, however, still comes from the supergravity analysis. The characterization of the nonsupersymmetric model in terms of p anti-D3 branes inside the conifold geometry suggests that, somewhere in the dual field theory, there should be a hint of an (unbroken) $SU(p)$ gauge symmetry. Indeed, our proposed dual interpretation in terms of a phase with a non-zero condensate for $\mathrm{tr}\,\mathcal{B}\tilde{\mathcal{B}}$ naturally leads to a breaking of the $SU(2M-p)$ gauge symmetry to $SU(p)$. It seems natural to identify the worldvolume theory of the p anti-D3 branes with the effective low energy description of this $SU(p)$ sector. In the following subsection, we will find an independent indication from the type IIA perspective that the nonsupersymmetric theory is described by an $SU(p) \times SU(M-p)$ gauge theory.

2.5.2 Type IIA Brane Configurations

The RG cascade in the KS system can also be understood via a dual type IIA perspective [19]. In the IIA description, one studies a theory on D4 branes suspended between NS 5 branes. Consider an NS 5 and an NS 5′ brane, the first filling out the 012345 directions, and the latter the 012389 directions in spacetime. Suppose them separated only along the x^6 direction, compactified on a circle. One can then stretch N D4 branes around the circle, and M D4 branes on one of the two segments. The former correspond to the N D3 branes and the latter to fractional branes. The resulting field theory has $SU(N+M) \times SU(N)$ gauge group and the matter stretching across the NS 5 and NS 5′ branes gives precisely the bi-fundamentals which arise in the KS field theory.

In this description, the forces on the branes are not perfectly balanced – the NS branes bend together on the segment with the additional fractional branes. This corresponds to the fact that the $SU(N+M)$ gauge theory has $N_f = 2N$ flavors and becomes strongly coupled as one flows to the IR. One can move the $5'$ brane through the NS 5 brane and around the x^6 circle to avoid this intersection; this will have the effect of reducing the gauge group to $SU(N) \times SU(N-M)$. This is the first step in the RG cascade, and it is repeated until the rank of the gauge groups is low enough that $N_f < N_c$ in one of the factors and the non-perturbative dynamics becomes more subtle.

The difference in our setup is the addition of p $\overline{D3}$ branes to the conifold. In the dual brane configuration, these should appear as p $\overline{D4}$ branes stretched between the NS 5 and the NS $5'$ branes in addition to those already present in the KS setup. There are now (at least) two obvious options:

• The p anti-branes can annihilate with the D4 branes in each segment, leaving an $SU(N+M-p) \times SU(N-p)$ gauge theory. This subsequently undergoes the RG cascade as described above; assuming $N = KM$ and $p \ll M$, the endpoint is a supersymmetric $SU(2M-p) \times SU(M-p)$ gauge theory. This is, of course, the dual gauge theory description of the final state (2.6).

• Alternatively, one can first go through the KS RG cascade with the D4 branes, leaving a pure $SU(M)$ gauge theory from the D4 brane sector. Then, including the p anti-branes, one finds a nonsupersymmetric theory with gauge group $SU(M-p) \times SU(p)$. Special cases of this theory were discussed in [35]. This is a type IIA dual description of our nonsupersymmetric configuration. It would be interesting to understand the vacuum structure and gauge symmetry breaking patterns found in §2.3 from the IIA perspective. The analysis in [35] finds evidence of a symmetry breaking pattern which depends sensitively on the radius of the x_6 circle, but is carried out far from the $p \ll M, N$ regime of interest to us here.

2.6 Concluding Remarks

We have found that the configuration of p $\overline{D3}$ branes probing the KS geometry constitutes a rich system with several interesting properties. The basic physics is apparent in Figure 2.4. For $p \ll M$, the system relaxes to a metastable nonsupersymmetric vacuum but eventually

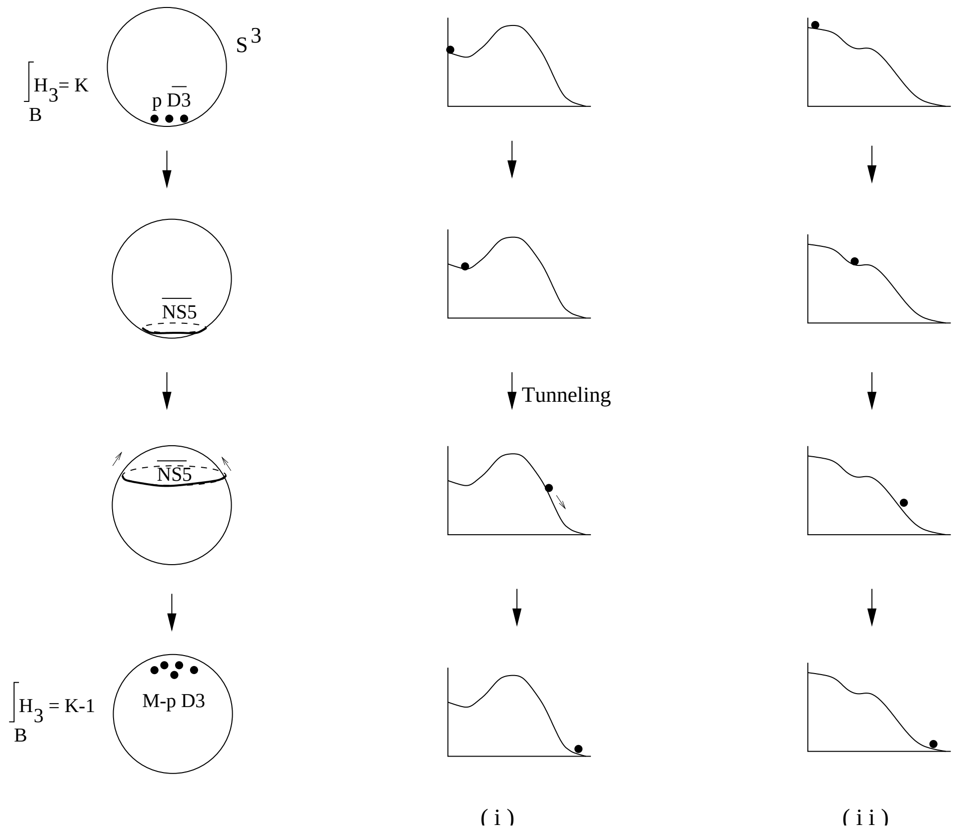

Figure 2.4: Schematic depiction of the brane/flux annihilation process for (i) subcritical and (ii) supercritical values for p/M.

tunnels to a supersymmetric final state. This decay has a strongly suppressed rate given in (2.64) and the nonsupersymmetric vacuum can therefore be made arbitrarily long-lived. For p/M larger than some critical value (of about 8% in our probe approximation) the decay takes place classically. In both cases, the decay is effected by "brane/flux annihilation" where the branes first form an NS 5-brane which later unwinds around the S^3, creating $M-p$ D3 branes in the process. An important remaining problem is to find a supergravity solution of the nonsupersymmetric minimum that includes the backreaction of the p NS 5-branes.

The same basic brane/flux transmutation process whereby fluxes are traded for $D3$ branes may also provide a new perspective on many of the dualities currently under study as geometric transitions. An analogue of our microscopic description of this process via NS5-brane nucleation may also play an important role in those transitions, which encode the information about D-brane gauge theories in terms of dual geometries with fluxes (see e.g. [21, 37]).

In most of this chapter, we have restricted our attention to the infrared physics of the model, implicitly assuming that it embedded in a non-compact warped geometry with fixed boundary conditions in the UV. It is an interesting question, however, to ask what happens when we embed our model in a true string compactification with a finite volume as constructed in [22]. In this case, the holographically dual gauge theory will be coupled to 4-d gravity. Looking at the form of the potentials in Figure 2.4, it is then natural to ask what type of cosmological evolutions are possible in this set-up. Indeed, this is a question to which we will turn in the next chapter.

Besides the ψ field, the string compactification will generally give rise to many other light moduli fields. In the basic model of [22], all can be made massive except for the Kähler modulus $u(x)$ controlling the overall volume of the 6d internal space Y. If we ignore the backreaction due to the branes and fluxes, it is defined by

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + e^{2u} g_{i\bar{j}} dz^i d\bar{z}^{\bar{j}}, \tag{2.71}$$

where ds^2 is the 10d string frame metric and $g_{i\bar{j}}$ the Ricci-flat metric on Y. Following the steps outlined in [22], one obtains for the 4d low energy effective action

$$S = \frac{1}{2\kappa_4^2} \int d^4x \, (-\tilde{g}_4)^{1/2} \left(\tilde{R}_4 - 6 \, (\partial_\mu u)^2 - a_0^2 \, e^{-6u} V_2(\psi) (\partial_\mu \psi)^2 + a_0^4 \, e^{-12u} \, V_{\rm tot}(\psi) \right), \tag{2.72}$$

with $\tilde{g}_4$ the 4d Einstein frame metric, and V_2 and $V_{\rm tot}$ as in (2.39) and (2.44)-(2.48). The coefficient a_0 is the warpfactor at the location of the anti-D3-branes.

The dynamics of this low energy field theory is dominated by the steep inverse-volume dependence evident in (2.72), which implies that the presence of the extra energy density in the anti-branes will quickly force the Calabi-Yau manifold to decompactify. In addition, it prevents the model from giving rise to any appreciable inflation. To stop this decompactification process, or to get an inflationary solution, it seems that one would have to find a novel means of stabilizing the Kähler moduli (for a discussion of moduli stabilization in roughly this context, see e.g. [22, 36] and especially the proposed de Sitter solution of [43]. In the next chapter, we will return to these and several related questions.

Lastly, the gravitational effect of brane nucleation processes that induce discrete flux jumps has recently been investigated in [38], as a possible dynamical mechanism for neutralizing the cosmological constant. In particular the set-up considered in the second reference appears closely related to ours.

Chapter 3

Inflatons

3.1 Introduction

There are few theoretical elements in modern cosmology so crucial as inflation [40]. As a mechanism which purports to solve a host of cosmological problems—the homogeneity of the universe, the miniscule monopole density, the scale-invariant spectrum of density perturbations—it now seems indispensable to any modern theory of the early universe. As a premise sufficiently generic to seem inevitable, it is deemed a necessary component of every cosmological particle theory. However, other than the empirical facts it was designed to corroborate, inflation has remained a success of theory more than observation, and more data is desperately needed to identify and constrain its source.

Yet the prospectus for verifying some form of inflation is rapidly improving. Over the next decade, a series of extensive and unprecedented experiments in observational cosmology—BOOMERANG, WMAP, PLANCK, the Sloan Digital Sky Survey—will begin returning precision data from the early universe, and a confirmation not only the existence but a specific model of inflation is expected.

For these reasons and more, then, one would expect inflation to emerge from a complete theory of quantum gravity, particularly string theory. Unfortunately, efforts along this direction have encountered a daunting series of obstacles. For starters, models based on string compactifications contain a host of light moduli fields which correspond to unfixed parameters of the underlying geometry. These fields, which would mediate long-range forces in the universe, must then be stabilized by some as-yet-unknown process which renders them massive in the present epoch. Secondly, in order to produce significant inflation, the inflaton must roll along a potential which is both flat enough to maintain an extended period of slow roll and contains a stable minimum in which reheating can occur. For string theory, this has proven to be a notoriously difficult problem, as quantum corrections are generically expected to break all accidental symmetries in the theory, and such a symmetry is usually necessary to produce the nearly-flat potential needed for slow roll. In fact, the challenge of stabilizing all compactification moduli—apart from inflation—has been met only in a select few models [42, 43].

As we have already intimated, most studies of inflation have focused more specifically on the scenario known as "slow roll"[1] , a particularly simple, solvable scenario in which it is assumed that the potential and kinetic energy satisfy the relations

$$V \gg K, \quad \dot{V} \gg \dot{K}, \quad V \gg |V'|, \quad \text{and} \quad V \gg |V''|, \tag{3.1}$$

where $V' = dV/d\phi$ and the last two inequalities follow from the additional assumption that there is but a single inflaton. Furthermore, it is typical to rewrite these constraints in terms of the inflaton potential itself as the condition that

$$\epsilon = \frac{M_{Pl}^2}{2}\left(\frac{V'}{V}\right)^2 \qquad \text{and} \qquad \eta = M_{Pl}^2 \frac{V''}{V} \tag{3.2}$$

are both much less than 1 [62]. In this approximation, the number of e-foldings is given by the integral over the inverse of η,

$$N_e = \frac{1}{M_{Pl}^2} \int d\phi \, \frac{V}{V'}, \tag{3.3}$$

and the Hubble expansion is exponential, with the universe experiencing a prolonged de Sitter phase.

[1]For an interesting exception to this trend, see [51].

In the models we will consider, the potential V will be supplied by the four-dimensional effective action of some compactification—usually with brane sources—and the vacuum energy of these ingredients will serve as the cosmological constant of the dimensionally-reduced supergravity. It is this energy which will source the Friedmann equations and drive the expansion of the universe, continuing until the slow roll conditions cease to hold and the inflaton begins to oscillate in the stable minimum of its potential. From there, the kinetic energy of the inflaton will redshift away, reheating the universe through its couplings to regular matter.

As to experiment, we note as a theoretical constraint that at least 60 e-foldings are presumed necessary to reproduce the observed Hubble horizon and flatness of the universe. In addition, the spectrum of gravitational perturbations produced by inflation is nearly scale-invariant, with tilt parameter

$$n = 1 - 6\epsilon + 2\eta \approx 1.13 \pm 0.08 \tag{3.4}$$

[41]. Ideally, this spectrum would be produced by an analysis of quantum inflaton fluctuations in the background of our effective supergravity, though we will not perform such a calculation here.[2] Nevertheless, agreement with experiment on this point is expected of any successful inflationary model.

