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Perturbative calculation of supersymmetric gradient flow in $\mathcal{N} = 1$ supersymmetric QCD

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We demonstrate perturbative calculations of supersymmetric gradient flow in four-dimensional $\mathcal{N} = 1$ supersymmetric quantum chromodynamics (SQCD). A remarkable property of the gradient flow is to make ultraviolet (UV) divergences of flowed field correlators milder. To illustrate this property, we calculate two-point functions for the flowed fields in SQCD at the one-loop level and investigate their UV divergence structure. After renormalizing the SQCD at the boundary, the two-point functions of flowed gauge supermultiplets are shown to be UV-finite. On the other hand, those for flowed matter supermultiplets require extra wave function renormalization, which are found to be the common factor for all the fields in the multiplets.

Subject Index B14, B16, B32

1. Introduction

Gradient flow [1,2] has been applied in various studies because of its remarkable renormalization property. In the case of Yang–Mills (YM) theory, after renormalizing the boundary theory, extra wave function renormalization is not required in the correlation functions of the flowed field [3–5]. Any composite operators of the flowed field become automatically renormalized quantities. Since ultraviolet (UV)-finite quantities are independent of the regularization method, it is possible to connect lattice regularization and other regularizations such as dimensional regularization. Therefore, the gradient flow can be used to represent physical quantities such as the energy-momentum tensor (EMT) in terms of flowed field and is particularly useful in the context of lattice gauge theory. In lattice quantum chromodynamics (QCD), this property is used to formulate and simulate numerically the EMT [6–10].

Recently, several novel approaches employing a gradient flow method have been proposed. One is for investigating the anti-de Sitter (AdS)/conformal field theory (CFT) correspondence, in which a bulk metric emerges as the AdS geometry from a boundary CFT using a flow equation [11–19]. The other is to formulate an exact renormalization group (ERG) in gauge theory with manifest gauge invariance [20–23]. A manifestly gauge-invariant Wilson action is constructed using a coarse-graining technique through the gradient flow. Its associated ERG differential equation is derived and is extended to the inclusion of matter fields.

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It would be interesting to extend the gradient flow to supersymmetric theories. The YM flow [2] and a fermion flow [4] have already been applied to supersymmetric theories [24–26]. To respect supersymmetry (SUSY), it is convenient to use the superfield formalism, which is called a SUSY flow. For $\mathcal{N}=1$ supersymmetric YM (SYM) theory, a SUSY flow is defined by the gradient of the SYM action with respect to a vector superfield [27]. This flow in the component fields can be written in a gauge-covariant and supersymmetric manner [28].

A similar approach can be considered in the case of $\mathcal{N}=1$ supersymmetric QCD (SQCD) [29]. The gradient flow of SQCD is given in terms of the component fields of the Wess–Zumino gauge. In this paper, we calculate all the two-point functions of the flowed fields in SQCD at the one-loop level. We find that after renormalizing the parameters in the boundary theory, the two-point functions of the flowed fields in the gauge supermultiplet are UV-finite, but the UV divergences remain for those in the matter supermultiplets.

This paper is organized as follows. In the following section, we review the SQCD action and its renormalization. The gradient flow of SQCD is given in Sect. 3. We show one-loop calculations of the two-point functions for the flowed fields in Sect. 4. The conclusion is given in Sect. 5.

2. $\mathcal{N}=1$ SQCD and its renormalization

We begin by reviewing the renormalization of the $\mathcal{N}=1$ SQCD to fix the notations used in this paper. See Appendix A for more details.

2.1 *SQCD action*

We consider $\mathcal{N}=1$ SQCD which is an $\mathcal{N}=1$ supersymmetric SU(N_c) gauge theory with N_f quarks in the fundamental representation of the gauge group. In the off-shell formulation, the theory consists of a gauge multiplet (A_μ, λ, D) and N_f matter multiplets $(\varphi^m_\pm, \psi^m_\pm, G^m_\pm)$ for $m = 1, 2, \ldots, N_f$, where $A^a_\mu(x)$ is a gauge field, $\lambda^a_\alpha(x)$ is a gaugino field, $D^a(x)$ is a real auxiliary field, $\varphi^m_{\pm,i}(x)$ are complex scalar fields, $\psi^m_{\alpha,i}(x)$ are quark fields ($\gamma_5\psi_\pm = \pm\psi_\pm$), and $G^m_{\pm,i}(x)$ are complex auxiliary fields for $a = 1, 2, \ldots, N_c^2-1$ and $i = 1, 2, \ldots, N_c$. The coordinates and spinor index (α), flavor index (m) are often abbreviated in this paper for notational simplicity. The fields of gauge multiplet are expressed as matrix-valued fields such as $A_\mu(x) = \sum_{a=1}^{N_c^2-1} A^a_\mu(x)T^a$ with group generators T^a.

The Euclidean SQCD action is then given by $S^{\mathrm{E}}_{\mathrm{SQCD}} = S^{\mathrm{E}}_{\mathrm{SYM}} + S^{\mathrm{E}}_{\mathrm{MAT}}$ with

$$S^{\mathrm{E}}_{\mathrm{SYM}} = \frac{1}{g_0^2}\int \mathrm{d}^4x\,\mathrm{tr}\left(\frac{1}{2}F^2_{\mu\nu} + \bar{\lambda}\,\not{D}\lambda + D^2\right), \tag{1}$$

$$\begin{aligned} S^{\mathrm{E}}_{\mathrm{MAT}} = \int \mathrm{d}^4x\Big\{ &|D_\mu\varphi_+|^2 + |D_\mu\varphi_-|^2 + \bar{\psi}\,\not{D}\psi + |G_+|^2 + |G_-|^2 - i\left(\varphi^\dagger_+ D\varphi_+ - \varphi^\dagger_- D\varphi_-\right) \\ &+ \sqrt{2}i\left(\bar{\psi}_+\lambda\varphi_+ + \bar{\psi}_-\lambda\varphi_- - \varphi^\dagger_+\bar{\lambda}\psi_+ - \varphi^\dagger_-\bar{\lambda}\psi_-\right) \\ &+ m_0\left(\bar{\psi}\psi - i\varphi^\dagger_- G_+ - iG^\dagger_-\varphi_+ - i\varphi^\dagger_+ G_- - iG^\dagger_+\varphi_-\right)\Big\}, \end{aligned} \tag{2}$$

where $\not{D} \equiv \gamma_\mu D_\mu$,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu], \tag{3}$$

and see Appendix A for the notation of gamma matrices γ_μ, the charge conjugation matrix C and the definition of Majorana fermion. Weyl fermion $\psi_\pm$ are defined by

$$\psi_\pm = P_\pm\psi, \qquad \bar{\psi}_\mp = \bar{\psi}P_\pm, \tag{4}$$

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where $P_\pm$ are chiral projection operators, $P_\pm = \frac{1}{2}(1 \pm \gamma_5)$.

The covariant derivative D_μ depends on representation of corresponding fields. We define

$$\begin{aligned} D_\mu X_{adj} &= \partial_\mu X_{adj} + i[A_\mu, X_{adj}] \qquad (X_{adj} = \lambda, D, F_{\rho\sigma}, c, \bar{c}), \\ D_\mu X &= \partial_\mu X + iA_\mu X \qquad (X = \varphi_\pm, \psi, G_\pm), \\ D_\mu \bar{X} &= \partial_\mu \bar{X} - i\bar{X}A_\mu \qquad (\bar{X} = \varphi_\pm^\dagger, \bar{\psi}, G_\pm^\dagger), \end{aligned} \tag{5}$$

where c, $\bar{c}$ are ghost fields introduced later. The action is invariant under an infinitesimal gauge transformation with a gauge transformation function $\omega(x)$,

$$\begin{aligned} \delta_\omega^g A_\mu &= -D_\mu \omega, \\ \delta_\omega^g X_{adj} &= i[\omega, X_{adj}], \\ \delta_\omega^g X &= i\omega X, \\ \delta_\omega^g \bar{X} &= \bar{X}(-i\omega). \end{aligned} \tag{6}$$

We can also show that the action is invariant under a transformation,

$$\begin{aligned} \delta_\xi A_\mu &= \bar{\xi}\gamma_\mu \lambda, \\ \delta_\xi \lambda &= -\frac{1}{2}\gamma_\mu\gamma_\nu \xi F_{\mu\nu} - \gamma_5 \xi D, \\ \delta_\xi D &= \bar{\xi}\gamma_5 \not{D}_\mu \lambda, \\ \delta_\xi \varphi_\pm &= \sqrt{2}\bar{\xi}_\mp \psi_\pm, \\ \delta_\xi \varphi_\pm^\dagger &= \sqrt{2}\bar{\psi}_\pm \xi_\mp, \\ \delta_\xi \psi_\pm &= \sqrt{2}(\not{D}\varphi_\pm \xi_\mp + iG_\pm \xi_\pm), \\ \delta_\xi \bar{\psi}_\pm &= \sqrt{2}(-\bar{\xi}_\mp \not{D}\varphi_\pm^\dagger + i\bar{\xi}_\pm G_\pm^\dagger), \\ \delta_\xi G_\pm &= \sqrt{2}\bar{\xi}_\pm(-i\not{D}\psi_\pm + \sqrt{2}\lambda\varphi_\pm), \\ \delta_\xi G_\pm^\dagger &= \sqrt{2}(iD_\mu \bar{\psi}_\pm \gamma_\mu - \sqrt{2}\varphi_\pm^\dagger \bar{\lambda})\xi_\pm, \end{aligned} \tag{7}$$

where ξ_α is a global anti-commuting parameter, and $\xi_\pm = P_\pm \xi$, $\bar{\xi}_\pm = \bar{\xi}P_\mp$. Note that δ_ξ is the modified SUSY transformation for the component field and preserves the Wess–Zumino gauge [29]. See also Ref. [30].

In the perturbation theory, we introduce the gauge-fixing term and the ghost term into the action. The total action is given by $S_{\text{tot}} = S_{\text{SQCD}}^{\text{E}} + S_{\text{gf}} + S_{c\bar{c}}$, where the gauge-fixing term S_{gf} with gauge parameter ξ and the ghost action $S_{c\bar{c}}$ are

$$S_{\text{gf}} = \frac{1}{g_0^2}\int \mathrm{d}^4x \left\{\frac{1}{2\xi_0}\partial_\mu A_\mu^a(x)\partial_\nu A_\nu^a(x)\right\}, \qquad S_{c\bar{c}} = \frac{1}{g_0^2}\int \mathrm{d}^4x \Big\{\bar{c}^a(x)(-\partial_\mu D_\mu)c^a(x)\Big\}, \tag{8}$$

where c and $\bar{c}$ are ghost and anti-ghost fields, respectively.

2.2 *Renormalization of $\mathcal{N} = 1$ SQCD*

In later sections, we will investigate the UV divergence structure of flowed correlation functions at the one-loop level. To this end, in this section we summarize the one-loop renormalization in $\mathcal{N} = 1$ SQCD using the dimensional regularization and the minimal subtraction (MS) scheme.

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The renormalization factors are defined as follows

$$
\begin{aligned}
g_0^2 &= \mu^{2\epsilon} g^2 Z_g, \\
\xi_0 &= \xi Z_A, \\
A_\mu &= Z_g^{1/2} Z_A^{1/2} A_{R,\mu}, \\
\lambda &= Z_g^{1/2} Z_\lambda^{1/2} \lambda_R, \\
D &= Z_g^{1/2} Z_D^{1/2} D_R, \\
c &= Z_c Z_g^{1/2} Z_A^{1/2} c_R, \\
\bar{c} &= Z_g^{1/2} Z_A^{-1/2} \bar{c}_R, \\
\psi &= Z_\psi^{1/2} \psi_R, \\
\varphi_\pm &= Z_\varphi^{1/2} \varphi_{R\pm}, \\
G_\pm &= Z_G^{1/2} G_{R\pm}
\end{aligned} \tag{9}
$$

The parameters g and ξ without a subscript 0, and the fields with a subscript R are renormalized quantities. Z_g is a renormalization factor for the vertex correction and $Z_{A,\,\lambda,\,D,\,\varphi,\,\psi,\,G,\,c}$ are wave function renormalization factors for the corresponding fields.

