What if the positron never existed ?
What if the antielectron is a negative mass electron ?

Daniel M Dubois[1,2] **and Gilles L Nibart**[3]

[1] Centre for Hyperincursion and Anticipation in Ordered Systems (CHAOS), Institute of Mathematics B37, Sart Tilman, BE-4000 LIEGE, Belgium
[2] Honorary Professor at HEC.LIEGE, Management School, Liège Université, 14 rue Louvrex, 4000 Liège, Belgium. Web site http://www.sia.hec.ulg.ac.be

E-mails: daniel.dubois@ulg.ac.be, ddubois.chaos@gmail.com

[3] Laboratoire de Physique Théorique Fondamentale en PACA,
5 Le Pountis, 06140 Coursegoules, France

E-mails: gilles.nibart@sfr.fr, lab.phys.theo@club-internet.fr

Abstract. Although the question of negative energy solutions of the Dirac equation did not receive any relevant answer, it has been incorrectly said that the Dirac Sea Hole theory predicted the existence of the positron. Now several authors have considered the possible existence of negative mass particles. us We show that the concept and the definition of the antielectron should be re-examined, and should be understand in a way which is fully compatible with Dirac negative energy states of electrons, i.e. with negative masses. The symmetry between positive and negative energy states in the Dirac equation leads to add the mass inversion M as a new symmetry to the CPT group, giving the MCPT group. We discuss the completeness of the Wigner theorem and the definitions of time reversal. Furthermore negative masses have been recently applied to Cosmology.

1. Introduction

While the solution of any differential equation of Classical Mechanics defines a trajectory as a whole, the trajectory of any particle can be conceived as successive positions **r** separated by 3D space intervals Δ**r** or **λ**, in successive times t separated by discrete time interval Δt and then the trajectory is build with a computational method using both forward and backward discrete time derivatives:

$$\frac{\Delta_f F}{\Delta t}=\frac{F(t+\Delta t)-F(t)}{\Delta t} \quad \text{and} \quad \frac{\Delta_b F}{\Delta t}=\frac{F(t)-F(t-\Delta t)}{\Delta t} \tag{1}$$

and similarly for space derivatives. In such a work [13] Daniel M. Dubois defined a weighted sum of these derivatives:

$$\Delta_w = w\Delta_f F(t)+(1-w)\Delta_b F(t) \tag{2}$$

he deduced from it a complex discrete derivative with the following weight:

$$w = (1 \pm i)/2 \tag{3}$$

and built generalized complex continuous derivatives. Applying this method to the incursive and the hyperincursive discrete oscillators he deduced a Schrödinger equation in a 1-dimensional space (equation (82b) in reference [13]):

$$-\delta i \hbar \frac{\partial \phi(x,t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \phi(x,t)}{\partial x^2} \tag{4}$$

where δ = ± 1. So we should use the double Schrödinger equation with ± i as shown below:

$$-i\hbar \frac{\partial \phi_1(x,t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \phi_1(x,t)}{\partial x^2} \tag{5a}$$

$$+i\hbar \frac{\partial \phi_2(x,t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \phi_2(x,t)}{\partial x^2} \tag{5b}$$

The major consequence is that we don't need to do the complex conjugation when reversing time.

In a next work Daniel M. Dubois [14] generalized the equations of Schrödinger and Klein-Gordon to 3 space dimensions. He also introduced the electromagnetic field into the Klein-Gordon equation.

In an other work [15], starting from two incursive discrete harmonic oscillators, Daniel M. Dubois showed that there is a rotation of the incursive discrete oscillators to recursive discrete oscillators. Then he built the hyperincursive discrete Klein-Gordon equation and shows that it bifurcates to the relativist quantum Majorana equation and the Dirac equation.

In a previous paper [37] Gilles L. Nibart and Daniel M. Dubois have shown that computational discrete derivative equations lead to negative mass, studied the problem of negative energy states which are related to a negative rest mass or a negative frequency, and they have predicted gravitational properties of negative mass particles.

We recall that the well known equation of Klein [26], [27], Gordon [21], Fock [18] and Kudar [30] have positive and negative energy solutions of the same absolute value:

$$E^{(+)} = +\sqrt{\boldsymbol{P}^2 + m^2} \quad \text{and} \quad E^{(-)} = -\sqrt{\boldsymbol{P}^2 + m^2} \tag{6}$$

so there is a perfect symmetry between negative energy states and positive energy states.
Unfortunately in the Kein-Gordon equation the presence probability density [36] is not defined positive:

$$\rho(\boldsymbol{r}, t) = \frac{i}{2m} \left[\Psi^* \frac{\partial \Psi}{\partial t} - \Psi \frac{\partial \Psi^*}{\partial t} \right] \tag{7}$$

As some authors think that mass and energy must always be positive, we recall that it is not a law of Physics: the energy is not defined positive. A good example is the atom where energy levels E_n of any electron on a stationary orbital are negative, as shown in the well known law:

$$E_n = -\frac{k}{n^2} \tag{8}$$

where k depends on the atomic number Z, the electron mass m_e and electric charge e, as:

$$k = \frac{Z^2 e^4 m_e}{8 h^2 \epsilon_0^2} \tag{9}$$

2. Symmetrical energy states in the free Dirac equation

Any fermion (e.g. an electron) without electromagnetic potentials is ruled by the free Dirac equation which was written in its original version as:

$$\left(i\hbar\frac{\partial}{\partial t}-c\,\boldsymbol{\alpha}.\,\boldsymbol{p}-c^2\beta m\right)\Psi=0 \tag{10}$$

or in its covariant version as:

$$\left(i\hbar\gamma^{\mu}\frac{\partial}{\partial x_{\mu}}-cm\right)\Psi=0 \tag{11}$$

with the standard notation of $\boldsymbol{\alpha}$, β, γ^{μ} matrix, and m the rest mass.

Firstly we must remark that the electric charge e does not appear in these equations (10) and (11). We must also remember that the 1928 Dirac article [10] did not predict the existence of a new particle, such as the positron, and he wrote "*half the solutions must be rejected as referring to the charge +e on the electron*".