Nevertheless, there do exist scenarios in string theory in which a suitable inflaton may be found. In the case of brane inflation [48], the inflaton is taken to be the relative position between a brane and its corresponding anti-brane, and as the two move slowly toward one another and annihilate, the worldvolume of each undergoes an extended period of inflation. The typical setup is that of a $D3-\overline{D3}$ pair, one of which is presumed to be our universe. In this case, for distances $r \gg l_s$, the inter-brane potential can be calculated in supergravity, with the result

$$V(r) \sim 2T_3 - \frac{CT_3}{r^4}, \tag{3.5}$$

C some constant of order 1 in Planck units. Thus potential becomes very flat at large

[2] In fact, there are proposed mechanisms to produce scale-invariant curvature fluctuations by fields other than the inflaton. In what follows, we reserve the right to invoke mechanisms such as [52] at will.

distances, growing increasingly steep as the branes approach.

We would like to know how far apart the branes must begin in order to generate sufficient inflation. Plugging (3.5) into (3.2), we derive[3]

$$\eta \sim \left(\frac{L}{r}\right)^6, \tag{3.6}$$

where L is the typical length scale of the compactification ($M_{Pl}^2 = M_{Pl,10}^8 L^6$) and we have neglected constant factors. Clearly, we need $r \gg L$ for slow roll, and we are thus forced into the absurd requirement that the branes be separated by a distance larger than the manifold which contains them.

However, exactly this problem was recently solved by an inventive mechanism in [49]. Instead of a standard compactification, we are instructed to consider the inclusion of five-form flux in our internal manifold. With this addition to the supergravity equations, there exist new solutions corresponding to warped compactifications [24], in which the transverse geometry is multiplied by an exponential "warp factor." As is well known, these compactifications are the inevitable result of three-brane sources and generically exhibit a large hierarchy of scales [22]. The best-known case of this is of course anti-de Sitter space, where the warping is proportional to R/r_0, with $R \sim 4\pi g_s N$—N the number of five-form flux units—and r_0 some minimum radius at which we assume the AdS space truncates smoothly. In these scenarios, the large warp factor implies a sizable redshift between Planck and throat-scale physics, and the effective string scale at the bottom of the throat is consequently reduced. In most of what follows, we will limit ourselves to the case in which this hierarchy is some exponential in ratios of fluxes.

Now let us consider brane inflation in such a warped scenario. First, recall from §2.2.2 that, when placed in such a flux background, D3 branes are governed by a no-force BPS condition[4] while anti-branes are drawn by the combined force of gravity and flux to the tip of the geometry. Clearly, the dynamics of our brane-antibrane pair will be modified in this background: While the anti-brane is held by a combination of forces at the tip, the brane

[3]We ignore ϵ. Generally, the $\eta \ll 1$ is the more restrictive condition.

[4]This can be easily verified by writing the Born-Infeld and Chern-Simons actions for a three-brane and expanding both in the low velocity approximation [49].

experiences a cancellation of these forces and remains free to roam the manifold. Thus the brane-antibrane symmetry of the problem is broken, and the characteristic scale of physics on each is drastically modified. Recalculating the inter-brane potential in supergravity with the inclusion of the anti-brane perturbation, we arrive at a new expression

$$V \sim 2T_3 \left(\frac{r_0}{R}\right)^4 \left(1 - \frac{1}{N}\left(\frac{r_0}{r}\right)^4\right), \tag{3.7}$$

where r is the relative position of the branes. Note that the first term, corresponding to the vacuum energy of the two branes, is now reduced by the fourth power of the redshift factor, and our slow-roll parameter η is suppressed by an exponential factor $(r_0/R)^4$. Thus warping has resolved the difficulty discovered above: by sequestering the physics on each brane to hierarchically-separated scales, the potential is warped away and η is made small by the redshift. As a result, the branes need no longer be separated by a distance larger than the manifold to achieve slow roll.

Unfortunately, embedding this setup in a full string theory context raises a serious additional problem, that of moduli stabilization. In [17], it was pointed out that the effective action for warped compactifications, considered as a function of the overall Kahler modulus ρ, contained terms inversely proportional to ρ^6 and ρ^{12}. Thus the minimization of the effective energy for such a model would lead to a maximization of the Kahler volume, and the manifolds themselves would be unstable to rapid decompactification.

Recently, however, a particular solution of this problem was conjectured in the work of Kachru, Kallosh, Linde, and Trivedi [43], who considered the addition of anti-branes to the supersymmetric AdS warped compactifications. Beginning with the well-known KS geometry, they discovered $AdS_5 \times T^{1,1}$ solutions to the supergravity equations with tunably small cosmological constants. Adding anti-branes to these solutions then contributed a positive vacuum energy, which, when added to the negative cosmological constant of AdS sufficed to give a net positive result. Thus the effective four-dimensional geometry of the AdS plus anti-brane geometries became de Sitter space, and, as was argued in [43], these approximate solutions had energies small and decay barriers wide enough to be metastable. Thus these setups represented long-lived de Sitter phases of string theory, in which the four-dimensional universe was expanding exponentially. Better still, in the class of models considered, the volume modulus was stabilized by the addition of the anti-brane potential, and thus the dS solution fixed all moduli. In this case, runaway decompactification was separated from the dS minimum by both a small energy and a wide potential barrier.

However, this is not the end of our dilemma. In the discussion of warped brane inflation above, it was tacitly assumed that all moduli had been stabilized. And while it is known from specific examples [50] and general suspicion [22] that the presence of fluxes stabilizes the dilaton and all complex structure moduli, we needed special measures to deal with even a single Kahler modulus. Thus any proposed inflationary potential must consider the size of the manifold itself to be a variable field, and any attempt to embed brane inflation in strings must take into account the need to stabilize all moduli.

In order to see why this a problem, consider the superpotential for a warped compactification, which takes the general no-scale form [53]

$$K(\bar{\rho}, \rho, \bar{\phi}, \phi) = -3\log(\bar{\rho} + \rho - k(\bar{\phi}, \phi)). \tag{3.8}$$

Here we see that the Kahler modulus and the complex coordinates ϕ of the Calabi-Yau are present nonlinearly in the geometry, and thus we expect that stabilizing the modulus ρ will impact the treatment of warped potentials dependent upon ϕ in brane inflation.

We can see this in another way as follows: The presence of a four-form flux in the theory[5] is four-dimensionally dual to an axion propagating in spacetime. This axion then couples to the worldvolume of the D3-brane, yielding a moduli space with the axion nontrivially fibered over the Calabi-Yau [49]. As a result, the moduli space metric of the D3-brane—the metric of the Calabi-Yau itself—is only derivable from a Kahler potential if the Kahler modulus ρ is some combination of the volume r and complex coordinates ϕ:

$$2r = \bar{\rho} + \rho - k(\bar{\phi}, \phi). \tag{3.9}$$

However, this leads to a conundrum: Imagine an arbitrary superpotential, dependent upon the chiral superfields ρ and ϕ. With standard sources, this will lead to a scalar potential of

[5]Specifically, one of the form $C_{\mu\nu pq}$, with two indices in spacetime and two in the compactification

the form X/r^α, with X holomorphic in ρ and ϕ. In every known example of brane inflation, the potential will exhibit this characteristic radial falloff, approaching zero as the distance becomes infinite.

Now suppose there exists a mechanism for stabilizing the modulus. Because this mechanism fixes ρ and not r, the terms in the potential, which have denominators given by (3.9), now have an expansion about fixed ρ given by

$$V(\rho,\phi) \approx \frac{Y(\rho)}{\rho^\alpha}\left(1+\alpha\frac{|\phi|^2}{2r}\right)+\frac{Z(\rho)}{r^\alpha}|\phi|^2, \tag{3.10}$$

with $Y = X(\rho, 0)$ and $Z \sim \partial_\phi \overline{\partial}_{\bar{\phi}} X$. Thus corrections to the potential will generically induce contributions of order 1 to the slow roll parameter η and slow roll inflation will not take place. Of course, there is the possibility that these corrections will cancel against each other, but this would necessarily require a fine-tuning in the potential.[6]

Thus we see that even though warped compactifications solve some of the problems of brane inflation, the need for moduli stabilization once again calls into question their viability. Fortunately, such scenarios do not exhaust the possibilities for inflation in string theory compactifications. In the next section, we shall discuss a new setting for brane inflation which avoids the difficulties of moduli stabilization while posing new challenges of its own. This model will make explicit and deliberate use of the nonabelian character of D-brane geometry, in particular the ideas of [61], in which a process of successive nonabelianizations takes place among branes in the presence of background fluxes. With this as context, we will discuss the process of graviton amalgamation and expansion in terms of the rolling of a collective coordinate along potential energy landscapes, finding evidence that such transitions are always classical processes. In addition, we will examine the process of branes finding each other across intervening distance before expanding once again into a nonabelian configuration. We will compute slow roll parameters for a simple example.

Following this, we will consider quantum modifications to the landscapes of §3.2, computing one-loop corrections to the potential and discussing their impact on giant inflaton scenarios. We will see there that, while such corrections do not destroy the classical decay

[6] For a discussion of exactly how much fine-tuning is required, see the discussion in Appendix F of [49].

trajectories they do hold the potential to make both ϵ and η large. This will serve as a correction to our earlier estimates, casting doubt upon the viability of our matrix model scenario.

Finally, we will embed these matrix models in warped geometries following [39] and discuss ways in which naive inflationary scenarios are thwarted in the process. Placing ourselves in the brane-flux annihilation scenario of [17], we will examine the formation and trajectory of the supersymmetry-restoring five-brane domain wall, linking our previous discussion to studies of the giant inflaton. There, our slow roll field will be a noncommutative pseudo-Goldstone boson of the $SO(4)$-invariant A-cycle, and the exit mechanism will involve the domain wall traversing the compact three-sphere. In this context, we will have occasion to discuss interactions among anti-branes, noting the way in which their flux-mediated potentials ruin the possibility of slow roll. We conclude with a number of cautionary statements about the search for inflation in warped compactifications.

3.2 Matrix Dynamics of Fuzzy Spheres

Let us now consider a very simple situation: a cluster of p anti-branes (though the distinction here will be unimportant) transverse to a background of three-form Ramond-Ramond flux. For the moment, we will ignore backreaction, taking our metric as flat Euclidean space. We make no further assumptions about embedding or context. Such a setup could just as easily be found in noncompact space as in the S^3 tip of the KS solution which we will evoke later. For the concerns of this section, the distinction will not enter. Our program will be to investigate the prospects for inflation from giant graviton dynamics in this background, and we will do so in a pair of related examples. In the first, following [61], we will begin with a pair of concentric gravitons and study its (classical) evolution to a single fuzzy sphere. In the second, we will consider a pair of branes initially separated in the flat background and compute their dynamics as they approach and subsequently nonabelianize. In each, we will be concerned with the emergence of flat, slow roll directions from the configuration space of large numbers of fields.

We begin with the well-known action for such branes given in [12]:

$$S = T_p \int dt \, \mathrm{Tr} \left(\frac{1}{2} \dot{X}_i^2 + \frac{1}{4} [X_i, X_j]^2 - \frac{i}{3} \kappa \epsilon_{ijk} X_i [X_j, X_k] \right). \tag{3.11}$$

Here, T_p is the brane tension (we will not, as in [12], assume D0-branes, though we will assume the branes are pointlike in the flux directions), the matrices X_i are $p \times p$ for the p branes, κ is related to the background three-form flux, and $2\pi\alpha'$ is set equal to 1. In keeping with the discussion of the Myers effect in §1, we derive the equations of motion

$$[X_j, ([X_i, X_j] - i\kappa\epsilon_{ijk}[X_j, X_k])] = 0, \tag{3.12}$$

for which we have both the trivial solution $X_i = 0$ and the giant graviton solution

$$[X_i, X_j] = i\kappa\epsilon_{ijk} X_k \quad \Rightarrow \quad X_i = \kappa \bigoplus_{r=1}^{s} J_r, \tag{3.13}$$

a sum of representations of $SU(2)$. Again, each of these representations will constitute its own giant graviton with radius proportional to its quadratic Casimir: $R^2 \sim \kappa^2 j_r(j_r + 1)$. Similarly, the energy is a sum over representations, each with individual energy $E = -T_p \, \kappa^4 j_r(j_r + 1)(2j_r + 1)$. As before, this energy takes its maximum value of zero for the trivial solution and its minimum value for the largest-dimension irreducible representation. All other such solutions, corresponding to nested configurations of fuzzy spheres, have energies intermediate between these two. We therefore expect transitions between such configurations, though there remains the natural question of whether such transitions are classically allowed.

As we will presently demonstrate, the answer is yes, despite the fact that fluctuation analysis of these solutions performed in [61] discovered no negative modes. There, it was found that the existence of marginal solutions to the equations (3.12) is a crucial component of the classical energy landscape, and that instabilities to giant graviton decay only appear as the system evolves along these flat directions. In some cases, the instability is present with even an infinitesimal deformation, while for others, the tachyon does not appear until the system has traveled a finite distance along the marginal path. In all cases, however, such instabilities do exist, and the decay proceeds without exception by *classical* processes.

To this end, let us reconsider the equations of motion (3.12), seeking new solutions of the form

$$Z_i \equiv X_i + c_{ia} Y_a, \qquad [Y_a, Y_b] = 0, \qquad [Y_a, X_i] = 0, \tag{3.14}$$

with X_i the nested $SU(2)$ configuration of (3.13). With such an ansatz, it is not difficult to verify that if the Y_a exist and satisfy (3.14), the Z_i solve (3.12) and the c_{ia} generate a continuous family of degenerate solutions. On this basis, we recognize the Y_a as marginal directions in the nonabelian D-brane geometry.