These Z factors are calculated at the one-loop level as follows.

$$
\begin{aligned}
Z_g &= 1 + \frac{g^2}{16\pi^2\epsilon}(-3N_c + N_f), \\
Z_A &= 1 + \frac{g^2}{16\pi^2\epsilon}\left(\frac{3-\xi}{2}N_c - N_f\right), \\
Z_\lambda &= 1 + \frac{g^2}{16\pi^2\epsilon}(-\xi N_c - N_f), \\
Z_D &= 1 + \frac{g^2}{16\pi^2\epsilon}(-N_f), \\
Z_c &= 1 + \frac{g^2}{16\pi^2\epsilon}\frac{3-\xi}{4}N_c, \\
Z_\varphi &= 1 + \frac{g^2}{16\pi^2\epsilon}(1-\xi)C_F, \\
Z_\psi &= 1 + \frac{g^2}{16\pi^2\epsilon}(-1-\xi)C_F, \\
Z_G &= 1,
\end{aligned} \tag{10}
$$

where C_F is the quadratic Casimir for fundamental representation. As is well known, the component fields in each multiplet do not share a common wave function renormalization factor in this setup. This situation drastically changes when considering flowed fields obeying the SQCD flow equations defined in the next section.

3. Gradient flow equation and its iterative expansion

We first review how to derive a supersymmetric gradient flow in $\mathcal{N} = 1$ SQCD in Minkowski spacetime according to Ref. [29], after which we move on to the Wess–Zumino gauge and perform a Wick rotation to Euclidean signature. The flow equations are finally given in terms of

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the component fields of the Wess–Zumino gauge in Euclidean spacetime. To avoid the difficulty of introducing mass terms into the flow equations, we adopt massless flow equations.[1] By expanding the equation iteratively, we obtain the flow vertices which are needed in the perturbative calculation presented in the next section.

3.1 *SQCD flow equation*

All the fields that appear below depend on a flow time $t \geq 0$. For notational simplification, the flow field corresponding to a boundary field is represented by the same symbol. For instance the flowed field corresponding to $A_\mu(x)$ is represented by $A_\mu(t, x)$ with the boundary condition $A_\mu(t = 0, x) = A_\mu(x)$. The vector superfield V and chiral superfield $Q_\pm$ are also t-dependent superfields while the definition of differential operators Q_α, $\bar{Q}_{\dot\alpha}$, D_α, $\bar{D}_{\dot\alpha}$ is unchanged, and those defined at the boundary $t = 0$ are used for the flow field as they are. See Appendix B for the notation of the superfield formalism.

The flowed vector superfield is defined by $V^\dagger(z, t) = V(z, t)$ where $z = (x_\mu, \theta_\alpha, \bar\theta_{\dot\alpha})$, and it is invariant under four-dimensional Lorentz transformations and transforms as $\delta_\xi V = (\xi^\alpha Q_\alpha + \bar\xi_{\dot\alpha}\bar{Q}^{\dot\alpha})V$ under the supersymmetry transformation. For the gauge multiplet, a supersymmetric gradient flow is defined in the Minkowski space as

$$\partial_t V^a = -\frac{1}{2} g^{ab}(V)\frac{\delta S_{\mathrm{SQCD}}}{\delta V^b}. \tag{11}$$

Here $g^{ab}(V)$ is a metric derived from an invariant norm

$$\|\delta V\|^2 = \frac{1}{2}\int \mathrm{d}^8 z \mathrm{tr}(e^{-2V}\delta e^{2V} e^{-2V}\delta e^{2V})(z), \tag{12}$$

where $\int \mathrm{d}^8 z \equiv \int \mathrm{d}^4 x \mathrm{d}^2\theta \mathrm{d}^2\bar\theta$. Note that this norm is invariant under both the supersymmetry transformation and the extended gauge transformation $e^{2V} \to e^{2\Lambda^\dagger} e^{2V} e^{2\Lambda}$ with a chiral superfield Λ. The metric g^{ab} can be read from $\|\delta V\|^2 = \int \mathrm{d}^8 z g_{ab}(V)\delta V^a \delta V^b$ and an identity $g^{ac} g_{cb} = \delta^a{}_b$.

The chiral superfields $Q_\pm$ which contain spinors are defined by superchiral conditions $\bar{D}_{\dot\alpha} Q_\pm = 0$, which transform by the same rule as V for supersymmetry transformation. For the matter multiplet, keeping the superchiral condition for flowed chiral superfields, the SUSY flow equations are given by

$$\partial_t Q_+ = -\frac{1}{4}\bar{D}\bar{D}\left(e^{-2V}\frac{\delta S_{\mathrm{MAT}}}{\delta Q_+^\dagger}\right), \tag{13}$$

$$\partial_t Q_- = -\frac{1}{4}\bar{D}\bar{D}\left(\frac{\delta S_{\mathrm{MAT}}}{\delta Q_-^\dagger} e^{2V}\right). \tag{14}$$

Note that the gradient of S_{MAT} is the same as S_{SQCD} since S_{SYM} does not contain $Q_\pm$. These equations are covariant under t-independent super and extended gauge transformations $Q_+ \to e^{-2\Lambda} Q_+$, $Q_- \to Q_- e^{2\Lambda}$.

We consider flow equations for the component fields in the Wess–Zumino gauge. The flow equations above, however, are not consistent with the gauge because the r.h.s. of Eqs. (11), (13) and (14) provide the breaking terms. In order to keep the Wess–Zumino gauge, the SUSY flow

[1] In Refs. [31,32] it is pointed out that the inclusion of terms with bare parameters, such as mass terms in SQCD, is an obstacle to the renormalizability of the flow theory.

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equations should be modified by adding extended gauge transformation as

$$
\begin{aligned}
\partial_t V^a &= -\frac{1}{2} g_{ab} \frac{\delta S_{\mathrm{SQCD}}}{\delta V^b} + \delta_\Lambda V^a, \\
\partial_t Q_+ &= -\frac{1}{4} \bar{D}\bar{D} \left(e^{-2V} \frac{\delta S_{\mathrm{SQCD}}}{\delta Q_+^\dagger} \right) + \delta_\Lambda Q_+, \\
\partial_t Q_- &= -\frac{1}{4} \bar{D}\bar{D} \left(\frac{\delta S_{\mathrm{SQCD}}}{\delta Q_-^\dagger} e^{2V} \right) + \delta_\Lambda Q_-,
\end{aligned}
\tag{15}
$$

where δ_Λ is an infinitesimal transformation derived from an extended gauge transformation. Taking the component fields of Λ so that $\partial_t C = \partial_t \chi = \partial_t M = \partial_t N = 0$ where C, χ, M, N are component fields of the vector superfield (see Appendix B), the Wess–Zumino gauge is kept for any nonzero flow time [28,29].

The action appearing on the r.h.s. of Eqs. (15) is arbitrary. We use S_{SYM} instead of S_{SQCD} for the first equation and $S_{\mathrm{MAT}}|_{m=0}$ instead of S_{SQCD} for the second and third equations. These choices significantly simplify the component field equations. Although we consider the massive SQCD at $t=0$, the flow equation has no mass terms. After the Wick rotation to Euclidean space, these choices give us the flow equations of the component fields in the Euclidean SQCD;

$$
\begin{aligned}
\partial_t A_\mu &= D_\nu F_{\nu\mu} + i\bar{\lambda}\gamma_\mu\lambda, \\
\partial_t \lambda &= \not{D}^2 \lambda - i[\gamma_5\lambda, D], \\
\partial_t D &= D_\mu D_\mu D + i(\bar{\lambda}\gamma_5 \not{D}\lambda + \bar{\lambda}\overleftarrow{\not{D}}\gamma_5\lambda), \\
\partial_t \varphi_\pm &= D_\mu D_\mu \varphi_\pm + \sqrt{2} i \bar{\lambda} P_\pm \psi, \\
\partial_t \varphi_\pm^\dagger &= D_\mu D_\mu \varphi_\pm^\dagger - \sqrt{2} i \bar{\psi} P_\mp \lambda, \\
\partial_t \psi &= \not{D}^2 \psi + i\sqrt{2} P_+ (\gamma_\mu \lambda D_\mu \varphi_+ + i\lambda G_+) + i\sqrt{2} P_- (\gamma_\mu \lambda D_\mu \varphi_- + i\lambda G_-) - iD\gamma_5\psi, \\
\partial_t \bar{\psi} &= \bar{\psi}\overleftarrow{\not{D}}^2 + i\sqrt{2}(D_\mu \varphi_-^\dagger \bar{\lambda}\gamma_\mu - iG_-^\dagger \bar{\lambda}) P_+ + i\sqrt{2}(D_\mu \varphi_+^\dagger \bar{\lambda}\gamma_\mu - iG_+^\dagger \bar{\lambda}) P_- + i\bar{\psi}\gamma_5 D, \\
\partial_t G_\pm &= D_\mu D_\mu G_\pm \mp 2iDG_\pm + \sqrt{2}\bar{\lambda} P_\mp \not{D}\psi - \sqrt{2}\bar{\lambda}\overleftarrow{\not{D}} P_\pm \psi + 2i\bar{\lambda} P_\mp \lambda \varphi_\pm, \\
\partial_t G_\pm^\dagger &= D_\mu D_\mu G_\pm^\dagger \mp 2iG_\pm^\dagger D + \sqrt{2}\bar{\psi}\overleftarrow{\not{D}} P_\pm \lambda - \sqrt{2}\bar{\psi} P_\mp \not{D}\lambda + 2i\varphi_\pm^\dagger \bar{\lambda} P_\pm \lambda.
\end{aligned}
\tag{16}
$$

These flow equations are gauge covariant. In addition, these flows are supersymmetric except up to gauge transformation;

$$
[\delta_\xi, \partial_t] = \delta_{\tilde{\omega}}^g, \qquad \tilde{\omega} = \bar{\xi}\not{D}\lambda,
\tag{17}
$$

where $\delta_{\tilde{\omega}}^g$ is the infinitesimal gauge transformation (6), and δ_ξ is the modified SUSY transformation (7).

In this paper, we define flow equations only for the gauge and matter fields but not for the ghost field. In the Yang–Mills flow, the $(D+1)$-dimensional action corresponding to the D-dimensional flow is constructed in Ref. [3]. In the $(D+1)$-dimensional theory, a term corresponding to the ghost field flows is added to make the theory BRST invariant. However, the flowed ghost loop does not exist from theoretical constraints, i.e., there is no physical contribution from flowed ghosts at nonzero flow time even if we consider the flow equations for ghosts.

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3.2 *Iterative expansion of the flow equations*

The t-independent gauge covariance of the flow equation for the gauge field in Eq. (16) implies that the gauge degrees of freedom are not suppressed by the flow as in the case of the YM flow [2]. To suppress the evolution, we consider the modified equations by adding the corresponding gauge transformation (6) with $\omega = -\alpha_0 \partial_\nu A_\nu$ where α_0 is a gauge parameter.

For example, the flow equation for the gauge field is modified to the following:

$$\partial_t A_\mu = D_\nu F_{\nu\mu} + i\bar{\lambda}\gamma_\mu \lambda + \alpha_0 D_\mu \partial_\nu A_\nu, \tag{18}$$

with the boundary condition $A_\mu(t=0, x) = A_\mu(x)$. Solving the linearized equation by the use of the heat kernel

$$K_t(z)_{\mu\nu} = \int_p \frac{e^{ipz}}{p^2} \left\{ \left(\delta_{\mu\nu} p^2 - p_\mu p_\nu\right) e^{-tp^2} + p_\mu p_\nu e^{-\alpha_0 t p^2} \right\}, \tag{19}$$

enables the flow equation (18) to be expressed in the integral form

$$A^a_\mu(t,x) = \int \mathrm{d}^D y \left\{ K_t(x-y)_{\mu\nu} A^a_\nu(y) + \int_0^t \mathrm{d}s K_{t-s}(x-y)_{\mu\nu} R^a_\nu(s,y) \right\}, \tag{20}$$

where R^a_ν represents the nonlinear terms given by

$$\begin{aligned} R^a_\mu(t,x) = &-f^{abc} \left\{ 2A^b_\nu(t,x)\partial_\nu A^c_\mu(t,x) - A^b_\nu(t,x)\partial_\mu A^c_\nu(t,x) + (\alpha_0 - 1) A^b_\mu(t,x)\partial_\nu A^c_\nu(t,x) \right\} \\ &- \frac{1}{2} f^{abc} \lambda^b(t,x) C^{-1} \gamma_\mu \lambda^c(t,x) + f^{abc} f^{cde} A^b_\nu(t,x) A^d_\nu(t,x) A^e_\mu(t,x). \end{aligned} \tag{21}$$

It is useful to make a Fourier transform to the momentum space, and we find the integral equation

$$A^a_\mu(t,p) = K_t(p)_{\mu\nu} A^a_\nu(p) + \int_0^t \mathrm{d}s K_{t-s}(p)_{\mu\nu} R^a_\nu(s,p), \tag{22}$$

where

$$K_t(p)_{\mu\nu} = \frac{1}{p^2} \left\{ (\delta_{\mu\nu} p^2 - p_\mu p_\nu) e^{-tp^2} + p_\mu p_\nu e^{-\alpha_0 t p^2} \right\}, \tag{23}$$

$$\begin{aligned} R^a_\mu(t,p) = &\frac{1}{2} \int_{q,r} (2\pi)^D \delta(-p+q+r) \\ &\times \left\{ X^{(2,0)}_{A,AA}(-p,q,r)^{abc}_{\mu\nu\rho} A^b_\nu(t,q) A^c_\rho(t,r) + X^{(2,0)}_{A,\lambda\lambda}(-p,q,r)^{abc}_{\mu} \lambda^b(t,q) \lambda^c(t,r) \right\} \\ &+ \frac{1}{3!} \int_{q_1,q_2,q_3} (2\pi)^D \delta(-p+q_1+q_2+q_3) \\ &\times X^{(3,0)}_{A,AAA}(-p,q_1,q_2,q_3)^{ab_1b_2b_3}_{\mu\nu_1\nu_2\nu_3} A^{b_1}_{\nu_1}(t,q_1) A^{b_2}_{\nu_2}(t,q_2) A^{b_3}_{\nu_3}(t,q_3). \end{aligned} \tag{24}$$

Here, we introduce the flow vertices $X^{(2,0)}_{A,AA}$, $X^{(2,0)}_{A,\lambda\lambda}$, and $X^{(3,0)}_{A,AAA}$ given explicitly in Appendix D.

