INTRODUCTION TO MASSIVE NEUTRINOS

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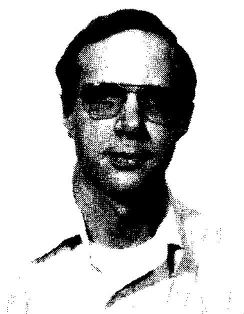

ABSTRACT

We discuss the theoretical ideas which make it natural to expect that neutrinos do indeed have mass. Then we focus on the physical consequences of neutrino mass, including neutrino oscillation and other phenomena whose observation would be very interesting, and would serve to demonstrate that neutrinos are indeed massive. We comment on the legitimacy of comparing results from different types of experiments. Finally, we consider the question of whether neutrinos are their own antiparticles. We explain what this question means, discuss the nature of a neutrino which is its own antiparticle, and consider how one might determine experimentally whether neutrinos are their own antiparticles or not.

I would like to begin this introduction to the subject of massive neutrinos by briefly discussing theoretical prejudices about neutrino mass. Do current theoretical views lead us to expect that neutrinos are indeed massive? If they do, do they tell us how heavy the neutrinos might be? Then, I would like to focus on the physical consequences of neutrino mass. How does neutrino mass lead to neutrino oscillation, and what is the physics of this process? How does one look for massive neutrinos in meson decay, or in neutrino decay? Need one worry about quantum-mechanical subtleties when comparing results from direct searches for neutrino mass with those from neutrino oscillation experiments? Finally, I would like to consider the question of whether neutrinos are their own antiparticles. In physical terms, what precisely does this question mean? Why is it that we do not already know the answer? How does one describe a neutrino which is its own antiparticle in field theory? Will accelerator experiments at higher energies help to tell us whether neutrinos are their own antiparticles? If not, what experiments _will_ help to answer this question?

Prejudices from Theory

From the standpoint of the grand unified theories (GUTS), which seek to unify the description of the weak, electromagnetic, and strong interactions, it is more natural for neutrinos to be massive than to be massless. The reason is trivial. In any grand unified theory, a given neutrino is put together with a charged lepton and with quarks in one large multiplet. Now, if the neutrino becomes a brother of a charged lepton, which, of course, is _massive_, and of quarks, which, of course, are also _massive_, then the neutrino itself would have to be exceptional to be _massless_. Naturally, this is not a proof that neutrinos have mass. One can construct grand unified theories in which the neutrinos are presumed to be massless, and their masslessness is put in by hand. People have constructed such GUTS. However, I think it is clear that, given the basic character of grand unified theories, this is not the natural thing to do. The natural thing in a GUT is for a neutrino to be massive, just as the other particles in its quark-lepton multiplet are.

Assuming, now, that neutrinos are indeed massive as the grand unified theories suggest, let us ask how heavy they are likely to be. Unfortunately, as far as gauge theories are concerned, the neutrino masses could be anything at all. If you force yourself to make an estimate of the neutrino mass scale which is natural in GUTS, you can do so by insisting that the neutrino mass term in the Lagrangian contain only known, rather tightly constrained ingredients. Let me explain.

There can be two types of neutrino mass terms in the Lagrangian for a theory. The first is the familiar Dirac mass term, of the form

$$\begin{aligned} L_D &= -M\bar{\nu}\nu = -M(\bar{\nu}_R \nu_L + \bar{\nu}_L \nu_R) \\ &= -M(\bar{\nu}_R \nu_L + \text{h.c.}) \end{aligned} \tag{1}$$

Here, M is the neutrino mass, and $\nu_{L,R}$ are the left- and right-handed projections of the neutrino field, given by

$$\nu_{L,R} = \frac{1 \pm \gamma_5}{2} \nu \ . \tag{2}$$

The second type of mass term is the Majorana mass term, which has the form

$$\begin{aligned} L_M &= -M(\overline{\nu^c})_R \ \nu_L + \text{h.c.}) \\ &= -M(\nu_L^T C \nu_L + \text{h.c.}) \ . \end{aligned} \tag{3}$$

Here ν^c, the so-called charge-conjugate field, is $C\bar{\nu}^T$, where $C = \gamma_2\gamma_4$ is the charge conjugation matrix and T denotes transpose. In going from the first line of Eq. (3) to the second, we have used the easily verified identities $(\nu^c)_R = (\nu_L)^c$ and $(\nu_L)^c = \nu_L^T C$. As we shall discuss later, a Majorana mass term leads to a neutrino which is its own antiparticle. Note that the Majorana mass term of Eq. (3) has been constructed entirely out of the familiar left-handed neutrino field ν_L. Now,[1] in the standard electroweak model, this field carries weak isospin $I_z = +1/2$, so that the mass term, as written, carries $I_z = +1$. Thus, in order to construct a term which is properly weak-iosopin invariant before symmetry breaking occurs, you have to introduce some additional fields which carry negative I_z. The neutral Higgs field ϕ^0 of the standard model has $I_z = -1/2$, so a term of the form

$$L \sim -(\phi^0)^2 \nu_L^T C \nu_L + \text{h.c.} \tag{4}$$

will do. When the Higgs develops a vacuum expectation value $\langle\phi^o\rangle$, this term will yield one of the Majorana mass term form $-\langle\phi^o\rangle^2 \nu_L^T C \nu_L$, with a constant multiplying $\nu_L^T C \nu_L$ as in Eq. (3). However, since $\langle\phi^o\rangle^2$ has the dimensions $(\text{mass})^2$, it must appear multiplied by an additional constant K with dimensions $(\text{mass})^{-1}$ if we are to have a mass term of the proper dimensions. Now, as explained in Ref. 1, the term (4) cannot occur in the _fundamental_ Lagrangian of the theory, but must be an _effective_ interaction induced by the exchange of some very heavy particles. These particles presumably have masses of order $M_{GUT} \sim 10^{15}$ GeV, the natural mass scale in grand unified theories. Then K must be $\sim f/M_{GUT}$, where f is some dimensionless effective coupling constant. That is, the effective interaction (4) leads to the Majorana mass term

$$L_M = -\frac{f}{M_{GUT}} \langle\phi^o\rangle^2 (\nu_L^T C \nu_L + \text{h.c.}) . \tag{5}$$

From Eq. (3), the mass of the neutrino is then

$$M \approx f \frac{\langle\phi^o\rangle^2}{M_{GUT}} . \tag{6}$$

Since $\langle\phi^o\rangle$ is known to be 300 GeV from the value of the Fermi coupling constant G_F, everything in this expression for the neutrino mass is known except for f, and we learn that

$$M \approx f(0.1 \text{ eV}) . \tag{7}$$

If f is smaller than unity, as many coupling constants are, the neutrino weighs less than 0.1 eV.

I repeat that this is only one estimate. Gauge theories can contain other mass terms with much less predictive power, and so the neutrino masses could be anything at all. However, as Eq. (7) illustrates, these masses could in particular be much less than those to which most current experiments are sensitive. Thus, if people at this Workshop have clever ideas on how to look for masses well below 1 eV , such ideas might be very helpful.