Of course, this new class of solutions reopens the question of stability, and as shown in [61], the inclusion of the deformations Y_a introduces tachyons into the previously-stable nested graviton configurations. For instance, in the 4×4 matrix $\frac{1}{2} \oplus 0 \oplus 0$ solution, which represents two free branes and a single spin-$\frac{1}{2}$ giant graviton, even an infinitesimal marginal deformation removes the flat directions found in the unperturbed stability analysis. Afterward, all modes are massive, with two stable and two tachyonic. Similar conclusions hold for the $1 \oplus 0$ and $\frac{1}{2} \oplus \frac{1}{2}$ configurations.

Let us then examine the dynamics of the decay process, focusing on the transition from multiple spheres to the single irrep. In what follows, we will consider a pair of simple examples, the $\frac{1}{2} \oplus \frac{1}{2} \to \frac{3}{2}$ and $0 \oplus 0 \to \frac{1}{2}$ transitions, each with its own set of initial conditions.[7] In both, we will see how the energy landscape alters our naive expectations of the system's dynamical behavior.

We begin, as in [61], with the ansatz

$$X_i(t) = \kappa \left(\tilde{J}_i + f(t)(J_i - \tilde{J}_i) \right), \tag{3.15}$$

imposing as boundary conditions the constraints

$$f(t_0) = 0, \qquad f(t_1) = 1. \tag{3.16}$$

Here $\tilde{J}_i$ is the initial configuration, a direct sum of $SU(2)$ representations, and J_i is the final

[7]While it would seem simpler to consider two sets of initial conditions for the $0 \oplus 0 \to \frac{1}{2}$ decay, the nested configuration in this case is classically unstable even without the inclusion of flat directions. The $\frac{1}{2} \oplus \frac{1}{2} \to \frac{3}{2}$ case is the simplest nontrivial case of the concentric analysis we would like to illustrate.

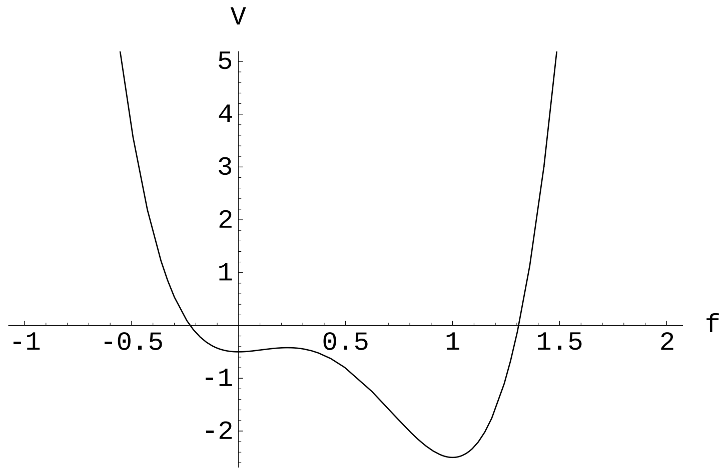

Figure 3.1: One-parameter potential for the $\frac{1}{2} \oplus \frac{1}{2} \rightarrow \frac{3}{2}$ transition. Note that without the marginal direction, the two spin-$\frac{1}{2}$ gravitons are stable against decay.

configuration, the maximal graviton irrep. We wish to study the dynamics of the transition between these two configurations, and if a classical trajectory should exist, $f(t)$ will be the parameter of its curve.

Plugging the ansatz (3.15) into (3.12) then gives an equation of motion $\ddot{f} = -V'(f)$, with

$$V(x) = -a + bx^2 - cx^3 + dx^4 \tag{3.17}$$

and a,b,c,d (all positive) dependent upon the final spin. Thus in Figure 3.1, we see that the $j = 3/2$ solution is the lower minimum, though no classical path connects it to the initial configuration. This partially substantiates our earlier remark—that in general these solutions are stable without flat directions—and we see that the search for a classical trajectory necessitates a more general ansatz.

Let us begin again, then, with the expanded ansatz

$$Z_i(t) = X_i(t) + \kappa g(t) d_i Y, \tag{3.18}$$

where $X_i(t)$ is again given by (3.15) and Y is the marginal direction which commutes with the X_i. Here d_i is an arbitrary constant, and g satisfies the boundary conditions

$$g(t_0) = 0 = g(t_1). \tag{3.19}$$

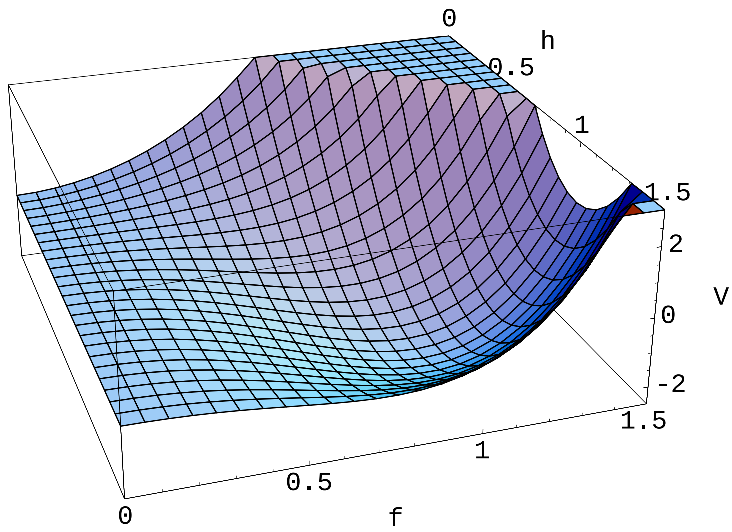

Figure 3.2: Plot of the $j = 3/2$ effective potential of (3.21). The variables (f, h) begin at $(0, 0)$ (the $\frac{1}{2} \oplus \frac{1}{2}$ configuration) in the upper left and roll to a minimum at $(1, 1)$ (the $j = 3/2$ configuration) in the lower right. The transition is purely classical.

As a specific example, let us specialize the 4×4 case of $\frac{1}{2} \oplus \frac{1}{2} \rightarrow \frac{3}{2}$. Evaluating $S = \int dt\,(K - V)$ on this ansatz then gives a kinetic term

$$K = 2\left((4 - \sqrt{3})\dot{f}^2 + 2\dot{f}\dot{g} + \dot{g}^2\right), \tag{3.20}$$

and diagonalizing (3.20) with the substitution $h = f + g$ gives an effective potential

$$V = -\frac{1}{2} + 4\left(2(h-1)^2 + (1 - \sqrt{3})\right) f^2 - 8(2 - \sqrt{3}) f^3 + (10 - 4\sqrt{3}) f^4, \tag{3.21}$$

depicted in Figure 3.2.

Examining this expression, it is clear that there is no real energy barrier to prevent the initial two-graviton configuration ($f = h = 0$) from rolling to the single-sphere irrep ($f = h = 1$). Thus the amalgamation is a purely classical process, proceeding by detour along the flat directions of the potential landscape. In an analogous fashion, this conclusion is expected to hold in the more generic case as well. There, we might also expect to find the decay taking place by a series of staggered stages, the system rolling between successive minima via a cascade of shallow ridges. However, we expect that in order to have inflation, at least one of these ridges must be of finite length and relatively shallow, a situation which fails to obtain in our present example. For slow roll to become feasible, we would like a potential in which the path of graviton amalgamation is narrow and sequestered by some long distance in field space from the emergence of a tachyon.

Fortunately, we have just such an example. Consider the situation in which, rather than beginning as concentric, a pair of graviton spheres are initially separated by a distance x_0. In this case, we expect that the branes will have to reach some minimum distance of approach before the amalgamation instability is present, and until they reach this nonabelianization range, the potential between them should be approximately flat. This is exactly the type of setup we expect for p point branes scattered randomly over some compact space, and their expansion into a maximal giant graviton should follow just such a process of "Brownian" clumping. As random density perturbations grow and the branes amalgamate, we expect the time scale for graviton formation to be much shorter than the time scale over which the branes find each other within the space. This appears to be a promising scenario for slow roll, provided we can successfully identify the inter-brane distance with a shallow potential, and it is exactly this prospect we wish to analyze below.

Now let us treat only the simplest example of such a scenario: two gravitons which find each other across some initial separation and subsequently abelianize. Moreover, in our setup, these two gravitons will be single branes, and we will study the $0 \oplus 0 \to \frac{1}{2}$ transition to a 2×2 graviton configuration.

Begin, then, with the action (3.11), supplying the ansatz (for later convenience, we also take the case of three-branes):

$$X_1(t) = \frac{\kappa\sigma_1}{2} f(t) \quad X_2(t) = \frac{\kappa\sigma_2}{2} f(t) \quad X_3(t) = \frac{\kappa\sigma_3}{2} g(t), \tag{3.22}$$

along with boundary conditions

$$f(0) = 0, \quad f(1) = 1, \quad g(0) = \frac{x_0}{\kappa}, \quad g(1) = 1. \tag{3.23}$$

Thus the initial center of mass separation is x_0 along the X_3 direction, and the branes end

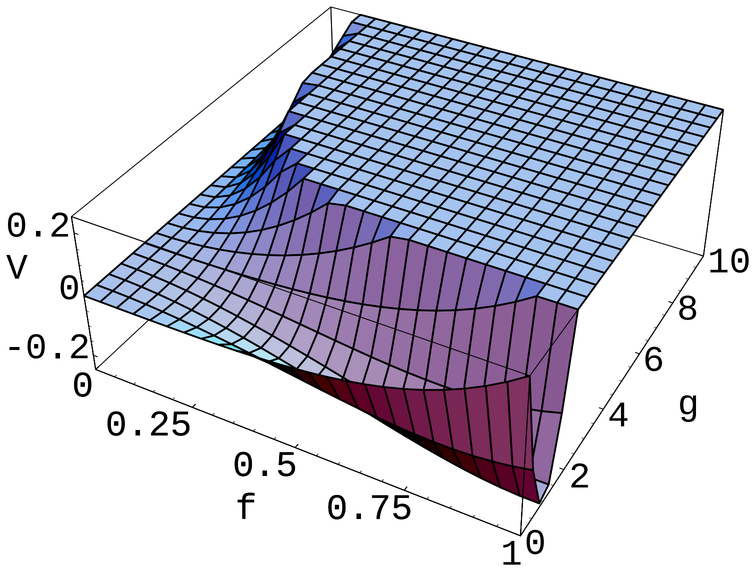

Figure 3.3: Potential landscape for a pair of pointlike branes. The two branes begin separated along the g direction at $f = 0$. The point which represents their center of mass then rolls around the corner and into the spin-$\frac{1}{2}$ graviton basin.

in the spin-$\frac{1}{2}$ giant graviton configuration at $t = 1$[8]. Plugging (3.22) into (3.11), we then derive an effective potential

$$V = T_3\kappa^4(\frac{1}{4}f^4 + (\frac{1}{2}g^2 - g)f^2) \tag{3.24}$$

plotted in Figure 3.3.[9] From the diagram, we see that two branes initially separated along the g axis will roll slowly along this direction until turning the corner, at last falling into the giant graviton basin.[10] As the branes move together, they experience a very shallow potential, until, at some minimum distance, they begin to sample each other's nonabelian

[8] Of course, we might consider more a more general ansatz for the X_i, but this parameterization will suffice because the barrier sequestering the flat direction from the instability is expected on physical grounds to be a generic result.

[9] In [61], it was necessary to first diagonalize the kinetic term. We have no such need here.

[10] To be precise, the marble roving over the landscape in our example represents the center of mass coordinate of the two branes.

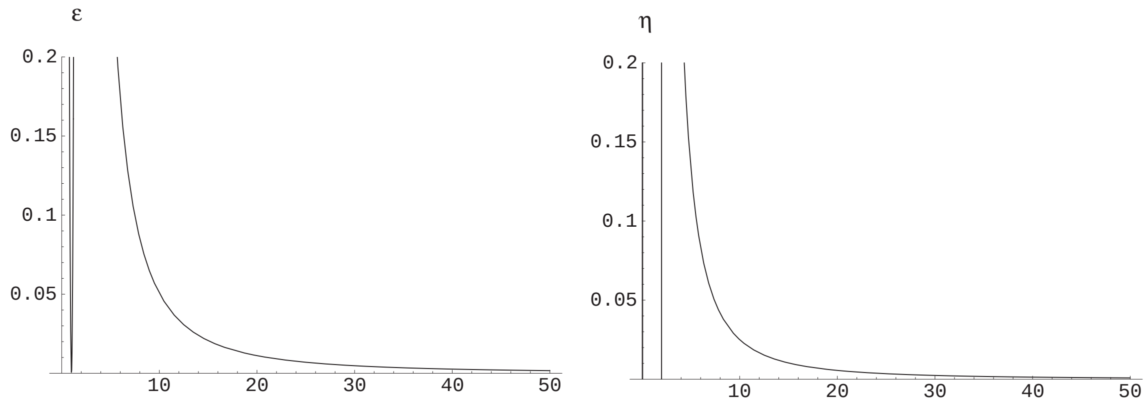

Figure 3.4: Plots of ϵ and η vs. $g_0 = x_0/\kappa$ for slow roll along the direction $f = 0.1$ in (3.24). Integrating from $g = 20$ to $g = 10$ with $M_{Pl} = 1$ gives well over 60 e-foldings.

natures, developing a steeper tachyonic direction. We also note that in the limit $x_0 \to 0$ of concentric spheres, we see a pure instability to nonabelianization.