Secondly we must remark that the rest mass m in the Dirac equation (10) or (11) may be either positive or negative. As the matrix β, γ^0 and ρ_3 evaluate to:

$$\beta=\gamma^0=\rho_3=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix} \tag{12}$$

and have a unitary square:

$$\beta^2=\left(\gamma^0\right)^2=\left(\rho_3\right)^2=\mathbf{1} \tag{13}$$

their eigenvalues are:

$$\beta_m=\pm 1 \tag{14}$$

The eigenvalue $\beta_m = +1$ corresponds to a quantum state which is associated with a positive rest mass, and the eigenvalue $\beta_m = -1$ corresponds to a quantum state which is associated with a negative rest mass. In the Dirac representation of the covariant equation (11) γ^0 is the time-like hermitian matrix, so it defines the sign of the energy with the same eigenvalues (14): the eigenvalue +1 corresponds to a positive energy and the eigenvalue −1 corresponds to a negative energy.

As the electric charge e does not appear in the equation (10) and (11), as the matrix β defines the sign of the rest mass in the Dirac equation (10) and γ^0 defines the sign of the energy in the equation (11), these equations does not contain anything which can define the sign of the electric charge.

The Dirac equation allows negative mass-energy solutions, as the Klein-Gordon equation does, with a perfect symmetry between negative energy states and positive energy states as shown in the figure 1 below. Therefore electrons can have quantum states with an equal probability for the negative or positive mass-energy of the same absolute value (6):

$$E^{(+)}=+\sqrt{\boldsymbol{P}^2+m^2}\quad\text{and}\quad E^{(-)}=-\sqrt{\boldsymbol{P}^2+m^2} \tag{15}$$

and the presence probability density is defined positive:

$$\rho(\boldsymbol{r},t)=\left(\Psi^{\dagger}\Psi\right) \tag{16}$$

where the exponent symbol † denotes the adjoint.

The negative energy states "were dismissed as *unphysical*, not recognized as being problems of relevance" [29] until Klein [28] showed that the Dirac theory allows transitions from positive energy states to negative energy states, and thus any electron should fall down deeper and deeper into lower energy levels, continuously emitting photons of energy

$$h\nu = E_{n+1} - E_n \tag{17}$$

as shown in figure 1 below.

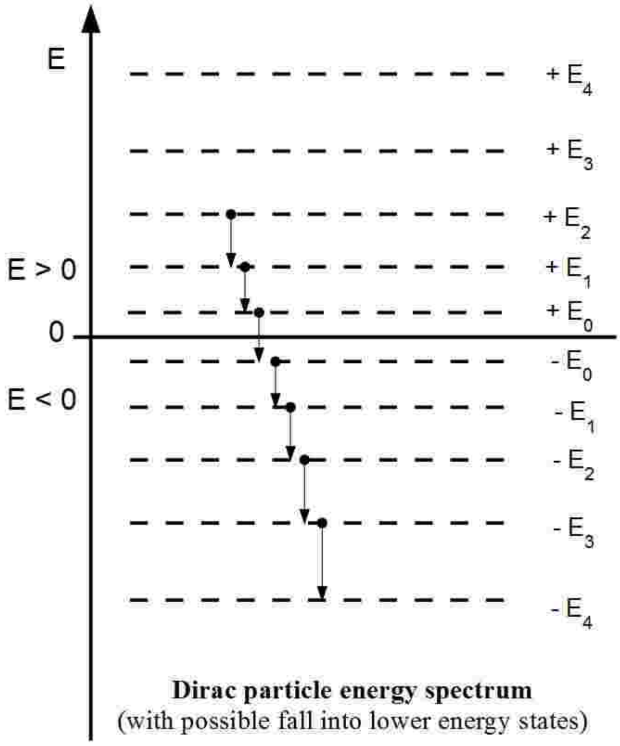

Figure 1

Moreover as we have explained the Dirac equation has introduced negative masses $-M$ because the matrix β has the two eigenvalues:

$$\beta_m = \pm 1 \tag{18}$$

which have to be related to positive and negative energies with:

$$E^{(+)} = +M\,c^2 \quad \text{and} \quad E^{(-)} = -M\,c^2 \tag{19}$$

where M is the relativist mass:

$$M = |m| \left(1 - v^2/c^2\right)^{1/2} \tag{20}$$

Consequently Dirac was much puzzling with negative energy solutions of his equation, and then he proposed [11] what Helge Kragh called "*a rather speculative theory about an infinite sea of electrons in negative energy state*s, only exceptionally interrupted by unoccupied states or holes" [29].

For Ettore Majorana (and some other authors at that time) "*it was an embarrassing fact that the original Dirac theory of the electron* [10] *introduced negative masses*" (citation from Fradkin [19]).

Then Ettore Majorana [32] proposed, as soon as 1932, a modified version of Dirac's equation which is relativistically invariant and which (a) avoids negative masses, (b) is a theory of arbitrary integer or half integer spin particles, (c) provides a mass spectrum for elementary particles, as explained by Fradkin [19]. In a later paper [33] starting from a Lagrange function, Majorana developed a field theory where the electron is represented by a complex field which is a combination of Hermitian and anti-Hermitian components.

Apart from the problem of negative energy states, the 1928 Dirac paper [10] has raised an other question which was forgotten. Starting from the Dirac equation with electromagnetic potentials:

$$\left[p_0+\frac{e}{c}A_0+\rho_1\left(\boldsymbol{\sigma},\boldsymbol{p}+\frac{e}{c}\right)+\rho_3 m c\right]\psi=0 \tag{21}$$

(with the standard notation of $\boldsymbol{\sigma}$, ρ_1, ρ_3 matrix) and applying a multiplication procedure he obtained a quadratic equation which is nearly similar to the Klein-Gordon equation but differs from it by the two extra terms:

$$\frac{eh}{c}(\boldsymbol{\sigma},\boldsymbol{H})+i\frac{eh}{c}(\boldsymbol{\sigma},\boldsymbol{E}) \tag{22}$$

Consequently the electron of Dirac's theory appears to have the magnetic moment

$$\mathbf{M_H}=\frac{eh}{2mc}\boldsymbol{\sigma} \tag{23}$$

which “is just that assumed in the spinning electron model”, but in equation (22) the Dirac electron also appears to have an electric moment which is purely imaginary:

$$\mathbf{M_E}=i\frac{eh}{2mc}\rho_1\boldsymbol{\sigma} \tag{24}$$

Dirac dismissed the electric moment as unphysical: he wrote “The electric moment, being a pure imaginary, we should not expect to appear in the model. *It is doubtful whether the electric moment has any physical meaning*”. The question of the electric moment should be raised gain after A. Proca [41] defined an electromagnetic moment for the electron from a specific Lagrangian function.