Solving the equation (22) iteratively, the flowed gauge field is obtained in powers of the boundary fields at $t=0$ as,

$$\begin{aligned} A^a_\mu(t,p) = K_t(p)_{\mu\nu} A^a_\nu(p) &+ \frac{1}{2} \int_0^t \mathrm{d}s K_{t-s}(p)_{\mu\nu} \int_{q,r} (2\pi)^D \delta(-p+q+r) \\ &\times \left\{ X^{(2,0)}_{A,AA}(-p,q,r)^{abc}_{\nu\rho\sigma} K_s(q)_{\rho\delta} K_s(r)_{\sigma\tau} A^b_\delta(q) A^c_\tau(r) \right. \\ &\left. + X^{(2,0)}_{A,\lambda\lambda}(-p,q,r)^{abc}_{\nu ij} K_s(q) K_s(r) \lambda^b_i(q) \lambda^c_j(r) \right\} + \cdots, \end{aligned} \tag{25}$$

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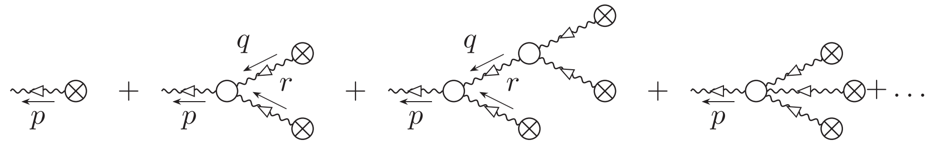
where $K_t(p) = e^{-tp^2}$. The solution is represented diagrammatically as

$$A^a_\mu(t,p) =$$



(26)

where open arrow lines represent the heat kernels K (or flow lines), open circles represent flow vertices X, and cross circles are boundary fields. The direction of momentum is opposite to that in Ref. [3], in the direction of the flow time. In perturbation theory, if rescaling the boundary fields of the gauge multiplet to the canonical normalization by the bare coupling such as $(A_\mu(x), \lambda(x), D(x)) \to (g_0 A_\mu(x), g_0\lambda(x), g_0 D(x))$, the iterative expansion (25) can be shown to be equivalent to the coupling expansion. Therefore, we treat the iterative solution as the coupling expansion in the calculation of the flowed field correlators. As a result of the above, the two-point function of the flowed gauge field is obtained as

$$\left\langle A^a_\mu(t,p)A^b_\nu(s,q)\right\rangle = (2\pi)^D\delta(p+q)\delta^{ab}g_0^2\left[(\delta_{\mu\nu}p^2 - p_\mu p_\nu)e^{-(t+s)p^2} + \xi_0 p_\mu p_\nu e^{-\alpha_0(t+s)p^2}\right] \times \frac{1}{(p^2)^2} + \mathcal{O}\left(g_0^4\right) \tag{27}$$

by combining the iterative expansion of the flowed fields and the coupling expansion of the boundary theory using the Feynman rule summarized in Appendix C. The other flow equations can be solved iteratively in the same way. These equations in integral form, the corresponding heat kernel, and flow vertices are given explicitly in Appendix D. The two-point functions are written as

$$\begin{aligned}
\left\langle\lambda^a(t,p)\lambda^b(s,q)\right\rangle &= (2\pi)^D\delta(p+q)\delta^{ab}g_0^2\frac{-i\,\not{p}C}{p^2}e^{-(t+s)p^2} + \mathcal{O}\left(g_0^4\right),\\
\left\langle D^a(t,p)D^b(q)\right\rangle &= (2\pi)^D\delta(p+q)\delta^{ab}g_0^2 e^{-(t+s)p^2} + \mathcal{O}\left(g_0^4\right),\\
\left\langle\varphi_{+,i}(t,p)\varphi^\dagger_{+,j}(s,q)\right\rangle = \left\langle\varphi_{-,i}(t,p)\varphi^\dagger_{-,j}(s,q)\right\rangle &= (2\pi)^D\delta(p+q)\delta_{ij}\frac{1}{p^2+m_0^2}e^{-(t+s)p^2} + \mathcal{O}\left(g_0^2\right),\\
\left\langle\psi_i(t,p)\bar{\psi}_j(s,q)\right\rangle &= (2\pi)^D\delta(p+q)\delta_{ij}\frac{-i\,\not{p}+m_0}{p^2+m_0^2}e^{-(t+s)p^2} + \mathcal{O}\left(g_0^2\right),\\
\left\langle G_{+,i}(t,p)G^\dagger_{+,j}(s,q)\right\rangle = \left\langle G_{-,i}(t,p)G^\dagger_{-,j}(s,q)\right\rangle &= (2\pi)^D\delta(p+q)\delta_{ij}e^{-(t+s)p^2} + \mathcal{O}\left(g_0^2\right).
\end{aligned} \tag{28}$$

Note that the commutation relation (17) also holds for the modified equations,

$$[\delta_\xi, \partial_t] = \delta^g_{\tilde{\omega}}, \qquad \tilde{\omega} = \bar{\xi}\,/\!\!\!D\lambda - \alpha_0\bar{\xi}\,/\!\!\!\partial\lambda, \tag{29}$$

so that the SQCD flow that we use here is supersymmetric in this sense.

4. One-loop renormalization of two-point function

In this section, we calculate two-point functions at the one-loop in SQCD and discuss the structure of their UV divergences. As in Sect. 2.2, we adopt the dimensional regularization and MS scheme. Feynman rules necessary for one-loop calculations are summarized in Appendixes C and D. In what follows, we set $\alpha_0 = 1$ for simplicity.

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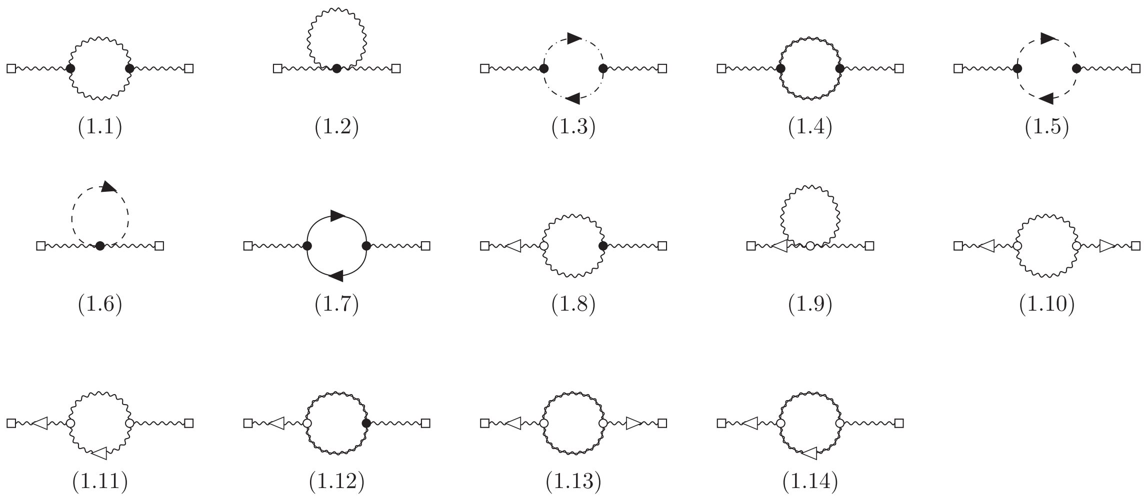

Fig. 1. One-loop diagrams contributing to the two-point function of the flowed gauge field. Flow diagrams are built from the conventional Feynman diagrams and flow diagrams. The former consist of boundary vertices (filled circles), gauge field propagators (wavy lines), ghost field propagators (directed dash-dotted lines), gaugino field propagators (solid lines with wavy lines), real scalar auxiliary field propagators (dotted lines), squark field propagators (dashed lines), quark field propagators (solid lines with filled arrow), and complex scalar auxiliary field propagators (circles lines). The latter are represented by flow vertices (open circles) and flow propagators (each field line with an open arrow). An open square indicates that the external lines are not amputated.

We first consider the divergence term of the flowed gauge multiplet and define its coefficients $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}$ through

$$\left\langle A^a_\mu(t,p)A^b_\nu(s,q)\right\rangle\Big|_{\text{pole}} = (2\pi)^D\delta(p+q)\frac{\delta^{ab}}{(p^2)^2}e^{-(t+s)p^2}\frac{\mu^{2\epsilon}g^4}{16\pi^2\epsilon}\left\{\left(\delta_{\mu\nu}p^2-p_\mu p_\nu\right)\mathcal{A}+\xi p_\mu p_\nu\mathcal{B}\right\}, \tag{30}$$

$$\left\langle \lambda^a(t,p)\lambda^b(s,q)\right\rangle\Big|_{\text{pole}} = (2\pi)^D\delta(p+q)\frac{-i\delta^{ab}(\not{p}C)}{p^2}e^{-(t+s)p^2}\frac{\mu^{2\epsilon}g^4}{16\pi^2\epsilon}\mathcal{C}, \tag{31}$$

$$\left\langle D^a(t,p)D^b(s,q)\right\rangle\Big|_{\text{pole}} = (2\pi)^D\delta(p+q)\delta^{ab}e^{-(t+s)p^2}\frac{\mu^{2\epsilon}g^4}{16\pi^2\epsilon}\mathcal{D}, \tag{32}$$

where $\epsilon = (4-D)/2$ in dimensional regularization (see Appendix E). One-loop diagrams contributing to the two-point functions of the flowed gauge multiplet are shown in Figs. 1, 2 and 3, respectively, except for diagrams with closed flow-line loops. Closed flow-line loops diagrams are set to zero [3]. Since the diagrams (1.1)–(1.7), (2.1)–(2.2), and (3.1) include only the boundary vertices, these diagrams are calculated by the ordinary calculation in the SQCD. The contribution to the diagram including the flow vertices, for example diagram (1.8), is explicitly

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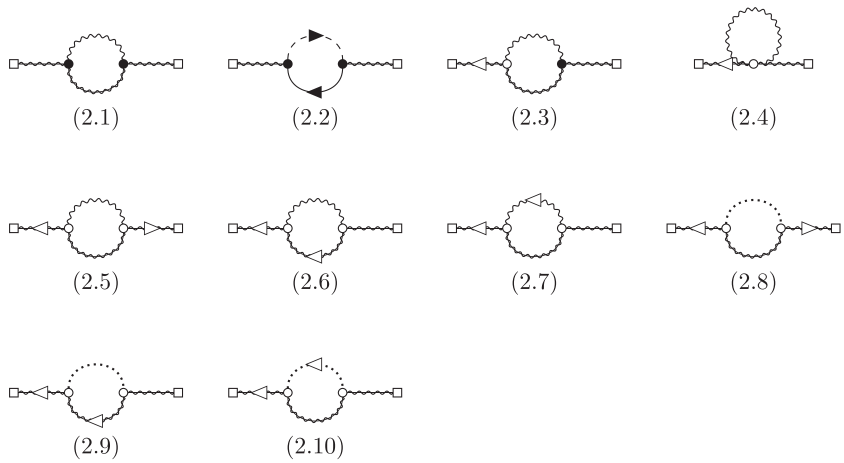

Fig. 2. One-loop diagrams contributing to the $\lambda\lambda$.

