Proc. 24th International Spin Symposium (SPIN2021)
JPS Conf. Proc. **37**, 020114 (2022)
https://doi.org/10.7566/JPSCP.37.020114

Twist-3 Gluon Fragmentation Contribution to Polarized Hyperon Production and its Frame Independence

Yuji Koike[1], Kenta Yabe[2] and Shinsuke Yoshida[3,4]

[1]*Department of Physics, Niigata University, Ikarashi 2-no-cho, Niigata 950-2181, Japan,*
[2]*Graduate School of Science and Technology, Niigata University, Ikarashi 2-no-cho, Niigata 950-2181, Japan,*
[3]*Guangdong Provincial Key Laboratory of Nuclear Science, Institute of Quantum Matter, South China Normal University, Guangzhou 510006, China,*
[4] *Guangdong-Hong Kong Joint Laboratory of Quantum Matter, Southern Nuclear Science Computing Center, South China Normal University, Guangzhou 510006, China*

E-mail: koike@phys.sc.niigata-u.ac.jp

(Received January 11, 2022)

Twist-3 gluon fragmentation function contribution to the transversely polarized hyperon production in the unpolarized proton-proton collision, $pp \to \Lambda^{\uparrow}X$, is formulated within the collinear factorization and the corresponding cross section is calculated in the leading order (LO) with respect to the QCD coupling constant. This has completed the LO cross section for this process. It is also shown that the QCD equation-of-motion relation and the Lorentz invariance relations are crucial to guarantee the frame independence of the cross section. The formula is relevant for the ongoing RHIC experiment.

KEYWORDS: hyperon polarization, twist-3 gluon fragmentation function, Lorentz invariance relation, QCD equation-of-motion relation

1. Introduction

Transverse polarization (∼ a few 10 %) of hyperons produced in unpolarized pp collision has been an important issue in QCD spin physics since its first discovery in 1970's. The parton model or QCD at twist-2 level gives essentially zero polarization, and it's been known that the twist-3 effect could give rise to such large transverse polarization. In this article we discuss the twist-3 polarized cross section for the process

$$N(p) + N(p') \to \Lambda(P_h, S_\perp) + X, \tag{1}$$

where p, p' and P_h are the 4-momenta of each particle and $S_\perp$ ($S_\perp^2 = -1$) is the transverse spin vector for the hyperon. The process (1) receives three types of twist-3 contributions: (i) Twist-3 unpolarized distribution function (DF) in one of the initial protons convoluted with the transversity fragmentation (FF) for $\Lambda^{\uparrow}$, (ii) Twist-3 quark FF for $\Lambda^{\uparrow}$ convoluted with the unpolarized DFs in the initial protons, and (iii) Twist-3 gluon FF for $\Lambda^{\uparrow}$ convoluted with the unpolarized DFs in the initial protons. The cross sections for (i) and (ii) were, respectively, derived in [1–3] and [4]. In our recent paper [5], we developed a formalism for calculating the contribution (iii) and presented the corresponding cross section, which completed the cross section for (1) in the leading order (LO) with respect to the QCD coupling. To guarantee the frame-independence of the cross section, the QCD equation-of-motion (EOM) relation and the Lorentz invariance relation derived in [6] were crucial.

Formulating the twist-3 gluon FF contribution is technically very complicated and is worth for studying since it constitutes a key element in high energy QCD. Besides the intrinsic importance of the formal development, it is also phenomenologically important given that gluons are ample in the collision environment and the twist-3 FF contribution has been shown to be important for A_N for $p^\uparrow p \to \pi X$ [7] which shows a rising behavior in the forward direction similarly to the polarization for (1). Therefore the above contributions (ii) and (iii) could be important as well, in particular, they mix under renormalization. This talk is a brief summary from [5, 6].