3. Is the Dirac sea hole theory consistent ?

The Dirac Sea is a theoretical model of vacuum. Although the Dirac equation is the theory of a single electron, the Dirac sea is a theory of N electrons so it may be considered as a quantum field theory. The Dirac a theory is thus inelegant as it breaks the symmetry between positive energy states which are accepted solutions of the Dirac equation and negative energy states which belong to vacuum.

Does the Dirac Sea make sense as a bare vacuum filled with an infinite negative charge density and an infinite negative energy ?

Let's apply the free Dirac equation (10) or (11) to the electron with its mass $m = m_e$ and also to the proton with its mass $m = m_p$, we may then consider the vacuum filled with both negative and positive charges so that the electric density is finite, but nevertheless the energy density remains infinite. This recall the problem of divergent terms in the quantum field theory.

To prevent electrons from falling down into negative energy states, Dirac put the Pauli exclusion principle forward, but this principle cannot work as vacuum is not an atom nor a cristal composed of atoms. The condition for vacuum to be completely filled was not defined at all, therefore there is no way to say there is a hole in any free place. Moreover the Dirac Sea of infinite extent may always receive new electrons and never be filled: it always contain an undefined number of holes. Dan Solomon has shown that “there exist states with less energy than that of the vacuum” [48], [49], [50]. He has also shown [51] with the perturbation method that “the conjecture that the Pauli principle prevents the existence of quantum states with less energy than that of the unperturbed vacuum state is not correct”.

The Dirac Sea contains electrons of negative energy states, but their number N is undefined, and the number N' of holes is quite unknown. Such a theory cannot rigorously predict the existence of positively charged particles as the proton, or the positron.

As a conclusion, the Dirac Sea theory cannot prevent positive mass electrons from falling into negative energy states. More generally it is true for any fermion (including the positron).

4. Should negative energies and masses be excluded ?

Are negative energies and masses really unphysical and should they be excluded from the quantum theory of the electron, as Majorana tried to do it ?

Firstly, we have to remark that the answer has already been given by Dirac [10]: negative energy and masses must not be excluded.

So he explained (W denotes the energy): "One gets over the difficulty on the classical theory by arbitrarily excluding those solutions that have a negative W. One cannot do this on the quantum theory, since in general a perturbation will cause transitions from states with W positive to states with W negative".

Secondly, in the same 1928 paper [10] (before the 1930 idea of Dirac Sea) he considered that the electron in a negative state would behave exactly as the now known positron. We demonstrate in the section 12., that this conjecture is quite correct. So Dirac wrote: "*Such a transition* [from a positive state to a negative state] *would appear experimentally as the electron suddenly changing its charge from* $-e$ *to* e, *a phenomenon which has not been observed*" – at that time.

However some possible transitions of electric charge inversion

$$-e \rightarrow +e \tag{25}$$

have been reported by Mark Rosen [44], [45] but the experimental evidence has to be confirmed in the conditions mentioned in his paper in the section "Experimental Verification". According to the author such a transformation should require a very strong magnetic field or electric field.

Thirdly, in a 1938 paper [12] Dirac wrote: "*We have the theory of the positron ... in which positive and negative values for the mass of an electron play symmetrical roles*". Thus Dirac thought to associate the positron to a negative mass, i.e. a negative energy state.

From this point of view the quantum transition (25) should be interpreted as:

$$E^{(+)} \rightarrow E^{(-)} \text{ and } +m_e \rightarrow -m_e \tag{26}$$

Moreover negative states play a fundamental role in the Zitterbewegung of the trajectory of any free electron. Feshbach and Villars [17] have explained it: "The narrowest packet that can be built up of positive states alone has a width of order $\hbar/mc$. To construct a δ function, negative and positive states must contribute with equal weight. It follows that eigenstates of the position operator **x** that is δ function $\delta(\mathbf{x} - \mathbf{x}_0)$ necessarily contain positive and negative components". An analytical study followed by numerical simulations has been done by Sang Tae Park [38]. The Zitterbewegung is now understood to be an interference effect between positive and energy parts of Dirac's equation solution, and the oscillation is composed of several frequencies.

J.P. Terletsky [52] wrote very interesting remarks about negative mass particles. Starting from the special theory of Relativity, he relates the 4-energy momentum ***P*** of the Relativity to the energy-momentum tensor with the integral:

$$P_k = \int_\alpha T_{k\alpha} d\sigma_\alpha \tag{27}$$

he shows that T_{44} defines the proper mass in any referential such as $T_{k4} = 0$ for (k = 1, 2, 3). Then Terletsky demonstrates that for the Dirac equation the following expression

$$T_{44} = \frac{1}{2}\left[\psi^* \frac{\partial \psi}{\partial t} - \frac{\partial \psi^*}{\partial x_4}\right] \tag{28}$$

is not defined positive, so the proper mass can be either positive or negative. Terletski alleges without demonstration that usual detectors (Wilson chamber, ionisation chamber, Geiger counter, Tcherenkov counter, etc...) are not able to record negative energy particles. To confirm his allegation the problem should be analyzed in details for every detector. In the case of the Wilson chamber see section 12..

Now negative masses are more usually considered in Cosmology: see section 14..

5. Is the antielectron obviously a positron ?

Since the 1933 discovery of antielectrons in cosmic rays by Carl Anderson [2] and his interpretation as positively charged electrons, it has been decided that the antielectron is a spin 1/2 particle with the same positive mass as the electron and a positive electric charge of the same absolute value as the electron. Therefore this definition of the positron became a consensus in the scientific community. It has also been said that the Dirac Sea hole theory has predicted the existence of the positron. But the following question has never been asked:

Is this point of view compatible with Dirac conceptions and with the Dirac equation with or without electromagnetic potentials ?

The electric charge e does not appear in the free Dirac equations (1) and (11), so there is no symmetry between -e and +e. The charge e was introduced in the equation (21) as a constant, not as an operator, so there is no symmetry between -e and +e.

Clearly all Dirac equations have the following symmetry:

$$E^{(+)} \longleftrightarrow E^{(-)} \tag{29a}$$

$$+m \longleftrightarrow -m \tag{29b}$$

With the citations from Dirac writings (section 4) we know that Dirac's concept of positron refers to the negative energy states which are solutions of the Dirac equation.