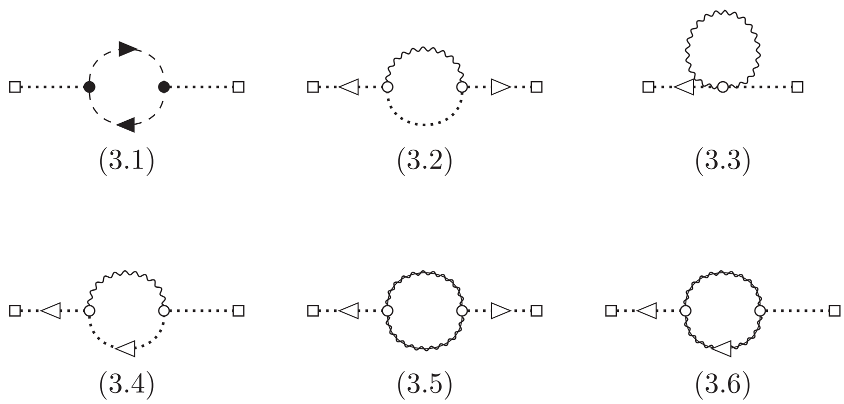

Fig. 3. One-loop diagrams contributing to the DD.

calculated as follows.

$$
\begin{aligned}
(\text{diagram 1.8})|_{\text{pole}} &= \int_0^t \mathrm{d}t' \int_q e^{-(t-t')p^2} \left[-i f^{acd} \left\{(2q-p)_\mu \delta_{\rho\sigma} - 2q_\sigma \delta_{\mu\rho} + 2(p-q)_\rho \delta_{\sigma\mu}\right\}\right] \\
&\quad \times e^{-t'(p-q)^2} \frac{\delta^{dd'} g_0^2}{((q-p)^2)^2} \left\{\delta_{\sigma\sigma'}(q-p)^2 - (1-\xi_0)(q-p)_\sigma (q-p)_{\sigma'}\right\} \\
&\quad \times \frac{i}{g_0^2} f^{b'c'd'} \left\{(2q-p)_{\nu'}\delta_{\rho'\sigma'} + (-q+2p)_{\rho'}\delta_{\sigma'\nu'} - (q+p)_{\sigma'}\delta_{\nu'\rho'}\right\} \\
&\quad \times e^{-t'q^2} \frac{\delta^{cc'} g_0^2}{(q^2)^2} \left\{\delta_{\rho\rho'} q^2 - (1-\xi_0) q_\rho q_{\rho'}\right\} \\
&\quad \times e^{-sp^2} \frac{\delta^{bb'} g_0^2}{(p^2)^2} \left\{\delta_{\nu'\nu} p^2 - (1-\xi_0) p_{\nu'} p_\nu\right\}\Big|_{\text{pole}} \\
&= \frac{\delta^{ab}}{(p^2)^2} e^{-(t+s)p^2} \frac{g_0^4}{16\pi^2\epsilon} \left\{\left(\delta_{\mu\nu} p^2 - p_\mu p_\nu\right) + \xi_0 p_\mu p_\nu\right\} \times \frac{3+3\xi_0}{2} N_c \qquad (33)
\end{aligned}
$$

The other diagrams are calculated in a similar way and all the divergent factors are summarized in Tables 1, 2 and 3. At all flow times, these singularities are canceled by the parameter renormalization required at flow time zero (10). This property for the gauge field is similar to

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Table 1. Contribution to AA from each diagram

Factor	Diagram								Total
	(1.1)–(1.7)	(1.8)	(1.9)	(1.10)	(1.11)	(1.12)	(1.13)	(1.14)	
$\mathcal{A}$	$\frac{3-\xi}{2}N_c - N_f$	$\frac{3+3\xi}{2}N_c$	$\frac{-9-3\xi}{4}N_c$	0	$\frac{9-\xi}{4}N_c$	N_c	$-N_c$	0	$3N_c - N_f$
$\mathcal{B}$	0	$\frac{3+3\xi}{2}N_c$	$\frac{-9-3\xi}{4}N_c$	0	$\frac{9-\xi}{4}N_c$	N_c	$-N_c$	0	$\frac{3+\xi}{2}N_c$

Table 2. Contribution to $\lambda\lambda$ from each diagram

Factor	Diagram									Total
	(2.1)–(2.2)	(2.3)	(2.4)	(2.5)	(2.6)	(2.7)	(2.8)	(2.9)	(2.10)	
$\mathcal{C}$	$-\xi N_c - N_f$	$(3+2\xi)N_c$	$(-3-\xi)N_c$	0	$\frac{3}{2}N_c$	$\frac{5}{2}N_c$	0	$-\frac{1}{2}N_c$	$-\frac{1}{2}N_c$	$3N_c - N_f$

Table 3. Contribution to DD from each diagram

Factor	Diagram						Total
	(3.1)	(3.2)	(3.3)	(3.4)	(3.5)	(3.6)	
$\mathcal{D}$	$-N_f$	ξN_c	$(-3-\xi)N_c$	0	$2N_c$	$4N_c$	$3N_c - N_f$

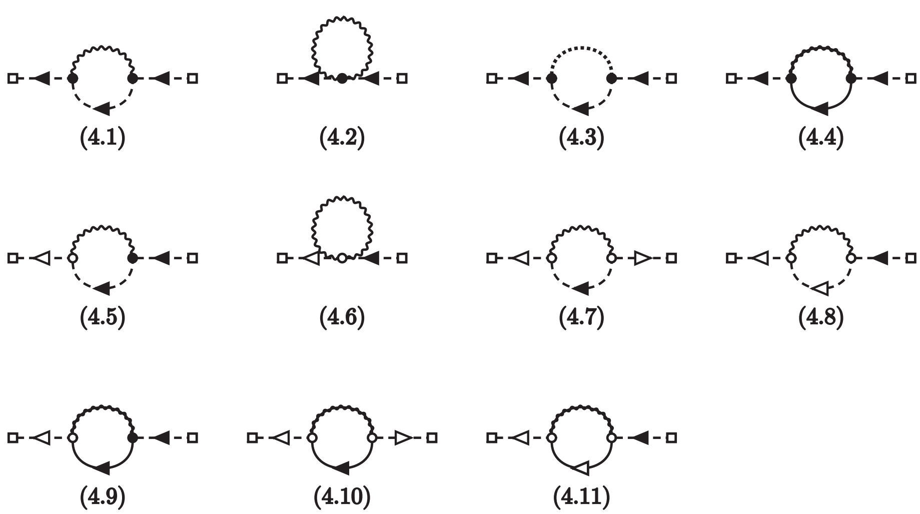

Fig. 4. One-loop diagrams contributing to the $\varphi\varphi^\dagger$.

that shown in the non-SUSY case [3]. On the other hand, gaugino and real auxiliary fields differ from the case of a fermion flow [4]. The non-SUSY fermion flow needs wave function renormalization, but our SUSY flow does not require extra renormalization.

Now we consider the matter supermultiplet (φ, ψ, G) and define $\mathcal{E}, \mathcal{F}, \mathcal{G}$ as

$$\left\langle \varphi_{\pm,i}(t,p)\varphi^\dagger_{\pm,j}(s,q)\right\rangle\Big|_{\text{pole}} = (2\pi)^D\delta(p+q)\delta_{ij}\frac{1}{p^2+m^2}e^{-(t+s)p^2}\frac{g^2}{16\pi^2\epsilon}\mathcal{E}, \tag{34}$$

$$\left\langle \psi_i(t,p)\bar{\psi}_j(s,q)\right\rangle\Big|_{\text{pole}} = (2\pi)^D\delta(p+q)\delta_{ij}\frac{-i\not{p}+m}{p^2+m^2}e^{-(t+s)p^2}\frac{g^2}{16\pi^2\epsilon}\mathcal{F}, \tag{35}$$

$$\left\langle G_{\pm,i}(t,p)G^\dagger_{\pm,j}(s,q)\right\rangle\Big|_{\text{pole}} = (2\pi)^D\delta(p+q)\delta_{ij}e^{-(t+s)p^2}\frac{g^2}{16\pi^2\epsilon}\mathcal{G}, \tag{36}$$

One-loop diagrams contributing to the two-point functions are shown in Figs. 4, 5 and 6, re-

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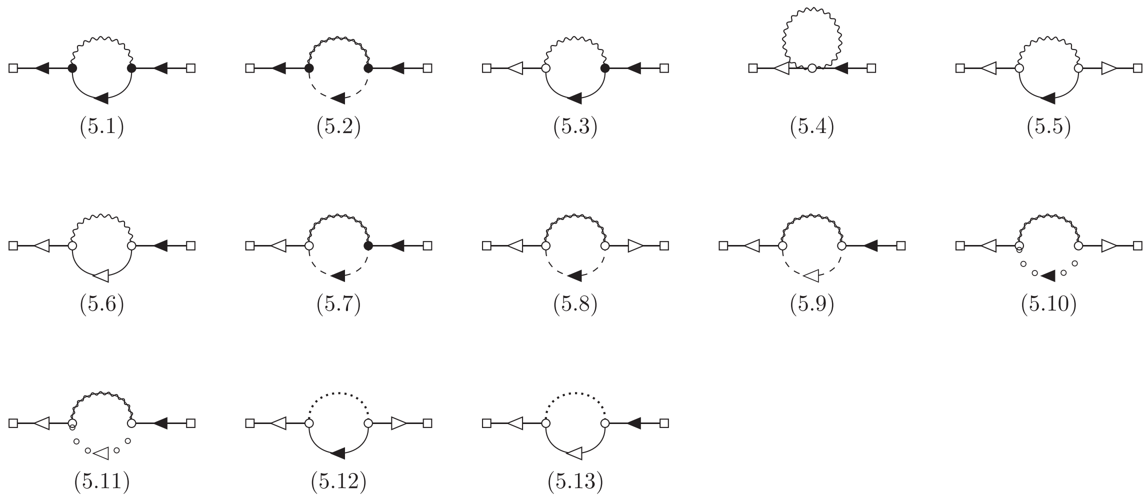

Fig. 5. One-loop diagrams contributing to the $\psi\bar{\psi}$.

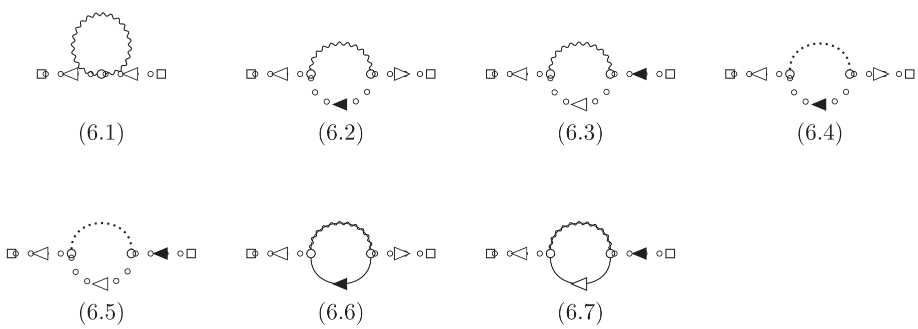

Fig. 6. One-loop diagrams contributing to the $GG^\dagger$.

Table 4. Contribution to $\varphi\varphi^\dagger$ from each diagram

	Diagram								
Factor	(4.1)–(4.4)	(4.5)	(4.6)	(4.7)	(4.8)	(4.9)	(4.10)	(4.11)	Total
$\mathcal{E}$	$(1-\xi)C_F$	$2\xi C_F$	$(-3-\xi)C_F$	0	0	$4C_F$	0	0	$2C_F$

Table 5. Contribution to $\psi\bar{\psi}$ from each diagram

	Diagram												
Factor	(5.1)–(5.2)	(5.3)	(5.4)	(5.5)	(5.6)	(5.7)	(5.8)	(5.9)	(5.10)	(5.11)	(5.12)	(5.13)	Total
$\mathcal{F}$	$(-1-\xi)C_F$	$(3+2\xi)C_F$	$(-3-\xi)C_F$	0	$\frac{3}{2}C_F$	$2C_F$	0	C_F	0	$-C_F$	0	$-\frac{1}{2}C_F$	$2C_F$

spectively. The diagrams (4.1)–(4.4) and (5.1)–(5.2) are calculated by the ordinary calculation in the SQCD. The other diagrams are calculated in the same way as Eq. (33) and all the divergent parts are summarized in Tables 4, 5 and 6.