It thus becomes feasible to ask (perhaps naively) how much inflation this scenario would produce. Without worrying for the moment whether κ is large or small or whether the requisite space for inflation would exist in a compact setting, we can certainly calculate the slow roll parameters ϵ and η for the potential landscape in Figure 3.3. Taking as our inflaton the g direction along $f = 0$, we expect slow inflation along the shallow valley trajectory, exiting as the field turns the corner at some minimum distance x_min. However, because $f = 0$ is an exactly marginal direction, some initial fluctuation is necessary before slow roll can begin. Thus we assume that the field is slightly perturbed from its minimum at $f = 0$ while keeping g large. Fortunately, even a small fluctuation in f will suffice to start the roll, and even a large fluctuation will not significantly alter its slowness.

As seen in Figure 3.4, this landscape produces very small η and ϵ over a large range of g_0. However, we have not said anything about whether κ is large or small, and thus it remains unclear whether g_0 can take values sufficiently large to produce 60 e-foldings is possible. Nevertheless, we can provide an estimate. If, as in the next section, we imagine embedding this landscape in the setup of [17], with p the number of anti-branes and the background geometry that of the S^3 tip, we know there exists a maximum value of $\sqrt{g_s M}$

for any coordinate distance x_0. Furthermore, matching coefficients to the matrix model of [17], $\kappa \sim M^{-1/2}$. This is a good sign. In the dimensionless units of our numerics, we can keep $x_0 \sim l_s$ (and the matrix model within its regime of validity) for $M \sim \mathcal{O}(100)$ and $g_s \sim \mathcal{O}(1/10)$. These numbers then give a g_0 of the order of 10-20, which looks promising.

However, such an analysis is really doomed from the start. In the above, we have assumed that the string scale at the tip is the unwarped string scale, and thus that factors of M_{Pl} in (3.2) are of order 1 in string units. Unfortunately, the conifold warping lowers exponentially the effective scale of inflation, and this suppression of the relevant mass translates to an exponential enhancement of the slow roll parameters. Thus, excepting cases of anomalously-mild warping, we are effectively redshifted out of the slow roll regime. In fact, this verdict—that all but the most highly-suppressed interactions among branes at the tip spoil inflation—is one which will be echoed in the analysis of [39] below.

Lastly, as a prelude to the corrective analysis of the next section, we would like to consider the addition of masses to the model (3.11). With a mass term $\mathrm{Tr}(X_i^2)$, the equations of motion (3.12) now read:

$$[X_j, ([X_i, X_j] - i\kappa\epsilon_{ijk}X_k)] - m^2 X_i = 0. \tag{3.25}$$

Perhaps surprisingly, these still permit a solution of the form (3.13), provided we rescale the coefficient κ as

$$\kappa \to (\kappa + \sqrt{\kappa^2 - 2m^2})/2. \tag{3.26}$$

As expected, this deformation generically lifts flat directions in the potential landscape, leaving only gauge transformation zero-modes. For this reason, we no longer anticipate a classical interpolating trajectory between graviton minima, and the only possible transition is one by quantum tunneling. Nevertheless, there does exist a special value $m = 2\kappa/3$ for which the model becomes supersymmetric. In this case, it is possible to rewrite the action in Bogomolnyi form: as a the sum of a perfect square which yields the first-order equations of motion and a total derivative piece which only depends on the difference between initial and final configurations. In this case, all giant graviton solutions have zero energy, and the classical transitions connecting them are contained in the moduli space of vacua.

More interesting, however, is the effect of a mass term on the non-concentric dynamics we have just considered. In our example, a mass term clearly lifts the $f = 0$ flat direction, and the center-of-mass field is then governed by a quadratic potential. Thus no fluctuation is necessary to start the rolling process, and the calculation of ϵ and η will be that of standard quadratic inflation [62]. There, the values of the slow roll parameters will be set by the ratio of mass-term and Planck scales, and without some restriction to small mass terms (or the unfeasible scenario of a trans-Planckian vev for the radial inflaton), it will no longer be possible to maintain the approximation of slow roll.

Nonetheless, it is an important observation that a mass term of the form $\mathrm{Tr}(X^2)$ will not destroy the classical instability. Because the branes are initially separated, they never find themselves inside the well at $X = 0$, and just as before, they will approach but ultimately avoid this point en route to the expanded graviton minimum. Note, however, that if the mass is too large, the graviton minimum might be raised above the level of the commuting solution, in which case one expects a quantum transition from the spherical to the commuting case.

Of course, we generically expect mass terms to be generated by quantum corrections to the potential. Our question then becomes one of the size of such terms, and we take it up in the next section in the computation of a one-loop correction to the action (3.11). We will compare this result to similar estimates provided by the giant inflaton scenario of [39].

3.3 A One-Loop Amendment

As already noted, the effect of a mass term on matrix models such as those studied above can be pronounced, with the capacity to destroy the prospects for inflation. In this section, we would like to estimate the size of such terms as generated by quantum effects. In [39], similar terms were considered in a closely-related setup, and their impact on the giant inflaton scenario was assessed. Here, after giving a treatment of our own, we will briefly discuss that scenario and compare our results with the approach presented there.

In order to make this a meaningful process, however, we must embed our matrix model in a compact framework. As suggested earlier, we will take this to be the brane-flux annihilation setup of [17]. Thus we will once again consider a cluster of p anti-D3-branes on the S^3 tip of the KS warped deformed conifold with radius $R \sim \sqrt{g_s M}$. These branes, in the presence of the background five-form flux, will held to the tip by a combination of flux and gravitational energetic considerations, and we will consider motion only along the directions of the sphere. Moreover, these anti-branes will also inherit a $\mathrm{Tr}(\Phi^3)$ Myers term from their interaction with the M units of Ramond-Ramond three-form flux through the sphere, an interaction which will generate the giant graviton instability.

Moreover, we will also assume that the overall volume of the compactification has been stabilized, by the mechanism of [43] or some other. The specific scheme will not be important for our purposes, but we will take as given a solution to the problem. As for the pitfalls of warped brane inflation discussed in [49], we will sidestep these by staging our nonabelianization on the compact A-cycle, a submanifold of constant Kahler form. Thus the potential for our inflaton, arising from the p^2 anti-brane degrees of freedom, will receive no corrections from moduli stabilization effects.

From this setup, we hope to extract an inflationary scenario as follows: Beginning with p anti-branes scattered over the sphere, we follow the process of brane amalgamation, allowing the branes to cluster and nonabelianize by a series of staggered classical decays. We hope that in this process there exist a number of long, marginal directions along which clumps of branes approach each other, driving inflation before the onset of giant graviton instabilities. At last, when the branes have assembled themselves into a single expanded graviton/brane, inflation will end when this object traverses the three-sphere as a supersymmetry-restoring domain wall [17]. This process, which leaves behind $M - p$ SUSY-preserving branes on the sphere, represents both a uniquely stringy exit mechanism and a fundamentally non-perturbative transition between cosmological phases. Such a scenario was studied from the supergravity point of view in [39], and the results of that work will be discussed below.[11]

To this end, we will return to the matrix model (2.24), this time including the kinetic

[11] For a related idea which follows along somewhat different lines, see [54].

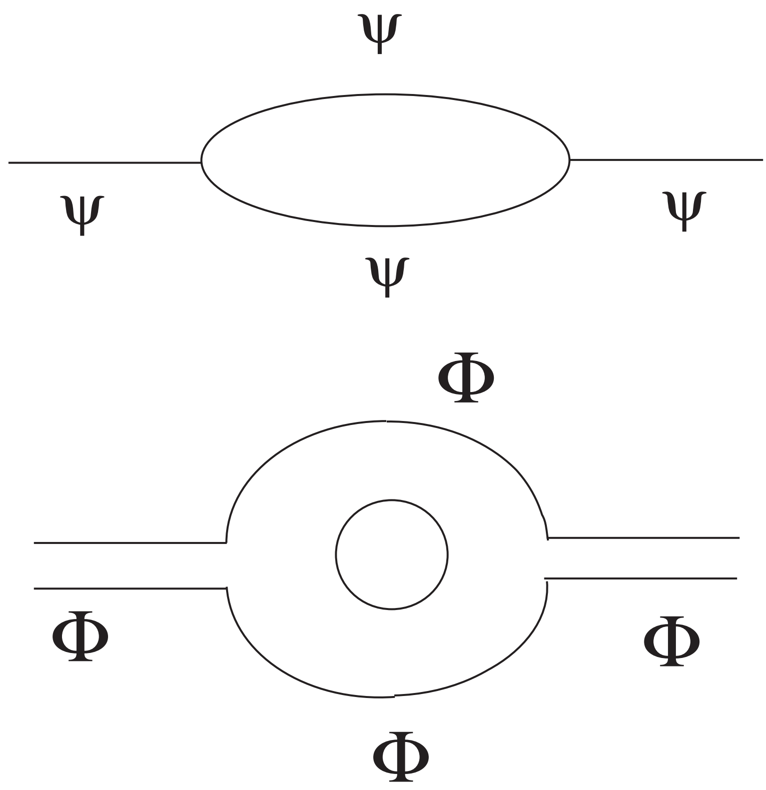

Figure 3.5: One loop corrections to the matrix model and ψ masses. In the first graph, the propagators are for a field $X \sim \Phi g_s^{-1/2}$. There is a factor of p from the trace around the loop. In the second diagram, the field in the loop is $\psi \sim R$.

term:

$$\mathcal{L} = \frac{\mu_3 p}{g_s} + \frac{\mu_3}{g_s}\left(\mathrm{Tr}((\dot{\Phi}^i)^2) + \frac{1}{g_s^2}\mathrm{Tr}([\Phi^i, \Phi^j]^2) + \frac{if}{3}\epsilon_{ijk}\mathrm{Tr}(\Phi^i[\Phi^j, \Phi^k])\right), \tag{3.27}$$

again with $f \sim F_{ijk} \sim 1/\sqrt{g_s^3 M}$. Of course, a straightforward calculation of the diagram in Figure 3.5 gives a mass correction to this model

$$m^2 \sim \frac{p}{16\pi^2}(f\sqrt{g_s})^2 \log(\Lambda/\mu) \sim \frac{p}{g_s^2 M}, \tag{3.28}$$

where p comes from the trace over the internal loop and the $\sqrt{g_s}$ results from properly normalizing the kinetic term. Such a mass is conceivably quite large: for subcritical numbers of anti-branes, p/M is of the order of a few percent, while g_s is expected to be of order

$\frac{1}{10}$ or less. This is obviously problematic from the standpoint of inflation, and stands to undermine our earlier optimism, even apart from concerns of warping.

Furthermore, we anticipate that the continuous NS5-brane description should tell a similar, if not identical, story. Beginning with (2.32), we can follow the calculation of §2.4.1, arriving once again at the effective potential (2.44). However, in considering the correspondence between this and the matrix configuration, we will be more interested in the small-radius potential (2.45). Once again, it will be worth our while to add the kinetic term, yielding a Lagrangian

$$\mathcal{L} = \frac{\mu_3 p}{g_s}\left((g_s M)(\partial_\mu \psi)^2 + \frac{4M}{3\pi p}\psi^3 - \frac{M^2}{2\pi^2 p^2}\psi^4\right), \tag{3.29}$$

without the constant potential.

As in [39], we might ask how large a mass correction is generated in this theory. To do this, we define a new, canonically-normalized field y and calculate the corrections to its mass from a similar one-loop term to that in Figure 3.5, yielding an answer

$$m^2 \sim f^2 \sim \frac{1}{g_s^2 M p^3}. \tag{3.30}$$

Certainly this is a much smaller result, down by a factor of p^4 from our matrix model estimate. And yet we expect that the two computations are morally equivalent, since the leading deformation to the theory of the NS5-brane should be related to the first-order correction to the matrix model. However, we have failed in this estimate to take into account two crucial factors. First, we have compared the weakly-coupled description of the anti-brane matrix theory to the strongly-coupled NS5-brane description. A priori, there is no expectation that these will produce the same numerical result, and we should take the agreement in g_s and M dependence as a gratuitous confirmation. In fact, this follows from the almost identical form of the actions: for small radius, $\sin^2\psi \sim \psi^2 \sim R^2 \sim \frac{1}{p}\mathrm{Tr}\left((\Phi^i)^2\right)$, and by defining the matrix variable

$$X^i = \sqrt{g_s M}\Phi^i \sim \frac{\sqrt{g_s M}\psi}{p} J^i, \tag{3.31}$$

where J^i is the $SU(2)$ generator and we have used $\mathrm{Tr}((J^i)^2) \sim p^3$ for large p, we can derive a complete term-by-term correspondence between (3.27) and (3.29).

The real reason, then, that these nearly identical actions do not lead to the same mass can be found in our second caveat: By using (3.29) to calculate (3.30), we have implicitly ignored the other $\mathcal{O}(p^2)$ degrees of freedom of the matrix model. Thus the effective action calculation (3.30) fails to take into account the complete phase space of contributing fluctuations and thus wildly underestimates the mass term. In effect, the fact that $\mathrm{Tr}(\Phi)^2 \sim p\psi^2$ means we should expect phase space enhancements by powers of p.

Unfortunately, as mentioned above, this only makes the prospects for inflation worse. Because g_s is assumed to be small with M large and p/M of the order a few percent, there is little hope that ϵ and η can be made small in the case that inflation is driven by the mass term above. Nevertheless, we have found an encouraging agreement between the matrix and Born-Infeld dynamics, and that lends credence to our use of the matrix model as an approximation.

All of this, of course, corroborates the pessimistic assessments of [39], which examined long-wavelength supergravity descriptions of the same brane setup. There, it was found that mass terms were generated from a host of sources, including $SO(4)$-breaking effects on the A-cycle from distant fluxes. These terms, generated by nonvanishing integrals over closed cycles not present in the KS solution,[12] yield mass terms in the supergravity equations according to

$$\nabla^2 A = \frac{g_s^2 |G|^2}{48}, \tag{3.32}$$

where the warp factor is written as e^A. Compared to the KS flux, which integrates to M over the A-cycle, these fluxes are suppressed by powers of the warp factor, and thus the masses which affect physics at the tip are exponentially small.