Therefore the antielectron should be conceived as a spin 1/2 particle with the same electric charge as the electron and a negative mass of the same absolute value as the electron.

6. About the several time reversal operators in quantum theories

Usual time reversal operators were defined from the idea that quantum transitions probabilities, quantum states and canonical commutators should be invariant through time reversal. An other idea is that time reversal should reverse the spin of fermions.

These ideas are postulates which cannot be tested as we are not actually able to reverse the time arrow, and this postulate leads to the well known paradox of particles which run backward in time.

The pioneers of Quantum Mechanics have introduced a unitary time evolution operator $\boldsymbol{U}$ as:

$$\boldsymbol{U}(t, t_0) = e^{\frac{-i}{\hbar}\boldsymbol{H}(t-t_0)} \tag{30}$$

where $\boldsymbol{H}$ is a Hamiltonian and t_0 the initial time of a quantum experiment. Here we see that the unitary time reversal operator which is known as the Racah operator [42]:

$$\boldsymbol{T_0}:\ t \rightarrow -t \tag{31}$$

may switch or not the sign of the energy depending on the definition of the Hamiltonian $\boldsymbol{H}$ as we can see below:

$$\boldsymbol{U}(-t, t_0) = e^{\frac{-i}{\hbar}\boldsymbol{H}(-t-t_0)} \tag{32}$$

For the Schrödinger equation the usual time reversal operator is the anti-unitary operator

$$\boldsymbol{T}_W = \boldsymbol{T_0}\boldsymbol{K} \tag{33}$$

which was defined so that the time independent Schrödinger equation is invariant, but the time dependent Schrödinger equation may be or not invariant depending on the definition of the Hamiltonian $\boldsymbol{H}$, as we have shown above. R. Shankar has proposed an other time operator:

$$\boldsymbol{T}_{Sh} = \boldsymbol{U_1} K \quad \text{with} \quad \boldsymbol{U_1} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{34}$$

but he did not applied it to the Dirac equation.

For the Dirac equation the usual time reversal operator [36] is the anti-unitary operator

$$\boldsymbol{T_D} = \gamma_0 \boldsymbol{K} \tag{35}$$

but from considerations of Pauli matrix Bryan Roberts [43] deduced an other anti-unitary operator

$$\boldsymbol{T_R} = \sigma_2 \boldsymbol{K} \tag{36}$$

with "an additional term σ_2 needed to reverse the sign of the [Pauli] matrices that don't have imaginary components" and he wrote: "This immediately implies the famous result that, for a spin ½ particle, applying time reversal twice fails to bring you back to where you started, but rather results in a global change of phase".

Consequently the time reversal operators (33), (34), (35), (36) are not valid for spin 1, 3/2, 2, … particles. So there is unfortunately no general theory of time reversal for all the quantum equations.

7. Complex conjugation and time reversal

Quantum theory uses complex numbers. The imaginary unit is defined by the equation below:

$$z^2 + 1 = 0 \tag{37}$$

which has the two well known solutions:

$$\sqrt{-1} = \pm i \tag{38}$$

We know that the complex conjugation

$$\boldsymbol{K} : i \rightarrow -i \tag{39}$$

defines an automorphism on the complex set $\mathbb{C}$ which means that $+i$ and $-i$ are equivalent imaginary units. So the Correspondence Principle in the Schrödinger representation should be extended symmetrically with both correspondences below:

$$E \rightarrow +i\hbar \frac{\partial}{\partial t} \quad \text{and} \quad \boldsymbol{p} \rightarrow -i\hbar \nabla \tag{40a}$$

$$E \rightarrow -i\hbar \frac{\partial}{\partial t} \quad \text{and} \quad \boldsymbol{p} \rightarrow +i\hbar \nabla \tag{40b}$$

In his 1931 German book [55] Eugen P. Wigner wrote both correspondences (40a, 40b) but his remark was just to explain that he started using the sign convention (40b) instead of (38a). This remark was fully removed from Griffin's translation to English [22].

With the Hamiltonian $\boldsymbol{H}$ we should consider the two conjugate Schrödinger equations as in equations (5a, 5b):

$$-i\hbar \frac{\partial}{\partial t} \Psi_1 - \boldsymbol{H}(r,t)\Psi_1 = 0 \tag{41a}$$

$$+i\hbar \frac{\partial}{\partial t} \Psi_2 - \boldsymbol{H}(r,t)\Psi_2 = 0 \tag{41b}$$

and so there is no need to insert the complex conjugation within the time reversal operator.

On the contrary Ahmed Zafar [57] proposed his own time reversal definition as the complex conjugation $\boldsymbol{K}$:

$$\boldsymbol{T_{Zafar}} = \boldsymbol{K} :\ i \rightarrow -i \tag{42}$$

Journal of Physics: Conference Series **2197** (2022) 012006 doi:10.1088/1742-6596/2197/1/012006

This is quite a nonsense. The complex conjugation just permutes the two Schrödinger equations (41a, 41b), although the unitary time reversal operator

$$\boldsymbol{T}_0 : t \to -t \tag{43}$$

which is known as the Racah operator [42] gives the two new Schrödinger equations:

$$-i\hbar \frac{\partial}{\partial t}\Psi_1 - \boldsymbol{H}(r,-t)\Psi_1 = 0 \tag{44a}$$

$$+i\hbar \frac{\partial}{\partial t}\Psi_2 - \boldsymbol{H}(r,-t)\Psi_2 = 0 \tag{44b}$$

where the Hamiltonian $\boldsymbol{H}(r, t)$ is not necessary an even function of the time variable t.

8. About the Wigner theorem

In his 1931 work [55] Eugen Wigner laid the foundations of modern quantum mechanics with the group theory of transformations such as rotations, translations and several symmetries (spin, parity, momentum). According to the so-called "Wigner theorem", any transformation of quantum states which preserves the probabilities are done by operators which are either unitary or anti-unitary.

The Wigner theorem is not complete, as it is restricted to bijective transformations of only one quantum state in separable Hilbert spaces.

Moreover Wigner proof of his theorem was not complete, so several authors have later work to demonstrate the Wigner theorem. The first proof for the bijective case was given by Lomont and Mendelson [31]. Several authors have proposed new proofs but again in the same restricted case. They are referenced in Gehér paper [20] which gives an elementary proof of the theorem for non-bijective transformations in separable and non-separable Hilbert spaces. Amaury Mouchet [35] has proposed an alternative proof based on complex analysis which applies to quantum field theory where Hilbert space are not separable.