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Table 6. Contribution to $GG^\dagger$ from each diagram

	Diagram							
Factor	(6.1)	(6.2)	(6.3)	(6.4)	(6.5)	(6.6)	(6.7)	Total
$\mathcal{G}$	$(-3-\xi)C_F$	ξC_F	0	$-C_F$	$-2C_F$	$4C_F$	$4C_F$	$2C_F$

As can be seen from the results, the remaining pole terms are all common in each component and are found to be proportional to $2C_F$. So, the two-point functions can be renormalized by renormalizing the fields according to

$$F = Z_F^{1/2} F_R, \qquad \bar{F} = Z_F^{1/2} \bar{F}_R, \qquad Z_F = 1 + \frac{g^2}{16\pi^2\epsilon} \times 2C_F, \tag{37}$$

where $F = \varphi_\pm, \psi, G_\pm$ and $\bar{F} = \varphi_\pm^\dagger, \bar{\psi}, G_\pm^\dagger$.

In QCD, correlation functions of flowed gauge fields are UV-finite and the flowed quark receives an extra wave function renormalization [3,4]. It is interesting to know that our results can be viewed as simple supersymmetric extensions of Refs. [3,4] at the one-loop level. In fact, two-point functions of fields in the gauge supermultiplet are UV-finite while matter multiplets receive an overall wave function renormalization. For Z factors at $t = 0$ presented in Eq. (10), supersymmetry is broken by the ghost and gauge fixing terms. Somewhat surprisingly, the SQCD flow respects supersymmetry in a sense that the flowed gaugino does not receive extra renormalizations as the gauge field does and the common Z factor (37) is obtained for all fields of matter multiplets at least at the one-loop level.

5. Conclusion

We have investigated the UV-divergence structure of a supersymmetric gradient flow in $\mathcal{N} = 1$ SQCD with $\mathrm{SU}(N_c)$ gauge group and N_f flavor fundamental quarks. We found that, at the one-loop level, two-point functions of fields in a flowed gauge multiplet were UV-finite while wave function renormalizations, which are common for all the fields, are needed for flowed matter multiplets.

These results can be viewed as a naive extension from the non-SUSY flows [2–4], in which correlation functions of the flowed gauge field are UV-finite and the flowed quark receives an extra wave function renormalization. In SQCD, the renormalization factors of the component fields in the Wess–Zumino gauge do not retain supersymmetry, but the UV-divergent part of flowed fields appears in a supersymmetric fashion after renormalizing the boundary theory. This interesting property would be a manifestation of supersymmetry in the SQCD flow which is defined in a supersymmetric way up to a gauge transformation.

In this paper, we only investigated the case of two-point functions at the one-loop level. In the YM flow, it is shown that UV divergences are absent in any flowed correlation function at all orders of perturbation theory [3,5]. To obtain an all-order proof in the SQCD flow, we need to construct $(D + 1)$-dimensional theory as Lüscher and Weisz did in the YM flow [3]. Our results provide useful information for such studies, and new and interesting results using the SUSY flow will be obtained in the future.

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Appendix A. Notation

The theory is defined on four-dimensional Euclidean space whose coordinates are expressed as x_μ where the Greek indices μ, ν, ρ, σ run from 0 to 3. We use α, β for the spinor index which runs from 1 to 4 and m, n for the flavor index which runs from 1 to N_f. The Einstein summation convention is used throughout this paper.

Gamma matrices γ_μ are Hermitian matrices that satisfy

$$\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}, \tag{A1}$$

and γ_5 is defined as

$$\gamma_5 \equiv \gamma_0\gamma_1\gamma_2\gamma_3 = \gamma_5^\dagger. \tag{A2}$$

The Majorana fermion λ^a (SU(N_c) adjoint rep.) is defined as

$$\lambda^a = -C(\bar{\lambda}^a)^T, \qquad \bar{\lambda}^a = (\lambda^a)^T C^{-1}, \tag{A3}$$

where the charge conjugation matrix C satisfies

$$C^T = -C, \qquad C^{-1}\gamma_\mu C = -\gamma_\mu^T, \tag{A4}$$

and we have $C^{-1}\gamma_5 C = \gamma_5^T$.

The group generators T^a are Hermitian and satisfy the standard relation,

$$[T^a, T^b] = if^{abc}T^c, \tag{A5}$$

and

$$\mathrm{tr}(T^aT^b) = T_F\delta^{ab}, \tag{A6}$$

$$(T^aT^a)_{ij} = C_F\delta_{ij}, \tag{A7}$$

$$f^{acd}f^{bcd} = C_A\delta^{ab}, \tag{A8}$$

with $T_F = \frac{1}{2}$, $C_F = \frac{N_c^2-1}{2N_c}$, and $C_A = N_c$ for SU(N_c). The fields of gauge multiplet are expressed as matrix-valued fields such as $A_\mu(x) = \sum_{a=1}^{N_c^2-1} A_\mu^a(x)T^a$.

Fourier transformations in the$D = 4 - 2\epsilon$ dimension are defined by

$$\phi(x) = \int_p e^{ipx}\phi(p), \quad \phi(p) = \int \mathrm{d}^D x\, e^{-ipx}\phi(x) \tag{A9}$$

where the momentum integral is abbreviated as

$$\int_p \equiv \int \frac{\mathrm{d}^D p}{(2\pi)^D}. \tag{A10}$$

Flow diagrams contain a linear combination of terms as

$$\int_0^t \mathrm{d}t' \int_q \frac{Q(q,p)}{q^2(q-p)^2} e^{-t'\{uq^2+v(q-p)^2\}}, \qquad u, v \in \{1, \alpha_0\}, \tag{A11}$$

where $Q(q, p)$ is a homogeneous polynomial in q and p. It must be eventually integrated over t from 0 to infinity, and then can be written as

$$\int_q \frac{1}{u+v}\frac{Q(q,0)}{(q^2)^2(q-p)^2}. \tag{A12}$$

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Appendix B. Superfield formalism in the Minkowski spacetime

We review the superfield formalism according to Ref. [30]. In the superfield formalism, a superfield $F(z)$ is introduced as a function of $z = (x_\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}})$ which transforms under a supersymmetry transformation as $\delta_\xi F(z) = (\xi^\alpha Q_\alpha + \bar{\xi}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}})F(z)$, where $\theta_\alpha, \bar{\theta}_{\dot{\alpha}}, \xi_\alpha, \bar{\xi}_{\dot{\alpha}}$ are two-component anti-commuting global parameters and $Q_\alpha, \bar{Q}_{\dot{\alpha}}$ are differential operators defined by

$$Q_\alpha = \frac{\partial}{\partial\theta^\alpha} - i(\sigma^\mu)_{\alpha\dot{\alpha}}\bar{\theta}^\alpha\partial_\mu, \quad \bar{Q} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + i\theta^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}\partial_\mu, \tag{B1}$$

where $\sigma^\mu = (-\mathbb{1}, \sigma^i)$ and $\bar{\sigma}^\mu = (-\mathbb{1}, -\sigma^i)$. For later convenience, we introduce the supersymmetric covariant derivatives as

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i(\sigma^\mu)_{\alpha\dot{\alpha}}\bar{\theta}^\alpha\partial_\mu, \quad \bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i\theta^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}\partial_\mu. \tag{B2}$$

Note that $\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = -\{D_\alpha, \bar{D}_{\dot{\beta}}\} = 2i\sigma^\mu_{\alpha\dot{\beta}}\partial_\mu$ and the other anti-commutation relations are zero.

The SQCD action involves chiral superfields $Q_\pm$ and vector superfields V. The chiral superfields $Q_\pm(z)$ are defined by $\bar{D}_{\dot{\alpha}}Q_\pm = 0$ and are expanded as

$$Q_\pm(y, \theta) = \phi_\pm(y) + \sqrt{2}\theta\psi_\pm(y) + \theta\theta F_\pm(y) \tag{B3}$$

where $y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$. Here $\phi_\pm$ are scalar fields, $\psi_\pm$ are two-component spinors, and $F_\pm$ are auxiliary fields. Similarly, $\bar{Q}_\pm$ satisfy $D_{\dot{\alpha}}Q^\dagger_\pm = 0$ and are expanded as

$$Q^\dagger_\pm(y^\dagger, \bar{\theta}) = \phi^\dagger_\pm(y^\dagger) + \sqrt{2}\bar{\theta}\psi^\dagger_\pm(y^\dagger) + \bar{\theta}\bar{\theta}F^\dagger_\pm(y^\dagger) \tag{B4}$$

where $y^{\dagger\mu} = x^\mu - i\theta\sigma^\mu\bar{\theta}$. The general form of a vector superfield defined by $V^\dagger = V$ is

$$\begin{aligned} V(x, \theta, \bar{\theta}) = C(x) &+ i\theta\chi(x) - i\bar{\theta}\bar{\chi}(x) \\ &+ \frac{i}{2}\theta\theta(M(x) + iN(x)) - \frac{i}{2}\bar{\theta}\bar{\theta}(M(x) - iN(x)) - \theta\sigma^\mu\bar{\theta}A_\mu(x) \\ &+ i\theta\theta\bar{\theta}\left(\bar{\lambda}(x) + \frac{i}{2}\bar{\sigma}^\mu\partial_\mu\chi(x)\right) - i\bar{\theta}\bar{\theta}\theta\left(\lambda(x) + \frac{i}{2}\bar{\sigma}^\mu\partial_\mu\chi(x)\right) \\ &+ \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}\left(D(x) + \frac{1}{2}\Box C(x)\right), \end{aligned} \tag{B5}$$

where C, M, N, A_μ, D are bosonic fields and χ, λ are fermionic fields. All components are in the adjoint representation of $SU(N_c)$ as $V = \sum_{a=1}^{N_c^2-1} V^a T^a$. The extended gauge transformation of chiral and vector superfields generated by a chiral superfield Λ are

$$e^{2V} \to e^{2V'} = e^{2\Lambda^\dagger}e^{2V}e^{2\Lambda}, \qquad Q_+ \to Q'_+ = e^{-2\Lambda}Q_+, \qquad Q_- \to Q'_- = Q_-e^{2\Lambda}. \tag{B6}$$

We can choose a special gauge called the Wess–Zumino gauge where the components C, χ, M, N are set to zero using an extended gauge transformation.

The SQCD action is given by $S_{\mathrm{SQCD}} = S_{\mathrm{SYM}} + S_{\mathrm{MAT}}$ with

$$S_{\mathrm{SYM}} = \frac{1}{2g_0^2}\int \mathrm{d}^4x\mathrm{tr}\Big(W^\alpha W_\alpha|_{\theta\theta} + \bar{W}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}}|_{\bar{\theta}\bar{\theta}}\Big), \tag{B7}$$

$$S_{\mathrm{MAT}} = \int \mathrm{d}^4x\Big\{\big(Q^\dagger_+e^{2V}Q_+ + Q_-e^{-2V}Q^\dagger_-\big)\Big|_{\theta\theta\bar{\theta}\bar{\theta}} + m_0\big(Q_-Q_+|_{\theta\theta} + Q^\dagger_+Q^\dagger_-|_{\bar{\theta}\bar{\theta}}\big)\Big\}, \tag{B8}$$

where

$$W_\alpha = -\frac{1}{8}\bar{D}\bar{D}e^{-2V}D_\alpha e^{2V}, \quad \bar{W}_{\dot{\alpha}} = \frac{1}{8}DDe^{2V}\bar{D}_{\dot{\alpha}}e^{-2V}. \tag{B9}$$

These actions (B.7), (B.8) are invariant under both the SUSY and extended gauge transformations. In the Wess–Zumino gauge, the SQCD action is written in

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$$S_{\rm SYM} = \frac{1}{g_0^2} \int \mathrm{d}^4x \mathrm{tr} \left(-\frac{1}{2} F_{\mu\nu}^2 - i\bar{\lambda} \not{D} \lambda + D^2 \right), \tag{B10}$$

$$\begin{aligned} S_{\rm MAT} = \int \mathrm{d}^4x \Big\{ & - |D_\mu \phi_+|^2 - |D_\mu \phi_-|^2 - i\bar{\psi} \not{D} \psi + |F_+|^2 + |F_-|^2 + \left(\phi_+^\dagger D \phi_+ - \phi_- D \phi_-^\dagger \right) \\ & + \sqrt{2} i \left(-\bar{\psi}_+ \lambda \phi_+ - \bar{\psi}_- \lambda \phi_-^\dagger + \phi_+^\dagger \bar{\lambda} \psi_+ + \phi_- \bar{\lambda} \psi_- \right) \\ & + m_0 \left(-\bar{\psi} \psi + \phi_- F_+ + F_- \phi_+ + \phi_+^\dagger F_-^\dagger + F_+^\dagger \phi_-^\dagger \right) \Big\} \end{aligned} \tag{B11}$$

in terms of a gauge multiplet (A_μ, λ, D) and N_f matter multiplets $(\phi_\pm, \psi_\pm, F_\pm)_f$ for $f = 1, 2, \ldots, N_f$. Here we used

$$\lambda = \begin{pmatrix} \lambda_\alpha \\ \bar{\lambda}^{\dot{\alpha}} \end{pmatrix}, \quad \bar{\lambda} = (\lambda^\alpha, \bar{\lambda}_{\dot{\alpha}}), \quad \psi = \begin{pmatrix} \psi_{+\alpha} \\ \bar{\psi}_-^{\dot{\alpha}} \end{pmatrix}, \quad \bar{\psi} = (\psi_-^\alpha, \bar{\psi}_{+\dot{\alpha}}), \quad \gamma_\mu^{(\rm M)} = \begin{pmatrix} 0 & \sigma_\mu \\ \bar{\sigma}_\mu & 0 \end{pmatrix} \tag{B12}$$

to move on to the notation with four-component spinors.

