Stability of global vertex solution in higher-dimensional spacetime

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Abstract

It is well known that higher-dimensional black objects with translational invariance are unstable, which is called Gregory-Laflamme instability. There is a question if this instability is eliminated by adding a scalar hair to the black objects. For the first step, we investigate a regular topological string solution and its stability in the 5-dimensional Einstein-Higgs system. Linear analysis shows that the string solution is stable against non-uniform perturbations.

1 Introduction

In higher-dimensional spacetime there is variety of black objects such as a black string, a black ring, and black branes aside from a black hole. The black string and black branes have translational invariance along one or some spatial direction(s). It is shown that these objects are unstable against the non-uniform perturbation, which is known as Gregory-Laflamme instability[1]. What is their final state? This is a question which attracted much attention in the last decade. Analysis beyond perturbation is necessary to answer this question. The numerical approach is the only method, and static solutions with nontrivial horizon geometry were constructed in 6-dimensional spacetime[2]. They are candidates for the final state. There are, however, other candidates. For instance, the horizon is pinched and continues to shrinks with infinite time[3].

Dynamical stability is the one of the aspects of the system. Thermodynamical stability is another aspect. It was proposed that dynamical stability is strongly related to thermodynamical stability, which is called Gubser-Mitra conjecture[4]. It states that for systems with a translational symmetry and an infinite extent dynamical Gregory-Laflamme instability arises precisely when the system is thermodynamically unstable. There are a lot of examples which support Gubser-Mitra conjecture in vacuum and electro-vacuum systems.

The black object has an event horizon, and we do not know what matters were distributed before the gravitational collapse. It is natural, however, to assume that the initial object has the same translational symmetry as the black objects. Topological defects such as a vertex (sting) and a domain wall are regular objects with the symmetry. Besides, they have the different type of stability, i.e., topological stability. Although perturbative analysis of dynamical stability shows local stability, topological stability indicates global stability in flat spacetime. When gravity is taken into account, global stability is not guaranteed. An event horizon may be formed in the middle of the transition to the "globally stable solution". Then all the energy density may be swallowed into the event horizon, and a vacuum black object remains.

In 4-dimensional spacetime, a static black hole solution with a scalar hair was discovered[5]. It is called a monopole black hole. Although its field configuration of far region is similar to the global monopole, the monopole black hole has an event horizon around the center. In 5-dimensional spacetime, a black string solution with the analogous scalar hair exists. Then what happens if a non-uniform perturbation is added to it? Which win, dynamical instability or topological stability? In this paper we investigate stability of the regular global string solution in 5-dimensional spacetime as the first step, because the above question is almost trivial if the global string is unstable against non-uniform perturbation.

The organization of this paper is as follows. In the next section, we construct the global string solution in 5-dimenion. In Sec. 3, we perform a perturbative analysis and give a result.

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2 Global String

We consider a real triplet scalar field Φ^a ($a = 1, 2, 3$) which has spontaneously broken internal $O(3)$ symmetry, and minimally couples to gravity. The action is

$$S = \int dx^5 \sqrt{-g} \left[\frac{R}{16\pi G} - \frac{1}{2} \partial_\mu \Phi^a \partial^\mu \Phi^a - \frac{\lambda}{4} (\Phi^a \Phi^a - v^2)^2 \right], \tag{1}$$

where R is the Ricci scalar of 5-dimensional spacetime. λ and v are the self-coupling constant and the vacuum expectation value (VEV) of the scalar field, respectively.

We shall assume that spacetime is static and has translational invariance along one of the spatial direction. The metric form is

$$ds^2 = -f(r)e^{-2\delta(r)}dt^2 + \frac{1}{f(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) + w(r)dz^2. \tag{2}$$

Basic equations becomes simpler by adopting function $z(r)$ defined by $g_{ww} = e^{z(r)}$. However, since the function $w(r)$ is useful when $g_{zz} \to 0$, we use $w(r)$.

The scalar field is assumed to have unit winding number and so-called hedge-hog configuration,

$$\Phi^a = h(r) \frac{x^a}{r}, \tag{3}$$

where x^a ($a = 1, 2, 3$) are the Cartesian coordinates for the fixed z.

There are two physical parameters λ and v in this system. By scaling the variables as

$$\bar{x}^\mu = v\sqrt{\lambda} x^\mu, \qquad \bar{\Phi}^a = \frac{\Phi}{v}, \tag{4}$$

the action can be rewritten as

$$\bar{S} = \int d^5x \sqrt{-g} \left[\frac{\bar{R}}{16\pi v^2} - \frac{1}{2} \partial_\mu \bar{\Phi}^a \partial^\mu \bar{\Phi}^a - \frac{1}{4} \left(\bar{\Phi}^2 - 1\right)^2 \right], \tag{5}$$

In this formula, the coupling constant λ is scaled out. The VEV appears only in the denominator of curvature term and affects the system only when self-gravity is taken into account.