While the Wigner theorem and its new versions define the invariance of quantum transition probabilities, the Uhlhorn theorem [54] states a much higher requirement of quantum mechanics: by a symmetry transformation n orthogonal vectors are transformed into n orthogonal vectors, and thus n independent states are transformed into n independent states. In the year 2007 a review of Wigner's theorem generalizations was published by Georges Chevalier [8].

9. Wigner's or Racah's time reversal operator

In the late English translation [22] of Wigner work the chapter 26 "*Time Inversion*", which does not appear in the table of contents of the German version [55], follows the next Wigner work [56].

It starts from the idea that "*the time behavior is described by the second Schrödinger equation*":

$$\frac{\partial \phi}{\partial t} = -\frac{i}{\hbar}\boldsymbol{H}\,\phi \tag{45}$$

and thus it introduces the antilinear time reversal defined as:

$$\boldsymbol{T}_W = \boldsymbol{T}_0 \boldsymbol{K} \tag{46}$$

but from the Schrödinger equations (41a, 41b) and the time reversed equations (44a, 44b) transformed by $\boldsymbol{T}_0$ we can see that the time dependent Schrödinger equation is not invariant by the Wigner time reversal operator.

Bryan W. Roberts [43] claims that the definition (46) of the anti-unitary time reversal operator "is perfectly appropriate, and is indeed forced by basic considerations about the nature of time in the quantum formalism". Starting from the position-momentum commutator

$$[\boldsymbol{Q}, \boldsymbol{P}] = (\boldsymbol{Q}\boldsymbol{P} - \boldsymbol{P}\boldsymbol{Q}) \tag{47}$$

and the canonical commutation relation

$$[\boldsymbol{Q}, \boldsymbol{P}] = i\hbar \tag{48}$$

and applying a time reversal operator ***T*** on both sides of (48) with the definition (47):

$$\boldsymbol{T}(\boldsymbol{Q}\boldsymbol{P} - \boldsymbol{P}\boldsymbol{Q})\boldsymbol{T}^{-1} = i\hbar \tag{49}$$

and developing:

$$(\boldsymbol{T}\boldsymbol{Q}\boldsymbol{T}^{-1})(\boldsymbol{T}\boldsymbol{P}\boldsymbol{T}^{-1}) - (\boldsymbol{T}\boldsymbol{P}\boldsymbol{T}^{-1})(\boldsymbol{T}\boldsymbol{Q}\boldsymbol{T}^{-1}) = i\hbar \tag{50}$$

the author demonstrates that T "can only be anti-unitary" from the common idea that "time reversal preserves position while reversing momentum":

$$\begin{aligned} \boldsymbol{T}\boldsymbol{Q}\boldsymbol{T}^{-1} &= \boldsymbol{Q} \\ \boldsymbol{T}\boldsymbol{P}\boldsymbol{T}^{-1} &= -\boldsymbol{P} \end{aligned} \tag{51}$$

Precisely the postulate of reversing the momentum operator originates from the idea of playing movies backwards considered in the framework of Classical Mechanics:

$$\boldsymbol{T}: t \to -t \Rightarrow m\vec{V}'(-t) = -m\vec{V}(t) \tag{52}$$

C. Callender [3] claimed that in Quantum Mechanics there is "*no need to switch sign of momentum and spin under time reversal, as momentum is a spatial derivative*":

$$\boldsymbol{P}_x = -i\hbar\frac{\partial}{\partial x} \tag{53}$$

Furthermore Andrew Holster [25] has analyzed and compared the solutions of the Schrödinger equation transformed by the Racah and Wigner time reversal operators. He has then criticized with key objections the common arguments given in education books (including Messiah) and Sachs arguments which all are in favour of the Wigner operator (46); and he has concluded that the Racah operator (43) should always be used.

10. Negative mass and time reversal

The classical idea in equation (52) presupposes that any rest mass is a positive constant and that time reversal inverse the momentum, but this equation might be differently interpreted with the mass inversion. The inversion of the classical momentum does not match the momentum operator (53) which is a space derivative.

Let's consider the usual time dependency function of a mass particle

$$\varphi(t) = \varphi_0 e^{-\frac{i}{\hbar}mc^2t} \tag{54}$$

applying the unitary time reversal operator (43) gives:

$$\varphi(-t) = \varphi_0 e^{-\frac{i}{\hbar}mc^2(-t)} = \varphi_0 e^{-\frac{i}{\hbar}(-m)c^2t} \tag{55}$$

so we may consider that the time reversal switches the sign of the mass (instead of the time arrow)

$$\boldsymbol{T}: t \to -t \Rightarrow m' = -m \tag{56}$$

and switches the sign of the velocity:

$$\boldsymbol{T}: t \to -t \Rightarrow \vec{V}'(-t) = -\vec{V}(t) \tag{57}$$

Consequently the classical momentum $\vec{P}$ remains unchanged by time reversal as the operator ***P*** in equation (53):

$$\boldsymbol{T}: t \rightarrow -t \Rightarrow \vec{P}'(-t) = (-m)(-\vec{V}(t)) = m\vec{V}(t) \tag{58}$$

Evans Boney [5] thinks that antiparticles cannot have a backward time arrow and so they must have a negative mass. But as antiparticles appear to have a positive energy in annihilation/creation experiments, he has proposed to modify the principle of mass-energy equivalence as:

$$E = |m| c^2 \tag{59}$$

but it is not compatible with the theory of Relativity.

Nathalie Debergh, Jean-Pierre Petit and Gilles D'Agostini [9] have argued for the unitary time reversal operator (43) and they have shown from the Dirac equation that “negative energies are acceptable provided the masses are simultaneously negative”.

11. Mass inversion and the MCPT group

K.H. Tzou [53] has considered the general solution of the Dirac equation, expressed as $\Psi(\chi, e, \boldsymbol{r}, t)$ where χ is a mass term, and analyzed the transformed solution generated by the transformations C, P, T, CP, CT, PT, CPT, the Lorenz transformation L combined with the mass inversion M. So he has shown that the mass inversion does not generate any new quantum state nor any new solution of the Dirac equation as negative mass/energy are already solutions.