An interesting question about neutrinos is whether they are their own antiparticles. Neutrinos might be Majorana particles, which means that $\bar{\nu} = \nu$, and lepton number is obviously violated. Or, they might be Dirac particles, which means that $\bar{\nu} \neq \nu$, and there is a conserved lepton number. I will explain the Majorana-Dirac distinction more carefully later, when we discuss the experimental possibilities for telling whether neutrinos are of Majorana or of Dirac character. However, as far as theoretical prejudices are concerned, grand unified theories are theories in which lepton number, like baryon number, is in general violated. It is violated, for instance, in the decay $n \rightarrow e^- K^+$. Consequently, one would naturally expect that in GUTS neutrinos would be of the lepton-violating Majorana variety, rather than of the Dirac variety. Of course, this is only a theoretical bias, and one can construct grand unified theories in which the neutrinos are Dirac particles, but I think it is more natural in such theories for them to be Majorana particles. Indeed, there has been a lot of discussion, in the context of various grand unified models, about the possibility that neutrinos are of Majorana character. For example, the Gell-Mann, Ramond, Slansky mechanism,[2] which protects neutrinos from becoming as heavy as quarks, is a mechanism in which the neutrinos are Majorana particles. Now the Gell-Mann, Ramond, Slansky mechanism leads one to expect naively that neutrino masses M are of order M_q^2/M_{GUT} , where M_q is a typical quark mass. If we take $M_q \sim 1$ GeV , then $M \sim 10^{-6}$ eV . Let me say yet again that neutrino masses could be anything, but in particular they could be much less than 1 eV , so ideas on how to look for such small masses would be welcome.

Phenomenology of Massive Neutrinos

I would like to turn now to a discussion of the phenomenological consequences of neutrino mass. One of the most interesting consequences is neutrino oscillation. Let us recall what that is. Imagine a world with just two kinds of neutrinos. (In other words, forget for the moment about the τ and its neutrino.) Suppose that these two neutrinos, which I shall call ν_1 and ν_2, are massive, and that in particular their masses M_1 and M_2 are different. Suppose that in the decay $\pi^+ \rightarrow e^+ + \nu_e$, the outgoing neutrino ν_e, which is called the "electron neutrino" by way of definition, is neither of the physical neutrinos ν_1 and ν_2 with definite mass, but a linear combination of them. That is,

$$\nu_e = \nu_1 \cos\theta + \nu_2 \sin\theta , \tag{8}$$

where θ is a mixing angle. Suppose finally that in the corresponding muonic decay, $\pi^+ \rightarrow \mu^+ + \nu_\mu$, the outgoing neutrino ν_μ, which is called the "muon neutrino" by definition, is the orthogonal linear combination. That is,

$$\nu_\mu = -\nu_1 \sin\theta + \nu_2 \cos\theta . \tag{9}$$

Note that in hypothesizing that the neutrino which accompanies a specific charged lepton in weak decays is a mixture of neutrino mass eigenstates, we are simply supposing that the leptons behave exactly as we know the quarks do. Namely, from low-energy data we know that, expressed in terms of quarks, the hadronic decays of the W-boson are of the form

$$W^- \rightarrow \bar{u} + d_c \tag{10}$$

and

$$W^- \rightarrow \bar{c} + s_c . \tag{11}$$

In the decay to the anti up-quark $\bar{u}$ (the analogue of the e^+ in $\pi^+ \rightarrow e^+\nu_e$), the accompanying quark d_c is a coherent mixture of the quarks d and s of definite mass:

$$d_c = d \cos\theta_c + s \sin\theta_c . \tag{12}$$

Here, θ_c is the Cabibbo angle. In the decay to the anti charmed-quark $\bar{c}$ (the analogue of μ^+), the accompanying quark s_c is the orthogonal linear combination:

$$s_c = -d \sin \theta_c + s \cos \theta_c \ . \tag{13}$$

Consider, now, a beam of neutrinos which have momentum p_ν and are born as muon neutrinos in pion decays of the dominant variety $\pi^+ \to \mu^+ + \nu_\mu$. If the suppositions we have made about neutrinos are right, then, at birth, each neutrino in the beam is the particular linear superposition of the neutrinos of definite mass given by Eq. (9). Now, if ν_1 and ν_2 have different masses, then at the given momentum p_ν the ν_1 and ν_2 components of the beam will travel at different speeds, the lighter component getting ahead of the heavier one. Thus, downstream of its point of birth, the beam will no longer consist of precisely the admixture (9) of ν_1 and ν_2 , but of some other admixture. That means that the beam is no longer purely ν_μ , but contains a ν_e component as well. Obviously, the ν_e-ν_μ composition of the beam will vary with distance.

This is the phenomenon of neutrino oscillation. Notice that for it to occur, two things must be true. First, the different neutrino mass eigenstates must have different masses, which implies that at least one of them must have a non-zero mass. Secondly, the neutrinos that go with particular charged leptons must be non-trivial mixtures of the mass eigenstates.

Let us now treat neutrino oscillation quantitatively, assuming that there are not just two but N different varieties ("flavors") of charged lepton: e , μ , τ , and however many more there may be. Correspondingly, we assume that there are N physical neutrinos (mass eigenstates) ν_m , with non-degenerate masses M_m . As before, we assume that the neutrinos ν_f of definite flavor (ν_e , ν_μ , etc.) are not the mass eigenstates, but linear combinations of them:

$$\nu_f = \sum_m U_{fm} \nu_m \ . \tag{14}$$

Here U is a unitary mixing matrix which can be taken to be real if CP is conserved.[3]

Consider a beam of neutrinos with a definite momentum p_ν, born in association with a particular charged lepton ℓ_f (such as a muon). If the mass resolution of the experiment is insufficient to tell which neutrino mass eigenstate is actually involved in each event, each neutrino in the beam will be a coherent superposition of the various ν_m. The ν_m component of the neutrino will have energy $E_m = (p_\nu^2 + M_m^2)^{1/2}$. At time $t = 0$, the moment of its birth, the neutrino is a ν_f; that is, it is the linear combination (14) of the ν_m. Thus, at $t = 0$, its wave function is

$$\psi(x,t = 0) = \sum_m U_{fm} \nu_m e^{ip_\nu x} . \tag{15}$$

After a time t, this wave function evolves into

$$\psi(x,t) = \sum_m U_{fm} \nu_m e^{ip_\nu x} e^{-iE_m t} , \tag{16}$$

since each ν_m has definite energy E_m. Assuming that the ν_m are very light ($M_m << p_\nu$), we may write $E_m \cong p_\nu + (M_m^2/2p_\nu)$. Under the same assumption, our neutrino, which was born as a ν_f at $t = 0$, will be traveling at roughly the speed of light. Thus, if it was born at $x = 0$, at time t it will be approximately at $x = t$. At that point the wave function is

$$\psi(t,t) \cong \sum_m U_{fm} \nu_m e^{-i(M_m^2/2p_\nu)t} . \tag{17}$$

This state is a superposition of all the flavors $\nu_{f'}$. If we use the inverse of Eq. (14), Eq. (17) becomes

$$\psi(t,t) \cong \sum_{f'} [\sum_m U_{fm} U_{f'm} e^{-i(M_m^2/2p_\nu)t}] \nu_{f'} . \tag{18}$$

Squaring the coefficient of $\nu_{f'}$ (and noting that $t = x$), we find that the probability of finding the neutrino to have flavor f' at a distance x from its source, if originally it had flavor f, is

$$P[f \rightarrow f';x] = \sum_m U_{fm}^2 U_{f'm}^2 + \sum_{m' \neq m} U_{fm} U_{fm'} U_{f'm} U_{f'm'} \cos 2\pi \frac{x}{\ell_{mm'}} . \tag{19}$$

Here the oscillation lengths $\ell_{mm'}$ are given by

$$\ell_{mm'} = 2\pi \frac{2p_\nu}{|M_m^2 - M_{m'}^2|} . \tag{20}$$

Note that the oscillating term in $P(f \to f';x)$ comes from interference between the different mass eigenstates in the neutrino wave function.

