p̄-Note 22?

High Energy Electron Cooling Revisited

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(Fermilab, Sept. 1982)

Electron Cooling in a Single Pass Mode

Initial Cooling Time, lower limit
Extremely Cold Electron Beam ($T_\parallel = T_\perp = 0$)

$$\tau = \frac{e\,\beta^4 \gamma^5 \theta^3}{4\pi\, r_e r_p\, J_e\, L\, \eta} \qquad \left(\frac{d\varepsilon}{dt} = -2\,\frac{\varepsilon_0}{\tau}\right)$$

(see p̄-Note 123)

$\beta \sim 1$
$\gamma = 1066.8$
$r_e = 2.8178 \times 10^{-15}$ m
$r_p = 1.5347 \times 10^{-18}$ m

L, Coulomb Log ~ 15

$$\eta = \frac{8\,m}{2000\,m \times \pi} = 0.0013 \quad \text{(medium straight section)}$$

$$J_e = \frac{I_e}{\pi a^2} = \frac{\text{Electron Beam Current}}{\text{Electron Beam Cross Section}}$$

$$\theta^2 = \theta_\perp^2 + \theta_\parallel^2/\gamma^2$$

$\theta_\perp^2 = 2\frac{\varepsilon}{\beta^*}$, same emittance in both planes

$\theta_\parallel = \frac{\Delta p}{p}$, momentum deviation

$$\theta_\parallel^2/\gamma^2 \ll \theta_\perp^2$$

then

$$\theta^3 = (2\varepsilon/\beta^*)^{3/2}$$

$\beta^* = 50$ m in both planes
$\varepsilon = 0.0077 \times 10^{-6}$ m·rad — $\theta = 17.55$ mra

Required Cooling Time : $\tau = 18$ hours

$$J_e = 1.75 \times 10^{10} \ A/m^2$$

Electron Cooling in a Storage Mode

C. Rubbia : "Relativistic Electron Cooling..." Proceedings of the Workshop on Produ[c]ing High Luminosity High Energy Proton-Antiproton Collisions - March 27-31, 1978 Berkeley, California

A. G. Ruggiero : "High-Energy Electron Cooling" Same Workshop and Proceedi[ngs]

Proceedings of the Workshop on "The Cooling of High Energy Beams" - November 3-4, 1978 - Madison, Wisconsin

D. Cline et al. : IEEE Trans. Nucl. Sci Vol. NS-26, No. 3, June 1979 page 3472

Equations for Equilibrium :

$$\begin{cases} \dfrac{d\varepsilon_p}{dt} = D_p \\[2ex] \dfrac{d\varepsilon_e}{dt} = D_e - \dfrac{2}{\tau}\,\varepsilon_e \end{cases}$$

We consider only betatron cooling (no p. Momentum spread very small for both beams -

From $\bar{p}$-Note # 227 (Sept. 1982)

$$D_p = 2.4 \times 10^{-13} \quad m/sec$$

$$\begin{cases} \dfrac{d\varepsilon_p}{dt} = -\dfrac{2}{\tau_p}\left(\varepsilon_p - \dfrac{m_e}{m_p}\,\dfrac{\beta_p^*}{\beta_e^*}\,\varepsilon_e\right) \\[2ex] \dfrac{d\varepsilon_e}{dt} = -\dfrac{2}{\tau_e}\left(\varepsilon_e - \dfrac{m_p}{m_e}\,\dfrac{\beta_e^*}{\beta_p^*}\,\varepsilon_p\right) \end{cases}$$

$$\begin{cases} \tau_p = \dfrac{e\,\beta^4\gamma^5\,\theta_p^3}{4\pi\, r_e r_p\, J_e\, L\, \eta_p} \\[3ex] \tau_e = \dfrac{e\,\beta^4\gamma^5\,\theta_e^3}{4\pi\, r_e r_p\, J_p\, L\, \eta_e} \end{cases}$$

$$\eta_e = B\,\eta_p \quad , \quad B = 3$$

$$J_e = \frac{I_e}{\pi\,\beta_e^*\,\varepsilon_e} \quad , \quad J_p = \frac{I_p}{\pi\,\beta_p^*\,\varepsilon_p}$$

Common cross-section : $\boxed{\beta_e^*\,\varepsilon_e = \beta_p^*\,\varepsilon_p}$

