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Phenomenology of Higgs Bosons Beyond the Standard Model

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Abstract

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After a long history of searches, a Higgs boson H was discovered by the ATLAS and the CMS experiments at the Large Hadron Collider (LHC) in 2012. Its properties fit well the ones predicted by the Standard Model (SM) of particle physics. However, the SM can not explain other established properties of Nature, such as the existence of Dark Matter. For this reason, models beyond the SM should be considered. Such models often predict the existence of several Higgs bosons and this thesis explores some of those models. In particular, the possibility to discover a charged Higgs boson, which would be a clear sign of physics beyond the SM, is studied.

A commonly studied extension of the SM is the framework of two-Higgs-doublet models (2HDMs), where there are five Higgs bosons. By confronting the parameter spaces of some 2HDMs with publically available data from the LHC, the prospects for finding the 2HDM Higgs bosons is presented through the calculation of production cross sections and decay branching ratios in various channels.

A new kind of 2HDM, called the Stealth Doublet Model is presented and the properties of the Higgs bosons are studied. In this model, it is shown that in particular the properties of the charged Higgs boson $H^{\pm}$ have new features not exhibited in earlier studied models. Within the parameter space compatible with the LHC results, the production cross section for $H^{\pm}$ can be sizeable enough to be experimentally observed.

Finally, the discovery prospects at the LHC, for a $H^{\pm}$ in the $pp \rightarrow tH^{\pm}$ process, with the decays $H^{\pm} \rightarrow HW^{\pm}$ and $H \rightarrow bb$, is studied in various models beyond the Standard Model. It is shown that for the supersymmetric models, this channel is beyond the discovery reach of the LHC. In some of the other studied models, in particular the Aligned 2HDM, the situation is improved and the channel is feasible.

Keywords: Particle physics, The Standard Model, Higgs bosons, Higgs mechanism, Beyond the Standard Model, Two Higgs Doublet Models, Charged Higgs boson, Higgs decays, Supersymmetry, Renormalization, LHC, Collider constraints

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Dedicated to my family and friends

List of papers

This thesis is based on the following papers, which are referred to in the text by their Roman numerals.

I R. Enberg, J. Rathsman, G. Wouda, Higgs properties in a softly broken Inert Doublet Model, *Journal of High Energy Physics (JHEP)* **1308** (2013) 079, Erratum, *JHEP*, in press (2015), [arXiv:1304.1714] [hep-ph].

II R. Enberg, J. Rathsman, G. Wouda, Higgs phenomenology in the Stealth Doublet Model, *Submitted (2014)*, [arXiv:1311.4367] [hep-ph]

III R. Enberg, W. Klemm, S. Moretti, S. Munir, G. Wouda, Charged Higgs boson in the $W^{\pm}$ Higgs channel at the Large Hadron Collider, *Submitted (2014)*, [arXiv:1412.5814] [hep-ph].

IV R. Enberg, O. Stål, G. Wouda, A light second Higgs doublet in light of LHC data, *In manuscript (2015)*.

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Contents

1. Introduction

Science is a combination of theory and experiment and the two together are how you make progress.

Lisa Randall

In the research area of particle physics, also called *high energy physics*, one studies the properties of the smallest building blocks of matter – *particles*. The properties one is interested in are which particles exist in Nature and their masses, electric charges, lifetimes and interactions. In addition, one studies possible underlying structures behind the properties of particles. Often, one adds the prefix *elementary* to particle physics to imply that these particles are believed to have no substructure or constituents. Currently, only upper limits on the size of elementary particles are known [1, 2].

Particle physics is both an experimental and theoretical science and much progress has been made since the first quarter of the 20th century. On the experimental side, the equipment (accelerators and detectors) has grown larger and today requires several thousands of physicists and engineers to build and run. Up to date one has found, depending how one counts, seventeen particles which are classified as elementary, and where applicable, also their corresponding antiparticles. The latest particle to be found was a *Higgs boson*, in the autumn of 2012 [3, 4, 5].

Theoretically, particles are described as the excitations of *quantum fields*. This framework, called *quantum field theory* (QFT), has also developed a lot since its original formulation in the first half of the 20th century. The starting point for QFT is a Lagrangian (or an action) of classical fields, which are then subject to quantization.[1]

We have today an almost complete understanding of three of the four known interactions in Nature and the existing particles. Some of the major unsolved questions are if there is a reason for why a kind of particle called *neutrinos* is massive, what the *dark matter* surrounding e.g. galaxies is composed of, and if it is possible to describe the electromagnetic, the weak and the strong force as three manifestations of one *grand unified* force? Another important area is the structure of the so called *Higgs sector* which will be the major topic of this thesis.

[1] In standard textbooks, *canonical quantization* and *path integral quantization* are the most common methods.

The fourth known force, gravity, has not yet been consistently described as a QFT of particles. Instead, one tries to describe the elementary constituents of matter in terms of *strings* and *branes*. In this framework, particles are not point-like but are described as excitations of these extremely small fundamental strings, which are attached to branes in extra dimensions. An important question here is how the known particles and interactions arise from this more fundamental theory.

The theory which describes the known particles and their interactions is called the *Standard Model* (SM). As mentioned above, there are several reasons for why this theory is believed to be incomplete and theories *beyond* the SM are required. In theoretical particle physics, models that often include several additional new particles are studied. The properties of these new particles and how they can be detected in experiments are predicted. Particle physics research in the interplay of theory and experiment is often called *particle phenomenology*. Today, the usage and construction of computer tools/programs of varying sophistication and complexity is an integral part of phenomenological research.

The goal of this first part of the thesis (*kappa*) is to provide an introduction to Higgs bosons, and to set a framework for the included research papers. It is organized as follows: Chapter 2 introduces gauge symmetry in particle physics and the Higgs mechanism. In chapter 3 the Electroweak Standard Model and its Higgs boson are discussed. This chapter also covers some of the shortcomings of the Standard Model. Chapter 4 discusses some of the models that are studied in the research papers of this thesis, in particular two-Higgs-doublet models. Chapter 5 covers aspects of renormalization in QFT, in particular within the context of Higgs bosons. In chapter 6, we will discuss how experimental data can constrain the parameter space of particle physics models. In the final chapter, we give a popular science summary in Swedish.

All chapters, except the summary in Swedish, assume that the reader has basic knowledge in quantum field theory and particle physics e.g. at the level of introductory university courses. The target audience is diploma-thesis students, peer doctoral-students and researchers in particle physics.

2. The physics of Higgs bosons

Higgs Boson?
Ask Me!

The text of a badge worn by some particle physicists,
distributed by the Royal Swedish Academy of Sciences

In this chapter, we will give a brief account of gauge symmetries in particle physics and how Higgs bosons arise through the *Higgs mechanism* when a gauge symmetry is *spontaneously* broken. Some useful and pedagogical references to this chapter are [6, 7, 8].

2.1 Gauge symmetries in particle physics

Consider the following Lagrangian of a complex scalar field $\phi(x)$,

$$\mathcal{L} = \partial_\mu \phi^\dagger \partial^\mu \phi - M^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2 \,, \tag{2.1}$$

with $M^2 > 0$, which has a U(1) global symmetry. The ϕ-field transforms under the U(1) symmetry according to

$$\phi \to e^{-\mathrm{i}\theta} \phi, \quad \phi^\dagger \to \phi^\dagger e^{\mathrm{i}\theta}. \tag{2.2}$$

This is a *global* symmetry since the transformation parameter θ does not depend on space-time. If we make θ *local*, i.e. let it depend on space-time[1], $\theta \to \theta(x)$, the kinetic term in (2.1) is no longer invariant. In order to make this Lagrangian *gauge invariant*, a spin-1 field A_μ, which has the U(1) gauge-transformation property

$$A_\mu(x) \to A_\mu(x) - \partial_\mu \theta(x), \tag{2.3}$$

is included. Upon replacing the ordinary derivative with the *covariant* derivative,

$$\partial_\mu \to \mathcal{D}_\mu = \partial_\mu - \mathrm{i} A_\mu \,, \tag{2.4}$$

[1] This is called *gauging* the symmetry.

and

$$\partial_\mu \phi \to \mathcal{D}_\mu \phi\,, \qquad\qquad \partial_\mu \phi^\dagger \to (\mathcal{D}_\mu \phi)^\dagger\,, \tag{2.5}$$

the modified Lagrangian becomes gauge invariant.[2] The field A_μ is called a *gauge*-field. We also need to include in the Lagrangian the kinetic term of A_μ

$$\mathcal{L}^{(A)}_{\text{kin}} = -\tfrac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}, \quad \text{where} \quad \mathcal{F}^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu, \tag{2.6}$$

which is U(1) gauge-invariant. An important consequence of gauge-symmetry is that the gauge-boson must be massless because a mass-term $m_A^2 A^\mu A_\mu$ is not invariant. The original Lagrangian described complex scalar particles interacting through the quartic interaction vertex $\lambda(\phi^\dagger\phi)^2$. This new, gauge-invariant, Lagrangian includes additional interactions, $\phi^\dagger(\partial^\mu\phi)A_\mu$ and $\phi^\dagger\phi A^\mu A_\mu$ whose strengths are uniquely determined by the gauge symmetry. This is why gauge symmetry has earned a special place in particle physics; it explains which gauge-particles should exist and their interactions with the *matter fields* ϕ. It should be noted that there are no self-interactions among the gauge-fields in this theory.

In the theory described above the resulting gauge-transformation of two consecutive transformations, with parameters θ_1 and θ_2, is independent of their ordering:

$$e^{-\mathrm{i}\theta_1}\left(e^{-\mathrm{i}\theta_2}\phi\right) = e^{-\mathrm{i}\theta_2}\left(e^{-\mathrm{i}\theta_1}\phi\right). \tag{2.7}$$

This is called *Abelian* gauge-theory and is the starting point for *Quantum electrodynamics* (QED) which describes the dynamics of electrically charged particles. In QED, the particles associated with the A_μ field are the photons γ.

Next, consider a Lagrangian of several complex scalar fields $\Phi_i(x)$ ($i = 1,\ldots,N$),

$$\mathcal{L} = \partial_\mu \Phi_i^\dagger \partial^\mu \Phi_i - m^2 \Phi_i^\dagger \Phi_i - \lambda(\Phi_i^\dagger \Phi_i)^2\,, \tag{2.8}$$

with repeated indices summed over. This Lagrangian is symmetric with respect to the SU(N) transformation

$$\Phi_i \to \mathcal{U}_{ij}\Phi_j, \quad \Phi_i^\dagger \to \Phi_j^\dagger \mathcal{U}_{ji}^\dagger. \tag{2.9}$$

An element of the transformation-matrix $\mathcal{U}$ can be written as

$$\mathcal{U}_{ij} = \exp\left(-\mathrm{i}(T^a)_{ij}\theta^a\right), \tag{2.10}$$

where the hermitian and traceless matrices T^a are called *generators*. The number of generators, and transformation parameters, in SU(N) are $N^2 - 1$. In the simplest case, which is SU(2), there are two fields Φ_1 and Φ_2 and the generators are the three Pauli matrices.

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[2]Gauge-invariance is a redundancy of the description of the fundamental fields of the theory. Different configurations, *gauges*, results in the same physics. Consider Electromagnetism: $-\vec{E} = \vec{\nabla}A_0 + \dot{\vec{A}}$ and $\vec{B} = \vec{\nabla}\times\vec{A}$. Clearly, $\vec{E}$ and $\vec{B}$ are invariant under (2.3).

By making the transformation parameters θ^a local, $\theta^a \to \theta^a(x)$, we again have to introduce the covariant derivative where N^2-1 number of massless gauge fields A^a_μ are required. A gauge-invariant kinetic term of A^a_μ should be included in the Lagrangian as well. An important application of this gauge-theory is *Quantum chromodynamics* (QCD) which is based on a Lagrangian of fermions Ψ_i called quarks, and SU(3) as the symmetry group:

$$\mathcal{L} = \mathrm{i}\bar{\Psi}_i\gamma^\mu\left[\partial_\mu - \mathrm{i}(T^a)_{ij}A^a_\mu\right]\Psi_j - m_q\bar{\Psi}_i\Psi_i, \tag{2.11}$$

where $i = 1,2,3$ is the *color* index of the quark. The terms inside the large parenthesis in (2.11) make up the covariant derivative $\mathcal{D}_{\mu ij}$, and A^a_μ are the gauge fields which are called gluons and there are 8 of them. This theory, where the fields Ψ_i are transformed into a linear combination of themselves, is *non-Abelian*, meaning that the order of two consecutive transformations matters. Comparing with the Abelian case, the non-Abelian A^a_μ gauge fields will have cubic and quartic self-interactions which are uniquely determined by the gauge symmetry. Non-Abelian SU(N) gauge-theories are often called Yang-Mills theories [9].

It should be noted that, in general, other continuous symmetries (*groups*) than SU(N) can be gauged. In addition, one can have the matter-fields Φ_i, Ψ_i to transform under a different *representation* of the gauge-group. Every continuous group can be characterized by its Lie algebra $[T^a,T^b] = \mathrm{i}f^{abc}T^c$, where the f^{abc} are called the *structure constants*. A representation of a given group satisfies the same Lie algebra. One particular representation is the *adjoint* representation: $(T^a)^{bc} = -\mathrm{i}f^{abc}$, i.e. the entries of the generators are determined by the structure constants, $(a,b,c) \in \{1,2,3\}$ in SU(2).

We have outlined that two of the known interactions in nature could be described as gauge-theories. As a matter of fact, the theoretical predictions of these two theories, QED and QCD, show tremendous agreement with experimental measurements [10, 11, 12, 13, 14, 15]. However, at this point it seems to be impossible to describe *weak interactions* as a gauge-theory by identifying the W-bosons as the corresponding gauge bosons. This is because the W-bosons are very heavy, but the gauge symmetry requires massless gauge-bosons. One possible solution is to postulate that the gauge symmetry for weak interactions is broken. The standard assumption is that it is broken *spontaneously* and this is the subject of chapters 2.2 and 3.

It should be noted that it is also possible to describe *gravity* as a gauge-theory of fields [16, 17]. The quantization of such a theory has not yet been successful since it seems impossible to *renormalize* it [18, 19]. Despite the fact that a QFT of gravity is not renormalizable, it is possible to study it, at least to some extent [20]. In section 5 we will discuss renormalization of QFTs in more detail.

For completeness, we will mention something regarding quantization of gauge-theories. In path-integral quantization, one integrates over all A^a_μ configurations. Because the action $\mathcal{S}[A]$ is gauge-invariant, one will integrate over infinitely many physically equivalent field-configurations. It is therefore desirable to count each such field-configuration only once by restricting the functional integration to physically *inequivalent* field-configurations. In quantum gauge-theories, one therefore needs to fix the gauge in order to perform calculations. According to the Faddeev-Popov prescription, one effectively adds the *Gauge-fixing* term $-(\partial^\mu A_\mu)^2/2\xi$ to the Lagrangian [21]. Additionally, *ghosts* – scalar fields with Fermi-Dirac statistics, must be included to cancel non-physical degrees of freedom. The gauge-symmetry at the classical level remains as a global symmetry that relates the matter-fields, the gauge-fields and the ghosts. This is called BRST symmetry, and is crucial to prove that e.g. the ghosts do not exist as physical states and to prove renormalizability of gauge-theories [22, 23, 24, 25].