2. Twist-3 gluon fragmentation functions

We first recall the complete set of the twist-3 gluon FFs relevant to (1) [6]. They are classified into *intrinsic*, *kinematical* and *dynamical* FFs. The intrinsic FFs are defined as

$$\widehat{\Gamma}^{\alpha\beta}(z) = \frac{1}{N^2-1}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0|F_a^{w\beta}(0)|hX\rangle\langle hX|F_a^{w\alpha}(\lambda w)|0\rangle$$
$$= -iM_h \epsilon^{P_h w S_\perp [\alpha} w^{\beta]} \Delta\widehat{G}_{3T}(z) + M_h \epsilon^{P_h w S_\perp \{\alpha} w^{\beta\}} \Delta\widehat{G}_{3\bar{T}}(z) + \cdots, \tag{2}$$

where the lightlike vector w satisfying $P_h \cdot w = 1$ is introduced, and the gauge links are suppressed for simplicity. $\widehat{G}_{3\bar{T}}(z)$ is naively T-odd and contributes to hyperon polarization. The kinematical FFs are defined from the correlators with the derivative as

$$\widehat{\Gamma}_\partial^{\alpha\beta\gamma}(z) = \frac{1}{N^2-1}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0|F_a^{w\beta}(0)|hX\rangle\langle hX|F_a^{w\alpha}(\lambda w)|0\rangle \overleftarrow{\partial}^\gamma \tag{3}$$
$$= -i\frac{M_h}{2} g_\perp^{\alpha\beta} \epsilon^{P_h w S_\perp \gamma} \widehat{G}_T^{(1)}(z) + \frac{M_h}{2} \epsilon^{P_h w \alpha\beta} S_\perp^\gamma \Delta\widehat{G}_T^{(1)}(z) - i\frac{M_h}{8}\left(\epsilon^{P_h w S_\perp \{\alpha} g_\perp^{\beta\}\gamma} + \epsilon^{P_h w \gamma\{\alpha} S_\perp^{\beta\}}\right) \Delta\widehat{H}_T^{(1)}(z) + \cdots,$$

where $\widehat{G}_T^{(1)}(z)$ and $\widehat{H}_T^{(1)}(z)$ are naively T-odd and contribute to the hyperon polarization. The dynamical FFs are defined from the correlators of three field-strengths as

$$\hat{\Gamma}_{FA}^{\alpha\beta\gamma}(\frac{1}{z_1},\frac{1}{z_2}) = \frac{-if_{abc}}{N^2-1}\sum_X \int \frac{d\lambda}{2\pi}\int\frac{d\mu}{2\pi} e^{-i\frac{\lambda}{z_1}} e^{-i\mu(\frac{1}{z_2}-\frac{1}{z_1})} \langle 0|F_b^{w\beta}(0)|hX\rangle\langle hX|F_a^{w\alpha}(\lambda w) gF_c^{w\gamma}(\mu w)|0\rangle$$
$$= -M_h\left(\widehat{N}_1\left(\frac{1}{z_1},\frac{1}{z_2}\right) g_\perp^{\alpha\gamma}\epsilon^{P_h w S_\perp \beta} + \widehat{N}_2\left(\frac{1}{z_1},\frac{1}{z_2}\right) g_\perp^{\beta\gamma}\epsilon^{P_h w S_\perp \alpha} - \widehat{N}_2\left(\frac{1}{z_2}-\frac{1}{z_1},\frac{1}{z_2}\right) g_\perp^{\alpha\beta}\epsilon^{P_h w S_\perp \gamma}\right), \tag{4}$$

and

$$\hat{\Gamma}_{FS}^{\alpha\beta\gamma}(\frac{1}{z_1},\frac{1}{z_2}) = \frac{d_{abc}}{N^2-1}\sum_X \int \frac{d\lambda}{2\pi}\int\frac{d\mu}{2\pi} e^{-i\frac{\lambda}{z_1}} e^{-i\mu(\frac{1}{z_2}-\frac{1}{z_1})} \langle 0|F_b^{w\beta}(0)|hX\rangle\langle hX|F_a^{w\alpha}(\lambda w) gF_c^{w\gamma}(\mu w)|0\rangle$$
$$= -M_h\left(\widehat{O}_1\left(\frac{1}{z_1},\frac{1}{z_2}\right) g_\perp^{\alpha\gamma}\epsilon^{P_h w S_\perp \beta} + \widehat{O}_2\left(\frac{1}{z_1},\frac{1}{z_2}\right) g_\perp^{\beta\gamma}\epsilon^{P_h w S_\perp \alpha} + \widehat{O}_2\left(\frac{1}{z_2}-\frac{1}{z_1},\frac{1}{z_2}\right) g_\perp^{\alpha\beta}\epsilon^{P_h w S_\perp \gamma}\right). \tag{5}$$

These dynamical FFs are complex functions and their T-odd imaginary parts contribute to the hyperon polarization.