The Euclidean theory is obtained by the Wick rotation given by

$$x^0 = t \to -ix^0 = -it, \quad A_0 \to iA_0, \tag{B13}$$

and replacements of auxiliary fields

$$D \to iD, \quad F_\pm \to iF_\pm, \quad F_\pm^\dagger \to iF_\pm^\dagger. \tag{B14}$$

The Euclidean gamma matrix γ_μ is defined from the Minkowski one $\gamma_\mu^{(\rm M)}$ as $\gamma_0 = \gamma_0^{(\rm M)}$, $\gamma_i = i\gamma_i^{(\rm M)}$. The Euclidean action $S_{\rm E}$ is read from the Minkowski action $S_{\rm M}$ as $iS_{\rm M} \to -S_{\rm E}$ after the Wick rotation. Redefining the component fields:

$$\varphi_+ \equiv \phi_+, \qquad \varphi_- \equiv \phi_-^\dagger, \qquad G_+ \equiv F_+, \qquad G_+ \equiv F_-^\dagger, \tag{B15}$$

we obtain the Euclidean action (1), (2).

Appendix C. Feynman rules

The Feynman rule necessary for one-loop calculations is summarized. For simplicity of explanation, we take $N_f = 1$ and drop the flavor index in the rule.

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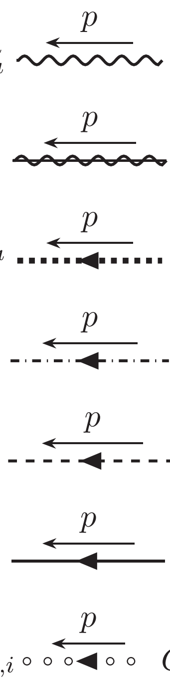
Feynman propagators are



$$
\begin{aligned}
A^a_\mu \quad A^b_\nu \quad &= \delta^{ab} g_0^2 \frac{1}{(p^2)^2}\Big\{\delta_{\mu\nu}p^2 - (1-\xi_0)p_\mu p_\nu\Big\}, \\
\lambda^a \quad \lambda^b \quad &= \delta^{ab} g_0^2 \frac{-i\not{p}C}{p^2}, \\
D^a \quad D^b \quad &= \delta^{ab} g_0^2, \\
c^a \quad \bar{c}^b \quad &= \delta^{ab} g_0^2 \frac{1}{p^2}, \\
\varphi_{\pm,i} \quad \varphi^\dagger_{\pm,j} &= \delta_{ij} \frac{1}{p^2+m_0^2}, \\
\psi_i \quad \bar{\psi}_j \quad &= \delta_{ij} \frac{-i\not{p}+m_0}{p^2+m_0^2}, \\
G_{\pm,i} \quad G^\dagger_{\pm,j} \quad &= \delta_{ij},
\end{aligned}
\tag{C1}
$$

and the other two-point functions are zero. In the momentum space, the interaction terms are

$$
\begin{aligned}
S_{\rm int} = &-\int_{p,q,r} (2\pi)^D \delta(p+q+r) \frac{1}{3!} V_{AAA}(p,q,r)^{abc}_{\mu\nu\rho} A^a_\mu(p) A^b_\nu(q) A^c_\rho(r) \\
&+ \int_{p,q_1,q_2,q_3} (2\pi)^D \delta(p+q_1+q_2+q_3) \frac{1}{4!} V_{AAAA}(p,q_1,q_2,q_3)^{abc}_{\mu\nu\rho} A^a_\mu(p) A^b_\nu(q_1) A^c_\rho(q_2) A^d_\sigma(q_3) \\
&- \int_{p,q,r} (2\pi)^D \delta(p+q+r) \frac{1}{2!} V_{\lambda A\lambda}(p,q,r)^{abc}_{\mu} \lambda^a(p) A^b_\nu(q) \lambda^c(r) \\
&- \int_{p,q,r} (2\pi)^D \delta(p+q+r) V_{\bar{c}Ac}(p,q,r)^{abc}_{\mu} \bar{c}^a(p) A^b_\nu(q) c^c(r) \\
&+ \int_{p,q,r} (2\pi)^D \delta(p+q+r) V_{\varphi^\dagger A\varphi}(p,q,r)^{a}_{\mu ij} \varphi^\dagger_{\pm,i}(p) A^a_\mu(q) \varphi_{\pm,j}(r) \\
&+ \int_{p,q_1,q_2,q_3} (2\pi)^D \delta(p+q_1+q_2+q_3) \\
&\times \frac{1}{2!} V_{\varphi^\dagger AA\varphi}(p,q_1,q_2,q_3)^{ab}_{\mu\nu ij} \varphi^\dagger_{+,i}(p) A^a_\mu(q_1) A^b_\nu(q_2) \varphi_{\pm,j}(q_3) \\
&- \int_{p,q,r} (2\pi)^D \delta(p+q+r) V_{\varphi^\dagger D\varphi}(p,q,r)^{a}_{ij} \varphi^\dagger_{\pm,i}(p) D^a(q) \varphi_{\pm,j}(r) \\
&+ \int_{p,q,r} (2\pi)^D \delta(p+q+r) V_{\bar{\psi}A\psi}(p,q,r)^{a}_{\mu ij} \bar{\psi}_i(p) A^a_\mu(q) \psi_j(r) \\
&+ \int_{p,q,r} (2\pi)^D \delta(p+q+r) V_{\bar{\psi}\lambda\varphi}(p,q,r)^{a}_{ij} \bar{\psi}_i(p) \lambda^a(q) \varphi_{\pm,j}(r) \\
&- \int_{p,q,r} (2\pi)^D \delta(p+q+r) V_{\varphi^\dagger\lambda\psi}(p,q,r)^{a}_{ij} \varphi^\dagger_{\pm,i}(p) \lambda^a(q) \psi_j(r)
\end{aligned}
\tag{C2}
$$

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where vertices are

$$
\begin{aligned}
V_{AAA}(p,q,r)^{abc}_{\mu\nu\rho} &= \frac{i}{g_0^2} f^{abc}\left\{(r-q)_\mu\delta_{\nu\rho} + (p-r)_\nu\delta_{\rho\mu} + (q-p)_\rho\delta_{\mu\nu}\right\},\\
V_{AAAA}(p,q_1,q_2,q_3)^{abcd}_{\mu\nu\rho\sigma} &= -\frac{1}{g_0^2}\Big\{ f^{abe}f^{ecd}(\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho}) + f^{ace}f^{edb}(\delta_{\mu\sigma}\delta_{\rho\nu} - \delta_{\mu\nu}\delta_{\rho\sigma})\\
&\quad + f^{ade}f^{ebc}(\delta_{\mu\nu}\delta_{\rho\sigma} - \delta_{\mu\rho}\delta_{\nu\sigma})\Big\},\\
V_{\lambda A\lambda}(p,q,r)^{abc}_{\mu} &= \frac{1}{g_0^2} f^{abc}(C^{-1}\gamma_\mu),\\
V_{\bar{c}Ac}(p,q,r)^{abc}_{\mu} &= \frac{i}{g_0^2} f^{abc} p_\mu,\\
V_{\varphi^\dagger A\varphi}(p,q,r)^{a}_{\mu ij} &= -(p+r)_\mu T^a_{ij},\\
V_{\varphi^\dagger AA\varphi}(p,q_1,q_2,q_3)^{ab}_{\mu\nu ij} &= -\{T^a, T^b\}_{ij}\delta_{\mu\nu},\\
V_{\varphi^\dagger D\varphi}(p,q,r)^{a}_{ij} &= \pm i T^a_{ij},\\
V_{\bar{\psi}A\psi}(p,q,r)^{a}_{\mu ij} &= -iT^a_{ij}\gamma_\mu,\\
V_{\bar{\psi}\lambda\varphi}(p,q,r)^{a}_{ij} &= -\sqrt{2} i P_\mp T^a_{ij},\\
V_{\varphi^\dagger\lambda\psi}(p,q,r)^{a}_{ij} &= +\sqrt{2} i C^{-1} P_\pm T^a_{ij}.
\end{aligned}
\tag{C3}
$$

Here we abbreviated $\pm$ of $\varphi_\pm$ for $V_{\varphi^\dagger_\pm A\varphi_\pm}$, $V_{\bar{\psi}\lambda\varphi_\pm}$, $V_{\varphi^\dagger_\pm\lambda\psi}$ if the same rule applies to $\varphi_\pm$.

Appendix D. Flow vertices

The formal solutions of flow equations are summarized below, and flow vertices are straightforwardly derived from interaction terms.

The formal solution of gauge field is given by Eqs. (22)–(24). Similarly, the formal solution for the gaugino field and the auxiliary field D are given in the following form:

$$
V^a(t,p) = K_t(p)V^a(p) + \int_0^t \mathrm{d}s K_{t-s}(p) R^a_V(s,p), \tag{D1}
$$

where $V = \lambda, D$. Here the flow propagator is defined as $K_t(p) = e^{-tp^2}$. It is also straightforward to give the formal solution for matter fields $F = \varphi_\pm, \psi, G_\pm$ and $\bar{F} = \varphi^\dagger_\pm, \bar{\psi}, G^\dagger_\pm$ as

$$
F_i(t,p) = K_t(p)_{ij}F_j(p) + \int_0^t \mathrm{d}s K_{t-s}(p)_{ij} R_{F,j}(s,p), \tag{D2}
$$

$$
\bar{F}_i(t,p) = \bar{F}_j(p)K_t(p)_{ji} + \int_0^t \mathrm{d}s \bar{R}_{\bar{F},j}(s,p) K_{t-s}(p)_{ji}, \tag{D3}
$$

where the flow propagator $K_t(p)_{ij} = \delta_{ij}e^{-tp^2}$.