When x is small compared to the oscillation lengths $\ell_{mm'}$, the effects of oscillation are not yet visible, and the neutrino has essentially its original flavor. When x is comparable to the oscillation lengths, the flavor content of the beam varies with distance in the characteristic oscillatory manner described by Eq. (19). When x is much larger than the oscillation lengths, then in practice the flavor content of the beam no longer varies with distance. The reason is that any actual beam has a finite momentum spread δp_ν, so that from Eq. (20) it also entails a finite spread in the oscillation length corresponding to any pair of mass eigenstates. Suppose, for example, that only two mass eigenstates, ν_1 and ν_2, participate appreciably in the oscillation, so that there is only one oscillation length $\ell = 4\pi p_\nu / |M_1^2 - M_2^2|$. This oscillation length will have a fractional spread $\delta\ell/\ell = \delta p_\nu / p_\nu$, and by the time x reaches

$$x_{wash} = \left(\frac{1}{\delta p_\nu / p_\nu}\right) \ell , \tag{21}$$

the oscillatory term in Eq. (19) will have washed out. Note, however, that after the oscillations in Eq. (19) have washed out,

$$P[f \to f';x] = \sum_m U_{fm}^2 U_{f'm}^2 . \tag{22}$$

While this transition probability no longer varies with distance, it still does reflect the presence of any non-trivial neutrino mixing. A neutrino born as a ν_μ can still be found on detection to be, for example, a ν_τ.

In terms of convenient units, the argument of the cosine in Eq. (19) is

$$2\pi \frac{x}{\ell_{mm'}} = 2.53 \left(\frac{\delta M_{mm'}^2}{1 eV^2}\right) \frac{(x/1\ \mathrm{km})}{(p_\nu / 1\ \mathrm{GeV})} , \tag{23}$$

where $\delta M^2_{mm'} = |M^2_m - M^2_{m'}|$. From this expression, one can draw the obvious conclusions about the sensitivity of accelerator experiments to neutrino oscillation. To mention a non-accelerator experiment, if one studies ~ 1 GeV neutrinos whose distance of travel before detection is the diameter of the earth ($x = 10^4$ km) , then one can evidently probe down to $\delta M^2_{mm'} \sim 10^{-4}\ eV^2$.[4]

Of course, sensitivity to neutrino oscillation depends not only on having an x which is not too small compared to the oscillation lengths, but also on having reasonable values of the U_{fm} (i.e., reasonably large "mixing angles"). Unfortunately, theory offers essentially no clues about the values of the neutrino mixing angles, any more than it predicts the neutrino masses. In the two-neutrino mixing example of Eqs. (8) and (9), if θ is equal to the Cabibbo angle, the amplitude of oscillation will be 20%. That is, there will be distances x at which the flux of neutrinos of the original flavor will be only 80% of its value at $x = 0$, the remaining 20% having gone into another flavor.

Neutrino oscillation is a lovely example of quantum mechanics in action. I would like to illustrate that.[5] Suppose that you perform the neutrino oscillation experiment sketched in Fig. 1. In the neutrino source region, you create neutrinos in association with muons through the decay of pions in flight. Then, downstream, you let the neutrinos interact with a target, and you insist that each neutrino produce, in particular, an electron. This experiment searches, in other words, for $\nu_\mu \longleftrightarrow \nu_e$ oscillations. Suppose that these are found and that, in particular, when the source-to-detector distance x is varied, the oscillatory x-dependence predicted by Eq. (19) is observed.

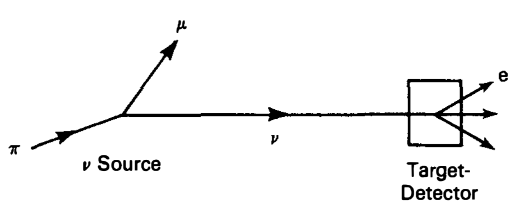

Fig. 1. A neutrino oscillation experiment.

Now imagine adding apparatus at the neutrino source which measures the momenta of the pion and muon in each event. Suppose that these momenta are measured so precisely that they determine the mass-squared of the neutrino, M_ν^2, with an error $\Delta(M_\nu^2)$ less than all $|M_m^2 - M_{m'}^2|$. Then you will know which physical (mass eigenstate) neutrino ν_m actually went down the beam line in each event of the experiment!

Clearly, if you know which mass eigenstate is actually involved in each event, the electron counting rate measured by the detector can no longer oscillate with x. Now only a single physical neutrino, as ordinary a beam particle as a proton, contributes to any given event. You do not have the coherent contributions from several ν_m whose interference with one another was the origin of oscillatory x-dependence.

This disappearance of oscillation with x as a result of momentum measurements which determine M_ν^2 raises an interesting question. Namely, what do these measurements <u>do</u> to destroy the oscillation pattern? The answer to this question is given by the uncertainty principle. To determine M_ν^2, you must measure the momenta of both the pion and the muon. Now, the more accurately the pion momentum is measured, the more uncertain the pion position will be. Consequently, the more uncertain the point where the neutrino is born will be. You might guess that just when the pion momentum is measured accurately enough for $\Delta(M_\nu^2)$ to be less than all $|M_m^2 - M_{m'}^2|$, the uncertainty in the neutrino source point will exceed all the oscillation lengths $\ell_{mm'}$. Obviously, any oscillation pattern will then be obliterated.

This guess is precisely correct. Indeed, we can easily show that, independent of the details of the neutrino source and detector, if measurements made at either place become sufficiently accurate to reveal which ν_m is involved in each event, then the consequent uncertainty in the neutrino source point, or the detection point is that is where the measurements are made, grows larger than all the oscillation lengths. In a given event, the neutrino mass M_ν is related to its energy E_ν and momentum p_ν by

$$M_\nu^2 = E_\nu^2 - p_\nu^2 \ . \tag{24}$$

If the measurements from which E_ν and p_ν are deduced are made with uncorrelated errors ΔE_ν and Δp_ν, then the error $\Delta(M_\nu^2)$ in the resultant value of M_ν^2 will be

$$\Delta(M_\nu^2) = [(2E_\nu)^2(\Delta E_\nu)^2 + (2p_\nu)^2(\Delta p_\nu)^2]^{1/2} . \qquad (25)$$

Now, if we want to know which ν_m is involved in each event, then we must have

$$\Delta(M_\nu^2) < |M_m^2 - M_{m'}^2| \qquad (26)$$

for all m,m'. From Eq. (25), we see that the error in p_ν must then satisfy

$$2p_\nu \Delta p_\nu < |M_m^2 - M_{m'}^2| . \qquad (27)$$

The uncertainty principle then implies that the neutrino source point, or its detection point if that is where the p_ν and E_ν measurements are made, will have an uncertainty Δx obeying

$$\Delta x > \frac{2p_\nu}{|M_m^2 - M_{m'}^2|} \qquad (28)$$

for all m,m'. From Eq. (20), we see that, apart from a factor of 2π, the right-hand side of this relation is precisely the oscillation length $\ell_{mm'}$.

Of course, if the neutrino masses M_m are of order 10 eV or less, and you perform an experiment with $p_\nu > 100$ MeV, there is no danger that you will <u>inadvertently</u> measure p_ν with an accuracy exceeding the critical value given by Eq. (27), thereby accidentally obliterating the oscillation pattern. On the other hand, you may be interested in the opposite limit, where neutrino masses are of order 100 MeV, and the measurements intended to determine these masses do reveal which ν_m is produced in each decay of the parent meson. To search for such heavy neutrinos, you would not look for an oscillation pattern. (Even if $p_\nu > 10$ GeV, when $M_m = 100$ MeV and $M_{m'} \ll M_m$, $\ell_{mm'} < 10^{-9}$ cm !) Rather, you would try to measure the charged particle momenta in decays such as $\pi^+ \to e^+ + \nu_m$ and $K^+ \to \mu^+ + \nu_m$ accurately enough to determine the masses of the ν_m . As we have been saying all along, the neutrino which accompanies the charged lepton of flavor f is, with amplitude U_{f1} , the mass eigenstate ν_1 , with amplitude U_{f2} , the mass eigenstate ν_2 , and so forth. Thus, if some of these mass eigenstates are heavy enough, then, for example, in the decay $K^+ \to \mu^+ + \nu_m$ of kaons at rest, the muon momentum spectrum would exhibit several visibly distinct monochromatic lines, corresponding to the masses of the different ν_m .[6]

Searches for heavy neutrinos in leptonic K and π decay have so far yielded limits of order $10^{-5} - 10^{-6}$ on $|U_{em}|^2$ and $|U_{\mu m}|^2$ in rather broad but not exhaustive regions of ν_m mass.[7] Should limits of this magnitude be considered very stringent? In the Kobayashi-Maskawa matrix, the quark sector analogue of the leptonic mixing matrix U considered here, some of the elements could be as small as $10^{-5} - 10^{-6}$. Thus, for all we know, there is mixing in the lepton sector below this level.