For the rms beam

$$\theta^2 = 2\,\varepsilon / \beta^* \qquad \text{either } e \text{ or } p$$

At the equilibrium

$$\begin{cases} \dfrac{d\varepsilon_p}{dt} = D_p - K_p \dfrac{\varepsilon_p - \frac{m_e}{m_p}\frac{\beta_p^*}{\beta_e^*}\varepsilon_e}{\varepsilon_e\left(\frac{\varepsilon_p}{\beta_p^*} + \frac{\varepsilon_e}{\beta_e^*}\right)^{3/2}} = (\\ \\ \dfrac{d\varepsilon_e}{dt} = D_e - \dfrac{2}{\tau}\varepsilon_e - K_e \dfrac{\varepsilon_e - \frac{m_p}{m_e}\frac{\beta_e^*}{\beta_p^*}\varepsilon_p}{\varepsilon_p\left(\frac{\varepsilon_p}{\beta_p^*} + \frac{\varepsilon_e}{\beta_e^*}\right)^{3/2}} = (\end{cases}$$

with

$$\begin{cases} K_p = \dfrac{8 r_e r_p I_e L \eta_p}{2^{3/2} e \beta^4 \gamma^5 \beta_e^*} \\ \\ K_e = \dfrac{8 r_e r_p I_p L \eta_e}{2^{3/2} e \beta^4 \gamma^5 \beta_p^*} \end{cases}$$

If ε_p and ε_e denote the emittance values at the equilibrium

$$\varepsilon_p = \frac{\varepsilon_e \varepsilon_o}{\varepsilon_e - \bar{\varepsilon}_e}$$

where

$$\bar{\varepsilon}_e = \frac{\tau D_e}{2}, \quad \text{natural emittance}$$

$$\varepsilon_o = \frac{1}{2}\tau_o D_p$$

$$\tau_o = \tau \frac{K_e}{K_p}\frac{m_p}{m_e}\frac{\beta_e^*}{\beta_p^*}$$

$$= \tau \frac{m_p}{m_e}\left(\frac{\beta_e^*}{\beta_p^*}\right)^2 \frac{I_p}{I_e} B$$

At the equilibrium:

$$\boxed{\varepsilon_p = \varepsilon_o + \frac{\beta_e^*}{\beta_p^*}\bar{\varepsilon}_e, \quad \varepsilon_e = \frac{\beta_p^*}{\beta_e^*}\varepsilon_o + \bar{\varepsilon}_e}$$

The following equation has also to be satisfied

$$\left(\frac{\varepsilon_p}{\beta_p^*}\right)^{3/2} = \frac{K_p}{D_p}\left(\frac{\beta_e^*}{\beta_p^*}\right)^4 \frac{1 - \frac{m_e}{m_p}\left(\frac{\beta_p^*}{\beta_e^*}\right)^2}{\left(1 + \beta_e^{*2}/\beta_p^{*2}\right)^{3/2}}$$

In the following we neglect intrabeam scattering in the electron beam.

Inserting numbers in the above equation gives

$$I_e = (21.2\ \text{KA})\ \frac{\left(1 + \beta_e^{*2}/\beta_p^{*2}\right)^{3/2}}{\left(\beta_e^*/\beta_p^*\right)^3}$$

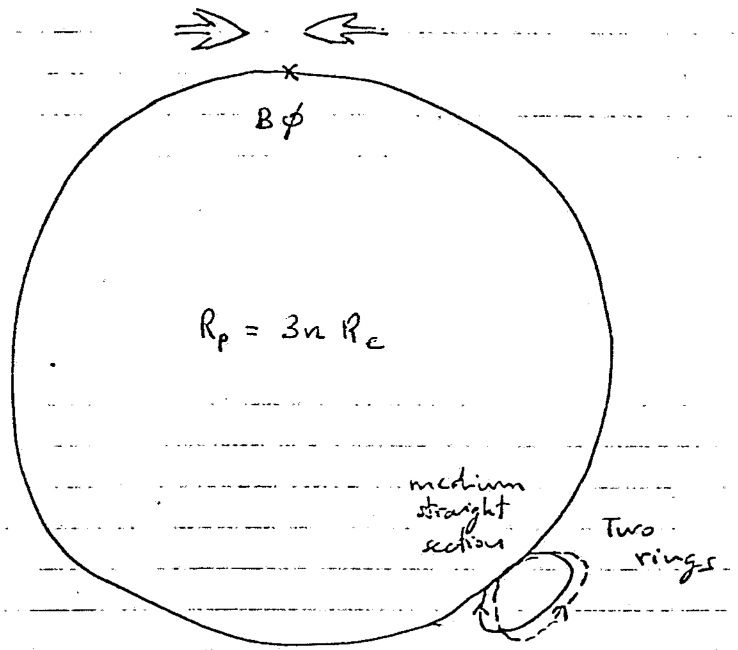
For extremely "cold" beam $\beta_e^*/\beta_p^* \to \infty$ and we recover the result we obtained for the Single Pass Mode

$$I_e = 21.2\ \text{KA} \qquad \text{(no surprise)}$$

Large energy electron storage rings like PEP and PETRA have peak curr[ents] of several thousand of ampères with extremely short bunches (psec), but it is very doubtful that one can get these peak current values at an ene[rgy] as low as 0.5 GeV.

All previous papers had very gross and serious mistakes. The conclusion here is that:

the electron current required to keep the proton beam emittance to some constant value is the same whether there is a single pass (gun) or a multiple passes (storage rings). The synchrotron radiation does not effect the cooling rate ∇



$E_p = 1000$ GeV

$E_e = 0.5$ GeV

No solenoid field
is required for e-beam