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The concrete forms of R_V, R_F, $\bar{R}_{\bar{F}}$ are defined as follows:

$$\begin{aligned}
R^a_\lambda(t,p) = &\int_{q,r}(2\pi)^D\delta(-p+q+r)\\
&\times\left\{X^{(1,1)}_{\lambda,\lambda A}(-p,q,r)^{abc}_\mu\lambda^b(t,q)A^c_\mu(t,r)+X^{(1,1)}_{\lambda,\lambda D}(-p,q,r)^{abc}\lambda^b(t,q)D^c(t,r)\right\}\\
&+\int_{q_1,q_2,q_3}(2\pi)^D\delta(-p+q_1+q_2+q_3)\\
&\times\frac{1}{2}X^{(2,1)}_{\lambda,\lambda AA}(-p,q_1,q_2,q_3)^{abcd}_{\mu\nu}\lambda^b(t,q_1)A^c_\mu(t,q_2)A^d_\nu(t,q_3),
\end{aligned}\tag{D4}$$

$$\begin{aligned}
R^a_D(t,p) = &\int_{q,r}(2\pi)^D\delta(-p+q+r)\\
&\times\left\{X^{(1,1)}_{D,AD}(-p,q,r)^{abc}_\mu A^b_\mu(t,q)D^c(t,r)+\frac{1}{2}X^{(2,0)}_{D,\lambda\lambda}(-p,q,r)^{abc}\lambda^b(t,q)\lambda^c(t,r)\right\}\\
&+\int_{q_1,q_2,q_3}(2\pi)^D\delta(-p+q_1+q_2+q_3)\\
&\times\left\{\frac{1}{2}X^{(2,1)}_{D,AAD}(-p,q_1,q_2,q_3)^{abcd}_{\mu\nu}A^b_\mu(t,q_1)A^c_\nu(t,q_2)D^d(t,q_3)\right.\\
&\left.+\ \frac{1}{2}X^{(2,1)}_{D,\lambda\lambda A}(-p,q_1,q_2,q_3)^{abcd}_\mu\lambda^b(t,q_1)\lambda^c(t,q_2)A^d_\mu(t,q_3)\right\},
\end{aligned}\tag{D5}$$

$$\begin{aligned}
R_{\varphi_\pm,i}(t,p) = &\int_{q,r}(2\pi)^D\delta(-p+q+r)\\
&\times\left\{X^{(1,1)}_{\varphi,A\varphi}(-p,q,r)^a_{\mu ij}A^a_\mu(t,q)\varphi_{\pm,j}(t,r)+X^{(1,1)}_{\varphi,\lambda\psi}(-p,q,r)^a_{ij}\lambda^a(t,q)\psi_j(t,r)\right\}\\
&+\int_{q_1,q_2,q_3}(2\pi)^D\delta(-p+q_1+q_2+q_3)\\
&\times\frac{1}{2}X^{(2,1)}_{\varphi,AA\varphi}(-p,q_1,q_2,q_3)^{ab}_{\mu\nu ij}A^a_\mu(t,q_1)A^b_\nu(t,q_2)\varphi_{\pm,j}(t,q_3),
\end{aligned}\tag{D6}$$

$$\begin{aligned}
\bar{R}_{\varphi^\dagger_\pm,i}(t,p) = &\int_{q,r}(2\pi)^D\delta(-p+q+r)\\
&\times\left\{X^{(1,1)}_{\varphi^\dagger,\varphi^\dagger A}(-p,q,r)^{aij}_\mu\varphi^\dagger_{\pm,j}(t,r)A^a_\mu(t,q)+X^{(1,1)}_{\varphi^\dagger,\bar{\psi}\lambda}(-p,q,r)^a_{ij}\bar{\psi}_j(t,r)\lambda^a(t,q)\right\}\\
&+\int_{q_1,q_2,q_3}(2\pi)^D\delta(-p+q_1+q_2+q_3)\\
&\times\frac{1}{2}X^{(2,1)}_{\varphi^\dagger,\varphi^\dagger AA}(-p,q_1,q_2,q_3)^{ab}_{\mu\nu ij}\varphi^\dagger_{j\pm}(t,q_3)A^a_\mu(t,q_1)A^b_\nu(t,q_2),
\end{aligned}\tag{D7}$$

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$$
\begin{aligned}
R_{\psi,i}(t,p) &= \int_{q,r} (2\pi)^D \delta(-p+q+r) \\
&\times \Big\{ X^{(1,1)}_{\psi,A\psi}(-p,q,r)^a_{\mu ij} A^a_\mu(t,q)\psi_j(t,r) + X^{(1,1)}_{\psi,\lambda\varphi}(-p,q,r)^a_{ij}\lambda^a(t,q)\varphi_{\pm,j}(t,r) \\
&+ X^{(1,1)}_{\psi,\lambda G}(-p,q,r)^a_{ij}\lambda^a(t,q)G_{\pm,j}(t,r) + X^{(1,1)}_{\psi,D\psi}(-p,q,r)^a_{ij}D^a(t,q)\psi_j(t,r)\Big\} \\
&+ \int_{q_1,q_2,q_3} (2\pi)^D \delta(-p+q_1+q_2+q_3) \\
&\times \Big\{ \frac{1}{2} X^{(2,1)}_{\psi,AA\psi}(-p,q_1,q_2,q_3)^{ab}_{\mu\nu ij} A^a_\mu(t,q_1)A^b_\nu(t,q_2)\psi_j(t,q_3) \\
&+ X^{(1,2)}_{\psi,\lambda A\varphi}(-p,q_1,q_2,q_3)^{ab}_{\mu ij}\lambda^a_j(t,q_1)A^b_\mu(t,q_2)\varphi_\pm(t,q_3)\Big\}, \qquad \text{(D8)}
\end{aligned}
$$

$$
\begin{aligned}
\bar{R}_{\bar\psi,i}(t,p) &= \int_{q,r} (2\pi)^D \delta(-p+q+r) \\
&\times \Big\{ X^{(1,1)}_{\bar\psi,A\bar\psi}(-p,q,r)^a_{\mu ij} A^a_\mu(t,q)\bar\psi_j(t,r) + X^{(1,1)}_{\bar\psi,\varphi^\dagger\lambda}(-p,q,r)^a_{ij}\varphi^\dagger_{\pm,j}(t,r)\lambda^a(t,q) \\
&+ X^{(1,1)}_{\bar\psi,G^\dagger\lambda}(-p,q,r)^a_{ij}G^\dagger_{\pm,j}(t,r)\lambda^a(t,q) + X^{(1,1)}_{\bar\psi,D\bar\psi}(-p,q,r)^a_{ij}\bar\psi_j(t,r)D^a(t,q)\Big\} \\
&+ \int_{q_1,q_2,q_3} (2\pi)^D \delta(-p+q_1+q_2+q_3) \\
&\times \Big\{ \frac{1}{2} X^{(2,1)}_{\bar\psi,AA\bar\psi}(-p,q_1,q_2,q_3)^{ab}_{\mu\nu ij} A^a_\mu(t,q_1)A^b_\nu(t,q_2)\bar\psi_j(t,q_3) \\
&+ X^{(1,2)}_{\bar\psi,\lambda A\varphi}(-p,q_1,q_2,q_3)^{ab}_{\mu ij}\lambda^a(t,q_1)A^b_\mu(t,q_2)\varphi_{\pm,j}(t,q_3)\Big\}, \qquad \text{(D9)}
\end{aligned}
$$

$$
\begin{aligned}
R_{G_\pm,i}(t,p) &= \int_{q,r} (2\pi)^D \delta(-p+q+r) \\
&\times \Big\{ X^{(1,1)}_{G,AG}(-p,q,r)^a_{\mu ij} A^a_\mu(t,q)G_{\pm,j}(t,r) + X^{(1,1)}_{G,DG}(-p,q,r)^a_{ij}D^a(t,q)G_{\pm,j}(t,r) \\
&+ X^{(1,1)}_{G,\lambda\psi}(-p,q,r)^a_{ij}\lambda^a(t,q)\psi_j(t,r)\Big\} \\
&+ \int_{q_1,q_2,q_3} (2\pi)^D \delta(-p+q_1+q_2+q_3) \\
&\times \Big\{ \frac{1}{2} X^{(2,1)}_{G,AAG}(-p,q_1,q_2,q_3)^{ab}_{\mu\nu ij} A^a_\mu(t,q_1)A^b_\nu(t,q_2)G_{\pm,j}(t,q_3) \\
&+ X^{(1,2)}_{G,\lambda A\psi}(-p,q_1,q_2,q_3)^{ab}_{\mu ij}\lambda^a(t,q_1)A^b_\mu(t,q_2)\psi_j(t,q_3) \\
&+ \frac{1}{2} X^{(2,1)}_{G,\lambda\lambda\varphi}(-p,q_1,q_2,q_3)^{ab}_{ij}\lambda^a(t,q_1)\lambda^b(t,q_2)\varphi_{\pm,j}(t,q_3)\Big\}, \qquad \text{(D10)}
\end{aligned}
$$

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$$
\begin{aligned}
\bar{R}_{G^\dagger_\pm,i}(t,p) = &\int_{q,r} (2\pi)^D \delta(-p+q+r) \\
&\times \Big\{ X^{(1,1)}_{G^\dagger,AG^\dagger}(-p,q,r)^a_{\mu ij} G^\dagger_{\pm,j}(t,r) A^a_\mu(t,q) + X^{(1,1)}_{G^\dagger,DG^\dagger}(-p,q,r)^a_{ij} G^\dagger_{\pm,j}(t,r) D^a(t,q) \\
&+ X^{(1,1)}_{G^\dagger,\bar{\psi}\lambda}(-p,q,r)^a_{ij} \bar{\psi}_j(t,r) \lambda^a(t,q) \Big\} \\
&+ \int_{q_1,q_2,q_3} (2\pi)^D \delta(-p+q_1+q_2+q_3) \\
&\times \Big\{ \frac{1}{2} X^{(2,1)}_{G^\dagger,G^\dagger AA}(-p,q_1,q_2,q_3)^{ab}_{\mu\nu ij} G^\dagger_{\pm,j}(t,q_3) A^a_\mu(t,q_1) A^b_\nu(t,q_2) \\
&+ X^{(1,2)}_{G^\dagger,\bar{\psi}A\lambda}(-p,q_1,q_2,q_3)^{ab}_{\mu ij} \bar{\psi}_j(t,q_3) A^b_\mu(t,q_2) \lambda^a(t,q_1) \\
&+ \frac{1}{2} X^{(2,1)}_{G^\dagger,\varphi^\dagger\lambda\lambda}(-p,q_1,q_2,q_3)^{ab}_{ij} \varphi^\dagger_{\pm,j}(t,q_3) \lambda^b(t,q_2) \lambda^a(t,q_1) \Big\},
\end{aligned}
\tag{D11}
$$

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where flow vertices are

$$
\begin{aligned}
X^{(2,0)}_{A,AA}(p,q,r)^{abc}_{\mu\nu\rho} &= -if^{abc}\{(q-r)_\mu\delta_{\nu\rho} - 2q_\rho\delta_{\mu\nu} + 2r_\nu\delta_{\mu\rho} + (\alpha_0-1)(r_\rho\delta_{\mu\nu} - q_\nu\delta_{\mu\rho})\},\\
X^{(2,0)}_{A,\lambda\lambda}(p,q,r)^{abc}_{\mu} &= -f^{abc}(C^{-1}\gamma_\mu),\\
X^{(3,0)}_{A,AAA}(p,q_1,q_2,q_3)^{ab_1b_2b_3}_{\mu\nu_1\nu_2\nu_3} &= f^{ab_1c}f^{b_2b_3c}(\delta_{\nu_1\nu_2}\delta_{\mu\nu_3} - \delta_{\nu_1\nu_3}\delta_{\mu\nu_2})\\
&\quad + f^{ab_2c}f^{b_3b_1c}(\delta_{\nu_2\nu_3}\delta_{\mu\nu_1} - \delta_{\nu_2\nu_1}\delta_{\mu\nu_3}) + f^{ab_3c}f^{b_1b_2c}(\delta_{\nu_3\nu_1}\delta_{\mu\nu_2} - \delta_{\nu_3\nu_2}\delta_{\mu\nu_1}),\\
X^{(1,1)}_{\lambda,\lambda A}(p,q,r)^{abc}_{\mu} &= if^{abc}\left\{2q_\mu + (1-\alpha_0)r_\mu + \frac{1}{2}(\not{r}\gamma_\mu - \gamma_\mu\not{r})\right\},\\
X^{(1,1)}_{\lambda,\lambda D}(p,q,r)^{abc} &= f^{abc}\gamma_5,\\
X^{(1,2)}_{\lambda,\lambda AA}(p,q_1,q_2,q_3)^{abcd}_{\mu\nu} &= -2f^{ace}f^{bde}\delta_{\mu\nu} - f^{abe}f^{cde}\gamma_\mu\gamma_\nu,\\
X^{(1,1)}_{D,AD}(p,q,r)^{abc}_{\mu} &= -if^{abc}\{2r_\mu + (1-\alpha_0)q_\mu\},\\
X^{(2,0)}_{D,\lambda\lambda}(p,q,r)^{abc} &= if^{abc}C^{-1}\gamma_5(\not{q} - \not{r}),\\
X^{(2,1)}_{D,AAD}(p,q_1,q_2,q_3)^{abcd}_{\mu\nu} &= (f^{abe}f^{cde} + f^{ace}f^{bde})\delta_{\mu\nu},\\
X^{(2,1)}_{D,\lambda\lambda A}(p,q_1,q_2,q_3)^{abcd}_{\mu} &= -(f^{abe}f^{cde} + f^{ace}f^{bde})C^{-1}\gamma_5\gamma_\mu,\\
X^{(1,1)}_{\varphi,A\varphi}(p,q,r)^{a}_{\mu ij} &= -2r_\mu T^a_{ij} - (1-\alpha_0)q_\mu T^a_{ij},\\
X^{(1,1)}_{\varphi,\lambda\psi}(p,q,r)^{a}_{ij} &= i\sqrt{2}T^a_{ij}C^{-1}P_\pm,\\
X^{(2,1)}_{\varphi,AA\varphi}(p,q_1,q_2,q_3)^{ab}_{\mu\nu ij} &= -\delta_{\mu\nu}\{T^a,T^b\}_{ij},\\
X^{(1,1)}_{\varphi^\dagger,\varphi^\dagger A}(p,q,r)^{a}_{\mu ij} &= -2r_\mu T^a_{ji} + (1-\alpha_0)q_\mu T^a_{ji},\\
X^{(1,1)}_{\varphi^\dagger,\bar\psi\lambda}(p,q,r)^{a}_{ij} &= -i\sqrt{2}T^a_{ji}P_\mp,\\
X^{(1,2)}_{\varphi^\dagger,\varphi^\dagger AA}(p,q_1,q_2,q_3)^{ab}_{\mu\nu ij} &= -\delta_{\mu\nu}\{T^a,T^b\}_{ji},\\
X^{(2,0)}_{\psi,A\psi}(p,q,r)^{aij}_{\mu ij} &= -\left\{2r_\mu + (1-\alpha_0)q_\mu + \frac{1}{2}(\not{q}\gamma_\mu - \gamma_\mu\not{q})\right\}T^a_{ij},\\
X^{(1,1)}_{\psi,\lambda\varphi}(p,q,r)^{a}_{ij} &= -\sqrt{2}P_\pm\not{t}T^a_{ij},\\
X^{(1,1)}_{\psi,\lambda G}(p,q,r)^{a}_{ij} &= -\sqrt{2}P_\pm T^a_{ij},\\
X^{(1,1)}_{\psi,D\psi}(p,q,r)^{a}_{ij} &= -i\gamma_5 T^a_{ij},\\
X^{(2,1)}_{\psi,AA\psi}(p,q_1,q_2,q_3)^{ab}_{\mu\nu ij} &= -\delta_{\mu\nu}\{T^a,T^b\}_{ij} - \gamma_\mu\gamma_\nu[T^a,T^b]_{ij},\\
X^{(1,2)}_{\psi,\lambda A\varphi}(p,q_1,q_2,q_3)^{ab}_{\mu ij} &= -\sqrt{2}P_\pm\gamma_\mu(T^aT^b)_{ij},
\end{aligned}
\tag{D12}
$$