The basic equations are

$$\frac{f'}{r} + \frac{w'f'}{4w} + \frac{w''f}{2w} - \frac{1}{r^2} + \frac{fw'}{rw} + \frac{f}{r^2} - \frac{w'^2 f}{4w^2} = -2\pi v^2(h^2-1)^2 - 4\pi v^2 f h'^2 - \frac{8\pi v^2 h^2}{r^2}, \tag{6}$$

$$\frac{w''}{2w} - \frac{w'^2}{4w^2} + \frac{2\delta'}{r} + \frac{w'\delta'}{2w} = -8\pi v^2 h'^2, \tag{7}$$

$$\frac{1}{r^2}\left(f - 1 + rf' - rf\delta'\right) - \frac{fw'}{2rw} + \frac{w'^2 f}{4w^2} - \frac{w'f'}{2w} + \frac{fw'\delta'}{2w} - \frac{w''f}{2w} = -8\pi v^2 \frac{h^2}{r^2}, \tag{8}$$

$$h'f' + h''f - h'f\delta' + \frac{2h'f}{r} - \frac{2h}{r^2} + \frac{h'fw'}{2w} - (h^2-1)h = 0, \tag{9}$$

where a prime denotes a derivative with respect to the radial coordinate. We have omitted the bar of the variables.

Basic equations are solved with suitable boundary conditions. Putting the regularity condition at the axis $r = 0$, we will obtain the self-gravitating global vertex solution. The variables are expanded as

$$f(r) = 1 - \frac{2\pi v^2}{9}(1 + 18{h_1}^2)r^2 + \dots, \qquad w(r) = w_0\left(1 - \frac{4\pi v^2}{9}r^2 + \dots\right),$$

$$\delta(r) = \delta_0 + \frac{\pi v^2}{9}(1 - 18{h_1}^2)r^2 + \dots, \qquad h(r) = h_1 r - \frac{h_1}{10}\left[1 - \frac{2\pi v^2}{9}(7 + 54{h_1}^2)\right]r^3 + \dots.$$

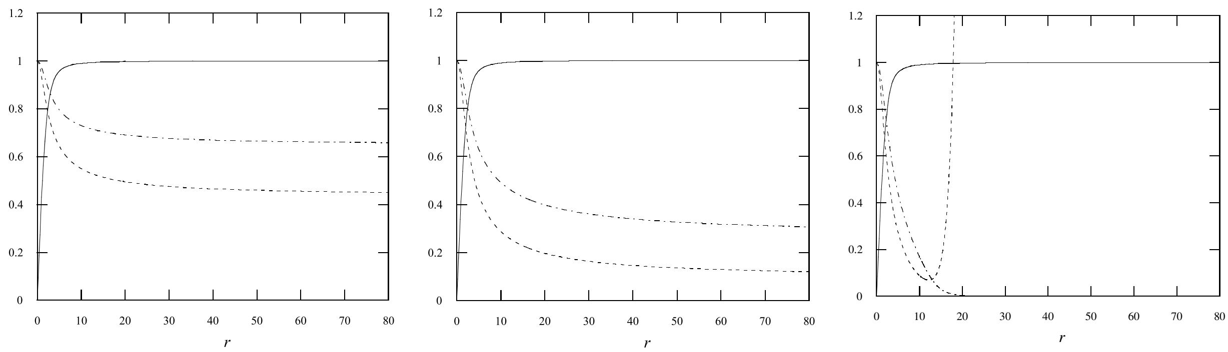

Figure 1: The field configurations of the static global string in 5-dimensional spacetime (left: $v = 0.15$, center: $v = 0.19$, right: $v = 0.20$). The solid, the dashed, and the dot-dashed lines show the field variables h, f, and w, respectively.

δ_0 and w_0 are not determined by the regularity condition. However, we can assume $\delta_0 = 1$ and $w_0 = 1$ without loss of generality because of scaling of the coordinates t and z. Therefore, the free parameter is just h_1.

At infinity $r \to \infty$, the variables are expanded as

$$f(x) = 1 - 8\pi v^2 + f_1 x + \left[\frac{1}{4}w_1^2(1-8\pi v^2) - \frac{1}{4}w_1 f_1 - \frac{40}{3}\pi v^2\right]x^2 + \cdots,$$

$$w(x) = w_\infty\left(1 + w_1 x + \frac{1}{2}w_2 x^2 + \cdots\right), \qquad \delta(x) = \delta_\infty + \frac{w_1}{2}x + \frac{1}{8}(3w_2^2 - 2w_1^2)x^2 + \cdots,$$

$$h(x) = 1 - x^2 - \frac{3 - 16\pi v^2}{2}x^4 + \cdots.$$

where $x := 1/r$, and

$$w_2 = \left[\frac{1}{8\pi v^2 - 1}\left(\frac{32\pi v^2}{3} + f_1 w_1\right) + w_1^2\right].$$

w_∞ and δ_∞ are determined by solving the basic equations from the axis to infinity. The solution is characterized by the boundary values f_1 and w_1. f_1 corresponds to the mass observed at infinity $r \to \infty$. w_1 has following physical meaning. Since there is the translational invariance along the z axis, the spacetime can be reduced to a 4-dimensional system by Kaluza-Klein dimensional reduction. Then the metric function $w(r)$ becomes a dilaton field. w_1 is related to the scalar charge of this dilaton field.