If neutrinos have mass, then one expects them to decay through the radiative process $\nu_m \to \nu_{m'} + \gamma$. This process is engendered by loop diagrams such as that in Fig. 2. In the standard model, the lifetime $\tau(\nu_m)$ for radiative decay is extremely long:[8]

$$\tau(\nu_m) > 10^{22} \text{ years} \times \left(\frac{30\text{eV}}{M_m}\right)^5 . \tag{29}$$

(Relation (29) assumes for simplicity that $M_{m'} << M_m$.) Thus, one need not worry that the composition of a laboratory neutrino beam will be altered by radiative decays. On the other hand, perhaps one can observe the photons coming from such decays of astrophysical neutrinos. To the author's knowledge, the best limit on $\tau(\nu_m)$ from searches for these photons is[9]

$$\tau(\nu_m) > (10^{16} - 10^{18}) \text{ years} \ , \ 3 < M_m < 10 \text{ eV} \ , \tag{30}$$

based on observations of the Coma and Virgo clusters of galaxies. Obviously, this limit is not yet at the level of the theoretical prediction.

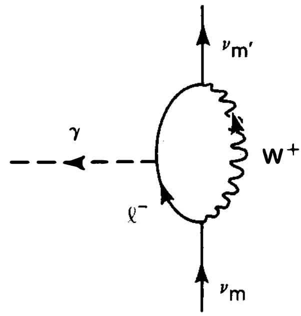

Fig. 2. A loop diagram for radiative decay of a neutrino. The symbol ℓ denotes a charged lepton.

If the mass difference $M_m - M_{m'}$ between two mass eigenstates exceeds the mass of an e^+e^- pair, then in addition to radiative decays, decays of the type $\nu_m \to \nu_e e^- e^+$ are possible. Such decays correspond to the diagram in Fig. 3, which evidently is identical to that for μ decay, except that ν_m replaces the μ and there is a factor of U_{em} at the $\nu_m e^- W$ vertex. Remembering that the μ decay process scales as M_μ^5, we see that

$$\tau(\nu_m \to \nu_e e^- e^+) = \tau(\mu)\left(\frac{M_\mu}{M_m}\right)^5 \frac{1}{|U_{em}|^2} . \qquad (30)$$

From a search for this decay in a wide band neutrino beam at CERN, the CHARM collaboration has placed an upper limit on $|U_{em}|^2$.[10] For $100 \text{ MeV} < M_m < 490 \text{ MeV}$, this limit is down in the range $10^{-5} - 10^{-6}$, comparable to the limits from π_{e2} decay for lower masses. The CHARM group was able to obtain an even more stringent limit assuming that the heavy ν_m is essentially ν_τ and making an assumption about the rate of ν_τ production in a beam dump. However, the limit I have quoted here is more independent of assumptions.

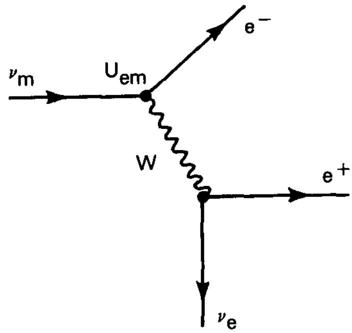

Fig. 3. The decay $\nu_m \to \nu_e e^- e^+$. As shown, the $\nu_m e^- W$ coupling is proportional to U_{em} .

Since they are to be covered in other talks at this Workshop, I shall not discuss the very important experiments which seek to measure neutrino masses directly, either in tritium beta decay, or in electron capture associated with internal Bremsstrahlung. For the same reason, I shall not discuss the astrophysical consequences of neutrino mass, such as the possibility that neutrinos constitute the non-luminous matter in the universe.

I would, however, like to comment on the possible implications of existing tritium results when these are compared with results on neutrino oscillation. When just two mass eigenstates, ν_1 and ν_2 , participate significantly in an oscillation, the U matrix takes the simple form

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} , \tag{31}$$

and Eq. (19) becomes

$$P[f \to (f' \neq f);x] = \sin^2 2\theta \sin^2 \pi \frac{x}{\ell_{12}} , \tag{32}$$

with

$$P[f \to f;x] = 1 - P[f \to (f' \neq f);x] .$$

Negative oscillation results are then expressed as correlated limits on the mixing angle θ and the quantity $\delta M_{12}^2 = |M_1^2 - M_2^2|$ appearing in the oscillation length $\ell_{12} = 4\pi p_\nu/|M_1^2 - M_2^2|$. In particular, assuming that just two mass eigenstates are involved, the Gösgen reactor experiment on oscillation in a $\bar{\nu}_e$ beam finds[11] that unless $\sin^2 2\theta \lesssim 0.2$, $\delta M_{12}^2 \lesssim 0.04\ eV^2$. Now, let us suppose that, as suggested by the ITEP studies of tritium beta decay,[12] $M_{\bar{\nu}_e} \approx 30$ eV. If we continue to assume that "$\bar{\nu}_e$" is comprised essentially of just two mass eigenstates, and that the masses of both of these are near 30 eV, then the Gösgen result leaves us with two possibilities. Either the mixing is small, or else

$$\frac{|M_1 - M_2|}{M_1 + M_2} \lesssim 10^{-5} . \tag{33}$$

The latter degeneracy would be interesting, to say the least.[13]

During the Workshop, it was pointed out by Bergkvist[14] and by Robertson[15] that the ITEP group finds its data to be better fitted by two quite different masses than by two roughly equal ones. The best fits are achieved with $M_1 = (80-115)$eV and $M_2 = (0-20)$eV.[14],[15] Then $\delta M_{12}^2 \approx 10^4 eV^2$. Now, in the Gösgen experiment, p_ν is of order 4MeV, and the detector is positioned at either 38m or 46m from the reactor core. Thus, if $\delta M_{12}^2 = 10^4 eV^2$, the Gösgem detector is more than 3×10^4 oscillation lengths from the $\bar{\nu}_e$ source! It was recalled by Workshop participants that, many oscillation lengths from the neutrino source, a neutrino wave packet will break up into non-overlapping pieces. The author was asked to talk briefly on the question of whether this breakup would make it impossible for the Gösgen experiment to see oscillation or to say anything about neutrino masses and mixing if $\delta M_{12}^2 \approx 10^4 eV^2$.

I believe the answer to this question is as follows.[16] At a given momentum, the pieces of a neutrino wave packet corresponding to different mass eigenstates ν_m travel at different speeds. For a highly relativistic ν_m, spreading of its piece of the wave packet can be shown to be negligible. Thus, the wave packet for a neutrino born as, say, a $\bar{\nu}_e$ will indeed split into nonoverlapping pieces when its various ν_m components become more widely separated than the original length h of the packet.[17] Of course, when they no longer overlap, the different ν_m components cannot interfere to make the flavor content of the beam vary with distance.