3. Twist-3 gluon fragmentation function contribution to $pp \to \Lambda^\uparrow X$

The purely gluonic twist-3 FFs contribution to (1) comes from diagrams in Fig. 1. It can be written in a gauge invariant form as [5]

$$E_h \frac{d\sigma(p,p',P_h;S_\perp)}{d^3P_h} = \frac{1}{16\pi^2 S_E}\int_0^1 \frac{dx}{x} f_1(x)\int_0^1 \frac{dx'}{x'} f_1(x')\left[\Omega^\mu{}_\alpha \Omega^\nu{}_\beta \int_0^1 dz\, \mathrm{Tr}\left[\hat{\Gamma}^{\alpha\beta}(z) S_{\mu\nu}(P_h/z)\right]\right.$$

$$-i\,\Omega^{\mu}{}_{\alpha}\Omega^{\nu}{}_{\beta}\Omega^{\lambda}{}_{\gamma}\int_0^1 dz\,\mathrm{Tr}\left[\hat{\Gamma}^{\alpha\beta\gamma}_{\partial}(z)\,\frac{\partial S_{\mu\nu}(k)}{\partial k^{\lambda}}\bigg|_{c.l.}\right]+\Re\Big\{i\,\Omega^{\mu}{}_{\alpha}\Omega^{\nu}{}_{\beta}\Omega^{\lambda}{}_{\gamma}\int_0^1\frac{dz}{z}\int_z^{\infty}\frac{dz'}{z'}\left(\frac{1}{1/z-1/z'}\right)$$
$$\times\mathrm{Tr}\left[\left(-\frac{if^{abc}}{N}\hat{\Gamma}^{\alpha\beta\gamma}_{FA}\left(\frac{1}{z'},\frac{1}{z}\right)+d^{abc}\frac{N}{N^2-4}\hat{\Gamma}^{\alpha\beta\gamma}_{FS}\left(\frac{1}{z'},\frac{1}{z}\right)\right)S^{L}_{\mu\nu\lambda,abc}(z',z)\right]\Big\}\Big], \tag{6}$$

where $\hat{\Gamma}^{\alpha\beta}(z)$, $\hat{\Gamma}^{\alpha\beta\gamma}_{\partial}(z)$, $\hat{\Gamma}^{\alpha\beta\gamma}_{FA}$ and $\hat{\Gamma}^{\alpha\beta\gamma}_{FS}$ are defined in (2)-(5), $f_1(x)$ is the unpolarized quark or gluon distribution in the initial proton, $\Omega^{\mu}_{\alpha}=g^{\mu}_{\alpha}-P^{\mu}_{h}w_{\alpha}$, $S_{\mu\nu}(k)$ is the partonic hard part for the two-body gluon FF matrix element shown in Fig. 1(a), and $S^{L}_{\mu\nu\lambda,abc}(z',z)$ is the partonic hard part for the three-body gluon FF matrix element shown in Fig. 1(b). Calculating the LO diagrams for the partonic hard part, one obtains the cross section in terms of the intrinsic, kinematical and dynamical FFs. Each partonic hard cross section obtained this way depends on the kinematic factors $\epsilon^{wP_hp'S_\perp}$ and $\epsilon^{wP_hpS_\perp}$, whose w-dependence brings the frame dependence of the cross section. To circumvent this situation, we recall that the twist-3 gluon FFs are subject to EOM relations and the Lorentz invariance relations (LIRs). The former reads