This study leads to define the MCPT transformation group.

Benoit Guay [23] has explained that the mass inversion “does not reverse the mass by acting directly on it”. We can be “acting with an electromagnetic potential upon a quantum state where the mass is $+m$ so that there is a non negligible probability that it it is changed into another quantum state where the mass is $-m$”. And reciprocally.

12. About Wilson chamber experiments

On the photographs with a Wilson chamber, electron and antielectron tracks appear to be arcs of circle. The circular trajectories are produced by a great uniform magnetic field $\boldsymbol{B}$. When a particle having the electric charge q enter the chamber with the velocity $\boldsymbol{V}$ the magnetic force is:

$$\boldsymbol{F_B} = q\,\boldsymbol{V} \wedge \boldsymbol{B} \tag{60}$$

As the system is built to make the magnetic field orthogonal to the initial velocity of particles:

$$\boldsymbol{B} \perp \boldsymbol{V} \tag{61}$$

the trajectory is circular with a centrifugal force $\boldsymbol{F}_C$ defined by:

$$\boldsymbol{F_C}(t) = -m\frac{\boldsymbol{V}^2}{r}\boldsymbol{u}(t) \tag{62}$$

where $\boldsymbol{u}$ is the current unit radius vector oriented to the circle centre.

From the equilibrium of the forces

$$\boldsymbol{F_B} + \boldsymbol{F_C} = 0 \tag{63}$$

we can deduce the current radius vector of the circular trajectory:

$$\boldsymbol{r}(t) = \frac{m}{q}\frac{V}{B}\boldsymbol{u}(t) \tag{64}$$

so we can define algebrically an oriented curvature radius R as:

$$R=\frac{m}{q}\,\frac{V}{B} \tag{65}$$

When both an electron and an antielectron cross a Wilson chamber they leave two circular tracks which are curved in opposite direction. Let's suppose they have the same initial velocity *V*, we then have (with the electron labelled #1 and the antielectron #2):

$$R_2=-R_1 \tag{66}$$

therefore their mass and electric charge are related by the following ratios:

$$\frac{m_2}{q_2}=-\frac{m_1}{q_1} \tag{67}$$

For the electron we have $m_1=m_e>0$ and $q_1=-e<0$ so the antielectron has the ratio:

$$\frac{m_2}{q_2}=-\frac{m_e}{(-e)} \tag{68}$$

Mathematically simplifying the right expression in equation (68) by removing minus signs leads to the definition of a positively charged electron: the positron. This has been the choice of Carl Anderson [2] because he took for granted that any mass and energy must be positive. It is not required by the laws of Physics, and it does not match the negative energy solutions of the Dirac equation.
From equation (68) as it is written we can define the antielectron as a negative mass electron:

$$q_2=-e \text{ and } m_2=-m_e \tag{69}$$

Despite Terletsky [52] allegation, the Wilson chamber can well be used to detect negative mass particles and record their trajectories. While it creates condensation centres, a charged particle with positive mass looses a very small amount of energy and its speed slows down very little; on the contrary a charged particle with negative mass accelerates as he explained. So the trajectories are not quite circular, but this tiny perturbation is currently neglected.

13. About anti-hydrogen free fall experiments

After Galileo, every heavy body falls with the same speed. This means that the gravity acceleration is independent of the gravitational mass of any falling body.

So the naïve intuition that a negative mass should be repelled by Earth gravitation is quite misleading, as we demonstrate it below. Let's consider a test body (particle, atom, or molecule) with its gravitational mass m_g and its inertial mass m_i. The gravitational mass M_g of the Earth produces a gravitational force F_g which is:

$$F_g=G\frac{m_g M_g}{r^2} \tag{70}$$

then the test body undergoes an acceleration γ such as:

$$F_g=m_i\,\gamma \tag{71}$$

so the acceleration evaluates to:

$$\gamma=\frac{m_g}{m_i}\,\frac{G M_g}{r^2} \tag{72}$$

and from the mass equivalence principle

$$m_i = m_g \tag{73}$$

we deduce the acceleration of any negative or positive mass as being:

$$\gamma = \frac{G M_g}{r^2} \tag{74}$$

Consequently any mass cannot be detected as negative in free-fall experiments. However amazingly an antihydrogen free-fall experiment has been built at the CERN [7]. If the antihydrogen occurred to be repelled by Earth gravitation it cannot mean that the antihydrogen mass is negative, but it would show that the mass equivalence principle (73) does not hold for antiparticles.

From other point of views, two different experiments have been built: the project of the Alpha collaboration [1] and the GBAR project [6].

14. Negative mass cosmologies

New works in Cosmology are now published with some propositions to include negative masses in several new models of the Universe. H. Bondi [4] has studied the possible properties of negative and positive mass in the framework of the general theory of Relativity, but he found that the runaway phenomenon is not compatible with Einstein's theory.

In J.S. Farnes model [16] the cosmological constant Λ appears to be equivalent to a distribution of negative masses, and to be compatible with the expanding Universe the negative masses must be continuously created at a rate of

$$\Gamma(t) = 3H \tag{75}$$

and he has modified Einstein equations to include this matter creation. Moreover he has shown that negative masses and energies can flatten the rotation curves of galaxies.

Following Andreï Sakharov idea of a twin Universe [46], [47], Jean-Pierre Petit and Gilles d'Agostini [39] have proposed to represent the Universe as a 4-dimensional manifold which has two metrics: $g_{\mu\nu}^{(+)}$ for positive mass/energy and $g_{\mu\nu}^{(-)}$ for negative mass/energy, so the well known Einstein equation:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \chi T_{\mu\nu} \tag{76}$$

is split into two coupled field equations:

$$\begin{aligned} R_{\mu\nu}^{(+)} - \frac{1}{2} R^{(+)} g_{\mu\nu}^{(+)} &= \chi \left(T_{\mu\nu}^{(+)} + T_{\mu\nu}^{(-)} \right) \\ R_{\mu\nu}^{(-)} - \frac{1}{2} R^{(-)} g_{\mu\nu}^{(-)} &= -\chi \left(T_{\mu\nu}^{(+)} + T_{\mu\nu}^{(-)} \right) \end{aligned} \tag{77}$$

They call it the Janus cosmological model. They have shown that it avoids the runaway paradox and it perfectly fits available observational data [40].