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$$X^{(1,1)}_{\bar{\psi},A\bar{\psi}}(p,q,r)^a_{\mu ij} = -\left\{2r_\mu - (1-\alpha_0)q_\mu + \frac{1}{2}(\not{q}\gamma_\mu - \gamma_\mu\not{q})\right\} T^a_{ji},$$
$$X^{(1,1)}_{\bar{\psi},\varphi^\dagger\lambda}(p,q,r)^a_{ij} = \sqrt{2}C^{-1}\not{r}P_\mp T^a_{ji},$$
$$X^{(1,1)}_{\bar{\psi},G^\dagger\lambda}(p,q,r)^a_{ij} = \sqrt{2}C^{-1}P_\mp T^a_{ji},$$
$$X^{(1,1)}_{\bar{\psi},\bar{\psi}D}(p,q,r)^a_{ij} = i\gamma_5 T^a_{ji},$$
$$X^{(2,1)}_{\bar{\psi},AA\bar{\psi}}(p,q_1,q_2,q_3)^{ab}_{\mu\nu ij} = -\left[\delta_{\mu\nu}\{T^a,T^b\}_{ji} - \gamma_\nu\gamma_\mu[T^a,T^b]_{ji}\right],$$
$$X^{(1,2)}_{\bar{\psi},\lambda A\varphi}(p,q_1,q_2,q_3)^{ab}_{\mu ij} = \sqrt{2}C^{-1}\gamma_\mu P_\mp(T^bT^a)_{ji},$$
$$X^{(1,1)}_{G,AG}(p,q,r)^a_{\mu ij} = -\{2r_\mu + (1-\alpha_0)q_\mu\}T^a_{ij},$$
$$X^{(1,1)}_{G,DG}(p,q,r)^a_{ij} = \mp 2iT^a_{ij},$$
$$X^{(1,1)}_{G,\lambda\psi}(p,q,r)^a_{ij} = -i\sqrt{2}C^{-1}(\not{q}-\not{r})P_\pm T^a_{ij},$$
$$X^{(2,1)}_{G,AAG}(p,q_1,q_2,q_3)^{ab}_{\mu\nu ij} = -\{T^a,T^b\}_{ij}\delta_{\mu\nu},$$
$$X^{(1,2)}_{G,\lambda A\psi}(p,q_1,q_2,q_3)^{ab}_{\mu ij} = i\sqrt{2}(T^aT^b + [T^a,T^b])_{ij}C^{-1}\gamma_\mu P_\pm,$$
$$X^{(2,1)}_{G,\lambda\lambda\varphi}(p,q_1,q_2,q_3)^{ab}_{ij} = 2i\{T^a,T^b\}_{ij}C^{-1}P_\mp,$$
$$X^{(1,1)}_{G^\dagger,G^\dagger A}(p,q,r)^a_{\mu ij} = -\{2r_\mu - (1-\alpha_0)q_\mu\}T^a_{ji},$$
$$X^{(1,1)}_{G^\dagger,G^\dagger D}(p,q,r)^{aij} = \mp 2iT^a_{ji},$$
$$X^{(1,1)}_{G^\dagger,\bar{\psi}\lambda}(p,q,r)^a_{ij} = -i\sqrt{2}(\not{q}+\not{r})P_\pm T^a_{ji},$$
$$X^{(2,0)}_{G^\dagger,G^\dagger AA}(p,q_1,q_2,q_3)^{ab}_{\mu\nu ij} = -\{T^a,T^b\}_{ji}\delta_{\mu\nu},$$
$$X^{(1,2)}_{G^\dagger,\bar{\psi}A\lambda}(p,q_1,q_2,q_3)^{ab}_{\mu ij} = -i\sqrt{2}(T^bT^a - [T^b,T^a])_{ji}\gamma_\mu P_\pm,$$
$$X^{(2,1)}_{G^\dagger,\varphi^\dagger\lambda\lambda}(p,q_1,q_2,q_3)^{ab}_{ij} = 2i\{T^b,T^a\}_{ji}C^{-1}P_\pm.$$

Appendix E. Dimensional regularization

We summarize useful formulas of the dimensional regularization. In the perturbative calculation with the dimensional regularization, we encounter the following integral,

$$I_{\beta,n}\left(p^2,m_1^2,m_2^2\right) \equiv \int_0^1 \mathrm{d}x x^n g_\beta(\Delta), \tag{E1}$$

where

$$g_\beta(\Delta) \equiv \frac{1}{(4\pi)^{D/2}}\frac{\Gamma(2-D/2-\beta)}{\Delta^{2-D/2-\beta}}, \quad \Delta\left(p^2,m_1^2,m_2^2\right) = p^2x(1-x) + m_1^2(1-x) + m_2^2x. \tag{E2}$$

The integral $I_{\beta,n}$ satisfies the recurrence relations

$$nI_{\beta,n-1} - 2aI_{\beta-1,n+1} - bI_{\beta-1,n} - g_\beta(a+b+c) + \delta_{n0}g_\beta(c) = 0 \qquad (n \geq 0), \tag{E3}$$

$$\left(2-\frac{D}{2}-\beta\right)I_{\beta,n-1} - aI_{\beta-1,n+1} - bI_{\beta-1,n} - cI_{\beta-1,n-1} = 0 \qquad (n \geq 1), \tag{E4}$$

where $a = -p^2,\ b = p^2 - m_1^2 + m_2^2,\ c = m_1^2$.

The formulas of dimensional regularization are summarized as follows: for $\alpha, \beta = 1, 2$,

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$$\int_q \frac{1}{\left(q^2+m_1^2\right)^\alpha \left((q-p)^2+m_2^2\right)^\beta} = K_0^{\alpha\beta}\left(p^2, m_1^2, m_2^2\right), \tag{E5}$$

$$\int_q \frac{q_\mu}{\left(q^2+m_1^2\right)^\alpha \left((q-p)^2+m_2^2\right)^\beta} = p_\mu K_1^{\alpha\beta}\left(p^2, m_1^2, m_2^2\right), \tag{E6}$$

$$\int_q \frac{q_\mu q_\nu}{\left(q^2+m_1^2\right)^\alpha \left((q-p)^2+m_2^2\right)^\beta} = p_\mu p_\nu K_2^{\alpha\beta}\left(p^2, m_1^2, m_2^2\right) + \frac{1}{2}\delta_{\mu\nu} L_0^{\alpha\beta}\left(p^2, m_1^2, m_2^2\right), \tag{E7}$$

$$\int_q \frac{q_\mu q_\nu q_\rho}{\left(q^2+m_1^2\right)^\alpha \left((q-p)^2+m_2^2\right)^\beta} = p_\mu p_\nu p_\rho K_3^{\alpha\beta}\left(p^2, m_1^2, m_2^2\right) + \frac{1}{2}\delta_{(\mu\nu} p_{\rho)} L_1^{\alpha\beta}\left(p^2, m_1^2, m_2^2\right), \tag{E8}$$

$$\int_q \frac{q_\mu q_\nu q_\rho q_\sigma}{\left(q^2+m_1^2\right)^\alpha \left((q-p)^2+m_2^2\right)^\beta} = p_\mu p_\nu p_\rho p_\sigma K_4^{\alpha\beta}\left(p^2, m_1^2, m_2^2\right) + \frac{1}{2}\delta_{(\mu\nu} p_\rho p_{\sigma)} L_2^{\alpha\beta}\left(p^2, m_1^2, m_2^2\right)$$
$$+ \frac{1}{4}\delta_{(\mu\nu}\delta_{\rho\sigma)} M^{\alpha\beta}\left(p^2, m_1^2, m_2^2\right), \tag{E9}$$

where

$$\delta_{(\mu\nu} k_{\rho)} = \delta_{\mu\nu} k_\rho + \delta_{\nu\rho} k_\mu + \delta_{\rho\mu} k_\nu, \tag{E10}$$

$$\delta_{(\mu\nu} k_\rho k_{\sigma)} = \delta_{\mu\nu} k_\rho k_\sigma + \delta_{\nu\rho} k_\mu k_\sigma + \delta_{\rho\mu} k_\nu k_\sigma + \delta_{\nu\sigma} k_\mu k_\rho + \delta_{\sigma\mu} k_\nu k_\rho + \delta_{\rho\sigma} k_\mu k_\nu, \tag{E11}$$

$$\delta_{(\mu\nu}\delta_{\rho\sigma)} = \delta_{\mu\nu}\delta_{\rho\sigma} + \delta_{\mu\rho}\delta_{\nu\sigma} + \delta_{\mu\sigma}\delta_{\nu\rho}. \tag{E12}$$

Here K, L, M are defined by

$$K_m^{\alpha\beta} = \int_0^1 \mathrm{d}x (1-x)^{\alpha-1} x^{\beta-1+m} g_{2-\alpha-\beta}(\Delta), \tag{E13}$$

$$L_n^{\alpha\beta} = \int_0^1 \mathrm{d}x (1-x)^{\alpha-1} x^{\beta-1+n} g_{3-\alpha-\beta}(\Delta), \tag{E14}$$

$$M^{\alpha\beta} = \int_0^1 \mathrm{d}x (1-x)^{\alpha-1} x^{\beta-1} g_{4-\alpha-\beta}(\Delta), \tag{E15}$$

where $m = 0, 1, \ldots, 4$ and $n = 0, 1, 2$.

In order to pick up the UV divergences quickly, we denote a divergent part of X as $[X]$. For example, since

$$g_0(\Delta) = \frac{1}{(4\pi)^2}\left(\frac{4\pi}{\Delta}\right)^\epsilon \Gamma(\epsilon) = \frac{1}{(4\pi)^2}\left(\frac{1}{\epsilon} + \mathcal{O}(1)\right), \tag{E16}$$

we have $[g_0(\Delta)] = \frac{1}{(4\pi)^2}\frac{1}{\epsilon}$ and

$$[I_{0,n}] = \frac{1}{n+1}\frac{1}{(4\pi)^2}\frac{1}{\epsilon}. \tag{E17}$$

Using the recurrence relations (E.3), (E.4), and (E.17), we can easily count the divergence of each graph.

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