$$\frac{1}{z}\Delta\widehat{G}_{3\bar{T}}(z)=\int_0^{1/z}d\left(\frac{1}{z'}\right)\left(\frac{1}{\frac{1}{z}-\frac{1}{z'}}\right)\Im\left\{2\widehat{N}_1\left(\frac{1}{z'},\frac{1}{z}\right)+\widehat{N}_2\left(\frac{1}{z'},\frac{1}{z}\right)-\widehat{N}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right)\right\}$$
$$+\frac{1}{2}\left(\widehat{G}^{(1)}_{T}(z)+\Delta\widehat{H}^{(1)}_{T}(z)\right)-\frac{2}{C_F}\Im\int_0^{1/z}d\left(\frac{1}{z_1}\right)\widetilde{D}_{FT}\left(\frac{1}{z_1},\frac{1}{z_1}-\frac{1}{z}\right), \tag{7}$$

and the latter read

$$\begin{aligned}\frac{1}{z}\frac{\partial}{\partial(1/z)}[\widehat{G}^{(1)}_{T}(z)] &= 2\widehat{G}^{(1)}_{T}(z)-\frac{4}{C_F}\int_0^{1/z}d\left(\frac{1}{z'}\right)\Im\widetilde{D}_{FT}\left(\frac{1}{z'},\frac{1}{z'}-\frac{1}{z}\right)\\ &+ 4\int_0^{1/z}d\left(\frac{1}{z'}\right)\frac{1}{\frac{1}{z}-\frac{1}{z'}}\Im\left\{\widehat{N}_1\left(\frac{1}{z'},\frac{1}{z}\right)-\widehat{N}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right)\right\}\\ &+ \frac{2}{z}\int_0^{1/z}d\left(\frac{1}{z'}\right)\frac{1}{\left(\frac{1}{z}-\frac{1}{z'}\right)^2}\Im\left\{\widehat{N}_1\left(\frac{1}{z'},\frac{1}{z}\right)+\widehat{N}_2\left(\frac{1}{z'},\frac{1}{z}\right)-2\widehat{N}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right)\right\},\end{aligned} \tag{8}$$

and

$$\begin{aligned}\frac{1}{z}\frac{\partial}{\partial(1/z)}[\Delta\widehat{H}^{(1)}_{T}(z)] &= 4\Delta\widehat{H}^{(1)}_{T}(z)-\frac{8}{C_F}\int_0^{1/z}d\left(\frac{1}{z'}\right)\Im\widetilde{D}_{FT}\left(\frac{1}{z'},\frac{1}{z'}-\frac{1}{z}\right)\\ &+ 8\int_0^{1/z}d\left(\frac{1}{z'}\right)\frac{1}{\frac{1}{z}-\frac{1}{z'}}\Im\left\{\widehat{N}_1\left(\frac{1}{z'},\frac{1}{z}\right)+\widehat{N}_2\left(\frac{1}{z'},\frac{1}{z}\right)\right\}\end{aligned}$$

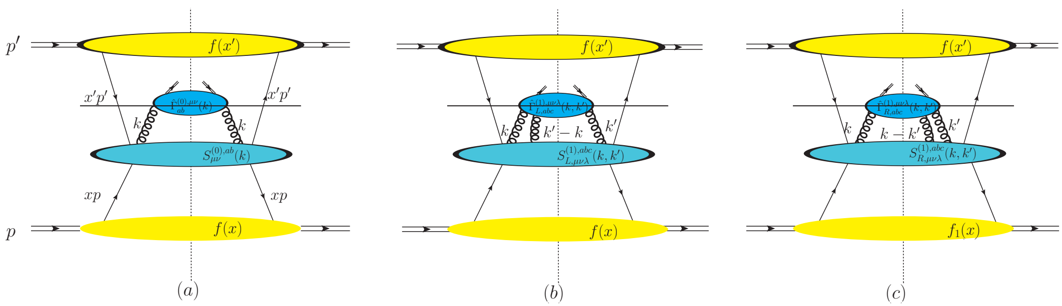

Fig. 1. Diagrams representing the twist-3 gluon fragmentation contribution to $pp\to\Lambda^{\uparrow}X$.