The more important story in this context is that of the interaction among anti-branes. Whereas in supersymmetric backgrounds, probe branes which preserve the supersymmetries of the ambient space are governed by a BPS no-force condition, supersymmetry-breaking probes, as we have seen, *do* experience net forces, often quite strong. There is still some question, however, as to whether *mutually* BPS objects, even those which break the back-

[12]These fluxes were, however, assumed to preserve the imaginary self-dual condition. In the example given there, they were proportional to the holomorphic three-form.

ground supersymmetry, feel any force from one another. Obviously, should such a force exist, it must be the result of supersymmetry-breaking effects in the background.

This is in fact the case, as argued in [39]. There, ISD fluxes on the A-cycle, in combination with gravitational effects, mediate a net *attractive* force between anti-branes, leading to strong dynamics which destroy the potential for slow roll. According to the authors, the gravitational warping caused by a large number of clustered anti-branes on the sphere traps flux in the branes' own throat. This pinned flux then screens the charge of the anti-branes, giving them at long distance a net *positive* three-brane charge. Thus a wandering anti-brane on the S^3 will experience the clump as an object with positive three-brane charge and will be attracted as if the configuration were a normal three-brane. As a result, there exists an effective potential

$$V(X) = \frac{2\pi}{M}\frac{a_0^4}{X^2}, \tag{3.33}$$

where X is the normalized rms brane separation $X^2 = \frac{1}{p^2}\sum_{i\neq j}(y_i - y_j)^2$ on the sphere. Unfortunately, as is the case with all dynamics at the bottom of the throat, the η parameter for this potential is made large by the warping which exponentially lowers the Planck scale.

More problematically, this is a generic result. The picture which emerges from the analysis of [39] is one of roaming anti-branes which are quickly drawn together on the sphere and form an expanding NS5-brane which traverses the S^3 as in [17]. All of these processes, it would seem, are much too fast to produce significant inflation.

However, there is a final optimism in the evolution of the five-brane trajectory. Recalling the effective potential (2.44) plotted in Figure 2.2, we might wonder if the transition through a near-critical inflection point might provide velocities sufficiently small for slow roll. In fact, numerical results presented in [39] do manage some 60 e-foldings for values of

$$B \equiv \frac{a_0^2 M_s^2}{M_{Pl}^2} \tag{3.34}$$

of order 1. Therefore, in cases of very mild warping, $a_0 \sim 1$, we might hope to discover inflation either in amalgamation dynamics of gravitons on the sphere or in the expansion of the nonabelianized NS5 domain wall. However, such a regime lies beyond the range of many of our approximations, which relied on an approximate treatment of highly-warped geometries, assuming fluxes $K \sim g_s M$ or greater. Our understanding of mildly-warped geometries is comparatively weaker, and it seems that until we gain control of this mild warping regime, the case for a giant inflaton will remain suggestive at best.

Chapter 4

Twisted Bits

4.1 Introduction

As it is conjectured, AdS-CFT is not merely a correspondence between supergravity and gauge theory, but between gauge theory and strings [18]. However, results beyond the supergravity approximation, including α' corrections and the regime of small 't Hooft coupling, have proven more difficult to calculate. For this reason, there has been keen interest in finding sectors of conformal field theory which would correspond to stringy effects, as well as theories which capture the full string theory dynamics [66, 67].

In addition, there have long been efforts to find and classify exactly solvable string theories, both as a means of understanding strings in nontrivial[1] backgrounds and as a testing ground for ideas beyond the arena of flat space [68, 69]. It has been hoped that such backgrounds, where they exist, would further our understanding of strings at strong coupling.

Thus it is unsurprising that plane waves [74] have proven a subject for repeated and serious investigation as exactly solvable backgrounds of string theory. While it has long been known [69] that the NS version is a solvable background, not until the analysis of [76] was it understood that the same was true of its RR counterpart. There, it was found that the

[1] Especially spaces with nonvanishing Ramond-Ramond flux.

complicated interaction of the Ramond-Ramond flux with the worldsheet fermions reduces to a simple, diagonal mass term in light cone gauge, and the string worldsheet reduces to a theory of equi-massive fermions and bosons.

This presents an intriguing possibility: since plane waves are known to emerge as the generic outcome of so-called Penrose limits of spacetimes [75], it becomes possible, by taking such a limit of AdS, to find a correspondence between some subsector of the gauge theory and strings with massive worldsheets. Such a procedure, undertaken in the work of Berenstein, Maldacena and Nastase [70], thus provided a novel means of comparison between large-N gauge theory and lightcone string field theory. Using this idea, subsequent authors have found agreement over several orders of approximation for three and four-point amplitudes [94, 82, 92, 73].

The plan of this chapter, then, is as follows: In §4.2, we will review essential elements of what has come to be known as the BMN correspondence, outlining the dictionary between gauge theory computations and the strings themselves. This will allow us to review several proposed checks on the theory, as well as noting in advance some historical subtleties in the literature. Then, in §4.3, we will review the so-called "string bit" formalism of Verlinde, a framework reminiscent of matrix string theory [4] which effectively lies between the gauge and string field approaches. Lastly, in §4.4, we will use string bits to perform a nontrivial test of the correspondence, finding exact agreement among all three approaches.

4.2 The BMN Correspondence

We begin with the Penrose limit of AdS_5. Starting with the global metric,

$$ds^2 = R^2 \left[-dt^2 \cosh^2 \rho + d\rho^2 + \sinh^2 \rho d\Omega_3^2 + d\psi^2 \cos^2 \theta + d\theta^2 + \sin^2 \theta d\Omega'^2_3 \right], \qquad (4.1)$$

we imagine zooming in on a null geodesic situated at $\rho = 0$ and $\theta = 0$, moving in the ψ direction. Defining, $\tilde{x}^{\pm} = (t \pm \psi)/2$, we do this by rescaling coordinates

$$x^+ = \tilde{x}^+, \quad x^- = R^2 \tilde{x}^-, \quad \rho = \frac{r}{R}, \quad \theta = \frac{y}{R} \qquad (4.2)$$

and taking the limit $R \to \infty$. This serves to boost the graviton along the x^+ direction while scaling out the details of the metric transverse to its worldline, leaving us with the much simpler effective metric

$$ds^2 = -4dx^+dx^- - \mu^2|\mathbf{z}|^2(dx^+)^2 + d\mathbf{z}^2 \tag{4.3}$$

with background flux

$$F_{+1234} = F_{+5678} \sim \mu. \tag{4.4}$$

Here $\mathbf{z}$ represents the $SO(8)$-invariant space transverse to the light cone, and μ is the vestige of the AdS five-form. This is the solution for a plane wave geometry, studied in [74].

However, we would also like to match gauge theory charges to dynamical quantities in spacetime. To do this, we identify $E = i\partial_t = \Delta$ and $J = -i\partial_\psi$ and perform the same scaling as before, giving

$$2p^- = -p_+ = \Delta - J \tag{4.5}$$

$$2p^+ = -p_- = \frac{\tilde{p}_-}{R^2} = \frac{\Delta + J}{R^2}, \tag{4.6}$$

where we anticipate identifying Δ with the conformal dimension of an operator in gauge theory and $p^- \geq 0$ by the BPS condition. Recalling that $R^2 \sim \sqrt{N}$ for AdS-CFT and demanding that these charges remain finite in the limit, we then find ourselves restricted to the sector with $J \sim \sqrt{N}$ and $\Delta - J$ fixed.[2]

Now let us return to the other half of the correspondence, namely string theory on an RR plane wave background. After fixing light cone gauge and specializing to (4.3), we then arrive at the worldsheet action [76]

$$S = \frac{1}{2\pi\alpha'} \int dt \int_0^{2\pi\alpha' p^+} d\sigma \left[\frac{1}{2}\dot{z}^2 - \frac{1}{2}z'^2 - \frac{1}{2}\mu^2 z^2 + i\overline{S}(\partial\!\!\!/ + \mu\Gamma^{1234})S\right], \tag{4.7}$$

where S is Majorana on the worldsheet and has positive chirality under $SO(8)$ rotations of the z^i. Note also that the length of the string is proportional to its momentum p^+, in

[2] As we have already argued in §1, this regime is intermediate between that of gauge theory degrees of freedom and giant gravitons. According to [78], giant gravitons correspond to certain "subdeterminant" operators in gauge theory, the maximal example of which is $\mathcal{O}_{gg} \sim \det X$. That is, such operators are the orthogonal basis of states in the sector where $J \sim N$.

keeping with the proposed UV-IR correspondence, and that the presence of Γ^{1234} breaks the bosonic $SO(8)$ to $SO(4) \times SO(4)$.

Of course, this action has a straightforward quantization (even the zero modes are now harmonic oscillators), and in analogy with the flat space lightcone string, it has a Hamiltonian

$$2p^- = -p_+ = H = \sum_{n=-\infty}^{\infty} N_n \sqrt{\mu^2 + \frac{n^2}{(\alpha' p^+)^2}}. \qquad (4.8)$$

Here we see that the theory is controlled by the parameter ${\lambda'}^{-1} \mu p^+ \alpha'$, and that varying it takes us between two distinct limits. In the first, $\lambda' \gg 1$, the effect of μ is small, and we recover the spectrum in flat space. This corresponds to scaling out flux from the plane wave background. In the second case, $\lambda' \ll 1$, the second term under the square root becomes unimportant, and the string becomes an infinite series of nearly degenerate oscillators. This is the case in which the RR flux dominates over the curvature, and is the limit in which most tractable computations can be performed.

Let us see what this means for gauge theory. Rewriting (4.8) as

$$(\Delta - J) = \sum_{n=-\infty}^{\infty} N_n \sqrt{1 + \frac{4\pi g N}{J^2} n^2}, \qquad (4.9)$$

where we have scaled out $\mu \to 1$ and used the definition of p^+ in (4.5) along with $R^4 = 4\pi g N {\alpha'}^2$. Notice that $\lambda' = \frac{4\pi g N}{J^2}$ remains finite in this limit,[3] and that it, not the 't Hooft coupling, is now the relevant parameter.

Turning again to the gauge theory, we see that the relevant basis of operators is a restricted sector of what was $\mathcal{N} = 4$ super Yang-Mills. According to the limit we have taken, keeping the momenta (4.5) finite requires both $\Delta - J$ fixed and $J \sim \sqrt{N}$. Thus we are interested in gauge theory operators with large R-charge J but finite twist. Classifying these operators by their $\Delta - J$ eigenvalues and writing Z for the complex combination of $SO(6)$ scalars whose J-charge is 1, the first state we can write down has $\Delta - J = 0$

$$\frac{1}{\sqrt{J} N^{J/2}} \mathrm{Tr}\,[Z^J] \quad \longleftrightarrow \quad |0, p_+\rangle \qquad (4.10)$$

[3] We have also kept g fixed.

and should be identified as the ground state of a single string with light cone momentum p_+ [70]. Each Z, of which there are p^+, is a single "bit" of the string, and excited states can be built from (4.10) by acting on it with operators of nonzero $\Delta - J$. These operators—either broken $SO(6)$ generators, covariant derivatives, or supersymmetry generators—will replace Z's with other complex scalars, $D_i Z$'s, or fermions, and the new state will have twist 1. In fact, in our limit of large N, such excitational impurities can act several times in succession, producing a string with multiple modified bits.[4] Thus new operators will be of the form (writing ϕ and ψ as the other $SU(3)$ scalars)

$$\mathrm{Tr}\,[\phi^r Z^{J-r}], \quad \mathrm{Tr}\,[\phi^r Z^k \psi^s Z^{J-r-s-k}], \quad \mathrm{Tr}\,[\phi Z \psi Z \phi \ldots], \quad \text{etc.} \tag{4.11}$$

Of course, in the large-N limit, planarity implies that we cannot reorder these "words" at will. The only symmetry we can impose is cyclicity of the trace.

And yet the above examples are insufficient to serve as string states at nonzero momentum. The correct prescription, according to [70], is to insert each impurity with a position-dependent phase:

$$|1, p_+\rangle \quad \longleftrightarrow \quad \frac{1}{\sqrt{J}} \sum_{l=1}^{J} \frac{1}{\sqrt{J} N^{(J+1)/2}} \mathrm{Tr}\,[Z^l \phi Z^{J-l}] e^{2\pi i n l/J}. \tag{4.12}$$

Such a state, however, clearly vanishes by cyclicity of the trace, a statement we shall justify more easily below. The next attempt, then, consists of two inserted impurities and takes the form

$$|1, p; p_+\rangle \quad \longleftrightarrow \quad \frac{1}{\sqrt{J}} \sum_{l=1}^{J} \frac{1}{\sqrt{J} N^{J/2+1}} \mathrm{Tr}\,[\phi Z^l \psi Z^{J-l}] e^{2\pi i n l/J}. \tag{4.13}$$

This does not vanish, and represents a string with relative momentum between the ϕ and ψ impurities.

We understand this as follows: In the case of the massive worldsheet, the lightcone spectrum is generated by eight bosonic and eight fermionic operators, a pair for each of the transverse directions. These correspond in the gauge theory case to the four non-Z scalars, the four covariant derivatives, and the eight fermionic partners to the scalars. Acting on the

[4]Along with the large-N limit, however, we will restrict the numbers of these impurities such that they remain dilute along the string.

worldsheet vacuum with an operator $(a_p^i)^\dagger$ thus produces a state which must be projected out of the spectrum, as it fails to satisfy the condition $L_0 - \bar{L}_0 = 0$. Two impurities, on the other hand, may be generated by acting with $(a_p^i)^\dagger (a_{-p}^j)^\dagger$, which obeys the $L_0 - \bar{L}_0$ constraint. Thus our putative single-impurity state above was forced to vanish while our two-impurity state survived, and this projection of the spectrum was accomplished by the cyclicity of the trace. In this way, we build up the Fock space of the plane wave string, reproducing its spectrum from the gauge theory side.