In the next section we will demonstrate how massive gauge-bosons can be accommodated by means of the *Higgs-mechanism*.

2.2 The Higgs mechanism

In this section, we will discuss how massive gauge-bosons can be realized and still maintain the gauge-invariance of the Lagrangian. This is achieved by the *Higgs mechanism* and will lead to the existence of a new class of particles, *Higgs bosons*.

2.2.1 Abelian Higgs-mechanism

As our starting point, we consider the QED Lagrangian of fermions and scalars

$$\mathcal{L}_{\text{QED}} = \mathrm{i}\bar{\Psi}\gamma^\mu \mathcal{D}^\Psi{}_\mu \Psi - m\bar{\Psi}\Psi + (\mathcal{D}^\Phi{}_\mu \Phi)^\dagger \mathcal{D}^{\Phi\mu}\Phi - \mathcal{V}(\Phi) - \tfrac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}, \quad (2.12)$$

where

$$\mathcal{V}(\Phi) = M^2\Phi^\dagger\Phi + \tfrac{1}{4}\lambda(\Phi^\dagger\Phi)^2\,, \quad (2.13)$$

is the scalar potential with $\lambda > 0$ to have the potential bounded from below (in order to have a physically meaningful theory),

$$\mathcal{D}^{\Psi(\Phi)}{}_\mu = \partial_\mu - \mathrm{i}q_{\Psi(\Phi)}A_\mu \quad (2.14)$$

is the covariant derivative, and $q_{\Psi(\Phi)}$ the is *charge* of the fermion and scalar fields respectively.

For $M^2 > 0$ in the scalar potential (2.13), the Lagrangian (2.12) describes the theory of (electrically) charged fermions with charge q_Ψ and mass m, scalars with charge q_Φ and mass M, which interacts with a massless gauge-boson A [3].

[3]For instance electrons and positrons $e^\pm$, and charged pions $\pi^\pm$, interacting via exchange of photons γ.

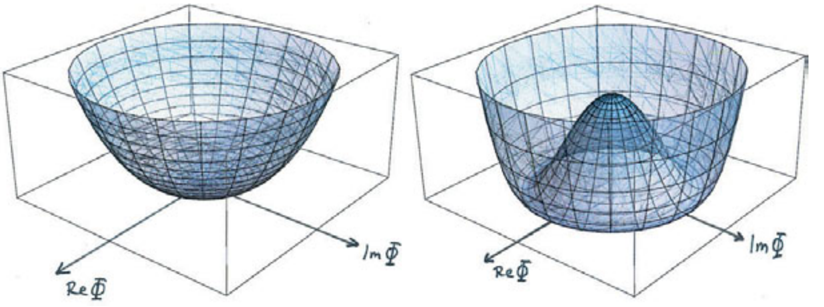


Figure 2.1. The Higgs potential for $M^2 > 0$ (left) and $M^2 < 0$ (right) in the Abelian Higgs-model.

If we let $M^2 < 0$ in (2.13), this would naively describe a charged scalar with imaginary mass $\mathrm{i}|M|$. In this case, we clearly need a different interpretation.

The resolution can be found in the scalar potential. For $M^2 > 0$ the shape of the potential is depicted in Fig. 2.1(a), the ground state is located for zero value of the scalar field. For the situation $M^2 < 0$, where the potential has the shape as in Fig. 2.1(b), something interesting occurs. The ground state is now described by a class of minima states

$$\langle\Phi\rangle_0 \equiv \Phi_{\text{min}} = \tfrac{1}{\sqrt{2}} v e^{\mathrm{i}\omega}, \tag{2.15}$$

where $v = \sqrt{4|M^2|/\lambda}$ is called the *vacuum expectation value*, and ω is a phase corresponding to the angular position in the potential. It is always possible to make a redefinition of Φ to make the value v real since (2.12) possesses a global U(1) symmetry in addition the U(1) gauge symmetry.

Let us rewrite Φ, called the *Higgs field*, with the ansatz

$$\Phi(x) = \frac{1}{\sqrt{2}}(v + H(x))e^{-\mathrm{i}G(x)/v}, \tag{2.16}$$

where H describes quanta (particles) in the radial direction of the potential and G quanta in the angular direction. By inserting this ansatz in (2.12) it can be shown that the gauge-boson field A has acquired a mass

$$m_A = v q_\Phi. \tag{2.17}$$

The term H of the Higgs-field Φ is called the *Higgs boson*. The Higgs boson has a mass $m_H = \sqrt{2}|M|$. The interpretation of the field G, called *Goldstone boson* [26] will be explored in section 2.2.3.

To summarize, the Lagrangian (2.12) is gauge-invariant, but the ground state $\langle\Phi\rangle_0$ is not. This is called *spontaneous symmetry breaking* of the gauge symmetry, or *the Higgs mechanism* [27, 28, 29]. The outcome of this procedure is that the gauge-boson has acquired a mass and the seemingly unphysical scalar field Φ (it naively had an imaginary mass) has “turned” into a physical degree of freedom H with mass m_H. The Higgs boson H of this model has charge q_Φ and has interactions with the gauge-field accordingly. The model described in this section describes "massive QED". It was originally introduced in the context of non-relativistic quantum field theory of solids in order to explain certain features of superconductivity [30].

2.2.2 Non-Abelian Higgs-mechanism

The Higgs mechanism for non-Abelian gauge-theories follows the previous analysis for the Abelian case. In the non-Abelian case, we now have the freedom to choose which representation of the given gauge-group the Higgs field Φ transforms under. As an example, we let the gauge-group be SU(2) and we let the Higgs-field transform in the adjoint representation [31]. Hence, the Higgs field of this model is a real 3-component field $\Phi = (\phi_1, \phi_2, \phi_3)^T$.

The ground state of the corresponding Higgs potential can be choosen to be

$$\langle\Phi\rangle_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix}. \tag{2.18}$$

This is because the Lagrangian also has a global SU(2) symmetry which makes it possible to align the entire vev in one of the three directions in group space. This is analogous to defining the vev of the previously discussed Abelian model to be real.

Only two of the gauge-bosons, A^1 and A^2, become massive in this model, whereas A^3 remains massless. This can be understood as the vev $(0,0,v)^T$ being invariant under SU(2) transformations in the 3-direction ($\theta^1 = \theta^2 = 0$). The corresponding gauge-boson is therefore massless. The Higgs boson of this model has interactions with A^1 and A^2 but not with A^3.

One important example of a non-abelian Higgs-sector is the Electroweak Standard Model (to be discussed in Chapter 3), where the gauge-group is SU(2)⊗U(1). There are four gauge-bosons accompanied with this symmetry group, three of them should be massive (the W^+, W^- and the Z-bosons) and one should be massless (the photon γ).

Before discussing the Standard Model, we will mention a few topics regarding quantizing spontaneously broken gauge-theories and explicit gauge-symmetry breaking.

2.2.3 Quantum Higgs-mechanism

We will consider the Abelian Higgs-model. The discussion for the non-Abelian case follows closely. Instead of the ansatz in (2.16), we expand the Higgs field Φ as

$$\Phi = \frac{1}{\sqrt{2}}(v + H + \mathrm{i}G), \tag{2.19}$$

where G is the Goldstone boson. Upon expanding the gauge-kinetic term for the Higgs field we obtain a peculiar cross-term between the gauge boson field and G: $q_\Phi v A^\mu \partial_\mu G$. Now, let the gauge-fixing term in the Lagrangian be modified according to

$$-(\partial_\mu A^\mu)^2/2\xi \to -(\partial_\mu A^\mu - \xi q_\Phi v G)^2/2\xi . \tag{2.20}$$

This gauge-fixing is called *R_ξ-gauge* and ξ is a parameter to be chosen freely, no physical processes should however depend on it.

It is interesting to examine what has happened with the Goldstone boson in R_ξ-gauge. The crossterm $q_\Phi v A^\mu \partial_\mu G$ has disappeared from the Lagrangian and G has become the sought longitudinal component of the gauge-boson A, propagating with a mass $\sqrt{\xi} m_A$.

For tree-level calculations, a gauge called *unitary gauge* is useful since the Goldstone boson and the ghosts never appear in the Feynman diagrams. External gauge bosons are treated as having three polarization states. The unitary gauge corresponds to $G = 0$, or $\xi \to \infty$ in R_ξ-gauge. In loop-calculations, unitary gauge is not useful because the A-propagator in this gauge does not have well suited properties in the high-momentum limit $k^2 \to \infty$, making renormalization cumbersome. In R_ξ-gauge, with finite ξ, the A-propagator falls of as $1/k^2$ which is desirable.

2.2.4 Explicit gauge-symmetry breaking

We will now give a short digression regarding the effect of breaking the gauge-symmetry explicitly by a mass term for the gauge fields, without invoking the Higgs-mechanism. The gauge-boson propagator becomes

$$\mathrm{i}\Delta^A(k)_{\mu\nu} = \frac{g_{\mu\nu} - k_\mu k_\nu / m_A^2}{k^2 - m_A^2} , \tag{2.21}$$

and is the same as we would get in the previously mentioned unitary gauge. In the high-momentum limit, $\Delta^A(k)_{\mu\nu}$ goes to a constant, ${m_A}^{-2}$, and renormalization is not transparent.

In addition, the matrix elements $\mathcal{M}$ of scattering processes involving gauge-bosons, e.g. $AA \to AA$ in a non-Abelian theory, are proportional to the scattering energy $\sqrt{s}$. This leads to violation of unitarity above a certain $\sqrt{s}$. The inclusion of the Higgs boson in the Feynman diagrams of $\mathcal{M}$ saves unitarity, provided that m_H is not too large, see chapter 3.3.

3. The electroweak Standard Model

I do not keep up with the details of particle physics.

Murray Gell-Mann

We will now discuss a model, *the Glashow-Weinberg-Salam model*, best known as the *Standard Model* (SM) [32, 33, 34]. This model describes the weak and electromagnetic (electroweak) properties of the known matter particles, the leptons and quarks. The presentation given here will be focused on the Higgs- and gauge bosons. The SM has indeed been successful but it has some shortcomings which will be discussed in section 3.4.

The gauge group of the Electroweak SM is $SU(2)_L \otimes U(1)_Y$, where L denotes that only left-handed fermions transform under the $SU(2)_L$ subgroup. The subscript Y denotes the *hypercharge*. As we will see, the electric charge is a derived quantity in this model. More detailed descriptions can be found in e.g. [35] and [8].

3.1 Spontaneous symmetry breaking and gauge bosons

The Higgs field Φ of the SM transforms under $SU(2)_L$ as a doublet, and has hypercharge $Y = \frac{1}{2}$. Its gauge- properties read

$$\Phi(x) \to \exp(\mathrm{i}T^a \omega^a(x)) \exp(\mathrm{i}\chi(x))\Phi(x), \tag{3.1}$$

where Φ is a complex 2-component field, and ω^a and χ are the gauge transformation parameters of $SU(2)_L$ and $U(1)_Y$ respectively. The covariant derivative of the Higgs field Φ is

$$\mathcal{D}_\mu^\Phi = \left(\mathbf{1}\partial_\mu - \mathrm{i}g_2 T^a W_\mu^a - \mathrm{i}\mathbf{1}g_1 Y B_\mu\right), \tag{3.2}$$

where $g_{2,1}$ are the gauge-couplings[1] and W^a, B the gauge-fields. When Φ develops a vev[2]

$$\langle\Phi\rangle_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}, \tag{3.3}$$

[1] The gauge-couplings were previously called “charge”.

[2] The vev can be chosen to be real since the Lagrangian is invariant under global $U(1)_Y$ transformations.

where $v \approx 246$ GeV, the following linear combinations of gauge-bosons will acquire mass

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp \mathrm{i} W_\mu^2\right), \quad m_W = g_2 \frac{v}{2}, \tag{3.4}$$

$$Z_\mu = \cos\theta_\mathrm{W} W_\mu^3 - \sin\theta_\mathrm{W} B_\mu, \quad m_Z = \frac{g_2}{\cos\theta_\mathrm{W}}\frac{v}{2} = \frac{m_W}{\cos\theta_\mathrm{W}}, \tag{3.5}$$

where $\cos\theta_\mathrm{W} = g_2/\sqrt{g_2^2+g_1^2}$. The *Weinberg-angle* θ_W denotes the magnitude of mixing between the W^3 and B gauge-fields. The field orthogonal to Z_μ is massless

$$A_\mu = \sin\theta_\mathrm{W} W_\mu^3 + \cos\theta_\mathrm{W} B_\mu, \quad m_A = 0. \tag{3.6}$$

The existence of a massless gauge-field in this model can be inferred by considering a gauge-transformation $\omega^1 = \omega^2 = 0$, $\omega^3 = \chi$, which leaves the vev (3.3) invariant. Gauge transformations induced by the linear combination of the generators

$$Q \equiv \tfrac{1}{2}T^3 + Y, \tag{3.7}$$

are not spontaneously broken by the Higgs field and the corresponding gauge-boson remains massless. The $W^\pm$ and Z bosons were discovered at the UA1 and UA2 experiments at CERN in 1983 [36, 37, 38, 39], with the correct relation between m_W and m_Z as in (3.5) [40].

It is instructive to write the Higgs field in components

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}G^+ \\ v + H + \mathrm{i}G^0 \end{pmatrix}, \tag{3.8}$$

where H is the SM Higgs-boson whereas G^0 and $G^\pm$ are the Goldstone bosons that will provide the longitudinal degrees of freedom to the Z and $W^\pm$ bosons respectively.

If the A-field is identified as the photon field, its interaction strength with the other fields should be interpreted as their electric charge

$$e \equiv g_1 \cos\theta_\mathrm{W}. \tag{3.9}$$

For instance, the $W^\pm$ bosons have indeed electric charge $\pm e$ and the Z-boson is electrically neutral. The electric-charge generator is therefore Q in (3.7). This is the reason for why the $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ gauge-theory is called *electroweak*. The weak and electromagnetic interactions of particles are two manifestations of a single interaction[3]. For scattering energies $s \gtrsim m_W^2$, the weak and electromagnetic interactions will be of similar importance.

[3]Some would call this a *unification*, but the electroweak theory still has two gauge couplings. A true unified theory should have only one, common, gauge coupling.