$$+\ \frac{4}{z}\int_0^{1/z} d\left(\frac{1}{z'}\right)\frac{1}{\left(\frac{1}{z}-\frac{1}{z'}\right)^2}\Im\left\{\widehat{N}_1\left(\frac{1}{z'},\frac{1}{z}\right)+\widehat{N}_2\left(\frac{1}{z'},\frac{1}{z}\right)\right\}, \qquad (9)$$

where $\widetilde{D}_{FT}$ is another dynamical FF defined from the $q\bar{q}g$-correlator as

$$\widetilde{\Delta}^{\alpha}_{ij}\left(\frac{1}{z_1},\frac{1}{z_2}\right) = \frac{1}{N}\sum_X\int\frac{d\lambda}{2\pi}\int\frac{d\mu}{2\pi}e^{-i\frac{\lambda}{z_1}}e^{-i\mu(\frac{1}{z_2}-\frac{1}{z_1})}\langle 0|F_a^{w\alpha}(\mu w)|hX\rangle\langle hX|\bar{\psi}_j(\lambda w)t^a\psi_i(0)|0\rangle$$
$$= M_h\left[\epsilon^{\alpha P_h w S_\perp}(\not{P}_h)_{ij}\widetilde{D}_{FT}\left(\frac{1}{z_1},\frac{1}{z_2}\right)+iS^{\alpha}_{\perp}(\gamma_5\not{P}_h)_{ij}\widetilde{G}_{FT}\left(\frac{1}{z_1},\frac{1}{z_2}\right)\right]. \qquad (10)$$

The relations (7), (8) and (9) show that the three types of twist-3 gluon FFs introduced in the previous section are related to the $q\bar{q}g$-type FFs. This suggests that in calculating the cross section one should also take into account the contribution shown in Fig. 2 together. We thus have included it following the formalism in [8]. Using (7), (8) and (9) to eliminate the intrinsic FF $\Delta\widehat{G}_{3\bar{T}}(z)$ and the derivative of

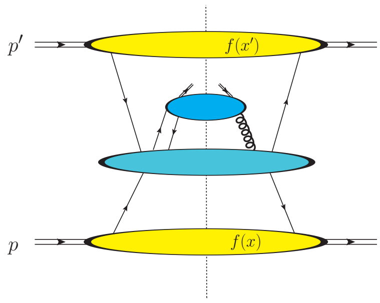

Fig. 2. Diagram for the $q\bar{q}g$-type correlation function in $pp \to \Lambda^{\uparrow}X$. Mirror diagram also contributes.

kinematical FFs, $\frac{\partial}{\partial(1/z)}[\widehat{G}_T^{(1)}(z)]$ and $\frac{\partial}{\partial(1/z)}[\Delta\widehat{H}_T^{(1)}(z)]$, one eventually obtains the twist-3 cross section from Figs. 1 and 2 as

$$E_h\frac{d\sigma(p,p',P_h;S_\perp)}{d^3P_h}$$
$$= \frac{M_h\alpha_s^2}{S_E}\int_0^1\frac{dx}{x}f_1(x)\int_0^1\frac{dx'}{x'}f_1(x')\int_0^1 dz\,\delta(s+t+u)\left(-2\frac{xx'}{z}\epsilon^{pp'P_hS_\perp}\right)$$
$$\times\ \left\{\widehat{G}_T^{(1)}(z)\hat{\sigma}_G+\Delta\widehat{H}_T^{(1)}(z)\hat{\sigma}_H\right.$$
$$+\ \int_0^{1/z}d\left(\frac{1}{z'}\right)\left[\frac{1}{1/z-1/z'}\Im\left(\widehat{N}_1\left(\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{N1}+\widehat{N}_2\left(\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{N2}+\widehat{N}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{N3}\right)\right.$$
$$+\ \frac{1}{z}\left(\frac{1}{1/z-1/z'}\right)^2\Im\left(\widehat{N}_1\left(\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{DN1}+\widehat{N}_2\left(\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{DN2}+\widehat{N}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{DN3}\right)$$
$$+\ \frac{1}{1/z-1/z'}\Im\left(\widehat{O}_1\left(\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{O1}+\widehat{O}_2\left(\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{O2}+\widehat{O}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{O3}\right)$$
$$+\ \left.\frac{1}{z}\left(\frac{1}{1/z-1/z'}\right)^2\Im\left(\widehat{O}_1\left(\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{DO1}+\widehat{O}_2\left(\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{DO2}+\widehat{O}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{DO3}\right)\right]$$
$$+\ \int_0^{1/z}d\left(\frac{1}{z'}\right)\frac{2}{C_F}\left[\Im\widetilde{D}_F\left(\frac{1}{z'},\frac{1}{z'}-\frac{1}{z}\right)\left(\hat{\sigma}_{DF1}+\frac{1}{z}\frac{1}{1/z-1/z'}\hat{\sigma}_{DF2}+\frac{z'}{z}\hat{\sigma}_{DF3}\right)\right.$$