Thus far, we have only established that the numbers of states in the string theory and gauge theory are equal. We have not, as yet, reproduced (4.9) beyond leading order for $\lambda' \ll 1$. However, in order to calculate the anomalous dimensions of our BMN operators beyond leading order, we must take into account interaction terms in the gauge theory, terms of the form

$$\mathcal{L} \quad \supset \quad g\mathrm{Tr}\left([Z,\phi^j][\bar{Z},\phi^j]\right) \quad + \quad \sqrt{g}\mathrm{Tr}\left([Z,\bar{\theta}]\theta\right) \tag{4.14}$$

with $g \sim g_s \sim g_{YM}^2$. These interactions, in diagrammatic form, operate by exchange or splitting of bits, and, at the planar level, they do so only between nearest neighbors. As such, they will renormalize the naive $\Delta - J$ calculation and reproduce the full square root of (4.9) as calculated in [70].

What's more, these basic interactions, when exponentiated, can be rewritten as the action of a scalar field which decomposes spatially as a sum of creation and annihilation operators acting on each bit. This field, as was shown in [70], then has an effective Hamiltonian

$$H \sim \int_0^{p^+} d\sigma\, \frac{1}{2}\left[\dot{X}^2 + X'^2 + X^2\right], \tag{4.15}$$

which is precisely that of a worldsheet embedding coordinate. It is this computation which justifies the identification of the single-trace operator as the discretized worldsheet of a string with length proportional to its light cone momentum, as well as the identification of the anomalous dimension of the gauge theory with the value of the light cone Hamiltonian.

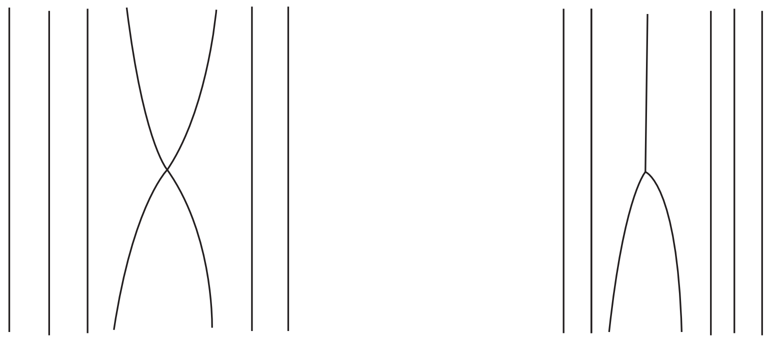

Figure 4.1: Exchange interactions on the BMN string. In (a), nearest neighbor bosons bits interact via the double-commutator, four-scalar interaction. In (b), a pair of fermionic bits combine to form a boson. The crossing-symmetric versions of these interactions take place as well.

However, we have said nothing of string interactions, those interchanges which would split and join the string worldsheet. Presumably, these are calculable in a light cone string field theory [109] in plane wave background, and the process should also be describable in the language of gauge theory. Subsequent to [70], this calculation of strings in light cone gauge was carried out [82, 92] and found to agree exactly[5] with the gauge theory result we now proceed to describe.

From the vantage of gauge theory, the process of string splitting appears to have a simple interpretation: If $\mathrm{Tr}\,[Z^J]$, or some other, more complicated impurity state, represents a single string, then multiple strings must live in the tensor product of such spaces, and a two-string state must take the form of a double-trace operator:

$$|2,k,y\rangle \quad \longleftrightarrow \quad \frac{1}{\sqrt{J_1(J-J_1)N^{J+2}}}\sum_{l=1}^{J_1} \mathrm{Tr}\,[\phi Z^l \psi Z^{J_1-l}]\,\mathrm{Tr}\,[Z^{J-J_1}]\,e^{2\pi i k l/J_1} \tag{4.16}$$

This suggestion was first put forward in [84], and was used there to calculate the three-point string vertex by the overlap of a long single-trace with two shorter double-trace operators. Actual string splittings, in this language, corresponded to non-planar diagrams, with the genus expansion parameter not the 't Hooft $1/N$ but its BMN modification $g_2 = J^2/N$. In this way, it was argued, the gauge theory two-point function would reproduce all multi-string

[5] After some mutual confusion among gauge theory, string-bit, and string field theory collaborations.

scattering amplitudes order-by-order in λ', provided that the pair of operators contained appropriate numbers of traces for the in and out Hilbert spaces of the strings. Unfortunately, such an approach proved inadequate for reasons we will discuss below, and its extension to finite g_2 proved one of the central achievements of the string bit model.

4.3 String Bits

As we have already seen, strings in plane wave backgrounds may be viewed as discretized worldsheets whose interactions are given by large-N Yang-Mills theory. In this limit, there are two new parameters, corresponding to the genus factor and the 't Hooft coupling of normal large-N gauge theory, which are modified in the correspondence to $g_2 \equiv J^2/N$ and $\lambda' = 8\pi^2 gN/J^2$.[6] Perturbation theory is thus a dual expansion in these two numbers, both of which have most often been taken in the literature as small [18]. Quantities which are well-defined on both sides of the correspondence—most interestingly string splitting and joining amplitudes—are then calculable in both gauge theory and string field theory in light cone gauge. There is, however, a third approach, based on the combinatorics developed for long string twisted sectors, which reproduces both of these and serves as a bridge between the two. It is this "string-bit" model which we intend to describe.

The inspiration for this approach is of course matrix string theory [79, 80], originally conceived as a IIB matrix formulation of non-perturbative string theory T-dual to the matrix theory of Type IIA. Matrix strings are thus governed by an $\mathcal{N} = 8$ $d = 2$ super Yang-Mills theory, the eigenvalues of its matrix-valued fields representing bit-like "short strings" which join under permutation symmetry into the "long strings" of conventional string theory. If we denote by $\mathcal{M}$ the Hilbert space of a single short string, the full multi-string space is then equivalent to the quotient of its N-fold product by an orbifold action:

$$\mathcal{H} = \mathrm{Sym}^N \mathcal{M} = \mathcal{M}^N / S_N, \tag{4.17}$$

where S_N is the permutation group on eigenvalues. For this reason, the correct classifica-

[6] Here we change the definition of λ' by a trivial constant factor

tion of states is based upon the enumeration of twisted sectors, and the full Hilbert space decomposes into a sum over conjugacy classes of the permutation group [81]:

$$\mathcal{H} = \bigoplus_{\gamma} \mathcal{H}_\gamma. \tag{4.18}$$

Most importantly, as argued in [79, 80], interactions take place by the splitting and joining of long string conjugacy classes, effected by an interaction which is composed of twist operators on the string worldsheet which interchange bosonic and fermionic matrix eigenvalues.

All of these elements will reappear in strikingly similar contexts in our consideration of string bits. In fact, as argued in [71], RG relations are expected to link the two as the string bit size becomes much larger than the length scale set by the worldsheet mass. Fortunately, in the opposite limit, even as we lose the power of worldsheet techniques, the combinatorial apparatus of the bit model provides us with an effective means of replicating gauge theory calculations.

The program of this section, then, is as follows: First, we review the basic formulation of the string bit model and its attendant discretized supersymmetry algebra, identifying the structure of the multi-string Hilbert space as a sum of permutation cycles. Further, we comment on the imposition of the $L_0 - \bar{L}_0$ condition, finding it as a direct result of the formalism. Then, we discuss the problem of operator mixing at finite g_2 and its solution in terms of a redefined inner product which contains terms of all orders in the genus expansion. Lastly, we introduce the framework for a supersymmetry algebra at finite g_2 containing string interactions.

We begin by discretizing the string worldsheet to a collection of J bits satisfying canonical commutation relations, labeled by an index n:

$$[\,p_n^i, x_m^j\,] = i\delta^{ij}\delta_{mn}\,, \qquad \{\theta_n^a, \theta_m^b\} = \frac{1}{2}\delta^{ab}\delta_{mn}\,, \qquad \{\tilde{\theta}_n^a, \tilde{\theta}_m^b\} = \frac{1}{2}\delta^{ab}\delta_{mn}. \tag{4.19}$$

For a conjugacy class γ of the permutation group corresponding to a single string, the action of the centralizer subgroup C_γ then acts on the bits according to

$$\{\,p_n^i, x_n^i, \theta_n^a\} \;\rightarrow\; \{\,p_{\sigma(n)}^i, x_{\sigma(n)}^i, \theta_{\sigma(n)}^a\}\,, \qquad \sigma \in C_\gamma. \tag{4.20}$$

The result is that the J bits are partitioned into groups of cyclic permutations, and the state γ becomes the product

$$(J_1)(J_2)\dots(J_s), \tag{4.21}$$

each cycle of some length J_ℓ. Interpreting J_ℓ as the discrete light cone momentum, invariance under the centralizer group then corresponds to translation of each bit along its string (but not between strings) by the operation of

$$U_\ell = e^{2\pi i(L_0^\ell - \bar{L}_0^\ell)/J_\ell}. \tag{4.22}$$

As with cyclicity of the trace in gauge theory, invariance under such a symmetry imposes for us the $L_0 - \bar{L}_0$ constraint.

In the same way, then, we should discretize the supersymmetry algebra of [76], graded by powers of the 't Hooft coupling $\lambda^2 = g_{YM}^2 N$:

$$Q_{0,1} = Q^{(0)} + \lambda\, Q^{(1)}\,, \qquad Q_{0,2} = \tilde{Q}^{(0)} - \lambda \tilde{Q}^{(1)}\,, \qquad H_0 = H^{(0)} + \lambda\, H^{(1)} + \lambda^2 H^{(2)} \tag{4.23}$$

As we saw in §4.2, the $H^{(1)}$ and $H^{(2)}$ terms contain gauge interactions which produce "hopping" of string bits at the nearest-neighbor level. In terms of the phase space variables, the interactions can be written explicitly:

$$Q^{(0)} = \sum_n (p_n^i \gamma_i\, \theta_n - x_n^i(\gamma_i \Pi)\tilde{\theta}_n)\,, \qquad Q^{(1)} = \sum_n (x^i_{\gamma(n)} - x^i_n)\gamma_i\, \theta_n \tag{4.24}$$

$$H^{(0)} = \sum_n (\frac{1}{2}(p_{i,n}^2 + x_{i,n}^2) + 2\mathrm{i}\, \tilde{\theta}_n \Pi \theta_n)\,, \tag{4.25}$$

$$H^{(1)} = -\sum_n \mathrm{i}(\theta_n \theta_{\gamma(n)} - \tilde{\theta}_n \tilde{\theta}_{\gamma(n)})\,, \qquad H^{(2)} = \sum_n \frac{1}{2}(x^i_{\gamma(n)} - x^i_n)^2\,. \tag{4.26}$$

Lastly, we will introduce the permutation operator Σ_{nm}, defined by its action on individual bits

$$\Sigma_{nm} X_m = X_n \Sigma_{nm}, \quad \Sigma_{nm} X_k = X_k \Sigma_{nm}, \qquad k \neq m, n \tag{4.27}$$

($X_n = \{p_n^i, x_n^i, \theta_n^a\}$), along with the permutation invariant operator

$$\Sigma \equiv \frac{1}{J^2} \sum_{n<m} \Sigma_{nm}. \tag{4.28}$$

At the conclusion of §4.2, we mentioned a puzzle which concerned the genus expansion: that while to lowest order, two-string states in string field theory corresponded to double-trace operators in Yang-Mills, and string splitting corresponds to what appears to be a three-point function, the situation is actually more complex. Here, we will elaborate on that somewhat cryptic claim, proceeding thereby to an evaluation of amplitudes in the string bit model.

Put simply, the conundrum is as follows: Even at $\lambda = 0$, when the Yang-Mills theory is non-interacting, there still exists nontrivial overlap between BMN operators. That is to say, even without interactions, there exists a genus expansion of correlation functions in g_2 to all orders. Thus correlation functions involving single BMN strings can and do involve double-trace operators as intermediate states, and such an effect induces operator mixing at large-N.

Clearly, this produces problems for the proposed identification of numbers of strings with numbers of traces. As $\lambda' \to 0$, string splitting effects vanish, and the gauge theory operator which represents a single string cannot be allowed to mix with those operators which correspond to two or more. The operators we identify with multi-string states must diagonalize the $\lambda' = 0$ gauge theory Hamiltonian, and for this reason, we must redefine our inner product at finite g_2.

To this end, then, we start by defining

$$\langle \psi_1 | \psi_2 \rangle_{g_2} = \langle \psi_1 | S | \psi_2 \rangle_0, \tag{4.29}$$

with S a weighted sum over permutations

$$S = \sum_\sigma N^{-2h(\sigma)} \Sigma_\sigma. \tag{4.30}$$

This operator, which has the large-N expansion

$$S = 1 + \frac{1}{N}\Sigma_2 + \frac{1}{N^2}\Sigma_3 + \dots \tag{4.31}$$

is a sum over successive n-permutation operators, each of which might be said to constitute a single splitting or joining of the bit string. For instance, the first term

$$\Sigma_2 = \sum_{n<m} \Sigma_{(nm)} \tag{4.32}$$

is the simple permutation discussed above, while the second nontrivial operator will either split the initial string into three or split and recombine the strands into a single outgoing cycle.

Unfortunately, expressions for the higher-order splitting-joining operators are unwieldy and computationally cumbersome. However, it was argued in [72] that all such higher operators can be written as a sum of powers of (4.32) and terms which become negligible in the large-N limit.[7] Thus we can, up to higher terms in $1/N$, rewrite the change of basis operator as

$$S = e^{g_2 \Sigma}\,, \qquad\qquad \Sigma \equiv \frac{1}{J^2}\,\Sigma_2\,, \tag{4.33}$$

using the total permutation operator of (4.28). This redefined inner product now gives the correct overlap between string states at zero coupling, successfully reproducing the genus expansion of the $1 \to 1$ string amplitude [72].