3.2 Fermions in the Standard Model

It was known, from e.g. radioactive decays, that only left-handed fermions, and right-handed anti-fermions, interact weakly. This is the reason for the choice of $SU(2)_L$: the weak interaction is *chiral*. A consequence of a chiral symmetry is that the fermions must be massless. This can be inferred from the free Dirac Lagrangian

$$\mathcal{L} = \mathrm{i}\bar{\Psi}\partial\!\!\!/\Psi - m\bar{\Psi}\Psi = \mathrm{i}\left(\bar{\Psi}_L\partial\!\!\!/\Psi_L + \bar{\Psi}_R\gamma^\mu\partial\!\!\!/\Psi_R\right) - m\left(\bar{\Psi}_L\Psi_R + \bar{\Psi}_R\Psi_L\right) \tag{3.10}$$

where Ψ is a 4-component Dirac-spinor. A chiral symmetry will be broken by the mass term. This means that a chiral gauge-theory must have both massless fermions and gauge bosons. In the SM, the Higgs field can be used to give masses to the fermions as well through Yukawa-couplings[4]

$$-\mathcal{L}^{\mathrm{SM}}_{\mathrm{Yuk}} = y^{(d)}_{ij}\bar{Q}_{Li}\Phi\Psi^{(d)}_{R\ j} + y^{(u)}_{ij}\bar{Q}_{Li}\Phi^c\Psi^{(u)}_{R\ j} + y^{(e)}_{ij}\bar{E}_{Li}\Phi\Psi^{(e)}_{R\ j} + \text{h.c.}, \tag{3.11}$$

where $\Phi^c = \mathrm{i}\sigma^2\Phi^*$, and $y^{(d,u,e)}$ are 3×3 Yukawa-coupling matrices, i, j are generation, or *flavor*, indices, e.g. $e_1 = e$, $e_2 = \mu$, $e_3 = \tau$. The left-handed fermions are contained in the $SU(2)_L$ doublets Q_L and E_L, e.g. $E_{Li} = (\Psi^{(\nu)}_{L\ i}, \Psi^{(e)}_{L\ i})^T$. Under a $SU(2)_L$ gauge-transformation,

$$Q_L \to \exp(\mathrm{i}T^a\omega^a(x))Q_L, \quad \bar{Q}_L \to \bar{Q}_L\exp(-\mathrm{i}T^{a\dagger}\omega^a(x)), \tag{3.12}$$

and similarly for E_L and $\bar{E}_L$. The right-handed fermions do not transform under $SU(2)_L$. By assigning appropriate hypercharges to the fermion doublets and singlets, see table 3.1, $\mathcal{L}^{\mathrm{SM}}_{\mathrm{Yuk}}$ becomes gauge-invariant.

The mass-eigenstates are obtained by performing bi-unitary transformations[5] and field redefinitions. This is necessary because the Yukawa-matrices $y^{(d)}, y^{(u)}$ and $y^{(e)}$ are in general not diagonal. This will lead to the fact that the propagating quark-states are not the same as the eigenstates of the weak interaction. It can be shown that, written in quark mass-eigenstates, the charged current (CC) Lagrangian has the form

$$\mathcal{L}_{\mathrm{CC}} = \mathrm{i}\frac{g_2}{\sqrt{2}}V^{\mathrm{CKM}}{}_{ij}\bar{\Psi}^{(u)}_{L\ i}\gamma^\mu\Psi^{(d)}_{L\ j}W^+_\mu + \dots. \tag{3.13}$$

The 3×3 matrix V^{CKM} is parametrized by four real parameters, where one of them can be set as an irreducible complex phase [41]. This leads to a rich CC phenomenology in the electroweak quark-sector, for instance *CP*-violation [42].

In addition to interacting weakly and electromagnetically, the quarks interact with the color force through gluon exchange. The color force will affect the electroweak sector when higher-order corrections are considered [43].

[4] It should be noted that at the time of the formulation of the SM, the neutrinos were assumed to be massless, see Chapter 3.4

[5] Singular value decomposition.

	Q_L	E_L	d_R	u_R	e_R	Φ	$\Phi^c = i\sigma^2\Phi^*$
Y	1/6	-1/2	-1/3	2/3	-1	1/2	-1/2

Table 3.1. *The hypercharges of the fermions and the Higgs field in the SM.*

3.3 The Higgs boson in the Standard Model

The SM Higgs-boson has the mass

$$m_H = \sqrt{2}|M| = v\sqrt{\frac{\lambda}{2}}. \tag{3.14}$$

The Higgs boson has cubic HHH and quartic $HHHH$ self-interactions, and couplings to gauge-bosons and fermions as

$$\delta_V \frac{m_V^2}{2v} V^\mu V_\mu H, \quad \delta_V \frac{m_V^2}{2v^2} V^\mu V_\mu HH, \quad -\frac{m_F}{v}\bar{\Psi}^{(F)}\Psi^{(F)}H \tag{3.15}$$

where $V = W^\pm, Z$ and $\delta_{W^\pm,Z} = 2, 1$ and F denotes all fermions (except the neutrinos which are assumed to be massless). In addition, the Higgs boson has quartic $HHVV$ couplings as well. It should be noted from (3.15) that the Higgs-boson coupling-strength are proportional to the mass of the particles it couples to. At higher-orders, the Higgs boson will couple to the massless photons and gluons.

The decay branching ratios of the Higgs boson in the Standard Model depends strongly on the Higgs mass. In Fig. 3.1 the branching ratios of the most relevant channels are shown. The dominating decay modes are $H \to b\bar{b}$ for $m_H \lesssim 135$ GeV and $H \to WW$ for $m_H \gtrsim 135$ GeV. The loop-induced decay $H \to \gamma\gamma$ has a maximal branching ratio $\sim 2\times10^{-3}$ for $m_H \approx 125$ GeV, and was one of the most important channels in the discovery of a Higgs boson at the Large Hadron Collider (LHC) [3, 44].

At the LHC, the production channel with the highest cross-section is $pp \to H$, where p denotes a proton. This channel proceeds via the partonic subchannels $gg \to H$ (via a loop of heavy quarks) and $b\bar{b} \to H$. Other relevant channels for producing Higgs bosons at the LHC include $pp \to Hq_i'q_j'$ (called vector-boson fusion) and $pp \to VH$, where $V = W, Z$ (called Higgs-strahlung) [45]. In Fig. 3.2 the Feynman-diagrams for the previous mentioned production channels are shown.

From the Lagrangian point of view, there is no restriction on m_H; it is a free parameter of the model. However, there are several theoretical constraints on m_H that needs to be considered. First, consider tree-level unitarity of boson scattering [46, 47, 48, 49, 50]. Unitarity means that the probability for a scattering process should be less than, or equal to, unity. In particular, this requires that the partial-wave amplitudes a_ℓ in $2 \to 2$ scattering cross-sections of WW, ZZ, WZ, HH, HZ and HW should all fulfil $|\mathrm{Re}(a_\ell)| < 1/2$ in the limit

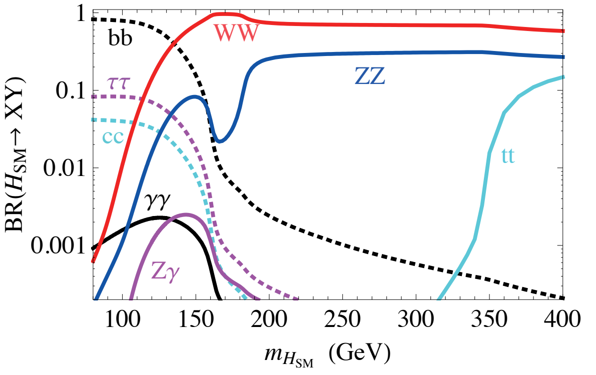


Figure 3.1. The branching ratios of the Standard Model Higgs boson [45].

where the scattering energy becomes infinitely large. This gives an upper bound on the Higgs-mass; $m_H \lesssim 700\,\text{GeV}$. However, for sufficient large values of m_H, the quartic coupling λ in the Higgs potential

$$m_H^2 = \frac{1}{2}v^2\lambda \quad \Rightarrow \quad \lambda = \frac{2m_H^2}{v^2} \tag{3.16}$$

may not longer be considered a "small" parameter, meaning that perturbation theory may no longer hold, the Higgs sector becomes strongly interacting [49, 51, 50, 52]. Consider e.g. the partial decay-width of H decaying into a pair of gauge bosons ($V = W^{\pm}, Z$) where higher-order loop-corrections from virtual Higgs-bosons are taken into account,

$$\Gamma(H \to VV) \approx \Gamma_0(H \to VV)\Big[1 + \alpha + \beta + \mathcal{O}(\lambda^3)\Big], \tag{3.17}$$

where Γ_0 is the tree-level result, $\alpha = 3\lambda/(4^3\pi^2)$ is the one-loop contribution and $\beta = 62\lambda^2/(4^6\pi^4)$ is the two-loop contribution. For $m_H \approx 1000$ GeV, one obtains $\alpha \approx \beta$ indicating breakdown of perturbation theory. It should however, be emphasized that the requirement of perturbativity of a QFT is not a requirement *per se*. However, because practically all calculations are done with perturbation theory, λ should not be "too" large. This requirement is called a perturbativity constraint.

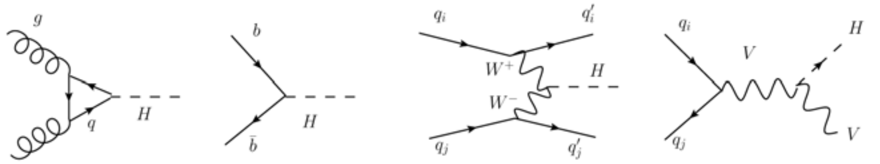


Figure 3.2. Feynman diagrams for the partonic sub-channels $gg \to H$, $b\bar{b} \to H$, $q_iq_j \to Hq'_iq'_j$ and $q_iq_j \to VH$.

An additional argument for an upper limit of m_H comes from the total decay-width of H. For large Higgs-boson masses $m_H \gg m_{W,Z,F}$, the partial widths grow as

$$\Gamma(H \to WW/ZZ) \sim m_H^3 , \qquad \Gamma(H \to \bar{F}F) \sim m_H . \tag{3.18}$$

At some point, the width of the Higgs boson will become so large that the particle description is no longer transparent. In particular, for $m_H \approx 1300$ GeV the total width is equal to the mass $\Gamma_H \approx m_H$. It is said that the Higgs boson becomes *obese* [8].

Perhaps the most physically motivated bound on m_H is the requirement that the Higgs potential should be stable and bounded from below, i.e. $\lambda > 0$ [53, 54, 55, 56, 57]. The idea is that at higher-orders, the $HHHH$-operator receives contributions from virtual Higgs-bosons, gauge-bosons and top-quarks. The coefficient of the $HHHH$-operator should be identified as the physical quartic coupling λ_{phys}, with $\lambda_{\text{phys}} > 0$.

The constraints of unitarity, perturbativity and stability are often applied to other models for electroweak symmetry breaking (EWSB), such as the two-Higgs-Doublet models described in the next chapter [58, 57, 59, 60, 61, 62, 63, 64].

3.4 Shortcomings of the SM

In this section we will discuss some of the shortcomings and limitations of the SM, rendering it implausible as the complete description of elementary-particle phenomena. It is widely assumed that the SM is only the low-energy theory of a more complete description of elementary particle interactions. In other words, the SM is an *effective field theory* (EFT) valid up to a certain energy scale Λ, called the *cut-off*. For energies larger than Λ, new physics, e.g. new particles and interactions, can start to have a considerable influence on SM observables. When calculating loop-diagrams, one integrates over all internal momenta which are not fixed by energy and momentum conservation. In an EFT, one has to stop the integration at the cut-off Λ.

Let us first consider the *fine-tuning* of the Higgs-boson mass, also called *the hierarchy problem* [8, 65, 66]. If one adopts the EFT point of view, the one-loop contribution (by virtual Higgs-boson exchange) to the physical Higgs-mass becomes

$$m^2_{H\text{phys}} = m_H^2 - \frac{3\Lambda^2}{8\pi^2 v^2} m^2_{H\text{phys}} , \tag{3.19}$$

where Λ is the cut-off scale, $m_{H\text{phys}}$ is the physical Higgs-boson mass and m_H is the parameter in the fundamental Lagrangian, i.e. the tree-level value.[6] Consider $m_{H\text{phys}} = \mathcal{O}(v)$ and $\Lambda = 10^{19}$ GeV, the energy-scale when quantum effects of gravity will be important [67].[7] In order to obtain $m^2_{H\text{phys}} = \mathcal{O}(v^2)$,

[6]In the next chapter, the notation will be changed due to renormalization.

[7]This scale is called the Planck-scale.

the fundamental parameter m_H^2 has to be $m_H^2 = O\Big((10^{19})^2/10\Big) + O(v^2)$, according to (3.19). In other words, a number $O(10^{37})$ with the last four or five digits *tuned* to match the value of $m_{H\text{phys}}^2 \approx v^2$. A way to reduce the amount of tuning in m_H is to consider very large $m_{H\text{phys}}$. As was outlined in the previous section, this seemed implausible due to the "theoretical constraints". A second approach is to lower the scale of new physics, the cut-off Λ.

There is of course nothing fundamentally wrong with having such an amazing cancellation. However, particle physicists strive to understand if there are underlying reasons for why the properties and interactions of particles are the way they are. Therefore, the fine-tuning of m_H in the SM has been a strong motivation for pursuing alternative models for EWSB. The research papers in this thesis have not primarily been motivated by the fine-tuning problem though.

In (3.19) the contributions from the gauge-bosons and the fermions (where the top-quark is the most important) were neglected. By including the effects of $W^{\pm}, Z$ and t in (3.19) the overall picture remains: m_H has to be *tuned*. It turns out that fermions always contribute destructively to $m_{H\text{phys}}^2$ which has been one of the strongest motivations for *Supersymmetry* (SUSY) in particle physics [68]. In SUSY, the Higgs boson has a fermionic partner called a *Higgsino* which contributes to the cancellation of the quadratic dependence of Λ in (3.19). In additions, there will be bosonic partners to the quarks (called squarks), the leptons (called sleptons) and fermionic partners to the gauge-bosons (called gauginos). An introduction to SUSY can be found in e.g. the following references [69, 70, 71, 72, 73].

The next imperfection of the SM concerns neutrino masses. By inspection of $\mathcal{L}_{\text{Yuk}}^{\text{SM}}$ in (3.11), the field $\Psi_R^{(\nu)}$ is absent. In its purest formulation, the SM therefore describes massless neutrinos. At the time when the SM was formulated, neutrinos were in fact believed to be massless indeed. When the Solar neutrino problem became more apparent, the solution called for neutrino masses [74, 75, 76]. It is fair to say that the inclusion of neutrino masses in the SM is perhaps not straightforward, so the SM could be extended [77]. The issue of Neutrino masses is very interesting in itself, but is not the topic of this thesis and we shall not discuss it further. The following references provides an interesting overview [78, 79, 80, 81]

We demonstrated that the SM provided a kind of unification between two interactions in Nature, the electromagnetic and the weak, into a single electroweak interaction. Is it possible to unify the color interaction of QCD with the electroweak ones at a high energy-scale ([82, 83, 84, 85])? At energies accessible at colliders today, the color force is much stronger than the electroweak one. The couplings g_{QCD} and g_2 becomes smaller at higher energies whereas g_1 grows larger. However, in the SM, all three of them never approach the same value at the same energy-scale, making unification questionable. Gauge-coupling unification is not a fundamental requirement, but is desirable for structural and aesthetic reasons. The energy dependence of couplings will be discussed in chapter 5.2.3.

We will now turn to more direct experimental evidence for why the SM should be extended. The fine-tuning problem and gauge-coupling unification can be considered as mere theoretical prejudices, and can therefore be discarded. The inclusion of neutrinos in the SM might also be considered a trivial problem. However, there is now strong evidence that the total matter-energy content of our universe is not constituted of *baryonic matter*, e.g. atoms in stars and dust [86, 87]. In fact, roughly one fourth of the universe is considered to be composed out of *Dark Matter* (DM) [88], and roughly two thirds *Dark Energy* [89], leaving only $\sim 5\%$ to baryonic matter. The DM should only interact weakly, and the only possible SM candidates, the neutrinos, can not contribute as much as and in such a way that is required [90]. Common extensions beyond the SM include weakly interacting massive particles (WIMPs) which are stable or long-lived with respect to cosmic time-scales [91].