$$+ \quad \Im\widetilde{G}_F\left(\frac{1}{z'},\frac{1}{z'}-\frac{1}{z}\right)\left(\hat{\sigma}_{GF1}+\frac{1}{z}\frac{1}{1/z-1/z'}\hat{\sigma}_{GF2}+\frac{z'}{z}\hat{\sigma}_{GF3}\right)\Big]\Big\}, \tag{11}$$

where $s = (xp+x'p')^2$, $t = (xp-P_h/z)^2$ and $u = (x'p'-P_h/z)^2$ are the partonic Mandelstam variables and $\hat{\sigma}_G, \hat{\sigma}_H, \cdots$, etc denote the partonic hard cross sections which are the functions of s, t and u and are listed in [5]. Here we note that the use of (7), (8) and (9) in (6) leads to the common combination of the kinematical factor in (11)

$$p \cdot P_h \epsilon^{wP_hp'S_\perp} - p' \cdot P_h \epsilon^{wP_hpS_\perp} = -\epsilon^{P_hpp'S_\perp}, \tag{12}$$

which is w-independent as shown in the right hand side of (12). We have shown that the EOM relation and LIRs are essential to guarantee the Lorentz invariance of the twist-3 cross section as shown for other processes [4, 9, 10]. Equation (11) is the final result for the twist-3 gluon FF contribution to $pp \to \Lambda^\uparrow X$.

4. Summary

In this talk, we presented the twist-3 gluon FF contribution to the polarized hyperon production in proton-proton collision $pp \to \Lambda^\uparrow X$. This has completed the LO twist-3 cross section for this process together with the known results in the literature. We have also shown that the EOM relation and LIRs are crucial to transform the cross section into the manifestly Lorentz invariant form. We hope RHIC will provide data for this process.

Acknowledgments

This work has been supported by the Grant-in-Aid for Scientific Research from the Japanese Society of Promotion of Science under Contract Nos. 19K03843 (Y.K.) and 18J11148 (K.Y.), National Natural Science Foundation in China under grant No. 11950410495, Guangdong Natural Science Foundation under No. 2020A1515010794 and research startup funding at South China Normal University.

References

[1] Y. Kanazawa and Y. Koike, Phys. Rev. D **64**, 034019 (2001).
[2] J. Zhou, F. Yuan and Z. T. Liang, 5Phys. Rev. D **78**, 114008 (2008).
[3] Y. Koike, K. Yabe and S. Yoshida, Phys. Rev. D **92**, 094011 (2015).
[4] Y. Koike, A. Metz, D. Pitonyak, K. Yabe and S. Yoshida, Phys. Rev. D **95**, 114013 (2017).
[5] Y. Koike, K. Yabe and S. Yoshida, Phys. Rev. D **104**, 054023 (2021).
[6] Y. Koike, K. Yabe and S. Yoshida, Phys. Rev. D **101**, 054017 (2020).
[7] K. Kanazawa, Y. Koike, A. Metz and D. Pitonyak, Phys. Rev. D **89**, 111501 (2014).
[8] K. Kanazawa and Y. Koike, Phys. Rev. D **88**, 074022 (2013).
[9] K. Kanazawa, A. Metz, D. Pitonyak and M. Schlegel, Phys. Lett. B **744**, 385 (2015).
[10] K. Kanazawa, Y. Koike, A. Metz, D. Pitonyak and M. Schlegel, Phys. Rev. D **93**, 054024 (2016).