Using Cartan's free coordinates calculus Patrick Marquet [34] has derived a differential form of Einstein's field equations from which he has deduced a dual field with a negative sign. The two opposite field equations describe the same twin universe.

Similarly but independently Frederic Henry-Couannier [24] has studied interactions between positive and negative energy fields in the framework of quantum field theory and has shown that positive/negative energy vacuum divergences may cancel exactly each other if they are maximally gravitationally coupled. Then he built a model of Universe with two time reversal conjugated metric tensors.

Remark: when a negative mass density ρ lower than a negative critical density, i.e.

$$\rho < -|\rho_c| \tag{78}$$

the highly strong gravitational field can produce matter aggregates and accretion disks, which by a nuclear fusion can be lit as negative mass stars. This might be observed in Astronomy.

15. Conclusion

The energy is not defined positive and there is no requirement for the mass to be positive. As the free Dirac equation has negative energy solutions it then predicts the existence of negative mass fermions. The Dirac Sea theory cannot prevent any positive mass fermion (including the positron) from falling into negative energy states, but the mass invariance can do it. The CPT group must be extended to the MCPT group including th mass inversion M.

In the Wilson chamber to the ratio *q/m* can be determined from the curvature of charged particle tracks, but it does not allow at all to decide both the sign of the electric charge and the sign of the mass. However the negative energy states predicted by the Dirac equation match the negative mass electrons – not the positrons.

Free fall experiments cannot detect antihydrogen or antiparticle as having a negative mass.

Negative masses might play a very important role in Cosmology.

We have stressed that the Wigner theorem is not complete and shown that the Schrödinger equation with a time dependent Hamiltonian $\boldsymbol{H}(t)$ or $\boldsymbol{H}(r, t)$ cannot be invariant through time-reversal as it depends on the definition of the considered Hamiltonian.

After Daniel M. Dubois [13] has shown that ± i must be used and leads to the two conjugate Schrödinger equations (5a, 5b), then consequently the two conjugate Schrödinger equations with a Hamiltonian $\boldsymbol{H}$ (41a, 41b) have to be studied for several different quantum systems. As a logic consequence we have proposed the new Extended Correspondence Principle (40a, 40b). So in a next work we will develop a new Quantum Mechanics.

References

[1] The Alpha Collaboration & Charman A 2013 Description and First Application of a New Technique to Measure the Gravitational Mass of Antihydrogen *Nature Communications* **4** 1785

[2] Anderson Carl D 1933 The Positive Electron *Physical Review* **41** 491-8

[3] Callender C 2000 Is Time Handed in a Quantum World? *Proc. Arist. Soc.* **121** 247-69

[4] Bondi H 1957 Negative Mass in General Relativity *Review of Modern Physics* **29** n°3 423-8

[5] Boney E 2019 Negative Mass Solutions to the Dirac Equation Move Forward in Time (preprint) DOI: 10.13140/RG.2.2.27190.34884

[6] Brook-Roberge D Perez P Banerjee D 2015 The GBAR Antimatter Gravity Experiment, *Hyperfine Interactions* **233** n°1 21-7

[7] Brusa R S et al 2017 The Aegis Experiment at CERN: Measuring Antihydrogn Free-fall in Earth's Gravitational Field to Test Wep With Antimatter *J. Physics: Conf. Ser.*, **791** 012014

[8] Chevalier G 2007 Wigner's Theorem and its Generalizations, in *Handbook of Quantum Logic and Quantum Structures: Quantum structures* ed K Engesser *et al* (Elsevier B V) 429-75

[9] Debergh N, Petit J-P & D'Agostini G 2018 On evidence for Negative Energies and Masses in the Dirac Equation Through a Unitary Time-Reversal Operator *Journal of Physics Communications* DOI: 10.13140/RG.2.2.29710.00322

[10] Dirac P A M 1928 The Quantum Theory of the Electron *Proceedings of the Royal Society* A **117** (778): 610-24 Bibcode:1928RSPSA.117..610D. doi:10.1098/rspa.1928.0023

[11] Dirac P A M 1930 A Theory of Electrons and Protons *Proc. R. Soc. Lond. A. Royal Society Publishing* **126** (801) 360-5 Bibcode:1930RSPSA.126..360D. doi:10.1098/rspa.1930.0013. JSTOR 95359

[12] Dirac P A M 1938 Classical Theory of Radiating Electrons *Proc. R. Soc. Lond.* A **167** 148-69 doi: 10.1098/rspa.1938.0124

[13] Dubois D M 1999 [13] Computational Derivation of Quantum and Relativist Systems with Forward-Backward Space-time Shifts, in Computing Anticipatory Systems, CASYS'98 Second International Conference, *AIP Conference Proceedings* **465** ed Daniel M Dubois 435-56

[14] Dubois D M 2000 Computational Derivation of Quantum Relativist Electromagnetic Systems with Forward-Backward Space-time Shifts, in Computing Anticipatory Systems, CASYS'99 Third International Conference, *AIP Conference Proceedings* **517** ed Daniel M Dubois 417-29

[15] Dubois D M 2020 Computing Hyperincursive Discrete Relativistic Quantum Majorana and Dirac Equations and Quantum Computation, in Frontiers in Quantum Computing ed Luigi Maxmilian Caligiuri *Nova Science Publishers, Inc*. 103-51 ISBN: 978-1-53618-515-7

[16] Farnes J S 2018 A Unifying Theory of Dark Energy and Dark Matter: Negative Masses and Matter Creation Within a Modified ΛCDM Framework *Astronomy & Astrophysics Manuscript* ESO **218** *arXiv*: 1712.07962v2

[17] Feshbach H & Villars F 1958 Elementary Relativistic Wave Mechanics of Spin 0 and Spin ½ Particles" *Reviews of Modern Physics* **30** n°1 24-45

[18] Fock V 1926 Zur Schrödingerschen Wellenmechanik *Zeitschrift für Physik* **38** 242-50

[19] Fradkin D M 2006 Comments on a Paper by Majorana Concerning Elementary Particles *EJTP* **3** n°10 305-14

[20] Gehér G P 2014 An Elementary Proof for the Non-Bijective Version of Wigner's Theorem *Physics Letters* A **378** n°30-31 2054-7 ISSN 0375-9601 doi.org/10.1016/j.physleta/2014.05.039