Having resolved the question of identifying gauge operators with strings, we are now left with another challenge—to incorporate string interactions into the supersymmetry algebra. Such a task will require evaluating matrix elements of the form

$$\langle\, \psi_2 | H \,|\, \psi_1 \rangle_{g_2} \;=\; \langle\, \psi_2 | S H \,|\, \psi_1 \rangle_0\,, \tag{4.34}$$

and for consistency, this will require hermiticity of H:

$$H = H^\dagger = S^{-1} H^{\dagger 0} S, \tag{4.35}$$

where $H^{\dagger 0}$ denotes the hermitian conjugate relative to the bare inner product. In addition, we will also require closure of the supersymmetry algebra:

$$\delta^{IJ} \{ Q^{\dot a}_I, Q^{\dot b}_J \} \;=\; \delta^{\dot a \dot b} H \,+\, J^{\dot a \dot b}\,, \tag{4.36}$$

[7] This is essentially a result of our "dilute impurity" approximation for BMN strings. In the large-N limit, the probability that successive splits or joins occur at the same site falls off as $1/N$.

where $J^{\dot{a}\dot{b}}$ is a suitable contraction of gamma matrices with the $SO(4) \times SO(4)$ Lorentz generators J^{ij} (see [76]).

To accomplish this, we will write the free supersymmetry generators in schematic form as a sum of left- and right-acting operators[8]

$$Q_0 = Q_0^{>} + Q_0^{<} \, . \tag{4.37}$$

While there exist explicit expressions for these charges in terms of creation and annihilation operators [72], it will suffice here to stipulate that $Q_0^{<}$ will only be modified to split or join strings to the right and $Q_0^{>}$ will only, when modified, induce string interactions on the left. In the gauge theory, these would correspond to terms of the form $\mathrm{Tr}\,(\theta[\bar{Z},\bar{\phi}])$, which have a double contraction with the incoming and single contraction with the outgoing state.

As we will see in §4.4 a consistent ansatz for these interacting supercharges can be found, one which shares, up to a sign, the original form posited by Verlinde [71]—a commutator of the free algebra with Σ. Moreover, we will address ourselves to the evaluation of correlations of multi-trace operators, relating them to the splitting and joining amplitudes of strings in light cone string field theory. We will find, as advertised, exact agreement among all three approaches.

4.4 Tracing the String

4.4.1 Introduction and Philosophy

The BMN correspondence [70] equates type IIB string theory on a plane wave background with a certain limit of $\mathcal{N}=4$ gauge theory at large R-charge J, where N is taken to infinity while the quantities

$$\lambda' = \frac{g_{\mathrm{YM}}^2 N}{J^2}, \qquad g_2 = \frac{J^2}{N} \tag{4.38}$$

are held fixed. The proposal is based on a natural identification between the basis of string theory states and the basis of gauge theory operators, and between the light-cone string

[8] In what follows, we will ignore nonlinear fermionic contributions to the supersymmetry generators, as they do not affect the relevant bosonic terms in the Hamiltonian.

Hamiltonian P^- and the generator Δ of conformal transformation in the gauge theory via[9]

$$\frac{2}{\mu}P^- = \Delta - J\,. \tag{4.39}$$

BMN argued, and it was subsequently confirmed to all orders in λ' [85, 112], that this identification holds at the level of free string theory ($g_2=0$).

This beautiful proposal equates two operators which act on completely different spaces: the light-cone Hamiltonian P^- acts on the Hilbert space of string field theory, and allows for the splitting and joining of strings, while $H \equiv \Delta - J$ acts on the operators of the field theory, and in general mixes single-trace operators with double- and higher-trace operators. Light-cone string field theory in the plane wave background has been constructed in [82, 92]. On the field theory side, a number of impressive papers [83, 90, 84, 114, 93, 98, 94, 115] have pushed the calculations to higher order in g_2 with the aim of showing that (4.39) continues to hold, thereby providing an equality between a perturbative, interacting string theory and perturbative $\mathcal{N}=4$ gauge theory. It is clear, however, that at finite g_2 the natural identification between single string states and single trace operators breaks down. For example, 1-string states are orthogonal to 2-string states for all g_2, but single-trace operators and double-trace operators are not. This raises the question how to formulate the BMN correspondence in the interacting string theory.

In order to prove that two operators in (4.39) are equal, it is sufficient to prove that they have the same eigenvalues. If they do, then there is guaranteed to exist a unitary transformation between the spaces on which the two operators act. A basis independent formulation of the BMN correspondence, therefore, is that the interacting string field theory Hamiltonian $\frac{2}{\mu}P^-$ and the gauge theory operator H must have the same eigenvalues.

While this is the minimum that we are allowed to expect from the BMN correspondence, we can hope to do better. Light-cone string field theory, as formulated in [113, 82, 92], comes

[9]The parameter μ can be introduced by performing a boost and serves merely as a bookkeeping device.

with a natural choice of basis: this string basis (of single and multiple strings) is neither the BMN basis (of single and multiple traces) nor the basis of eigenstates of the light-cone Hamiltonian. But how do we identify the string basis in the gauge theory?

One guess for the string basis was made in [84, 94], where it was argued that matrix elements of P^- between 1- and 2-string states should be equated with the coefficient of the three-point function of the corresponding BMN operators, multiplied by the difference in conformal dimension between the incoming and outgoing states. This proposal appeared to be supported by the subsequent string field theory calculation done in [92] (see also [106, 107, 110, 116, 118, 117]). It turns out, however, that the final step of the calculation in [92] suffered from a minus sign error (which we will correct below), which renders the alleged confirmation of this proposal invalid.

In this paper we propose a new, specific form for the transformation between the BMN basis and the string basis, valid to all orders in g_2. This basis transformation is trivial to write down, and has the pleasing feature that it does not depend on the conformal dimensions of the operators. In fact, our choice of transformation was already identified as a natural choice in [72], where it was shown that all computed amplitudes in gauge theory are reproduced via a relatively simple string bit formalism [77, 71, 103]. While most calculations in [72] were done in the BMN basis, it was pointed out that there exists a basis choice with the properties that (i) the inner product is diagonal, and (ii) the matrix elements of the supersymmetry generators Q are at most linear in g_2 (i.e. Q leads to only a single string splitting or joining). Here we will show that, when evaluated in this new basis, the matrix elements of the string bit Hamiltonian, which via the results of [72] may be identified with the gauge theory operator on the right-hand side of (4.39), agree precisely with the corrected answer of [92] for the matrix elements of the continuum string field theory Hamiltonian P^- appearing on the left-hand side!

The precise match between the three point functions means that, by combining the two formalisms, we can start filling in some important questions left open in [82] and [72]. A major technical obstacle in continuum light-cone string field theory is that higher order contact terms are needed for closure of the supersymmetry algebra, and that their value (at order g_2^2) affects the leading order shift in the eigenvalues of P^-. However, these contact terms are difficult to compute [100]. The supersymmetry algebra of the bit string theory, on the other hand, is known to all orders in g_2 but only to linear order in the fermions. It appears to be a fruitful strategy, therefore, to make use of the discretized theory to fix the order g_2^2 contact terms of the continuum theory, while the known non-linear fermionic form of the continuum interaction vertex may be of direct help in deriving the complete supersymmetry generators in the string bit formalism.

4.4.2 Identification of the String Basis in Gauge Theory

$\mathcal{N}=4$ gauge theory in the BMN limit comes with a natural choice of basis, which coincides with the natural string basis when $g_2=0$: an n string state corresponds to a product of n single trace BMN operators. We call this basis the BMN basis, denoted by $|\psi_n\rangle$. At non-zero g_2, the inner product (defined as the overlap as computed in the free gauge theory) becomes non-diagonal in this basis. The explicit form of the inner product is conveniently expressed in terms of the string bit language of [71, 72] as

$$\langle\psi_m|\psi_n\rangle_{g_2} = \left(e^{g_2\Sigma}\right)_{nm}, \qquad \Sigma = \frac{1}{J^2}\sum_{i<j}\Sigma_{ij} \tag{4.40}$$

where Σ_{ij} is the operator which interchanges the string bits via the simple permutation (ij). As explained in [71, 72], when acting on a BMN state $|\psi\rangle$ with n strings, Σ effectuates a single string splitting or joining.

This meaning of Σ in the gauge theory language can be made concrete as follows. Consider a long BMN string in its ground state. We can write the corresponding operator as

$$\mathcal{O}_J(\gamma) = \mathrm{Tr}(Z^J) = \sum_{\substack{i_1 \ldots i_J \\ \bar{i}_1 \ldots \bar{i}_j}} Z_{i_1\bar{i}_1}\, Z_{i_2\bar{i}_2} \ldots Z_{i_J\bar{i}_J}\; \delta^{\bar{i}_1 i_{\gamma(1)}}\, \delta^{\bar{i}_2 i_{\gamma(2)}} \ldots \delta^{\bar{i}_J i_{\gamma(J)}} \tag{4.41}$$

with $\gamma = (12 \ldots J)$ the cyclic permutation of J elements. The action of $\Sigma_{J_1 J}$, which implements the simple permutation $(J_1 J)$, is now defined as

$$\Sigma_{J_1 J}\, \mathcal{O}_J(\gamma) = \mathcal{O}_J(\gamma \circ (J_1 J)) \tag{4.42}$$

Since $\gamma \circ (J_1 J) = (1 \ldots J_1 {-} 1\, J)(J_1 \ldots J-1)$ we have that

$$\Sigma_{J_1 J}\, \mathcal{O}_J(\gamma) = \mathrm{Tr}(Z^{J_1})\mathrm{Tr}(Z^{J-J_1}), \tag{4.43}$$

showing that the simple permutation $\Sigma_{J_1 J}$ indeed induces a single splitting of a single trace into a double trace operator. It is easy to generalize this result to other operators, to show that Σ can either split a string or join two strings.

The identification of (4.40) with the inner product of the free gauge theory was motivated in [72] and explicitly verified for string ground states to all order in g_2 and for two-impurity states to order g_2^2.

States with different number of strings are therefore no longer orthogonal relative to (4.40). In the string field theory basis $|\tilde{\psi}_n\rangle$, on the other hand, the inner product should be diagonal for all g_2. The simplest basis transformation that achieves this goal is

$$|\widetilde{\psi}_n\rangle = (e^{-g_2\Sigma/2})_{nm}|\psi_m\rangle\,. \tag{4.44}$$

This is not the most general diagonalization, however, since we still have the freedom to redefine the new basis $|\widetilde{\psi}_m\rangle$ via an arbitrary unitary transformation [98]. The above redefinition (4.44), however, has the attractive feature that it is purely combinatoric and does not depend on the dynamics of the gauge theory. Furthermore, as we will see shortly, it has the desirable property that the (linearized) supersymmetry generators and light-cone Hamiltonian acquire a simple form in the new basis. We emphasize that the only way to check the proposal (4.44) for identifying the string field theory basis in the gauge theory is by comparing matrix elements of H calculated in the $|\widetilde{\psi}_n\rangle$ basis to those of $\frac{2}{\mu}P^-$ in light-cone string field theory. We show below that the proposal (4.44) passes this test.

In the following, we will study the consequences of this basis transformation for the specific class of two-impurity BMN states investigated in [83, 93, 84, 94, 98]. We will denote by $|1,p\rangle$ the normalized state corresponding to the single trace operator $\sum_l e^{2\pi ipl/J}\mathrm{Tr}(\phi Z^l \psi Z^{J-l})$, while $|2,k,y\rangle$ and $|2,y\rangle$ will denote the normalized states corresponding to the double trace operators $\sum_l e^{2\pi ikl/J_1}\mathrm{Tr}(\phi Z^l \psi Z^{J_1-l})\mathrm{Tr}(Z^{J-J_1})$ and $\mathrm{Tr}(\phi Z^{J_1})\mathrm{Tr}(\psi Z^{J-J_1})$ respectively, where $y = J_1/J$. The action of Σ on $|1,p\rangle$ reads

$$\Sigma\,|1,p\rangle = \sum_{k,y} C_{pky}\,|2,k,y\rangle \;+\; \sum_y C_{py}|2,y\rangle\,, \tag{4.45}$$

with

$$C_{pky} = \sqrt{\frac{1-y}{Jy}}\,\frac{\sin^2(\pi py)}{\pi^2(p-k/y)^2}, \qquad C_{py} = -\frac{\sin^2(\pi py)}{\sqrt{J}\pi^2 p^2}. \tag{4.46}$$

Via (4.44) we now introduce the corresponding two-impurity states in the string basis, which we will denote by $|\tilde{1},p,y\rangle$, $|\tilde{2},k,y\rangle$ and $|\tilde{2},y\rangle$, respectively. By construction, these form an orthonormal basis at finite g_2.

4.4.3 Interactions in the String Basis

In this section we obtain the matrix elements of the right-hand side of (4.39) in the string basis proposed in the previous section. For this we will employ the string bit model of [72], but by virtue of the established correspondence with the gauge theory amplitudes of [93, 94], the following calculation can also be viewed as a direct calculation within the gauge theory.