If there are just two contributing mass eigenstates and one corresponding oscillation length ℓ_{12}, the wave packet will have separated into nonoverlapping pieces by the time $x \sim x_{sep}$, where[16]

$$x_{sep} = (hp_\nu)\ell_{12} \quad . \tag{34}$$

Now, if h is bigger than 10^{-8} cm, the size of an atom, and $p_\nu > 1$ MeV, as in all current neutrino oscillation experiments, then $(hp_\nu) \gtrsim 10^3$. However, a typical neutrino beam has a finite momentum spread $(\delta p_\nu/p_\nu) > 1/10$. Thus (see discussion around Eq. (21)), the x-dependence of the flavor content of the beam will have become washed out by the time $x = x_{wash}$, where

$$x_{wash} = \left(\frac{1}{\delta p_\nu/p_\nu}\right)\ell_{12} < 10\ell_{12} \quad . \tag{35}$$

Hence, for any macroscopic value of h, separation of pieces of the wave packet at a given p_ν will not eradicate the oscillation with distance until long after it has disappeared anyway due to the broad p_ν spectrum of the beam.

At distances beyond the momentum-spread induced wash out, the transition probability $P[f \to f';x]$ is given by Eq. (22). What happens to this probability after the wave packet breaks up? After breakup, the different ν_m contribute to the event rate incoherently. What is the contribution of a given ν_m to an experiment in which the neutrinos are born in association with the charged lepton ℓ_f, but detected through their production of the differently-flavored charged lepton $\ell_{f'}$? It is the probability U^2_{fm} of creating the ν_m together with an ℓ_f, times the probability $U^2_{f'm}$ that this ν_m will produce an $\ell_{f'}$. Thus, the total event rate is $\sum_m U^2_{fm} U^2_{f'm}$. That is, Eq. (22)

describes $P[f \rightarrow f';x]$ correctly both after wash out and after wave packet breakup.

Obviously, through Eq. (22), data from a detector beyond the wash out or wave packet breakup point do provide limits on neutrino mixing, even though there is no x-dependence at such distances. (Of course, comparison of data from two distances which are both beyond the wash out point would show no effect; it is the comparison between one of these distances and $x = 0$ which counts.) In particular, one may ask whether the fits to the ITEP tritium measurements which imply that $\delta M_{12}^2 \approx 10^4 eV^2$ are consistent with the Gösgen finding that $\sin^2 2\theta \lesssim 0.2$ for large δM_{12}^2. The answer is that one of the two ITEP solutions quoted by Robertson[15] is consistent with the Gösgen limit, but, assuming that only two mass eigenstates are important, the other solution is not.

Are Neutrinos Majorana or Dirac Particles?

Let us turn now to the question of whether neutrinos are their own antiparticles, beginning with a careful statement of what one means by this question. Suppose there exists a massive neutrino ν_- with negative helicity (indicated by the subscript), as considered at the extreme left of Fig. 4(a). As Fig. 4(a) indicates, CPT-invariance then implies that there must also exist the CPT mirror-image of this particle, a positive-helicity antineutrino. In addition, if ν_- is massive, it travels slower than light, so that by travelling sufficiently fast you can overtake it. If you do, then in your frame it is going the other way, but still spinning the same way as in the original frame. Thus, the Lorentz transformation to your frame turns ν_- into a positive-helicity particle, depicted at the far right of Fig. 4(a). Now, this positive-helicity object may or may not be the same as the CPT mirror-image of ν_-. Let us suppose first that, as imagined in Fig. 4(a), it is not the same. Then it has its own CPT mirror-image, a negative-helicity antineutrino, and altogether there are four states with a common mass. This foursome is called a Dirac neutrino ν^D. In general, a Dirac neutrino will have a magnetic dipole moment, and perhaps even an electric dipole moment. Thus, in the Dirac case, ν_- can be converted into its opposite helicity partner ν_+ not just by a Lorentz transformation but also through the action of an external $\vec{B}$ or $\vec{E}$ field.

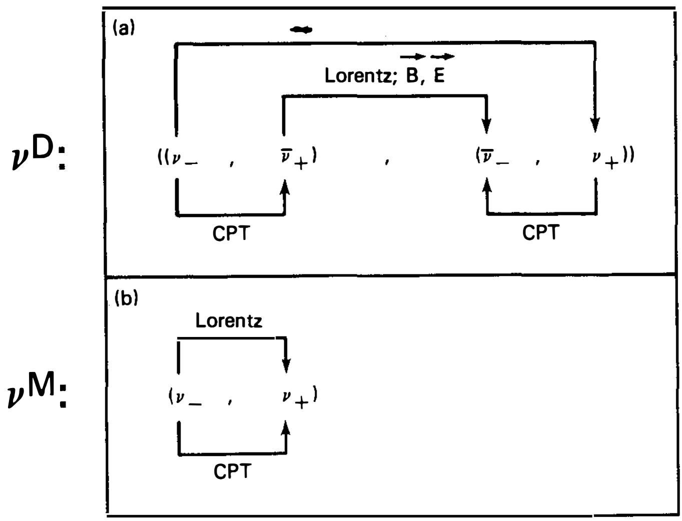

Fig. 4. (a) The four distinct states of a Dirac neutrino ν^D .
(b) The two distinct states of a Majorana neutrino ν^M .

The second possibility, pictured in Fig. 4(b), is that when you reverse the momentum of ν_- by overtaking it, the positive-helicity particle that you obtain is the same as the CPT mirror-image of ν_- . Then there are just two states with a common mass, and this pair of states is called a Majorana neutrino ν^M . In the rest frame, CPT applied to either of the two spin states of such a neutrino simply reverses the spin (due to the time reversal). By a 180° rotation, you can then reverse it again, returning the neutrino to its original state. It is in this sense --- CPT, followed by a 180° rotation --- that a Majorana neutrino is its own antiparticle.

Majorana neutrinos obviously lead to lepton number violation. Given the trivial effect of CPT on a Majorana neutrino, such a particle clearly carries no lepton number, and, brought to rest, has no memory of whether it was made in a decay such as $\pi^+ \to \mu^+ + \nu$, or one like $\pi^- \to \mu^- + \nu$. In subsequent interactions, this neutrino can produce either a μ^+ or a μ^-.

As long as the mass is not zero, a Dirac neutrino consists of four states $[(\nu_-^D, \bar{\nu}_+^D), (\bar{\nu}_-^D, \nu_+^D)]$, while in contrast a Majorana neutrino consists of but two (ν_-^M, ν_+^M). Nevertheless, as the mass goes to zero, the distinction between a Dirac and a Majorana neutrino gradually disappears. It appears that the situation can be summed up by what I like to call the

<u>Practical Dirac-Majorana Confusion Theorem:</u>[18]

Assume that all weak currents are left-handed. Assume further that experiments on a given neutrino are always done with one of two incoming states --- a state "ν_-" of negative helicity, or its CPT-conjugate, which we shall call "$\bar{\nu}_+$". (All neutrino experiments have, of course, been done this way.) Then, as the neutrino mass goes to zero compared to the other energy and mass scales in the problem, it gradually becomes more and more difficult to tell experimentally whether "ν_-" and "$\bar{\nu}_+$" are actually $(\nu_-^D, \bar{\nu}_+^D)$, two of the four states of a Dirac neutrino, or (ν_-^M, ν_+^M), the two states of a Majorana neutrino. When the mass vanishes, there is no physical distinction between these two cases.

When a neutrino is massless, it travels at the speed of light, so that one can no longer reverse its helicity by going to another Lorentz frame. Furthermore, if all weak currents are left-handed, the magnetic and electric dipole moments of a neutrino must vanish with its mass,[18] so that external $\vec{B}$ and $\vec{E}$ fields can no longer reverse its helicity either. Thus, the states $(\nu_-^D, \bar{\nu}_+^D)$ get disconnected from their opposite-helicity partners $(\bar{\nu}_-^D, \nu_+^D)$, and the latter states need not even exist. One just has two states, and whether one chooses to call them a Dirac neutrino and its antiparticle, or the two spin states of a Majorana neutrino, is purely a matter of semantics.

As the "confusion theorem" states, the approach to the massless limit is smooth (unfortunately), so that it will be hard to determine whether the light neutrinos with which nature confronts us are Majorana or Dirac particles. I would like to illustrate this state of affairs by considering neutral-current scattering.