The first step to obtain the static solution is choosing a value of h_1 at the axis. And secondary, we integrate numerically the basic equations from the axis to $r \to \infty$. The field variables diverge at finite r in the most cases, and hence, the value of h_1 should be tuned to satisfy the boundary condition at infinity by iterative method. In this sense, h_1 is a shooting parameter.

Fig. 1 shows the field configurations of the static global string in 5-dimensional spacetime. For the large VEV ($v \approx 0.20$), the metric function w vanishes and the numerical calculation stops at finite r.

3 Stability analysis

In this section, we analyze stability of the global string solution obtained in the previous section. The metric is perturbed as

$$g_{\mu\nu}(t, x^a) = \bar{g}_{\mu\nu}(x^a) + h_{\mu\nu}(t, x^a), \tag{10}$$

where $\bar{g}_{\mu\nu}(x^a)$ is the static solution and $h_{\mu\nu}(t, x^a)$ is perturbation function. Here, we define a new variable by

$$\psi_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}h, \tag{11}$$

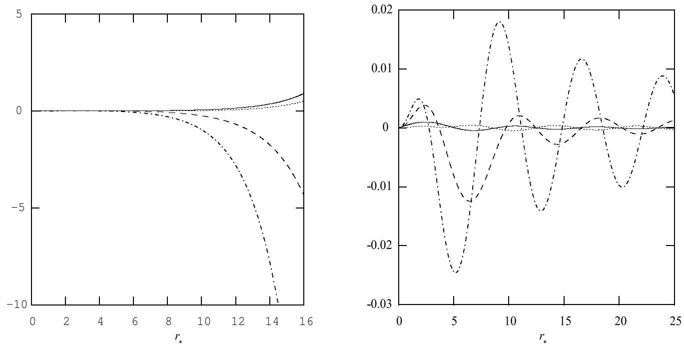

Figure 2: The configuration of the perturbation functions (left: $v = 0.15$, $k = 0.5$, $\sigma = 0$, right: $v = 0.15$, $k = 0.5$, $\sigma = 1.0$). The solid, the dotted, the dashed and the dot-dashed lines show the field variables η, N, L, and z, respectively.

where $h = h_\lambda{}^\lambda$, and adopt the gauge condition $\psi_\mu{}^\lambda{}_{;\lambda} = 0$. The perturbation of the scalar field is

$$\Phi^a(t, x^a) = \bar{\Phi}(x^a) + \delta\Phi^a(t, x^a), \tag{12}$$

where $\bar{\phi}(x^a)$ is the static solution.

We assume that perturbation does not depend on θ and ϕ and adopt the metric perturbation as

$$\psi_{\mu\nu} = e^{i(\sigma t + kw)} \begin{pmatrix} -fe^{-2\delta}N & iS_{tr} & 0 & 0 & S_{tz} \\ iS_{tr} & f^{-1}L & 0 & 0 & iS_{rz} \\ 0 & 0 & r^2T & 0 & 0 \\ 0 & 0 & 0 & r^2T\sin^2\theta & 0 \\ S_{tz} & iS_{rz} & 0 & 0 & wS_{zz} \end{pmatrix}, \tag{13}$$

where the functions N, L, T, S_{tr}, S_{rz}, S_{tz}, S_{zz} are the functions of r. The perturbation of the scalar field is assumed as

$$\delta\Phi^a = \eta(r)\frac{x^a}{r}. \tag{14}$$

The perturbation equations are obtained by substituting these ansätze. They are, however, tedious and we do not show them here explicitly.

The perturbation equations are integrated with the regular boundary condition at the axis. If there are bound states with $\sigma^2 < 0$, the perturbation grows exponentially with time, and the solution is found out to be unstable. By our analysis, however, we cannot find such modes. Configurations of the perturbation functions with $\sigma^2 = 0$ are shown in Fig. 2. In case where unstable modes exist, the perturbation functions with $\sigma^2 = 0$ usually have extremum points and nodes. But we cannot find them in Fig. 2. These facts imply that the static global solutions are stable against the perturbation assumed above. All the details will be reported elsewhere[6].

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