It was shown in [72] that the linearized (in the fermions) interacting supercharges in the string bit model can be written in the string basis as

$$Q = Q_0 + \frac{g_2}{2}[\widehat{Q}_0, \Sigma]\,, \qquad \widehat{Q}_0 = Q_0^{<} - Q_0^{>}\,, \tag{4.47}$$

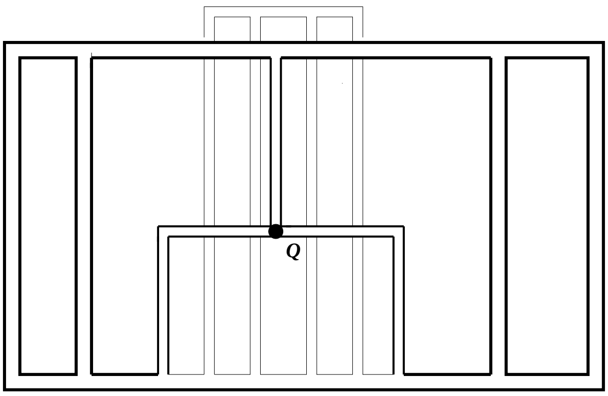

Figure 4.2: An insertion of the supercharge in the gauge theory will lead to a single splitting of the BMN string with which it has a double contraction.

where $Q_0 = Q_0^< + Q_0^>$ is the free supercharge of the bit string theory and the superscripts indicate the projection onto the term with fermionic creation ($<$) or annihilation ($>$) operators only. These charges generate the interacting superalgebra of string theory in the plane wave background, modulo higher order terms in the fermions. Our interest is to compute the matrix elements of H between the bosonic two-impurity states in the string basis.

The supercharge (4.47) truncates at linear order in g_2. As indicated in Figure 4.2, this truncation is expected from the BMN correspondence: matrix elements of the supercharges $Q = \mathrm{Tr}\,\theta[Z,\phi]$ in the gauge theory can lead (for connected diagrams, where Q has at least one contraction with either the "in" or "out" BMN state) to at most one single string splitting or joining [72]. For now, however, one may view (4.47) as a new starting point of the string bit model; in the remainder we will re-establish its equivalence with the perturbative gauge theory, by showing that, at least for the special class of two-impurity states, it leads to the same order g_2^2 mass renormalization (shift in conformal dimension) as computed in [93, 94].

From (4.47) we thus deduce that the interacting Hamiltonian truncates at order g_2^2:

$$H = H_0 + g_2 H_1 + g_2^2 H_2 \tag{4.48}$$

with[10]

$$H_0 = \{Q_0, Q_0\}\,, \qquad H_1 = \{Q_0, [\widehat{Q}_0, \Sigma]\}\,, \qquad H_2 = \frac{1}{4}\{[\widehat{Q}_0, \Sigma], [\widehat{Q}_0, \Sigma]\}\,. \tag{4.49}$$

From these expressions, it is straightforward to compute the matrix elements between the various two-impurity states. Using that $Q_0^>$ annihilates bosonic states, we find that

$$\langle\widetilde{\psi}_2|H_1|\widetilde{\psi}_1\rangle = \frac{1}{2}\langle\widetilde{\psi}_2|(H_0\Sigma + \Sigma H_0)|\widetilde{\psi}_1\rangle - 2\langle\widetilde{\psi}_2|Q_0^>\,\Sigma\, Q_0^<\,|\widetilde{\psi}_1\rangle. \tag{4.50}$$

Both matrix elements on the right-hand side have been computed in [72], with the result

$$\langle\tilde{2}, k, y|(H_0\Sigma + \Sigma H_0)|\tilde{1}, p\rangle \;=\; \lambda'(p^2 + k^2/y^2)\, C_{pky}\,, \tag{4.51}$$

$$\langle\tilde{2}, k, y|\, Q_0^>\Sigma\, Q_0^<\,|\tilde{1}, p\rangle \;=\; \frac{\lambda'}{2}\,(pk/y)\, C_{pky}\,, \tag{4.52}$$

where C_{pky} are the three point functions defined in eqn (4.46). Inserting the explicit expressions, we find

$$\langle\tilde{2}, k, y|H_1|\tilde{1}, p\rangle = \frac{\lambda'}{2}\sqrt{\frac{1-y}{Jy}}\frac{\sin^2(\pi p y)}{\pi^2}, \qquad \langle\tilde{2}, y|H_1|\tilde{1}, p\rangle = -\frac{\lambda'}{2}\frac{1}{\sqrt{J}}\frac{\sin^2(\pi p y)}{\pi^2}. \tag{4.53}$$

In a similar way one can obtain the order g_2^2 matrix elements between single string states. We postpone this discussion to later, and turn now to the continuum string field theory.

4.4.4 Light-Cone String Field Theory

We now investigate the matrix elements of the left-hand side of (4.39) in the continuum string theory, in order to compare with the gauge theory results of the previous section. In light-cone string field theory the cubic interaction is conveniently represented as a state in the three-string Hilbert space. If we restrict our attention to string states which have no fermionic excitations, this state can be expressed as

$$\frac{2}{\mu}|P_1^-\rangle = -\frac{y(1-y)}{2}\mathcal{P}|V\rangle. \tag{4.54}$$

[10]The notation in these equations is somewhat symbolic: the left-hand side in each equation is equal to the projection onto the $\delta^{\dot{a}\dot{b}}$ component of the anti-commutator on the right-hand side. Furthermore, as stated above, this formula for the Hamiltonian is valid only for computing matrix elements between bosonic states; for fermionic states, the non-linear fermionic corrections to (4.47) will become relevant.

Here $|V\rangle$ is a squeezed state in the 3-string Hilbert space,

$$|V\rangle = \exp\left[\frac{1}{2}\sum_{r,s=1}^{3}\sum_{m,n=-\infty}^{\infty} a^{I\dagger}_{m(r)}\overline{N}^{(rs)}_{mn}a^{I\dagger}_{n(s)}\delta_{IJ}\right]|0\rangle, \tag{4.55}$$

and the prefactor $\mathcal{P}$ is given by

$$\mathcal{P} = \sum_{r=1}^{3}\sum_{m=-\infty}^{\infty}\frac{\omega_{m(r)}}{\mu\alpha_{(r)}}a^{I\dagger}_{m(r)}a^{J}_{-m(r)}v_{IJ}, \tag{4.56}$$

where $v_{IJ} = \mathrm{diag}(1_4, -1_4)$, $\omega_{m(r)} = \sqrt{m^2 + \mu^2\alpha^2_{(r)}}$, and $\alpha_{(r)} = \alpha' p^+_{(r)}$, with the convention that $\alpha_{(r)}$ is negative for incoming strings 1 and 2 and positive for the outgoing string 3. In order to write $\mathcal{P}$ in this form, we have employed a very useful factorization identity derived in [119, 95]. The sign error in the original version of [92] amounts, after tracing through some changes of basis, to replacing the a_{-m} by a_m in (4.56), making manifest the incorrect claim that the prefactor gives the difference of energy between incoming and outgoing states.[11]

Implicit formulas for the matrix elements of $\overline{N}$, valid for all λ', were presented in [82]. While explicit formulas for the leading terms in an expansion around $\lambda' = 0$ are known, it is very difficult to extract exact formulas for higher-order terms [111]. At $\lambda' = 0$ the only nonzero Neumann matrices are

$$\overline{N}^{(13)}_{kp} = \frac{(-1)^{k+p+1}}{\sqrt{y}}\frac{\sin(\pi p y)}{\pi(p - k/y)}, \tag{4.57}$$

$$\overline{N}^{(23)}_{kp} = -\frac{1}{\sqrt{1-y}}\frac{\sin(\pi p(1-y))}{\pi(p - k/(1-y))}. \tag{4.58}$$

Using the above formulas, it is straightforward to derive the leading $\mathcal{O}(\lambda')$ contribution to the matrix elements

$$\frac{2}{\mu}\langle\tilde{2}, k, y|P^-_1|\tilde{1}, p\rangle = \frac{\lambda'}{2}(1-y)\frac{\sin^2(\pi p y)}{\pi^2}, \tag{4.59}$$

$$\frac{2}{\mu}\langle\tilde{2}, y|P^-_1|\tilde{1}, p\rangle = -\frac{\lambda'}{2}\sqrt{y(1-y)}\frac{\sin^2(\pi p y)}{\pi^2}.$$

[11] The error can also be understood as a missing factor of i in eqn (3.15) of [82], as pointed out in [95].

This result, which corrects the one originally reported in [92], is in precise agreement with (4.53) after taking into account the factor $\sqrt{Jy(1-y)}$ which arises because (4.53) is written in terms of unit normalized states while (4.59) is expressed in terms of continuum states satisfying $\langle i|j\rangle = p_i^+ \delta(p_i^+ - p_j^+)$.

4.4.5 Contact Terms and Mass Renormalization

Having established that the matrix elements of H evaluated in the $|\widetilde{\psi}_n\rangle$ basis agree at order g_2 with those of $\frac{2}{\mu}P^-$ in the natural string field theory basis, let us now revisit the issues of contact terms and the one-loop mass renormalization of the single-string state $|\tilde{1}, p\rangle$.

A significant advantage of the $|\widetilde{\psi}\rangle$ basis is that the matrix elements of the supercharge (4.47) terminate at order g_2. Therefore the Hamiltonian terminates at order g_2^2, with the term H_2 which comes from squaring the order g_2 term in the supercharge; there is no need for all of the higher-order contact terms which seem to plague continuum light-cone string field theory. Turning this observation around gives a definite prediction for string field theory: that the only contact term surviving in the large μ limit is the one which comes from squaring the cubic vertex in the dynamical supercharge.

The order g_2^2 matrix element between single string states has been computed in [72] (see equation (71)), with the result

$$\begin{aligned} g_2^2\langle\tilde{1}, q|H_2|\tilde{1}, p\rangle &= \frac{g_2^2}{4}\langle\tilde{1}, q|\,[\,Q_0^>, \Sigma][\,\Sigma, Q_0^<\,]\,|\tilde{1}, p\,\rangle \\ &= \frac{g_2^2\lambda'}{4}\sum_i (k^2/y^2 - \frac{1}{2}(p^2+q^2))C_{pi}C_{qi} \;\equiv \frac{1}{4}\frac{g_2^2\lambda'}{4\pi^2}\,B_{pq}. \qquad (4.60) \end{aligned}$$

Here the sum runs over intermediate 2-string states i of both types: $|\tilde{2}, k, y\rangle$ and $|\tilde{2}, y\rangle$ (and includes an integral over y). The explicit form of B_{pq} is as given in [93, 94].

To obtain the order g_2^2 mass renormalization of the state $|\tilde{1}, p\rangle$, we should add the matrix element (4.60) to the iterated H_1 interaction:[12]

$$\sum_i \frac{|\langle\, \tilde{1}, p\,|\, H_1 |\, i\,\rangle|^2}{E_p - E_i} \;=\; \frac{1}{4} g_2^2 \lambda' \sum_i \frac{(p-k/y)^4 C_{pi}^2}{p^2 - k^2/y^2} \tag{4.61}$$

$$= \frac{1}{4} g_2^2 \lambda' \sum_i (p^2 - k^2/y^2) C_{pi} C_{-pi} \tag{4.62}$$

$$= -\frac{1}{4} \frac{g_2^2 \lambda'}{4\pi^2} B_{p,-p}. \tag{4.63}$$

An additional subtlety in the calculation is that the states $|\tilde{1}, p\rangle$ and $|\tilde{1}, -p\rangle$ are degenerate at lowest order; we should check therefore whether we need to use degenerate perturbation theory. It is easy to see that

$$\sum_i \frac{\langle\, \tilde{1}, p\,|\, H_1 |\, i\,\rangle\langle\, i\,|\, H_1 |\, \tilde{1}, -p\rangle}{E_p - E_i} = -\frac{1}{4} \frac{g_2^2 \lambda'}{4\pi^2} B_{p,-p} \tag{4.64}$$

gives the same result as (4.61). The sum of the contact term H_2 and the iterated H_1 interaction is diagonal in the $\{|\tilde{1}, p\rangle, |\tilde{1}, -p\rangle\}$ basis, signalling that the degeneracy remains unbroken to this order.[13]

Putting everything together, we find that the order g_2^2 contribution to the eigenvalue is

$$\frac{1}{4} \frac{g_2^2 \lambda'}{4\pi^2} (B_{pp} - B_{p,-p}) = \frac{g_2^2 \lambda'}{4\pi^2} \left(\frac{1}{12} + \frac{35}{32\pi^2 p^2} \right). \tag{4.65}$$

This agrees precisely with the shift in anomalous dimension of the conformal eigen-operators, as reported in [93, 94].

4.4.6 Conclusion

We have proposed an explicit form (4.44) of the basis transformation that relates single and multi-trace BMN operators in the gauge theory to single and multi-string states of the dual string field theory in the plane wave background to all orders in g_2. This basis

[12] Here in the second step we use that $C_{-pky} = \frac{(p-k/y)^2}{(p+k/y)^2} C_{pky}$.

[13] In fact, supersymmetry requires these two states to be exactly degenerate [94]. The fact that we find degeneracy at this order is consistent with our observation that no additional contact terms are required for closure of the supersymmetry algebra.

transformation is natural from the point of view of the bit string theory of [72]: besides the fact that it diagonalizes the inner product, it has the property that the supersymmetry generators in the new basis truncate at linear order in g_2.

Our most encouraging result, however, is that the 3-point function in this basis precisely matches with the 3-string amplitude of the continuum string field theory [92]. In itself this match does not yet prove anything, because one can always find two bases that would lead to the same 3-point function. One also needs control over the order g_2^2 contact interactions before one can honestly compare the shift in the conformal dimensions in the gauge theory with the mass renormalization in the string theory [100]. However, we have more information than just the 3-point function: because the supersymmetry charge of the bit string model is linear in g_2, via the closure of the supersymmetry algebra we have a principle that uniquely determines the order g_2^2 contact interaction. It also tells us that any higher order contact terms are absent.

It would clearly be of interest to give a precise construction of the continuum limit of the bit string theory by taking the large J limit while keeping λ' fixed. The correspondence found in this paper is an encouraging indication that this continuum limit will coincide with the continuum light-cone string theory.

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