In the context of Higgs-physics, popular frameworks which strive to solve the DM problem include *Higgs-portals* [92, 93, 94, 95, 96, 97]. The main ideas in such models are to extend the SM to include fields that are singlets under $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$. Furthermore, symmetries that guarantee the stability of the lightest dark particle are implemented. The Higgs-boson will then serve as a mediator between the SM particles and the *dark sector*. As an example, consider the inclusion of a complex singlet scalar φ_D with a global U(1) symmetry [94]. The additional terms in the SM-Lagrangian are

$$\mathcal{L}_{\text{Dark}} = \partial_\mu \varphi_D^\dagger \partial^\mu \varphi_D - m_D^2 \varphi_D^\dagger \varphi_D - \lambda_D \varphi_D^\dagger \varphi_D \Phi^\dagger \Phi\,. \tag{3.20}$$

Another frequently studied model is the Inert Doublet Model (IDM), where there are two $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y = 2 \otimes \frac{1}{2}$ scalar doublets $\Phi_{1,2}$. A discrete symmetry among the doublets is implemented, making one of them SM-like [98, 99]. The mass-eigenstates of the second doublet will therefore only interact in pairs, and the lightest scalar will be stable and become a suitable DM candidate [99, 100, 101]. We will discuss this model in chapter 4.4 in the context of two-Higgs-doublet models.

Another direct observation where the SM falls short is the matter-antimatter asymmetry in the universe [102, 103]. This would require additional CP-violation and a different kind of electroweak phase transition than what is permitted in the SM [104, 105].

The last critical observation which demonstrates the limitations of the SM is the *cosmological constant* which is related to Dark Energy. In QFTs describing elementary particles, constant terms (vacuum energies) in the Lagrangians are often dropped because one is merely interested in energy differences, not the total energy. However, when coupled to gravity, all of these terms will contribute to the cosmological constant [106]. The fields of the SM fail to match the observed value of the cosmological constant by tens of orders of magnitude. This topic is very important and interesting in its own right and has a strong connection to quantum gravity.[8] However, we shall not discuss it further in this thesis.

A more fundamental question one might ask is if there are several generations of Higgs-fields Φ_i? After all, there is no established reason for why three known generations of matter particles, fermions, exists in Nature. Is there an underlying structure where complex Higgs-sectors emerge ([110])? What are the implications of a more complex Higgs-sector ([111])? How can such Higgs-sectors be verified and can they accommodate the recently found Higgs boson ([112, 113, 114, 115])? These kinds of questions are what serves as the motivation behind the research papers in this thesis.

[8] For instance, the framework of *Higgs-driven inflation* [107, 108, 109] is a lively research area.

4. Higgs sectors beyond the Standard Model

Things cannot be as simple as our Standard Model.

François Englert

In this chapter we will discuss a selection of Higgs sectors beyond the Standard Model which were investigated in the papers of this thesis. First, a class of models with two Higgs-doublets $\Phi_{1,2}$ with $Y = \frac{1}{2}$ is considered. These models are called Two-Higgs-doublet models (2HDM). For a comprehensive review, we refer to [116]. Finally, we will briefly discuss some basic features of Higgs sectors in two well studied supersymmetric models.

4.1 Two-Higgs-doublet model scalar potential

The most general, renormalizable, scalar potential with two Higgs-doublets with the same hypercharges, reads

$$\begin{aligned}\mathcal{V}_{2\Phi} = & m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - [m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}] \\ & + \tfrac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \tfrac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ & + \left\{\tfrac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + [\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)]\Phi_1^\dagger\Phi_2 + \text{h.c.}\right\}.\end{aligned} \tag{4.1}$$

Due to the hermiticity of $\mathcal{V}_{2\Phi}$, the parameters m_{11}^2, m_{22}^2 and $\lambda_{1,2,3,4}$ are real, and the remaining parameters are in general complex. Both doublets may develop a vev,

$$\langle\Phi_{1,2}\rangle_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_{1,2}e^{i\xi_{1,2}} \end{pmatrix}, \tag{4.2}$$

which leaves the previously introduced (eq. (3.7)) generator Q unbroken. By insertion of (4.2) into (4.1), the minimum conditions are obtained, which are lengthy and not relevant for the presentation given here. Only the relative phase $\xi_1 - \xi_2$ is observable due to possible field redefinitions. It is therefore customary to let only one of the vevs have a phase, e.g. $\xi_2 = \xi$ and $\xi_1 = 0$. The vevs fulfil $v_1^2 + v_2^2 = v^2$, and it is convenient to express them as $v_1 = v\cos\beta, v_2 = v\sin\beta$, where $\tan\beta = v_2/v_1$.

The two doublets can be seen as the components of a vector in Higgs-flavor space,

$$\vec{\Phi} = \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}. \tag{4.3}$$

A global U(2)-transformation in Higgs-flavor space is given by $\vec{\Phi}' = \mathcal{U}\vec{\Phi}$, where $\mathcal{U}$ is a 2×2 unitary matrix. Such a transformation can be seen as a change of basis, where the doublets change their vevs. One particularly useful class of bases are the *Higgs bases*, where only one of the doublets develops a vev, e.g. $v_1 = v$ and $v_2 = 0$. When working in the Higgs bases, it is common to employ a change of notation; $\Phi_i' \to \mathbf{H}_i$, $m_{ij}^{2\prime} \to M_{ij}^2$ and $\lambda_i' \to \Lambda_i$. The Higgs bases can be obtained from a generic basis according to

$$\vec{\mathbf{H}} = \mathcal{U}^{\mathbf{H}}\vec{\Phi} = \begin{pmatrix} e^{\mathrm{i}\chi}\cos\beta & e^{\mathrm{i}(\chi-\xi)}\sin\beta \\ -e^{-\mathrm{i}(\chi-\xi)}\sin\beta & e^{-\mathrm{i}\chi}\cos\beta \end{pmatrix}\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}, \tag{4.4}$$

where $0 \le \beta \le \pi/2$, $0 \le \chi \le \pi$ and $0 \le \xi \le 2\pi$.[1] By inserting the inversion of (4.4) into (4.1), one obtains the potential parameters in the Higgs bases expressed in the parameters of potential in the generic basis and the transformation matrix.

Because the two doublets $\Phi_{1,2}$ have identical quantum-numbers, the two Lagrangians

$$\begin{aligned} \mathcal{L}_{2\Phi} &= (\mathcal{D}^{\Phi\mu}\Phi_i)^\dagger(\mathcal{D}^{\Phi}{}_\mu\Phi_i) - \mathcal{V}_{2\Phi}, \\ \mathcal{L}_{2\Phi'} &= (\mathcal{D}^{\Phi\mu}\Phi_i')^\dagger(\mathcal{D}^{\Phi}{}_\mu\Phi_i') - \mathcal{V}_{2\Phi'}, \end{aligned} \tag{4.5}$$

are physically invariant. This fact should be carefully considered when one discusses the properties of the 2HDM scalar-potential [117, 118, 119, 120].

One particular property of $\mathcal{V}_{2\Phi}$ is related to the $\mathbb{Z}_2$-transformation

$$\mathbb{Z}_2\Phi_1 = +\Phi_1\,, \qquad \mathbb{Z}_2\Phi_2 = -\Phi_2\,. \tag{4.6}$$

The 2HDM-potential is invariant under this transformation if $m_{12}^2 = 0$ and if $\lambda_6 = \lambda_7 = 0$. The $\mathbb{Z}_2$-symmetry is broken *softly* by the dimension-2 operator $m_{12}^2\Phi_1^\dagger\Phi_2$ + h.c., meaning that at large energy-scales, $E^2 \gg |m_{12}^2|$, the symmetry is restored.[2] The dimension-4 operators with coefficients λ_6 and λ_7 in (4.1) breaks the $\mathbb{Z}_2$-symmetry *hard*, which means that the symmetry is broken at all scales. The requirement that $\mathcal{V}_{2\Phi}$ and $\mathcal{L}_{\text{tot}}$ respects this symmetry is particularly useful when considering the fermionic sector of 2HDMs, see section 4.3. However, due to the physical invariance of $\mathcal{L}_{2\Phi}$, the $\mathbb{Z}_2$-symmetry may be hidden in some basis Φ, but manifest in another basis Φ' according to

$$\mathbb{Z}_2\Phi_1' = +\Phi_1'\,, \qquad \mathbb{Z}_2\Phi_2' = -\Phi_2'\,. \tag{4.7}$$

[1] The Higgs-bases are related by the parameter χ; $\mathbf{H}_1 \to e^{\mathrm{i}\chi}\mathbf{H}_1$ and $\mathbf{H}_2 \to e^{-\mathrm{i}\chi}\mathbf{H}_2$.

[2] This is analogous to how phenomenological SUSY models are constructed. At the electroweak scale, SUSY is broken but should be restored at very high energies.

By employing the "basis-independent" methods of Refs. [117, 118], conditions for the existence of a basis Φ' where the $\mathbb{Z}_2$-symmetry is manifestly exact, or manifestly softly broken, can be obtained. This means that if e.g. the $\mathbb{Z}_2$-symmetry exists in one basis Φ', it will be hidden, but not broken, in other bases Φ. However, it should be stressed that it is the $\mathbb{Z}_2$-symmetry in (4.7) that is exact, not (4.6).

Additional properties of the 2HDM-potential includes the possibility to provide a solution to the strong CP-problem via *axions* [121, 122] and a source of CP-violation via the complex parameters $m_{12}^2, \lambda_{5,6,7}$ and the phase ξ [123, 124].

4.2 Mass-eigenstates and couplings in 2HDMs

In a 2HDM, there are eight degrees of freedom in the Higgs-sector in terms of fields. After spontaneous symmetry breaking, three of those become the electroweak Goldstone-bosons. The remaining five mass-eigenstates are the Higgs bosons of this model. There is one pair of electrically charged Higgs-bosons $H^\pm$, which are the orthogonal states to $G^\pm$. In the absence of CP-violation, there is one CP-odd Higgs-boson A, which is orthogonal to G^0. Finally, there are two CP-even Higgs-bosons h and H. In the presence of CP-violation, there are no Higgs-bosons with definite CP-properties. Instead there are, in addition to the charged pair $H^\pm$, three electrically neutral states H_1, H_2 and H_3 without definite CP transformation-properties. In what follows, we shall assume that CP is conserved.

It is instructive to expand the Higgs doublets in terms of the mass-eigenstates

$$\begin{aligned}
\Phi_1 &= \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}(G^+\cos\beta - H^+\sin\beta) \\ v\cos\beta - h\sin\alpha + H\cos\alpha + \mathrm{i}(G^0\cos\beta - A\sin\beta) \end{pmatrix}, \\
\Phi_2 &= \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}(G^+\sin\beta + H^+\cos\beta) \\ v\sin\beta + h\cos\alpha + H\sin\alpha + \mathrm{i}(G^0\sin\beta + A\cos\beta) \end{pmatrix},
\end{aligned} \tag{4.8}$$

in a generic basis. The tangent of the angle β is, as previously defined, the ratio of the vevs of the two doublets through $\tan\beta = v_2/v_1$. The angle α is the parameter of the orthogonal diagonalization of the CP-even mass-matrix. It is customary to define the range of α to have the relation $m_H \geq m_h$ fulfilled.

We will now give a short overview of the cubic couplings of the Higgs bosons in 2HDMs, which are the most relevant couplings for production and decays [119].

The CP-even h and H couples at tree-level to a pair of W and Z bosons and a pair of fermions, and at one-loop to gg, $\gamma\gamma$ and $Z\gamma$, in close analogy to the SM Higgs boson. In addition, h and H couples to AA, AZ and $H^\pm W^\mp$

in a 2HDM. Finally, there are cubic couplings amongst the CP-even bosons themselves: hhh, hHH, hhH and HHH.

Because the A boson is CP-odd, it does not couple to a pair of W or Z. For the same reason, it will not couple to a pair of hh, hH or HH. The A boson will couple to a pair of fermions though. At loop-level, A will couple to gg, $\gamma\gamma$ and $Z\gamma$ just as h and H. In addition to the couplings given for h and H above, A will couple to $H^\pm W^\mp$.

The charged Higgs-boson $H^\pm$ will couple to fermions in a similar way as the W-boson. There are no further couplings including $H^\pm$ which have not not already been given in this section. One should note that the couplings $H^\pm W^\mp\gamma$ and $H^\pm W^\mp Z$ are not possible at tree-level. The $H^\pm W^\mp\gamma$ coupling is zero at tree-level in all models including a charged Higgs boson due to the (classical) conservation of the electromagnetic current [125].[3] The $H^\pm W^\mp Z$ coupling is zero at tree-level due to the hypercharges and isospin representation of the field-components in $H^\pm$. In models including e.g. Higgs-triplets, where the Higgs-fields are transforming in the adjoint representation of $\mathrm{SU}(2)_L$, the $H^\pm W^\mp Z$ coupling exists at tree-level [125, 126]. Nevertheless, the $H^\pm W^\mp\gamma$ and $H^\pm W^\mp Z$ can be generated at one-loop level in 2HDMs though they have small branching ratios [127].