Before doing that, I would like to explain the difference between Majorana and Dirac fields. For those unfamiliar with Majorana particles, the subsequent discussion of neutral-current scattering will then serve as an introductory illustration of how such particles are handled in theoretical calculations.

The Dirac neutrino field ψ^D is given by

$$\psi^D = \sum_{\vec{p},s} \sqrt{\frac{M}{E_{\vec{p}}V}} \left(f_{\vec{p}s} u_{\vec{p}s} e^{ipx} + \bar{f}^\dagger_{\vec{p}s} v_{\vec{p}s} e^{-ipx} \right) . \tag{36}$$

Here M , $\vec{p}$, s , and $E_{\vec{p}}$ are, respectively, the neutrino mass, momentum, spin-projection, and energy, u and v are the usual Dirac spinors, f annihilates neutrinos, and $\bar{f}^\dagger$ creates antineutrinos. A related field which will be useful is the charge-conjugate of ψ^D , $(\psi^D)^c \equiv C(\bar{\psi}^D)^T$. The field $(\psi^D)^c$ is just ψ^D with $f_{\vec{p}s}$ and $\bar{f}^\dagger_{\vec{p}s}$ replaced, respectively, by $\bar{f}_{\vec{p}s}$ and $f^\dagger_{\vec{p}s}$; that is, with the roles of particle and antiparticle interchanged.

The Majorana neutrino field ψ^M , which describes the case where particle and antiparticle are identical, may be gotten from the Dirac field simply by setting $\bar{f}^\dagger_{\vec{p}s} = f^\dagger_{\vec{p}s}$. Thus,

$$\psi^M = \sum_{\vec{p},s} \sqrt{\frac{M}{E_{\vec{p}}V}} \left(f_{\vec{p}s} u_{\vec{p}s} e^{ipx} + \lambda f^\dagger_{\vec{p}s} v_{\vec{p}s} e^{-ipx} \right) , \tag{37}$$

where we have included a "creation phase factor" λ in the creation term. This arbitrary phase factor, which in general is present in a Majorana field, corresponds to the possibility of re-defining the one particle state by multiplying it by a phase factor. For some purposes, such as the treatment of neutrinoless double beta decay, it can be convenient <u>not</u> to set $\lambda \equiv 1$.[19]

One may easily show that ψ^M obeys the relation[20]

$$(\psi^M)^c \equiv C(\bar{\psi}^M)^T = \lambda^* \psi^M . \tag{38}$$

Conversely, if one expands any fermion field χ obeying Eq. (38)

in the general form (36), one quickly finds that Eq. (38) implies that $\bar{f}^{\dagger}_{\vec{p}s} = \lambda f^{\dagger}_{\vec{p}s}$. That is, χ is a Majorana field of the form (37), and describes a particle whose antiparticle is itself.

Earlier it was stated that the Majorana mass term (3) leads to a neutrino which is its own antiparticle. Why is that? First of all, any mass term in the Lagrangian, including a Majorana mass term, can be written in the familiar form $-M\bar{\psi}\psi$ whose interpretation we know because we know that it leads to the usual mass term in the Dirac equation. To cast the mass term of Eq. (3) in this canonical form, let us rewrite it in terms of the field ψ defined by[21]

$$\begin{aligned} \psi &= \nu_L + (\nu_L)^c \\ &= \nu_L + (\nu^c)_R \; . \end{aligned} \tag{39}$$

Obviously, the left-handed projection of ψ, ψ_L , is ν_L , while the right-handed projection, ψ_R , is $(\nu^c)_R$. Hence, the mass term (3) may be written as

$$L_M = -M(\bar{\psi}_R \psi_L + \text{h.c.}) = -M\bar{\psi}\psi \; . \tag{40}$$

Now, since $[(\nu_L)^c]^c = \nu_L$,

$$\psi^c = \psi \; . \tag{41}$$

That is, the field ψ that we would interpret, in view of Eq. (40), as the usual field operator for a spin-1/2 particle, obeys Eq. (38) with $\lambda^* = 1$. This implies, as we have said, that ψ is a Majorana field, and describes a neutrino whose antiparticle is itself.

Let us now contrast the behavior of Dirac and Majorana neutrinos in neutral-current reactions of the form $\nu + A \rightarrow \nu + B$, where A is some target, and B is any collection of outgoing particles.[22] At energies well below the Z^o mass, where all our neutral-current data have been gathered, the standard electroweak model describes any such reaction by an interaction of the form

$$H = [\bar{\psi}\gamma_\mu(1 + \gamma_5)\psi]\, M_\mu(A,B) \ . \tag{42}$$

Here ψ is the neutrino field, which could be either of Majorana or Dirac character, and the bracketed quantity is the neutrino neutral current. The operator $M_\mu(A,B)$ is a neutral current pertaining to A and B , whatever they may be.

From Eq. (36), we see that in the Dirac case the matrix element of the neutrino neutral current is (we set x = 0)

$$\langle\nu_f^D|\overline{\psi^D}\gamma_\mu(1 + \gamma_5)\psi^D|\nu_i^D\rangle = [\frac{M^2}{E_f E_i V^2}]^{1/2}\, \bar{u}_f\gamma_\mu(1 + \gamma_5)u_i \ . \tag{43}$$

Note that it is the field ψ^D which annihilates the incoming neutrino, and the field $\overline{\psi^D}$ which creates the outgoing one.

Turning to the Majorana case, we note first that the vector part of the neutrino neutral current, $\overline{\psi^M}\gamma_\mu\psi^M$, now vanishes. The reason is that, regardless of whether ψ is a Majorana or Dirac field, the vector current always satisfies the easily-verified relation

$$\overline{\psi^c}\gamma_\mu\psi^c = -\bar{\psi}\gamma_\mu\psi \ . \tag{44}$$

On the other hand, in the Majorana case Eq. (38) implies that

$$\overline{\psi^c}\gamma_\mu\psi^c = +\bar{\psi}\gamma_\mu\psi \ . \tag{45}$$

Thus,

$$\begin{aligned}\langle\nu_f^M|\overline{\psi^M}\gamma_\mu(1 + \gamma_5)\psi^M|\nu_i^M\rangle &= \langle\nu_f^M|\overline{\psi^M}\gamma_\mu\gamma_5\psi^M|\nu_i^M\rangle \\ &= [\frac{M^2}{E_f E_i V^2}]^{1/2}(\bar{u}_f\gamma_\mu\gamma_5 u_i - \bar{v}_i\gamma_\mu\gamma_5 v_f) \ ,\end{aligned} \tag{46}$$

where the second term in the second line arises from the fact that ψ^M , Eq. (37), as well as $\overline{\psi^M}$, can both annihilate and create neutrinos. Using $v_{\vec{p}s} = C\bar{u}^T_{\vec{p}s}$, we may rewrite Eq. (46) as

$$\langle\nu_f^M|\overline{\psi^M}\gamma_\mu(1 + \gamma_5)\psi^M|\nu_i^M\rangle = [\frac{M^2}{E_f E_i V^2}]^{1/2}\, 2\bar{u}_f\gamma_\mu\gamma_5 u_i \ . \tag{47}$$

On the surface, the Dirac and Majorana cases, Eqs. (43) and (47), appear to be quite different. Alas, as the "practical Dirac-Majorana confusion theorem" predicts, this difference evaporates as the neutrino mass goes to zero. Indeed, so long as the incoming (left-handed) neutrino is relativistic, its u spinor satisfies $\gamma_5 u_i \cong u_i$, so that the right-hand sides of Eqs. (43) and (47) cannot be distinguished. That is why the voluminous data on neutral-current scattering have told us nothing about whether neutrinos are Dirac or Majorana particles.