[21] Gordon W 1927 Der Comptoneffekt Nach der Schrödingerschen Theorie *Zeitschrift für Physik* **40** 117-33 (received 1926)

[22] Griffin J J 1959 translation from German of Wigner E P 1931 Group Theory and its Application to the Quantum Mechanics of Atom Spectra *New-York Academic Press* 233-6

[23] Guay B 2020 Dirac Field of Negative Energy and Primordial Antimatter Incursion *International Journal of Modern Theoretical Physics* **9** n°1 1-15 ISSN: 2169-7426

[24] Henry-Couannier F 2004 Energies and Time Reversal in Quantum Field Theory and General Relativity, the Dark Side of Gravity", hal-00001476v1

[25] Holster A 2003 The Quantum Mechanical Time Reversal Operator *ATASA Research, Te Kuiti, New Zealand* http://philsci-archive.pitt.edu/1449/

[26] Klein O 1926 Quantentheorie und Fünfdimensionale Relativitätstheorie *Zeitschrift für Physik* **37** 895-906

[27] Klein O 1927 Elektrodynamik und Wellenmechanik vom Standpunkt des Korrespondenzprincips *Zeitschrift für Physik* **41** 407-42

[28] Klein O 1928 Die Reflexion von der Elektronen an einem Potentialsprung Nach der Relativistischen Dynamik von Dirac *Zeitschrift für Physik* **53** 157-65

[29] Kragh H 1981 The Genesis of Dirac's Relativistic Theory of Electrons (communicated by Pedersen O & van der Waerden B L) Archive for History of Exact Sciences **24** *Springer-Verlag*

[30] Kudar J 1926 *Ann. phys.* **81** 632

[31] Lomont J A & Mendelson 1963 The Wigner Unitary-Antiunitary Theorem *Ann. Math.* **78** 548-59

[32] Majorana E 1932 Teoria Relativistica di Particelle con Momento Intriseco Arbitrario *Nuovo Cimento* **9** 335-44

[33] Majorana E 1937 A Symmetric Theory of Electrons and Positrons *Il Nuovo Cimento* **14** 171-84

[34] Marquet P 2019 Twin Universes: a New Approach *Progress in Physics* **15** n°2 64-7

[35] Mouchet A 2013 An Alternative Proof of Wigner Theorem on Quantum Transformations Based on Elementary Complex Analysis *Physics Letters* A **377** 2709-11 *arXiv*:quant-ph/1304.1376v2

[36] Messiah A 1995 *Mécanique Quantique* (Paris: Dunod)

[37] Nibart G L & Dubois D M 2016 Advanced Waves, Absorber Theory, Quantum Equations and Negative Mass in Unified Field Mechanics II Formulations and Empirical Tests *Proceeding of the Xth Symposium Honoring Noted French Mathematical Physicist Jean-Pierre Vigier, Porto Novo Italy, World Scientific Publishing Co* 322-30 ISBN 978-981-3232-03-7

[38] Park S T 2012 Propagation of a Relativistic Electron Wave Packet in the Dirac Equation *Physical Review A* **86** n°062105 1-12

[39] Petit J-P & d'Agostini G 2014 Cosmological Bimetric Model With Interacting Positive and Negative Masses and Two Different Speeds of Light in Agreement With the Observed Acceleration of the Universe *Modern Physics Letters* A **29** n°34 1450182

[40] Petit J-P & d'Agostini G 2018 Constraints on Janus Cosmological Model From Recent Observations of Supernovae Type Ia *Astrophysics and Space Science* **363** n°7 139

[41] Proca A 1936 Wave Theory of Positive and Negative Electrons *J. Phys. Radium* **7** n°8 347-53 DOI: 10.1051/jphysrad:0193600708034700

[42] Racah 1937 *Nuovo Cimento* **14**

[43] Roberts B W 2016 Three Myths About Time Reversal in Quantum Theory *arXiv*::physics.hist-ph/1607.07388v1

[44] Rosen M D 2015 Transforming an Electron into a Positron: A New Paradigm for Physics https://www.researchgate.net/publication/273318602_Transforming_an_Electron_into_a_Positron_A_New_Paradigm_for_Physics

[45] Rosen M D 2020 New Physics of the Electron Altering its Charge State, session Y12: Unconventional Ideas in Theory *APS April Meeting,Washington D.C.* **65** n°2 April 18-21

[46] Sakharov A D 1967 *ZhETF Pis'ma* **5** 32

[47] Sakharov A D 1967 Violation of CP Invariance, C Asymmetry and Baryon Asymmetry of the Universe *JETP Letters* **5** n°1 24-6

[48] Solomon D 2005 Some Differences Between Dirac's Hole Theory and Quantum Field Theory *Can. J. Phys*. **83** 257-71 *arXiv*:quant-ph/ 0506271

[49] Solomon D 2006 Some New Results Concerning the Vacuum in Dirac Hole Theory *Physc. Scr.* **74**, 117-22 *arXiv*:quant-ph/0607037

[50] Solomon D 2007 Quantum States With Less Energy Than The Vacuum in Dirac Hole Theory *arXiv*:quant-ph/0702271

[51] Solomon D 2008 Dirac's Hole Theory and the Pauli Principle: Clearing up the Confusion *arXiv*:quant-ph/0801.4140

[52] Terletsky J P 1962 Masses Propres Positives, Négatives et Imaginaires*J. Phys. Radium* **23** n°11 910-20 10.1051/jphysrad:019620023011091000; jpa-00236718

[53] Tzou K H 1959 Inversion de Masse et Solutions des Equations de Dirac*J. Phys. Radium* **20** n°12 933-6 10.1051/jphysrad:019590020012093300 HAL ID: jpa-00236169

[54] Uhlhorn U 1962 Representation of Symmetry Transformations in Quantum Mechanics *Arkiv for Fysik* **23** n°30 307-40

[55] Wigner E P 1931 Gruppentheorie und ihre Anwendung auf die Quanten Mekanik der Atomspektren F*riedrich* Vieweg *und Sohn*, Braunschweig, Deutschland ASIN B000K1MPEI

[56] Wigner E P 1932 Ueber die Operation der Zeitumkehr in der Quantenmechanik *Göttinger Nachrichten, Math-Phys* **1932** 546-*59*

[57] Zafar A 2003 P, T, PT, and CPT Invariance of Hermitian Hamiltonians *arXiv* :quant-ph/0302084v2