4.3 Fermions in 2HDMs

The inclusion of fermions in 2HDMs is not as straightforward as in the case of the SM. By assuming no sources of CP-violation apart from the CKM-matrix, the general Yukawa-Lagrangian for a 2HDM reads [119]

$$-\mathcal{L}_{2\Phi,\mathrm{Yuk}} = {y_1^{(d)}}_{ij}\bar{Q}_{Li}\Phi_1{\Psi_R^{(d)}}_j + {y_1^{(u)}}_{ij}\bar{Q}_{Li}\Phi_1^c{\Psi_R^{(u)}}_j + {y_1^{(e)}}_{ij}\bar{E}_{Li}\Phi_1{\Psi_R^{(e)}}_j + {y_2^{(d)}}_{ij}\bar{Q}_{Li}\Phi_2{\Psi_R^{(d)}}_j + {y_2^{(u)}}_{ij}\bar{Q}_{Li}\Phi_2^c{\Psi_R^{(u)}}_j + {y_2^{(e)}}_{ij}\bar{E}_{Li}\Phi_2{\Psi_R^{(e)}}_j + \mathrm{h.c.}. \tag{4.9}$$

To obtain the mass-matrix and mass-eigenstates for the fermions, we define

$$\kappa_0^{(F)} = y_1^{(F)}\cos\beta + y_2^{(F)}\sin\beta, \tag{4.10}$$

$$\rho_0^{(F)} = -y_1^{(F)}\sin\beta + y_2^{(F)}\cos\beta, \tag{4.11}$$

where $F = u,d,e$ and $\kappa_0^{(F)}$ is orthogonal to $\rho_0^{(F)}$ with respect to transformations in Higgs-flavor space. By insertion of $\kappa_0^{(F)}$ and $\rho_0^{(F)}$ in $\mathcal{L}_{2\Phi,\mathrm{Yuk}}$ only terms that include $\kappa_0^{(F)}$ will contribute to the fermion mass-matrix. The next step is

[3]The photon couples only to particle and antiparticle pairs at tree-level.

to perform a similar bi-unitary transformations of the fermion-fields as in the case of the SM

$$\kappa^{(F)} = \left(V_L^{(F)}\right)^\dagger \kappa_0^{(F)} V_R^{(F)} = \frac{\sqrt{2}}{v} M^{(F)}, \tag{4.12}$$

where $V_L^{(F)}$ and $V_R^{(F)}$ are 3×3 unitary matrices. The resulting matrices $\kappa^{(F)}$ are diagonal and are proportional to the mass-matrices $M^{(F)}$. There is one big difference with respect to the SM though. The Yukawa-matrices $\rho^{(F)} = \left(V_L^{(F)}\right)^\dagger \rho_0^{(F)} V_R^{(F)}$ are not in general diagonal, leading to *Flavor changing neutral currents* (FCNC) at tree-level. This feature can be read off from the 2HDM Yukawa-Lagrangian in the physical basis

$$\begin{aligned}
-\mathcal{L}_{2\Phi,\text{Yuk}} = {} & \tfrac{1}{\sqrt{2}} \bar{\Psi}^{(u)}{}_i \left[\kappa^{(u)} \sin(\beta-\alpha) + \rho^{(u)} \cos(\beta-\alpha)\right]_{ij} \Psi^{(u)}{}_j h \\
& + \tfrac{1}{\sqrt{2}} \bar{\Psi}^{(u)}{}_i \left[\kappa^{(u)} \cos(\beta-\alpha) - \rho^{(u)} \sin(\beta-\alpha)\right]_{ij} \Psi^{(u)}{}_j H \\
& - \tfrac{\mathrm{i}}{\sqrt{2}} \bar{\Psi}^{(u)}{}_i \gamma_5 \rho^{(u)}{}_{ij} \Psi^{(u)}{}_j A \\
& + \left\{ \bar{\Psi}^{(u)}{}_i \left[V^{\text{CKM}} \rho^{(u)} P_R - \rho^{(d)} V^{\text{CKM}} P_L\right]_{ij} \Psi^{(d)}{}_j H^+ + \text{h.c.} \right\},
\end{aligned} \tag{4.13}$$

where we have omitted the lepton couplings and the couplings of the neutral Higgs-bosons to the down-type quarks for clarity.

The existence of FCNC at tree-level in the 2HDM is somewhat problematic [110, 128]. The off-diagonal elements in $\rho^{(F)}$ will contribute to e.g. the mass splitting of neutral mesons, ΔM. Experimentally they are found to be very small $\Delta M \sim \mathcal{O}(10^{-15})$ GeV and are in general consistent with the predictions of the SM predictions. In the SM, contributions to ΔM proceeds via loop-diagrams with e.g. virtual W-bosons and are therefore kept small. To keep the 2HDM contributions to ΔM consistent with experiment, the off-diagonal elements of $\rho^{(F)}$ have to be set to very small values, e.g. $\rho^{(d)}{}_{13} \lesssim 10^{-4}$, see [116, 129] and references therein.

From a minimalistic point of view, it is desirable to have a mechanism which maintains the 2HDM contributions to ΔM naturally small. One such solution is to postulate that the previously introduced $\mathbb{Z}_2$ Higgs-flavor symmetry is realized [130, 131]. In addition, the fermions are assigned with Higgs-flavor quantum-numbers as well. By a suitable assignment, each generation of fermions can couple to only one doublet in (4.9). The result is that the physical $\rho^{(F)}$ matrices in (4.13) will be diagonal and the previously introduced $\tan\beta$ will be promoted to a physical parameter. There are four distinct such assignments, leading to four "types" of 2HDMs with $\mathbb{Z}_2$-symmetry, and the resulting $\rho^{(F)}$ matrices are shown in in Table 4.1.

A solution to the tree-level FCNC problem without invoking a symmetry is the alignment approach [132]. In the Aligned 2HDM (A2HDM), the Yukawa matrices in (4.9) are postulated to be proportional to each other $y_2^{(F)} = \xi^{(F)} y_1^{(F)}$. This arrangement will guarantee that the physical $\rho^{(F)}$ are diagonal. In the

	$\rho^{(u)}$	$\rho^{(d)}$	$\rho^{(e)}$
Type-I	$\kappa^{(u)}\cot\beta$	$\kappa^{(d)}\cot\beta$	$\kappa^{(e)}\cot\beta$
Type-II	$\kappa^{(u)}\cot\beta$	$-\kappa^{(d)}\tan\beta$	$-\kappa^{(e)}\tan\beta$
Type-III (Y)	$\kappa^{(u)}\cot\beta$	$-\kappa^{(d)}\tan\beta$	$\kappa^{(e)}\cot\beta$
Type-IV (X)	$\kappa^{(u)}\cot\beta$	$\kappa^{(d)}\cot\beta$	$-\kappa^{(e)}\tan\beta$
Aligned	$\kappa^{(u)}\tan\beta^{(u)}$	$\kappa^{(d)}\tan\beta^{(d)}$	$\kappa^{(e)}\tan\beta^{(e)}$

Table 4.1. *The different assignments of Yukawa-couplings in CP-conserving 2HDMs. One should note the alternative notations for Type-III (Flipped) and Type-IV (Lepton-specific) 2HDMs, which are often encountered in the literature.*

A2HDM, the $\mathbb{Z}_2$-symmetry is not required and the Yukawa sectors of the previously mentioned four types of 2HDM can be achieved as limits of the A2HDM. For instance, the type-I 2HDM corresponds to the limits $1/\xi^{(F)} \to 0$, and type-II as $(1/\xi^{(u)}, \xi^{(d)}, \xi^{(e)}) \to (0,0,0)$.

In the A2HDM, there is no physical meaning of $\tan\beta = v_2/v_1$, because the Yukawa sector is independent of it. It is customary to introduce the alternative parameters $\beta^{(F)}$, called alignment angles, according to

$$\rho^{(F)} = \tan\beta^{(F)}\kappa^{(F)}, \tag{4.14}$$

where

$$\tan\beta^{(F)} = \frac{\xi^{(F)} - \tan\beta}{1 + \xi^{(F)}\tan\beta}. \tag{4.15}$$

With this formulation, one can instead of $\xi^{(F)}$ give $\tan\beta^{(F)}$ and $\tan\beta$ as input when calculating e.g. partial decay widths of Higgs bosons (which are physical observables). This is very convenient because, in the literature and numerical tools for 2HDMs, couplings etc. are often given with respect to a specific basis, i.e. $\tan\beta$ value [133].

It should be noted that the Yukawa couplings in the A2HDM are in general not stable against radiative corrections [134]. This is due to the lack of a symmetry which could prevent large higher-order corrections. This means that the values of the physical couplings have to be fine-tuned.[4] Nevertheless, the A2HDM provides a usable phenomenological framework.

4.4 Fermiophobic Doublet models

In this section, we will briefly discuss 2HDMs were some of the physical scalars do not have coupling to fermions at tree-level. Therefore, they are often called *fermiophobic*.

[4]This is in close analogy with the fine tuning of m_H in the SM.

4.4.1 The Inert Doublet model

The starting point of the *Inert Doublet model* (IDM) is to consider only one of the doublets being responsible for EWSB. The second assumption is to have the previously mentioned $\mathbb{Z}_2$-symmetry exact. In terms of 2HDM nomenclature, the IDM is a 2HDM where $m_{12}^2 = 0$, $\lambda_{6,7} = 0$ and the Higgs-basis is the physical basis, $\tan\beta = 0$. The fermionic sector of the IDM is identical to the SM fermion sector. Taking Φ_1 to give rise to EWSB and Φ_2 to be the *inert* doublet, the doublets and mass-eigenstates become

$$\Phi_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}G^+ \\ v+h+\mathrm{i}G^0 \end{pmatrix}, \quad \Phi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}H^+ \\ H+\mathrm{i}A \end{pmatrix}. \tag{4.16}$$

Due to the $\mathbb{Z}_2$-symmetry, the lightest particle among H, A and $H^\pm$ will be stable and can therefore contribute to the Dark Matter content in the Universe [99]. In the IDM, the couplings must include an even number of inert scalars. Specifically, H, A and $H^\pm$ will not couple to fermions at any order in perturbation theory because of the exact $\mathbb{Z}_2$-symmetry. The inert scalars H, A and $H^\pm$ will have couplings including the gauge-bosons W and Z via the covariant derivative for Φ_2. This is contrary to the Higgs-portal models where the additional scalars are gauge-singlets, see chapter 3.4. The IDM in light of the first runs of the LHC was studied in [135, 100, 101].

4.4.2 The Fermiophobic Aligned model

In the limit where the alignment angles in the A2HDM go to zero, $\beta^{(F)} \to 0$, the $H^\pm$ and the A will not couple to fermions at tree-level because of $\rho^{(F)} \to 0$ in (4.14). In other words, they become fermiophobic [136, 137]. If a given basis ($\tan\beta$) is adopted, this fermiophobic limit can be rephrased in terms of the alignment parameters as $\xi^{(F)} \to \tan\beta$, according to (4.15). Because of the lack of $\mathbb{Z}_2$-symmetry, the CP-even states h and H will become an admixture of states from Φ_1 and Φ_2 and will both couple to fermions, contrary to the IDM. In some sense, the fermiophobic A2HDM is a generalization of the IDM and was explored in paper I and II. In these papers, the model is called the Stealth Doublet Model.

Owing to the lack of $\mathbb{Z}_2$-symmetry, the $H^\pm$ and A bosons will acquire fermionic couplings induced at one-loop level through their couplings with h and H. In paper II we computed the branching ratios of $H^\pm$ and A. It was shown that, in particular, the dominating decay of $H^\pm$ with low mass is $W^\pm\gamma$. Furthermore, we found that the branching ratio of $H^\pm \to W^\pm Z$ could be $\sim 1\%$. This is an enhancement of ~ 2 orders of magnitude compared to what is found in $\mathbb{Z}_2$-symmetric 2HDMs [127].

4.5 Higgs sectors in Supersymmetry

In this section, we will give a short introduction to the Higgs sectors in the two of the most studied SUSY models. General and useful references are [70, 73, 138, 139]

4.5.1 The MSSM Higgs sector

As mentioned in Chapter 3.4, SUSY will provide a supersymmetric partner to each particle in the SM. These *superpartners* will have opposite Fermi-Dirac statistics, i.e. the fermions in the SM will have scalar superpartners called *sfermions* and the Higgs boson will have a fermion superpartner called *Higgsino*, and so forth. In the minimal supersymmetric Standard Model (MSSM), the mininal amount of new fields and parameters are implemented.

To make SUSY work out, one needs at least two Higgs doublets. One reason is because in SUSY, charge conjugated fields can not be included in the Lagrangian. By inspection of (3.11), the field Φ^c is required in order to be able to generate masses for the up-type quarks. The MSSM requires one doublet Φ_1 which gives masses to the leptons and the down-type quarks, and one second doublet Φ_2 with opposite hypercharge which gives masses to the up-type quarks. In this aspect, the MSSM Higgs sector is a Type-II 2HDM. By comparing with the formalism of 2HDMs discussed previously in this chapter, the opposite hypercharges of the doublets will act as a $\mathbb{Z}_2$-symmetry. In addition, SUSY requires that the quartic couplings in the Higgs-potential should depend solely on the $SU(2)_L \otimes U(1)_Y$ gauge couplings g_2 and g_1. Specifically, one has

$$\lambda_1 = \lambda_2 = \frac{g_2^2 + g_1^2}{4}, \qquad \lambda_3 = \frac{g_2^2 - g_1^2}{4} \qquad \text{and} \qquad \lambda_4 = -\frac{g_2^2}{2}. \tag{4.17}$$

The mass relations, at tree-level, are quite constrained in the MSSM. They read

$$m_A^2 = \frac{m_{12}^2}{\cos\beta \sin\beta} \qquad m_{H^\pm}^2 = m_A^2 + m_W^2, \tag{4.18}$$

$$m_{h,H}^2 = \frac{1}{2}\left[m_A^2 + m_Z^2 \mp \sqrt{\left(m_A^2 + m_Z^2\right)^2 - 4m_A^2 m_Z^2 \cos^2 2\beta}\right]. \tag{4.19}$$

One should now note two important things. The first thing to note is that the MSSM Higgs-sector are determined by two parameters at tree-level, conventionally taken to be $\tan\beta$ and m_A. Secondly, the mass of the lightest CP-even state h is bounded from above according to

$$m_h \leq m_Z |\cos 2\beta| \leq m_Z, \tag{4.20}$$

which is valid at tree-level. However, loop corrections to m_h will increase its upper bound and also introduce a dependence on additional MSSM parameters, see e.g. [140]. The masses of the other Higgs bosons in the MSSM will also receive substantial loop corrections. The Higgs masses in the MSSM are usually computed from the other MSSM parameters by usage of numerical tools such as FeynHiggs [141].

The MSSM can provide a possible solution to the Dark Matter problem; the lightest superpartner can be stable [142]. Furthermore it can provide a unification of the gauge-couplings at a high energy scale called the GUT-scale M_{GUT} [85]. Finally, the MSSM can solve the previously discussed fine-tuning of the Higgs mass [70].

The MSSM suffers from another fine-tuning problem though. In the MSSM Lagrangian, there is an dimension-2 operator $\mu^2(|\Phi_1|^2+|\Phi_2|^2)$ where the vaule of μ should be around the electroweak scale because the operator is involved in the process of EWSB. But, from an argument similar to the Higgs mass in the SM, there is nothing that protects it from being around the, say, GUT or the Plank scale. This is called the μ-problem of the MSSM [143].

4.5.2 The NMSSM Higgs sector

The *Next to the minimal supersymmetric Standard Model* (NMSSM) attempts to solve the μ-problem of the MSSM by "replacing" the μ-parameter with λS, where S is a gauge-singlet scalar field and λ a dimensionless coupling [138, 139]. When S develops a vev v_S, an effective $\mu_{\mathrm{eff}} = \lambda v_S$ is generated and can be around the EW scale without any fine tuning.

From a phenomenological point of view one has, apart from additional input parameters and less constrained Higgs masses, an even richer spectrum of Higgs bosons and superparticles (singlinos) in the NMSSM compared to the MSSM [144]. The physical Higgs states of the NMSSM are the three CP−even Higgses H_1, H_2, H_3, the two CP-odd A_1, A_2 and a pair of charged Higgs bosons $H^\pm$.

The MSSM and NMSSM Higgs sectors in light of the first runs of the LHC was studied in [114, 115]. In paper III, we investigated the discovery prospects of a heavy $H^\pm$ decaying into $H_i W^\pm$ with the assumption that H_i is the Higgs boson discovered by the LHC, see chapter 6. In the same paper, other models without SUSY were studied.

5. Loops and renormalization

Look at the beta function of quantum chromodynamics.
The sign's been changed.

Sheldon Cooper, theoretical physicist in the sitcom *The big bang theory*.

In this chapter, we will discuss a few topics regarding higher-order calculations in perturbative QFTs. After a canonical introduction, we will discuss the issue of tadpole-renormalization which is related to the vev of e.g. the Higgs-field. We will end this chapter with a discussion on how unstable particles can be treated in QFT. Some introductory and insightful references to these subjects can be found in the textbooks [6, 7, 145, 146].