Are there ways of getting around the "Dirac-Majorana confusion theorem"? Suggestions would certainly be welcome. I myself would like to suggest a possibility whose consideration is instructive. Suppose the dominant mass eigenstate in ν_μ has a mass greater than 10 keV.[23] Then, in a 600 GeV pion beam, such as will be available at the Fermilab Tevatron, all neutrinos from the decay $\pi^+ \to \mu^+ + \nu_\mu$ will have laboratory momenta in the forward hemisphere with respect to the beam direction. Consider, now, a ν_μ which is emitted exactly backward, in the pion rest frame, by a pion in such a beam. As depicted in Fig. 5(a), in the pion rest frame the neutrino will have negative helicity (apart from a negligible correction of order M_{ν_μ}/E^*, where E^* is the energy of the ν_μ in the pion rest frame). However, in the lab. frame the decay will appear as shown in Fig. 5(b). The momentum of the ν_μ will point forward, and its helicity will be <u>positive</u>. Now, if this right-handed ν_μ is a Dirac particle, then it will not interact, because the weak currents are left-handed.[24] On the other hand, if it is a Majorana particle, then, on account of its helicity, it is just what we usually call a $\bar{\nu}_\mu$. Thus, it will interact, but, being a $\bar{\nu}_\mu$, will produce a μ^+, rather than a μ^-. Hence, by determining whether the mounic neutrinos emitted backward in the pion rest frame fail to interact or produce positive muons when they strike a target, we can find out whether they are Dirac or Majorana particles. Also, by observing that some fraction of the neutrinos emitted by fast positive pions exhibit either one of these anomalous behaviors, with the expected dependence on the pion energy, we can demonstrate that the ν_μ does have a non-zero mass. Will such an experiment work?

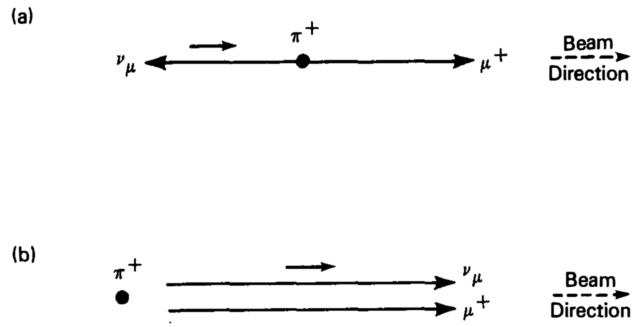

Fig. 5. The decay $\pi^+ \to \mu^+ + \nu_\mu$ of a fast pion into a massive neutrino as seen in (a) the pion rest frame, and (b) the lab. frame. The arrow over the neutrino line represents its spin.

Unfortunately, it will not work. First of all, suppose the neutrinos must travel down a neutrino beam line which is, say, 1 km. long before they intercept the detector which is, say, of order 1 m. wide. Then, to be detectable, they must have laboratory momenta which are not only in the forward hemisphere with respect to the beam direction, but in the forward-most milliradian. Now, let θ^* be the angle, with respect to the beam direction, at which a neutrino is emitted in the pion rest frame. Further, let β_π be the speed of the pion in the lab., and β^* the speed of the ν_μ in the pion rest frame. For the neutrinos with $\theta^* = \pi$ to go forward in the lab., we must have $\beta_\pi > \beta^*$, and we may write

$$\frac{\beta_\pi}{\beta^*} = 1 + \varepsilon \;, \qquad (48)$$

where the small quantity ε is given by

$$\varepsilon \simeq \frac{1}{2}\left[\left(\frac{M_{\nu_\mu}}{E^*}\right)^2 - \left(\frac{M_\pi}{E_\pi}\right)^2\right] . \qquad (49)$$

If $E_\pi = 600$ GeV , the neutrinos which are in the forward-most milliradian in the lab. are those with $\theta^* \lesssim \frac{5}{6}\pi$, or with $(\pi-\theta^*) \lesssim 4\varepsilon$. Now, the neutrinos with $(\pi-\theta^*) \lesssim 4\varepsilon$ are far too small a fraction of the neutrino flux to be useful. Those with $\theta^* \lesssim \frac{5}{6}\pi$ are a large fraction of the flux, but, as we shall see, the lab. frame helicities of these neutrinos differ only infinitesimally from their pion rest frame helicity of -1 ! Thus, any of these neutrinos will interact like a normal ν_μ , regardless of its Dirac or Majorana character.

The lab. frame helicity h of a neutrino whose pion rest frame helicity h^* is -1 may be found by Lorentz transforming to the lab. frame the left-handed spinor u^* which describes the neutrino in the pion frame. With much less work, it may also be found from a general expression in the literature[25] describing the Lorentz transformation properties of helicity. Either way, one finds that

$$h = -\frac{E_\pi p^* + p_\pi E^* \cos\theta^*}{M_\pi p} , \tag{50}$$

where p^* and p are, respectively, the magnitudes of the neutrino momentum in the pion rest frame and in the lab., and p_π is the momentum of the pion in the lab. From Eq. (50) it is trivial to show that so long as $E^* \cong p^*$, $E_\pi \cong p_\pi$, and θ^* is not near π ,

$$h \cong -1 . \tag{51}$$

Thus, the neutrinos with $\theta^* \lesssim \frac{5}{6}\pi$ will have quite ordinary behavior. Turning to the few neutrinos with θ^* very near π , we expand Eq. (50) in $(\pi-\theta^*)$, assuming that $M_{\nu_\mu} > 30$ keV , so that these neutrinos are forward-going and relativistic in the lab.[24] This yields

$$h \cong 1 - \frac{(\pi-\theta^*)^2}{\varepsilon} . \tag{52}$$

When $\theta^* = \pi$, h is indeed $+1$ as we have said, and it remains near this value until $(\pi-\theta^*) \sim \sqrt{\varepsilon}$. Hence, the uselessly few neutrinos with $(\pi-\theta^*) \lesssim 4\varepsilon$ do have h near $+1$. Equation

(52) suggests that the point beyond which h is quite far from $+1$ is $(\pi-\theta^*) \sim \sqrt{\varepsilon}$. Now, it is easy to show that when even the neutrinos with $\theta^* \approx \pi$ are forward-going and relativistic in the lab., the second term on the right-hand side of Eq. (49) may be neglected, so that $\sqrt{\varepsilon} \sim M_{\nu_\mu}/E^*$. Thus, the point beyond which h is quite far from $+1$ is $p_\perp \sim M_{\nu_\mu}$, where $p_\perp$ is the transverse momentum of the neutrino.

In retrospect, it should not have surprised us that a relativistic neutrino with θ^* not near π has almost exactly the same helicity in the lab. as in the pion rest frame, even though its momentum may point in very different directions in the two frames. We know, after all, that this would be true for a massless particle. Now, the Lorentz transformation from the pion rest frame to the lab. may be viewed as a succession of small transformations, throughout each of which our neutrino remains relativistic. Thus, it behaves very much like a massless particle. This picture only breaks down when θ^* is very near π ($p_\perp \lesssim M_{\nu_\mu}$), so that we are considering a neutrino whose momentum points in almost <u>exactly opposite</u> directions in the two frames of interest. The direction of the momentum of a massless particle can never be <u>exactly</u> reversed by a Lorentz transformation. However, the momentum of a massive particle can be <u>exactly</u> reversed, so when θ^* is sufficiently near π, the massless and massive cases are quite different.[26]

A basic attribute of any particle is its electromagnetic properties. Let us contrast Dirac and Majorana neutrinos in this regard. Of course, they are both electrically neutral, but perhaps they can have dipole moments.