5.1 Infinities and regularization

Consider the Lagrangian of a real scalar field

$$\mathcal{L} = \tfrac{1}{2}\partial^\mu\phi\partial_\mu\phi - \tfrac{1}{2}m^2\phi^2 - \tfrac{1}{4!}\lambda\phi^4\,, \tag{5.1}$$

which in some sense is a toy model for the Higgs Lagrangian. Consider the free-field Feynman propagator

$$\Delta_0^\phi(k^2) = \frac{\mathrm{i}}{k^2 - m^2 + \mathrm{i}\epsilon}\,, \tag{5.2}$$

which is related to the 2-point function $\mathcal{G}_\phi^{(2)}(k_1,k_2)$ up to four-momentum conservation. What are the effects of interactions? There are several Feynman-diagrams that contribute to the propagator, see Fig. 5.1. In fact, there are an infinite number of diagrams, but they can be systematically accounted for by noting that some of them are *one-particle-irreducible* (1PI), see Fig. 5.2. This means that the diagram will be connected even if one line is broken. Denoting the 1PI by a filled circle, attached to lines, the *physical* propagator is nothing but a geometric series in terms of 1PIs

$$\tilde{\Delta}^\phi(k^2) = \frac{\mathrm{i}}{k^2 - m^2 + \mathrm{i}\epsilon - \Sigma(k^2)}\,. \tag{5.3}$$

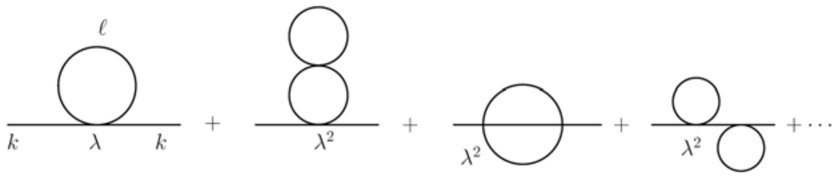


Figure 5.1. Some of the Feynman diagrams that contribute to the ϕ-propagator.

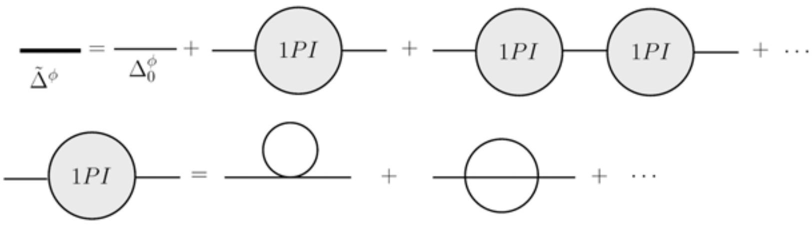


Figure 5.2. The physical propagator expressed in a geometric series of 1PI diagrams.

where $\Sigma(k^2)$ is the value of the 1PI-diagrams. If λ is considered to be a small parameter, the first diagram in Fig. 5.1 is sufficient to calculate for our purposes. If $\tilde{\Delta}^\phi(k^2)$ is the physical propagator, it should have a pole at $k^2 = m^2_{\text{phys}}$. By inspection of (5.3), we see that the pole is not located at m^2, but rather at

$$m^2_{\text{phys}} = m^2 + \Sigma(m^2_{\text{phys}})\,. \tag{5.4}$$

The parameter m in the Lagrangian is *not* the physical mass in general.

Let us now calculate $\Sigma(k^2)$. We should integrate over all internal momenta which are not fixed by energy- and momentum conservation

$$\Sigma(k^2) \sim \lambda \int \frac{\mathrm{d}^4\ell}{(2\pi)^4} \frac{1}{\ell^2 + m^2} = \infty\,, \tag{5.5}$$

as we integrate over $\ell \to \infty$. A similar result is obtained if we consider higher-order corrections to the $\phi\phi\phi\phi$ interaction-vertex, the diagrams depicted in Fig. 5.3, i.e. $\lambda_{\text{phys}} \sim \infty$. Either QFT is fundamentally flawed and can not describe anything of physical importance and should therefore be abandoned, or something important is missing. In some sense, the divergent behaviour in QFT is related to the fact that the elementary-particles (fields) are point-like and can interact at arbitrary close distances corresponding to $\ell \to \infty$. The

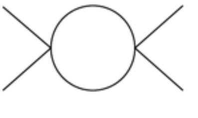


Figure 5.3. A higher order Feynman diagram that contribute to the $\phi\phi\phi\phi$ interaction vertex and λ_{phys}.

"cure" for the divergent behaviour in QFT is *renormalization* and will be discussed in the next section. The starting point is to extract a finite quantity in e.g. $\Sigma(k^2)$ in (5.5). Schematically, the procedure of *regularization* goes as

$$\text{“}\infty = \Xi + \infty\text{”}, \tag{5.6}$$

where Ξ is the (finite) sought quantity. There are several ways to regularize (5.5). We shall here describe *dimensional* regularization, where the space-time dimensions are treated as a continuous parameter $d = 4 - \varepsilon_d$. The limit $\varepsilon_d \to 0$ will represent $\ell \to \infty$ in the original integral. In d space-time dimensions, $[\lambda] = 4 - d$ ([ ] denotes the mass-dimension) to have the action $\mathcal{S} = \int \mathrm{d}^d x \mathcal{L}$ fulfill $[\mathcal{S}] = 0$. To account for this, an arbitrary superficial parameter μ called the *renormalization scale*, with $[\mu] = 1$ in all space-time dimensions, is introduced according to

$$\lambda_d = \lambda \mu^{\varepsilon_d}, \tag{5.7}$$

in $d = 4 - \varepsilon_d$ dimensions. Since μ is superficially introduced, no physical quantities should depend on it. The result of (5.5) in $4 - \varepsilon_d$ dimensions can be shown to be

$$\Sigma(k^2) \sim \lambda m^2 \left[\frac{1}{\varepsilon_d} + \log\left(\frac{\mu^2}{m^2} \right) \right], \tag{5.8}$$

which diverges as $\varepsilon_d \to 0$. However, we have separated the divergence as an isolated term according to (5.6). In the next section, we will describe how this separation can be used to provide well defined results in higher-order QFT calculations.

5.2 Renormalization

5.2.1 Renormalized perturbation theory

In order to make sense of the finite quantity obtained by the method of regularization in the previous section, we postulate that the fundamental quantities of the *classical* Lagrangian is not measurable.[1] It is said that the quantities are *bare* and they will be denoted by a subscript "0". The bare Lagrangian $\mathcal{L}_0$ is shifted into a *renormalized* Lagrangian $\mathcal{L}$ and a *counterterm* Lagrangian $\mathcal{L}_{\text{c.t.}}$ according to

$$\begin{aligned} \mathcal{L}_0 &= \tfrac{1}{2}\partial^\mu \phi_0 \partial_\mu \phi_0 - \tfrac{1}{2} m_0^2 \phi_0^2 - \tfrac{1}{4!}\lambda_0 \phi_0^4 \\ &= Z_\phi \tfrac{1}{2}\partial^\mu \phi \partial_\mu \phi - Z_{m^2} \tfrac{1}{2} m^2 \phi^2 - Z_\lambda \tfrac{1}{4!}\lambda \phi^4 \\ &\equiv \underbrace{\tfrac{1}{2}\partial^\mu \phi \partial_\mu \phi - \tfrac{1}{2} m^2 \phi^2 - \tfrac{1}{4!}\lambda \phi^4}_{\mathcal{L}} + \underbrace{\tfrac{1}{2}\delta_\phi \partial^\mu \phi \partial_\mu \phi - \tfrac{1}{2}\delta_{m^2} m^2 \phi^2 - \tfrac{1}{4!}\delta_\lambda \lambda \phi^4}_{\mathcal{L}_{\text{c.t.}}}, \end{aligned} \tag{5.9}$$

[1] All Lagrangians discussed up to this points have been considered classical, in the sense that it is the Lagrangian used in the path-integral.

where δ_i are called counterterms. The counterterms contain the infinities of $\mathcal{L}_0$ and have Feynman-rules associated with them. They may have finite parts as well which will define different *renormalization schemes* and is the topic of the next section. The outcome of renormalization is that we can only speak of observables at different energies, or scales. For instance, define the vertex-counterterm δ_λ to make the renormalized 4-point function $\tilde{\mathcal{G}}_\phi^{(4)}(k_1,k_2,k_3,k_4)$ fulfil

$$\tilde{\mathcal{G}}_\phi^{(4)}(k_1',k_2',k_3',k_4') \equiv \mathrm{i}\lambda\,\delta^{(4)}(k_1'+k_2'+k_3'+k_4')\,, \tag{5.10}$$

for a set of reference momenta k_1', k_2', k_3' and k_4'. This procedure *defines* the renormalized λ. It is now possible to make theoretical predictions and compare with experimental data for any set of momenta k_1'', k_2'', k_3'' and k_4''.

In renormalized perturbation theory, one uses the renormalized quantities in the perturbative expansion. The counterterms will absorb the infinities, order by order, up to a given precision in λ. The condition for a theory to be *renormalizable* is that there should be a finite number of counterterms to absorb the infinities. A necessary condition for a theory to be renormalizable is that the mass-dimensions of the coupling constants are positive or zero. If an infinite number of counterterms are needed to absorb the infinities in divergent diagrams, the theory is said to be *non*-renormalizable. A non-renormalizable theory has coupling constants with negative mass-dimensions. From a aesthetic point of view, non-renormalizable theories may seem unattractive since they require an infinite number of counterterms, operators and coefficient to be fixed by experiments. Pragmatically, non-renormalizable theories can provide valuable and testable quantum-corrections, especially in the context of effective field theories, and should therefore not be discarded *per se*. The SM is a renormalizable theory, but as we saw in Chapter 3.4, it is not complete.

5.2.2 Renormalization schemes

As we saw in section 5.1, interactions will shift the location of the pole in $\tilde{\Delta}^\phi(k^2)$ at $k^2 = m^2_{\text{phys}}$ to be located away from the (bare) Lagrangian parameter m_0^2. This remains true even after renormalization

$$m^2_{\text{phys}} = m^2 + \tilde{\Sigma}(m^2_{\text{phys}}) = m^2 + \tilde{\Sigma}(m^2)\,, \tag{5.11}$$

where we have used $m^2_{\text{phys}} = m^2 + \mathcal{O}(\lambda)$. The pole will not, in general, be located at $k^2 = m^2$. However, we now have a set of counterterms to absorb parts of the self-energy $\Sigma(k^2)$. Let us define the renormalized self-energy

$$\tilde{\Sigma}(k^2) = \Sigma(k^2) - \delta_\phi k^2 + \delta_m m^2\,, \tag{5.12}$$

to fulfil

$$\tilde{\Sigma}(m^2_{\text{phys}}) = \tilde{\Sigma}(m^2) + \mathcal{O}(\lambda^2) = 0\,. \tag{5.13}$$

This fixes the counterterms to be $\delta_{m^2} = -\Sigma(m^2)$ and $\delta_\phi = 0$ to $O(\lambda)$. With these definitions of δ_{m^2} and δ_ϕ, the Lagrangian parameter m^2 is renormalized to be the physical mass, $m^2 = m^2_{\text{phys}}$ to $O(\lambda)$. This renormalization-scheme is called *on-shell* renormalization (OS) and the physical mass is sometimes called *pole mass*.

Another class of frequently used renormalization schemes are the *minimal-subtraction* schemes (MS). In these schemes, only divergent quantities are absorbed by the counterterms, and possibly some finite terms as the Euler-constant γ_E and factors of π, which are always present in dimensional regularization. When one renormalization scheme has been adopted, it should be used throughout the entire perturbative calculation to prevent inconsistencies.

5.2.3 Running parameters

The renormalized quantities, the fields, masses, coupling constants and propagators, in different schemes are related, since they came from the same bare $\mathcal{L}_0$. In particular, the bare quantities themselves are independent of the renormalization scale μ. This seemingly simple fact has an important consequence, that the renormalized quantities in $\mathcal{L}$ depend on the energy-scale and are said to be *running*.

Again, consider the ϕ^4 model. We have

$$\phi_0 = \sqrt{Z_\phi}\,\phi \quad \text{and} \quad \lambda_0 = Z_\lambda Z_\phi^{-2}\,\lambda\,, \tag{5.14}$$

where $Z_i = 1 + \delta_i$. Explicitly, in the MS-scheme, the counterterms read at one loop

$$\delta_Z = 0 \quad \text{and} \quad \delta_\lambda = \frac{3\lambda}{16\pi^2 \varepsilon_d}\,, \tag{5.15}$$

which makes ϕ and λ finite but μ-dependent. By noting that λ_0 is independent of μ, one can evaluate

$$\mu\frac{\mathrm{d}\lambda_0}{\mathrm{d}\mu} = \mu\frac{\mathrm{d}}{\mathrm{d}\mu}\left(\mu^{\varepsilon_d}\lambda Z_\lambda Z_\phi^{-2}\right) = 0\,. \tag{5.16}$$

The solution reads, to $O(\lambda^2)$

$$\mu\frac{\mathrm{d}\lambda}{\mathrm{d}\mu} = \frac{3\lambda^2}{16\pi^2} \equiv \beta(\lambda) > 0\,, \tag{5.17}$$

where we have introduced the β-function and taken $\varepsilon_d \to 0$. This shows that when the renormalization scale μ in increased, the renormalized Lagrangian parameter λ grows. By solving the differential equation (5.17), we obtain the scale-dependence, running, of λ between two scales μ_1 and μ_2

$$\lambda(\mu_2) = \frac{\lambda(\mu_1)}{1 - \frac{16\pi^2}{3}\lambda(\mu_1)\log(\mu_2/\mu_1)}\,. \tag{5.18}$$

Let us consider the running of the electromagnetic charge in QED. At one-loop order, the running of $\alpha \equiv e^2/4\pi$ has the same form as (5.18) with the replacements $\lambda \to \alpha$ and $16\pi^2 \to 2$. The charge grows as the renormalization scale is increased. In particular, define $\alpha(m_e)^{-1} = 137.04$, i.e. $\mu_1 = m_e$, which is motivated by atomic physics. At a higher scale, approriate for electroweak physics, $\mu_2 = m_Z$, we obtain $\alpha(m_Z)^{-1} \approx 129$. This behaviour has been verified experimentally [147, 148].

In QCD, the β-function is *negative* [149]. This means that, contrary to QED, α_{QCD} becomes smaller at higher scales, and grows stronger by lowering the scale [150]. This has important consequences if one would like to study bound states of quarks, hadrons. At scales suitable for this task, α_{QCD} is large enough to make perturbation theory inadequate.

In addition to the running of couplings, the renormalized masses are scale-dependent as well. Recall the relation between the physical (pole) mass and the renormalized mass in (5.11). Consider quarks, they do not form asymptotic states, since they *hadronize* at very small length-scales. In this sense, a physical quark-mass is ambiguous. Nevertheless, in perturbation theory one simply defines the pole-mass as the pole of the renormalized quark-propagator. The running of the renormalized quark-mass is encoded in the *anomalous dimension*

$$\frac{\mu}{m_q}\frac{\mathrm{d}m_q}{\mathrm{d}\mu} = -\frac{2}{\pi}\alpha_{\mathrm{QCD}} \equiv \gamma_{m_q}\,, \tag{5.19}$$

at one-loop order in the MS-scheme [151]. By noting that γ_{m_q} is negative, the renormalized quark mass shows similar behaviour as α_{QCD}. At high energies, m_q becomes smaller, and vice versa. This has important consequences for Higgs-boson phenomenology. In renormalized perturbation theory, the Higgs boson should couple to the renormalized quark-mass through the yukawa couplings, which is smaller than the corresponding pole-mass [8].

5.2.4 Tadpole-renormalization

Consider the Abelian Higgs-model of Chapter 2.2.1. By expanding the bare potential $\mathcal{V}_0$ in terms of v_0, H_0 and G_0, we obtain a cubic vertex $H_0H_0H_0$. At higher orders, this vertex gives via the tadpole-diagram in Fig. 5.4, $\langle H_0\rangle_0 = v_H + \infty$. The quantity $\langle H_0\rangle_0$ requires renormalization. The procedure of renormalization can potentially give a contribution to the vev of the renormalized H-field, $\langle H\rangle_0$. At the classical level, all fields are defined to have zero vev, except the Higgs-field Φ_0. This was accounted for in the Φ_0 ansatz in (2.19) as $\Phi_0 = \frac{1}{\sqrt{2}}(v_0 + H_0 + \mathrm{i}G_0)$, where $\langle H_0\rangle_0 = 0$ at tree-level.[2] This can be interpreted as the renormalized and physical vevs not in general being equal to the classical vev v_0.

[2]The Goldstone-boson G will never acquire a vev because it is odd under CP-transformations.

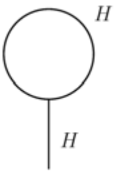


Figure 5.4. A tadpole diagram that could contribute to the vev of the H-field.

The condition that $\mathcal{V}_0$ is minimized at v_0 can be rephrased as

$$\mathcal{T}_0 = -|M_0^2|v_0 + \tfrac{1}{4}\lambda_0 v_0^3 = 0 \;\Rightarrow\; v_0 = \sqrt{\tfrac{4|M_0^2|}{\lambda_0}}\,, \tag{5.20}$$

where $\mathcal{T}_0$ is the linear (tadpole) term in

$$\mathcal{V}_0 = \mathcal{T}_0 H_0 - \tfrac{1}{2}m_{H0}^2 {H_0}^2 + \ldots\,, \tag{5.21}$$

where we have omitted terms involving G_0 and cubic and quartic terms. By shifting the bare potential, employing [152]

$$\begin{aligned} \Phi_0 &= \sqrt{Z_\Phi}\,\Phi\,, \qquad |M_0^2| = Z_\Phi^{-1}(|M^2| + \delta_{M^2})\,, \\ \lambda_0 &= Z_\Phi^{-2}(\lambda + \delta_\lambda)\,, \quad v_0 = v + \delta_v\,, \end{aligned} \tag{5.22}$$

we obtain the renormalized potential $\mathcal{V} = \mathcal{V}_0 - \mathcal{V}_{\text{c.t.}}$. The renormalized potential is formally equal to $\mathcal{V}_0$ but with all the bare quantities replaced with renormalized ones. The renormalized potential $\mathcal{V}_{\text{ren}}$ is defined to be minimized at $v = \sqrt{4|M^2|/\lambda}$. At one-loop order, only linear counterterms are kept,[3] and $\delta_\mathcal{V}$ contains the tadpole c.t. according to

$$\mathcal{V}_{\text{c.t.}} = v\delta_\mathcal{T} H + \ldots = v(v^2\delta_\lambda + \tfrac{1}{2}\lambda v \delta_v - 2v^2\delta_{M^2})H + \ldots. \tag{5.23}$$

Let $\delta_\mathcal{T}$ be chosen to fulfil

$$\tilde{\mathcal{G}}_H^{(1)}(k) \sim v(\mathcal{T} + \delta_\mathcal{T}) = 0\,, \tag{5.24}$$

where $\tilde{\mathcal{G}}_H^{(1)}(k)$ is the renormalized 1-point function for the H-field, and $\mathcal{T}$ denotes the sum of all one-loop tadpole-diagrams. In this scheme, $\langle H\rangle_0 = 0$ is maintained and tadpole-diagrams do not contribute to e.g. self-energies. This renormalization scheme was used in paper II.

The tadpole-renormalization of the SM is outlined at one-loop order in the textbook [153] and at two-loop order in [154].

[3] The counterterms are divergent, but are formally $O(\lambda)$.

5.2.5 Unstable particles and mixing

Only a few elementary particles that we know of are stable. In addition, two particles can mix if they have the same quantum numbers. In this section we will discuss some of the properties of unstable particles and mixing from the point of view of renormalization.

Consider the Lagrangian of the CP-even scalar sector of a 2HDM

$$\begin{aligned}\mathcal{L}_{h,H} &= \tfrac{1}{2}\partial^\mu h \partial_\mu h + \tfrac{1}{2}m_h^2 h^2 + \tfrac{1}{2}\partial^\mu H \partial_\mu H + \tfrac{1}{2}m_H^2 H^2 \\ &\quad + g_1 h^3 + g_2 H^3 + g_3 h^2 H + g_4 h H^2 + \ldots + \mathcal{L}_{\text{c.t.}}\,.\end{aligned} \tag{5.25}$$

The $\ldots$ denotes quartic terms in h and H. We have omitted the interactions of h and H with all the other particles for the sake of argument. The tadpole-terms are assumed to be renormalized to zero in accordance with the prescription outlined in the previous section. If $m_H > 2m_h$, the decay $H \to hh$ is possible due to the interaction term $g_3 h^2 H$. The H-boson is an *unstable* particle. In the S-matrix formalism, *in*- and *out*-states are defined in the $t \to \pm\infty$ limit. This means that, strictly speaking, the decay of an unstable particle can not be evaluated as an $1 \to N$ scattering event. Fortunately, there is an unambiguous way to find the decay-width Γ of, in this case, the H-boson. Consider the self-energy Σ_H, which at second order in perturbation theory is represented by a Feynman-diagram with a loop of h-bosons. There are two additional diagrams contributing, one which contains two H in the loop, and one with a h and a H. It can be shown that the self-energy develops a finite imaginary part

$$\text{Im}\left[\Sigma_H(p^2)\right] \neq 0\,, \tag{5.26}$$

for $p^2 > 4m_h^2$. By adopting the *real pole mass*-scheme,

$$m_{H\text{phys}}^2 = m_H^2 + \text{Re}\left[\tilde{\Sigma}_H(m_{H\text{phys}}^2)\right]\,, \tag{5.27}$$

the full H propagator becomes [4]

$$\tilde{\Delta}^H(p^2) = \frac{\text{i}}{p^2 - m_H^2 - \text{i}\,\text{Im}\left[\tilde{\Sigma}_H(p^2)\right]}\,. \tag{5.28}$$

Consider the elastic cross section σ of $h(p_1)h(p_2) \to h(p_3)h(p_4)$. For $s = (p_1 + p_2)^2 \approx 4m_H^2$, the s-channel exchange of a H-boson is the dominant contribution to σ

$$\sigma^{-1} \sim |s - m_H^2 - \text{i}\,\text{Im}\left[\tilde{\Sigma}_H(s)\right]|^2\,. \tag{5.29}$$

One then identifies $\text{Im}\left[\tilde{\Sigma}_H(s)\right]$ with the *resonance width* of the H-boson. In the rest-frame of the H-boson, we have

$$\Gamma_H = \text{Im}\left[\tilde{\Sigma}_H(m_H^2)\right]/m_H\,. \tag{5.30}$$

[4]Due to the hermiticity of $\mathcal{L}_{h,H}$ in (5.25), the conterterms must be real and can only, in any renormalization scheme, absorb parts of $\text{Re}\left[\Sigma_H(p^2)\right]$.

The result (5.30) is general and can be proven by using the *Cutkosky rules* [155]. If the resonance is *narrow*, i.e. $\Gamma \ll m$, the approximation $\mathrm{Im}\left[\tilde{\Sigma}(s)\right] = \mathrm{Im}\left[\tilde{\Sigma}(m^2)\right]$ is valid [7].

In phenomenological calculations, one often uses a propagator of the form

$$\mathcal{P}(s) = \frac{\mathrm{i}}{s - m^2 - \mathrm{i}m\Gamma} \tag{5.31}$$

where Γ is the decay-width in the rest-frame of the unstable particle.

In the previous discussion, we neglected an important issue, the fact that h and H can *mix*. Due to the $h-H$ mixing, the following counterterms must exist

$$\mathcal{L}_{\mathrm{c.t.}} = \delta_{hH}\,\partial^\mu h \partial_\mu H + \delta_{m^2_{hH}}\, hH + \ldots\,, \tag{5.32}$$

if we are to renormalize the self-energies of h and H.[5] In addition, there will be off-diagonal self-energies $\Sigma_{hH}(k^2) = \Sigma_{Hh}(k^2)$, which should be renormalized as well. By expressing the renormalized self-energies as

$$\tilde{\Sigma}_i(k^2) = \Sigma_i(k^2) - k^2\delta_{Z_i} + \delta_{m_i^2}\,, \tag{5.33}$$

where $i = h, H$ and hH, the inverse of the full propagator matrix becomes

$$\tilde{\Delta}^{h,H}(k^2)^{-1} = \begin{pmatrix} k^2 - m_h^2 - \tilde{\Sigma}_h(k^2) & -\tilde{\Sigma}_{hH}(k^2) \\ -\tilde{\Sigma}_{hH}(k^2) & k^2 - m_H^2 - \tilde{\Sigma}_H(k^2) \end{pmatrix}. \tag{5.34}$$

By inversion of (5.34), one obtains the full propagators which have poles located at the physical masses of the h and H-bosons. This formalism was applied to the CP-even Higgs-bosons in the MSSM in [156], where one-loop effects of their masses was studied. In paper II we considered the effects of $H^\pm - W^\pm$ and $A - Z$ mixing to the tree-level forbidden decays of $H^\pm$ and A into a pair of fermions.

[5] If the mixing is present at tree-level, we have not correctly identified h and H as the mass-eigenstates of the theory and an orthogonal transformation on $(h, H)^T$ should be applied.

6. Higgs bosons at colliders

Reality is what kicks back when you kick it.
This is just what physicists do with their particle accelerators.

Victor J. Stenger

In this chapter we will discuss how the results from Higgs-boson searches at the LHC [3, 4, 5] can be applied to study different scenarios for EWSB (Higgs sectors). In particular, we will discuss the implications of *exclusion limits* [157] and *signal strength* measurements [158]. We will review two public codes, HiggsBounds [159] and HiggsSignals [160], which tests the compatibility of arbitrary Higgs sectors with measurements from the LHC and other colliders. An early summary of the status of the LHC Higgs search can be found in [161].

6.1 Exclusion limits

In order to establish the existence of a new particle, one must understand what the experiments would record with and without the hypothesised particle. In the following, we will take the Standard Model Higgs boson as an example. The properties of this Higgs boson are determined by its mass m_H, see Chapter 3.3. Let us assume that one would like to discover it as being produced in the gg-process and decaying into a pair of photons ($\gamma\gamma$). The basic observable would be the invariant mass-distribution of two high-energetic photons and it should have a narrow peak centred at m_H. This distribution is often referred to as the *signal*. Furthermore, there are several other processes not involving a Higgs boson, that can yield two high-energetic photons at a collider such as the LHC. However, the invariant mass-distribution of such photons (*background*) will basically have an inverse power-law behaviour without any prominent peaks. In addition, the background distribution is independent of m_H.

The signal and background distributions for a given observable are obtained by means of dedicated Monte Carlo simulations and are finally compared to the actual experimental measurements. Negative search results are often summarized in terms of *exclusion plots* as illustrated in Fig. 6.1, which serves to find regions in m_H where the SM Higgs boson is unlikely to exist.[1] In this fictitious figure, which was made for illustration purposes only [164], there are many features. First of all, the *y*-axis displays the 95% confidence limit on the cross section of a given process normalized to the SM one, $\sigma/\sigma_{\text{SM}}$. The dotted black line is the expected exclusion in the absence of a signal with its 68% (1σ) and 95% (2σ) confidence intervals depicted in green and yellow bands respectively. The solid black line is the observed and experimentally established 95% confidence limit on $\sigma/\sigma_{\text{SM}}$. Whenever the solid black line is below the red line (indicating $\sigma/\sigma_{\text{SM}} = 1$), the SM Higgs-boson hypothesis in this process is *excluded* with 95% statistical confidence.

To summarize, the regions 140 GeV $\lesssim m_H \lesssim$ 280 GeV and 370 GeV $\lesssim m_H \lesssim$ 550 GeV are excluded at 95% confidence level (C.L.). This means that if the SM Higgs boson exists, it is unlikely to have a mass within these regions based on the fictitious analysis of Fig. 6.1.

Several searches for Higgs boson have been performed by experiments at the Tevatron, LEP and the LHC. In order to apply those analyses to arbitrary Higgs sectors, the public code HiggsBounds was constructed [159]. In the following, we shall briefly describe the philosophy of HiggsBounds. For a complete description of the usage of HiggsBounds, we refer to the manual [159].

The first step of HiggsBounds is to establish that the Higgs Bosons in a model to be tested are SM-like enough [165]. This is because many of the

[1] Exclusion plots for particles besides the SM also exists, e.g. charged Higgs bosons [162] and supersymmetric particles [163].

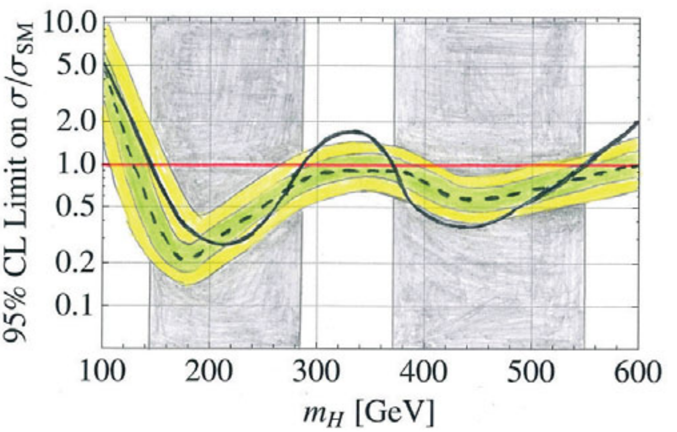


Figure 6.1. A fictitious exclusion plot for the SM Higgs-boson cross-section ratio $\sigma/\sigma_{\text{SM}}$ versus its mass. The solid black line is the observed limit, and the dotted black line is the expected limit without the SM Higgs-boson (background). The green and yellow bands shows the 68% and 95% confidence levels for the expected limit. The shaded grey regions display the mass intervals where the SM Higgs-boson is excluded with 95% confidence level.

analyses that HiggsBounds use have originally been performed within the SM framework, i.e. they are not model independent. The second step is to find the analysis A_i, applied to a given Higgs boson H_j, which has the best expected exclusion limit for a given quantity (e.g. cross section times branching ratios). When this analysis has been found, HiggsBounds applies the observed exclusion limit of that analysis. The 95% C.L. interpretation is kept because only one analysis is used, even though there might be several Higgs-bosons in the considered model. HiggsBounds was used in all papers of this thesis.