Suppose a Majorana neutrino had a magnetic dipole moment μ_{Mag} and an electric dipole moment $\mu_{E\ell}$. If this neutrino were at rest in a combination of static, uniform magnetic and electric fields, its interaction energy would be $-\mu_{Mag} \langle \vec{s} \cdot \vec{B} \rangle - \mu_{E\ell} \langle \vec{s} \cdot \vec{E} \rangle$. Here $\vec{s}$ is, or course, the neutrino spin operator. Now, under CPT, the $\vec{B}$ and $\vec{E}$ fields go into themselves. However, as we have said, the effect of CPT on a Majorana neutrino is to reverse its spin. Consequently, the dipole interaction energy changes sign under CPT. Thus, if CPT invariance holds, μ_{Mag} and $\mu_{E\ell}$ must vanish.[18]

By contrast, a Dirac neutrino is permitted to have dipole moments. Thus, as mentioned earlier, its helicity can be reversed by the torque,

$$\frac{d\vec{s}}{dt} = \vec{\mu}_{Mag} \times \vec{B} , \tag{53}$$

exerted by an external magnetic field.[27],[28] This helicity-reversal could be detected using the fact that a left-handed neutrino interacts, while a right-handed neutrino does not. Unfortunately, however, the magnetic dipole moment of a neutrino is expected to be quite small. The standard model predicts that for a neutrino of mass M [29]

$$\mu_{Mag} = \frac{3eG_F M}{8\pi^2\sqrt{2}} \simeq 3 \times 10^{-19}\left(\frac{M}{1eV}\right)\mu_{Bohr} . \tag{54}$$

Here μ_{Bohr} is the Bohr magneton. Shrock has considered the possibility of laboratory experiments to detect such a tiny dipole moment, and has concluded that they would not be feasible.[30]

There is only one experimental approach which currently shows promise as a means for determining whether neutrinos are Majorana or Dirac particles. This approach, which is being vigorously pursued, is the search for neutrinoless double beta decay [$(\beta\beta)_{0\nu}$].[31],[32] In this process, a nucleus would decay to another with two additional protons through the emission of two electrons <u>unaccompanied</u> by any antineutrinos:

$$(A,Z) \longrightarrow (A,Z+2) + 2e^- + 0\bar{\nu} . \tag{55}$$

The decay would be engendered by the diagram in Fig. 6, in which a pair of W bosons, emitted by two neutrons in the parent nucleus, produces the outgoing electrons through the exchange of a virtual neutrino. Of course, we expect that there exist several neutrino mass eigenstates ν_m. Thus, as Fig. 6 indicates, the amplitude for the process is actually a sum over the contributions of all of them, the coupling of a given ν_m to an electron being described by the mixing matrix element U_{em}. Notice that the exchanged neutrino is emitted together with an e^- at one vertex, and then absorbed to produce another e^- at a second vertex. Considering the first vertex, we would conclude that this "neutrino" is actually a $\bar{\nu}$, but considering the second we would conclude that it is a ν . Thus, for $(\beta\beta)_{o\nu}$ to occur, we must have $\bar{\nu} = \nu$. That is, the exchanged neutrinos ν_m must be Majorana particles.

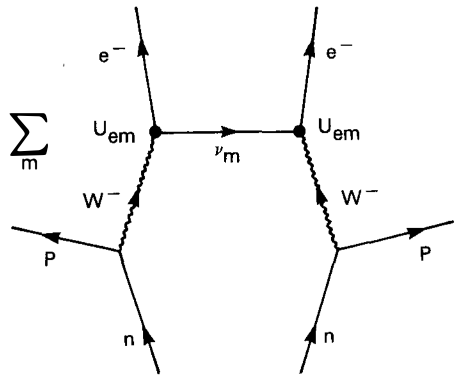

Fig. 6. Neutrinoless double beta decay.

In addition, for $(\beta\beta)_{o\nu}$ to occur, the ν_m must be massive, or else there must be right-handed currents, or both. If all weak currents are left-handed, then each ν_m , behaving like an antilepton at the vertex where it is emitted, is created in a predominantly right-handed state. However, at the vertex where it is absorbed, it behaves like a lepton, and so the current would prefer to absorb it from a left-handed state. The amplitude for the exchanged ν_m to be created left-handed is of order M_m/E_m , so the contribution of this ν_m to $(\beta\beta)_{o\nu}$ is proportional to M_m . However, if there is a right-handed current, then it can readily absorb a right-handed ν_m , and the ν_m contribution no longer depends on a non-zero M_m . Tests which could determine whether $(\beta\beta)_{o\nu}$, once it is observed, is resulting from non-zero masses or from right-handed currents have been proposed by Rosen.[33]

Assuming that there are no right-handed currents, and that to a good approximation CP is conserved in the lepton sector, as it is in the quark sector, the $(\beta\beta)_{o\nu}$ amplitude $A[(\beta\beta)_{o\nu}]$ can be shown to have the form

$$A[(\beta\beta)_{o\nu}] = [\sum_m \frac{\tilde{\eta}_{CP}(\nu_m)}{i} |U_{em}|^2 M_m]\bar{A} \equiv M_{eff}\bar{A} . \qquad (56)$$

In correspondence with Fig. 6, this amplitude is a sum over the contributions of the different ν_m . The contribution of each ν_m is proportional to its mass as we have explained, to the square of its coupling to an electron, and, finally, to its CP-parity $\tilde{\eta}_{CP}(\nu_m)$.[34],[35],[3],[36] Note that a Majorana neutrino, being an eigenstate of CPT (followed by a rotation), can have a well-defined value of CP when CP is conserved. However, it turns out that its CP-parity must be imaginary![36],[3] Thus, it is $\tilde{\eta}_{CP}(\nu_m)/i$ which is real. The quantity $\bar{A}$ in Eq. (56) is independent of m , and includes the very non-trivial nuclear matrix element governing neutrinoless double beta decay. Owing to the factor $\tilde{\eta}_{CP}(\nu_m)/i = \pm 1$ in Eq. (56), $(\beta\beta)_{o\nu}$ measures an effective mass, M_{eff} , which is a sum over actual neutrino masses M_m weighted by factors which can be either negative or positive.[37] Thus, an experimental upper limit on $A[(\beta\beta)_{o\nu}]$, translated into an upper limit on M_{eff} through one's knowledge of the relevant nuclear matrix element $\bar{A}$, does not necessarily imply an upper limit on the mass of any one neutrino.

Regardless of whether neutrinos are of Majorana character or not, nuclear double beta decay with the emission of antineutrinos to conserve lepton number,

$$(A,Z) \longrightarrow (A,Z+2) + 2e^- + 2\bar{\nu}_e , \qquad (57)$$

certainly does occur. Note, however, that four leptons are emitted in this decay mode, while only two would be emitted in the neutrinoless double beta decay in which we are interested. As a result, phase space favors the latter decay mode by six orders of magnitude! It is this circumstance which makes $(\beta\beta)_{o\nu}$ an especially promising reaction in which to look for evidence that neutrinos are Majorana particles, even though the reaction is suppressed by the small values of the neutrino masses, and vanishes, in conformity with the "Dirac-Majorana confusion theorem," when the masses go to zero.

The best current limit on M_{eff} comes from a geochemical study which compares the rates for double beta decay (with or without neutrinos) of two different tellurium isotopes.[38] From the geochemical data and ratios of nuclear matrix elements,[39] it is found that[40]

$$M_{eff} < 5.6 \text{ eV} . \tag{58}$$

Hopefully, future experiments will probe the region of even smaller masses.

Summary

From the standpoint of grand unified theories, it is more natural for neutrinos to be massive than massless. Thus, it is a good idea to have this Workshop! However, the neutrinos may be rather light compared to 1 eV . I think it would be worthwhile to give some thought to this possibility. If neutrinos do have mass, then there are fascinating physical consequences, such as neutrino oscillation, and a possibly very important role for neutrinos in astrophysics. Impressive efforts, both past and planned, to measure neutrino masses are discussed in these Proceedings. Laudable attempts to find out whether neutrinos are indeed their own antiparticles, a physically very interesting possibility, are also described.

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