6.2 Signal strengths

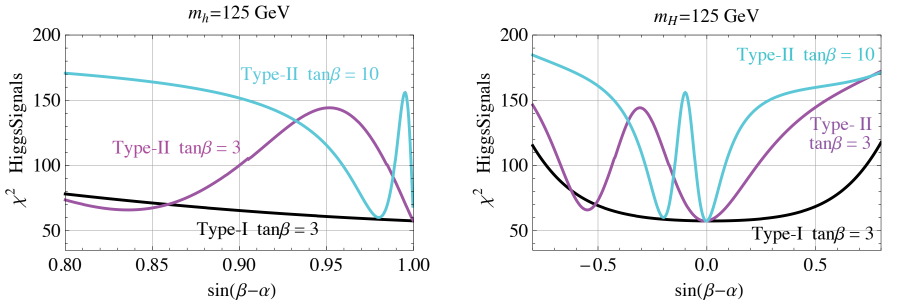
In the previous section, we outlined the implications of negative results in the search for Higgs bosons, namely exclusion limits. We will now discuss the impact of the Higgs boson discovery at the LHC [3, 4, 5] on Higgs sectors. Apart from surviving the constraints of exclusion, any model prediction should in addition fit the mass of the recently discovered Higgs boson $\mathcal{H}$ ($m_{\mathcal{H}} \approx 125$ GeV) and the *signal strengths*. The signal strength of a Higgs Boson H decaying into a final state Y is basically defined as

$$\mu_Y = \frac{\sum_i \sigma_i(pp \to H + X_i) \times \mathrm{BR}(H \to Y)}{\sum_i \sigma_i(pp \to H_{\mathrm{SM}} + X_i) \times \mathrm{BR}(H_{\mathrm{SM}} \to Y)}, \tag{6.1}$$

where i denotes the different production mechanisms, e.g. gg-fusion, vector boson fusion, and so on. By definition, the SM Higgs boson would give $\mu_Y = 1$ for all decay channels Y. The ATLAS and the CMS experiments present their results for μ_Y in two ways. The first way is to present all signal strengths μ_Y (and their corresponding errors) in a diagram for a fixed mass $m_{\mathcal{H}}$. The second way is to give each μ_Y as a function of $m_{\mathcal{H}}$, $\mu_Y(m_{\mathcal{H}})$.

In order to give a qualitative statement on how well (or badly) a particular parameter point in a given model fits the data, one performs a so called Chi-Squared test (χ^2). The authors of HiggsBounds have made another public code, HiggsSignals, which calculates the χ^2 for a parameter point in any model [165, 160]. HiggsSignals can perform this in two ways, using the publically available data from ATLAS and CMS. Either it can use the diagrams with all μ_Y at a given $m_{\mathcal{H}}$ (the *peak-centred* method) or it can use the $\mu_Y(m_{\mathcal{H}})$ distributions (the *mass-centred* method). In addition, HiggsSignals also gives the possibility to combine these two methods and also accounts for the various mass measurements. Finally, HiggsSignals also has a way of dealing with the possibility that several (almost) mass-degenerate Higgs bosons contribute to the various μ_Y [166]. For more details of the implementation and methods of HiggsSignals, we refer to the manual [160].

As a toy-example, we give the χ^2 distribution from HiggsSignals in Fig. 6.2 for the 2HDM Type-I and -II as a function of $\sin(\beta - \alpha)$ with the other parameters kept fixed. As can be seen in Fig. 6.2(a), where h is to be identified with $\mathcal{H}$,



(a) (b)

Figure 6.2. The χ^2_{tot} distributions obtained by HiggsSignals (peak-centred method) for 2HDM Type-I and -II. In the left figure, the lightest CP-even Higgs boson h is the candidate for the observed Higgs boson. The masses of the other states are taken as $m_H = m_A = m_{H^\pm} = 300$ GeV. In the right figure, the candidate for the observed Higgs boson is instead H. The masses of the other states are taken as $m_h = 85$ GeV, $m_H = m_A = m_{H^\pm}$. In both figures, we have $m_{12}^2 = m_A^2 \cos\beta \sin\beta$. No constraints have been applied.

the χ^2 has a minimum at $\sin(\beta-\alpha) = 1$ for both Type-I and -II. There is also a minimum around $\sin(\beta-\alpha) = 0.83$ ($\sin(\beta-\alpha) = 0.98$) for $\tan\beta = 3$ ($\tan\beta = 10$) in the Type-II model. In Fig. 6.2(b), where instead H is identified with $\mathcal{H}$, there are minima at $\sin(\beta-\alpha) = 0$ (for Type-I and -II) and at $\sin(\beta-\alpha) = -0.55$ ($\sin(\beta-\alpha) = -0.2$) for $\tan\beta = 3$ ($\tan\beta = 10$) in the Type-II model. Some of these features can easily be understood because the limit $\sin(\beta-\alpha) \to 1$ corresponds to give h SM-like properties, and similarly $\sin(\beta-\alpha) \to 0$ makes H SM-like. As a reference, the SM Higgs boson with a mass $m_{H_{\mathrm{SM}}} = 125.3$ GeV yields $\chi^2 = 57.1$ with this implementation of HiggsSignal. In paper IV we study 2HDMs in more detail with the usage of HiggsBounds and HiggsSignals.

7. Summary in Swedish – Sammanfattning på Svenska

If you can't explain something simply,
you don't know enough about it.

Albert Einstein

Denna avhandling är en vetenskaplig publikation som sammanfattar flera års arbete. I detta avslutande kapitel ges en populärvetenskaplig presentation av avhandlingens ämnesområde, frågeställningar och resultat.

Inom forskningsområdet *elementarpartikelfysik* (även kallat högenergifysik) studeras (det man känner till som) materiens minsta beståndsdelar. Man vill ta reda på vilka elementarpartiklar som existerar, deras egenskaper och hur de växelverkar med varandra. Elementarpartikelfysik är både en teoretisk och experimentell verksamhet. Den experimentella utrustningen är bland de mest omfattande och imponerande inom all naturvetenskaplig forskning. Tusentals fysiker och ingenjörer har jobbat med acceleratorerna och detektorerna under flera decennier. Den senast upptäckta elementarpartikeln är en *Higgsboson* vid LHC-experimenten år 2012. Denna partikels egenskaper är en av avhandlingens huvudområden.

Teoretiskt sett beskrivs elementarpartiklar som excitationer (vibrationer) av *kvantfält*. En analogi till denna beskrivning är atomer i ett fast material (t.ex. en bit metall). Atomerna i materialet växelverkar med varandra med en kraft som påminner mycket om en elastisk fjäder. Denna kraft gör att atomerna kan vibrera individuellt kring sina jämviktslägen. En yttre störningskraft kan dessutom förmå atomerna att vibrera tillsammans som ett kollektiv. Detta fenomen är inget annat än en ljudvåg som fortplantar sig genom materialet. Ljudvågen kan i sin tur matematiskt beskrivas som en slags partikel (*kvasipartikel, fonon*) eftersom den följer en partikellik rörelseekvation. Ljudvågen kan dessutom kollidera och spridas mot andra ljudvågor i materialet.

Elementarpartiklar skall i analogin därför identifieras med ljudvågorna, de kollektiva vibrationerna hos vibratorerna (atomerna). Vilken roll vibratorerna spelar i denna analogi är dock oklart. Inom *strängteori* så beskrivs elementarpartiklarna som vibrationer hos extremt små endimensionella strängar som vibrerar i extra, hoprullade, dimensioner.

En viktig del av den teoretiska elementarpartikelfysiken är att försöka systematisera och klassificera elementarpartiklarna, både de som man hittills har upptäckt och nya, postulerade, partiklar. Ofta använder man *symmetrier* som vägledning i detta arbete. Ett exempel är *paritet* som säger att partiklarnas egenskaper skall vara desamma om alla tre rumsdimensioner speglas.

En speciell symmetri är *gaugesymmetri*, som är en slags matematisk symmetri. I den matematiska beskrivningen av ett fenomen så har man fler frihetsgrader i beskrivningen än vad som är de fysikaliska frihetsgraderna (detta kallas redundans). Två kvantfält som är relaterade via gaugesymmetrin beskriver därmed samma fysikaliska verklighet. Genom att kräva gaugesymmetri i beskrivningen av elementarpartiklarna så visar det sig att de fysikaliska krafterna som vi idag känner till manifesterar sig som ett utbyte av masslösa partiklar som kallas kraftutbytespartiklar (gaugebosoner). Kravet på gaugesymmetri medför även att bl.a. de möjliga typerna av växelverkningar är entydigt bestämda. Det faktum att man får en sammanhållen beskrivning av elementarpartiklar och krafter med en symmetriprincip har varit mycket attraktivt och vägledande inom den teoretiska forskningen. Dessutom uppvisar teorier med gaugesymmetri matematiska egenskaper som möjliggör beräkningar av partiklarnas egenskaper med väldigt hög precision.

Det fanns dock ett stort problem med att beskriva en av de krafter vi känner till med gaugesymmetri, den svaga kraften. Till denna kraft behöver man kraftutbytespartiklar (W och Z-bosonerna) som är massiva, men gaugesymmetrin kräver masslösa sådana. Under 1960-talet påvisade François Englert, Robert Brout och Peter Higgs en mekanism som möjliggör gaugesymmetri och massiva kraftutbytespartiklar. Denna mekanism kallas för *Higgsmekanismen*.

I Higgsmekanismen postulerar man existensen av ett (eller flera) så kallade *Higgsfält* vars lägsta energitillstånd (vakuum) är nollskilt. Higgsfältet bidrar därför till energidensiteten på ett icke trivialt sätt även i avsaknad av excitationer av detta. Higgsfältet tillskrivs gaugesymetriegenskaper som möjliggör växelverkan mellan detta vakuum och vissa gaugebosoner. I denna växelverkan får de gaugebosoner som växelverkar med Higgsfältet massa. Ju starkare en gaugeboson växelverkar med vakuumet desto större massa får den.

Genom att lägga till Higgsfältet så bibehålls gaugesymmetrin samtidigt som kraftutbytespartiklarna får massa. Vakuumet är dock inte gaugesymmetriskt, man säger därför att gaugesymmetrin är spontant bruten. Partiklarna som är excitationer av Higgsfältet kallas *Higgsbosoner*. Att hitta Higgsbosoner är därför nödvändigt för att ge stöd åt Higgsmekanismen. En annan viktig konsekvens av Higgsmekanismen är att man fortfarande kan beräkna partiklarnas egenskaper med väldigt hög precision.

Det fanns dessutom problem med att beskriva den svaga kraften med gaugesymmetri och samtidigt ha massa hos de partiklar som bygger upp atomer, elektronerna (leptonerna) och kvarkarna. Higgsmekanismen möjligör även en lösning på detta problem genom att låta dessa *materiepartiklar* växelverka med Higgsfältet.

En analogi till Higgsmekanismen är ljus (fotoner) som i avsaknad av materia är masslösa och färdas med hastigheten c_0 = 299 792 458 m/s. I närvaro av materia är ljusets hastighet lägre än c_0. Man kan därför se det som att ljus effektivt sett får massa i materia. Beroende på materian och ljusets våglängd så är ljuset hastighet olika och karakteriseras av materiens brytningsindex. Ljuset kan på så sätt ha olika effektiv massa i olika medier.

I denna analogi så spelar vakuumet hos Higgsfältet ungefär samma roll som materian och brytningsindexet blir istället partikelberoende eftersom vakuumet antas vara konstant i hela universum. Exempelvis skulle både W och Z-bosonerna vara masslösa i avsaknad av Higgsvakuumet. Men, i närvaro av detta så blir Z-bosonen något tyngre än W-bosonen eftersom den växelverkar lite starkare med vakuumet.

Det finns, teoretiskt sett, en nästintill oändlig valmöjlighet att implementera Higgsfält för att ge massa åt W och Z bosonerna. Den enklaste modellen kallas idag för *Standardmodellen* och den har varit framgångsrik i att beskriva vad som sker i de experimenten som gjordes innan LHC, så när som på avsaknaden av direkta bevis på existensen av dess Higgsboson.

Trots de övriga framgångarna hos Standardmodellen så var man ändå inte helt tillfredsställd med den av andra skäl än frånvaron av Higgsbosoner i de tidigare experimenten. Några av dessa skäl är massor hos en viss typ av partiklar som kallas *neutriner*, avsaknaden av möjlighet att kunna beskriva mörk materia, asymmetrin mellan materia och antimateria, för att nämna några stycken. I modeller för att lösa några av dessa "problem" finns det ofta en eller flera Higgsbosoner vars egenskaper kan skilja sig från Standardmodellens Higgsboson.

I denna avhandling har aspekter hos några av dessa modeller studerats. I synnerhet har modeller med fler än en Higgsboson studerats, särskilt elektriskt laddade Higgsbosoner $H^{\pm}$. En viktig upptäckt i avhandlingen var att i en av modellerna (som undersöks i forskningsartiklarna I och II) så kan det dominerande sönderfallet hos den laddade Higgsbosonen vara $H^{\pm} \to W\gamma$, d.v.s. till en W-boson och en foton. Denna sönderfallskanal har i tidigare studerade modeller varit mycket osannolik. Dessutom kan inte några av Higgsbosonerna i denna modell produceras på liknande sätt som i tidigare studerade modeller.

I mitten av år 2012 annonserade ATLAS och CMS experimenten vid den nu ledande partikelacceleratorn LHC att man upptäckt en Higgsboson. Man har i dagsläget uppmätt flera av denna Higgsbosons egenskaper och den visar sig vara mycket lik den som förutsägs av Standardmodellen. Dock är dessa mätningar inte helt uteslutande bevis i favör för Standardmodellen. I avhandlingens forskingsarbeten har flera modeller studerats med hänsyn till resultaten från LHC-experimenten. Dessa modeller beskriver den uppmätta data mycket bra inom de delar av deras parameterrum som ger en standardmodellik Higgsboson.

Det mest slående beviset för en modell bortom Standardmodellen vore nog upptäckten av ytterligare en Higgsboson. Genom att studera de delar av modellernas parameterrum som är kompatibla med LHC-resultaten har förutsägelser gjorts, baserat på dessa modeller, för vad man kan upptäcka i framtiden när experimentet vid LHC fortsätter.

I ett av forskningsarbetena detaljstuderades möjligheten att upptäcka den tidigare nämnda laddade Higgsbosonen $H^{\pm}$ i en specifik produktions- och sönderfallskanal. Med hjälp av datorsimuleringar uppskattades hur denna kanal skulle kunna manifestera sig vid LHC-experimenten. Det visade sig att några av modellerna som förutsäger den undersökta kanalen är bortom LHC-experimentens förmåga att upptäcka. I några andra modeller är dock utsikterna något mer lovande. Förhoppningen är att experimentalister skall bli inspirerade av våra resultat och leta efter den laddade Higgsbosonen i samma kanal som vi studerade.

Om man inte lyckas hitta fler partiklar vid LHC än denna mycket standardmodellika Higgsboson så kommer man troligtvis behöva ompröva en mycket stor del av partikelfysiken i och med att Standardmodellen inte förmår, som tidigare nämnts, förklara vissa viktiga fenomen så som mörk materia. Det är en synnerligen spännande tid framöver i detta forskningsfält och det har varit ett stort privilegium att få bidra med denna avhandling.

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