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GRAVITATIONALLY MEDIATED ENTANGLEMENT AND DECOHERENCE

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ABSTRACT

Presently contemplated experiments propose to test whether or not gravity itself can serve as a mediator for quantum entanglement. The detection of such gravitationally mediated entanglement would provide the first example of an observed phenomenon that cannot be explained within the framework of classical general relativity, and would constitute the first experimental test of quantum gravity. This dissertation, based on a series of published articles [Danielson et al., 2022a,b, 2023, 2025b], develops the implications of gravitationally mediated entanglement should it prove to exist in Nature. By analyzing an apparent paradox between causality and complementarity in a gedankenexperiment, it is shown that the experimental discovery of gravitationally mediated entanglement may be viewed as implying the existence of the graviton. A similar gedankenexperiment outside a black hole then shows that a quantum superposition maintained outside a black hole must undergo a constant rate of decoherence, so that a black hole will eventually decohere any quantum superposition in its exterior. This occurs because of an unavoidable accumulation of soft, entangling gravitons on the black hole horizon, in a direct mathematical analog of the gravitational memory effect at null infinity. A similar quantum gravitational decoherence effect is shown to arise in the presence of a cosmological horizon, and more generally in the vicinity of any Killing horizon. The resulting decoherence rates are predicted in each case. A fully local account of this decoherence is developed, and reveals the soft radiation can be viewed as the result of extremely low frequency vacuum fluctuations interacting with the superposition. The decoherence in the presence of black holes is contrasted with the decoherence that would arise in the spacetime of a star, in a thermal bath, or in the presence of an ordinary material body.

To my parents, Nancy Haubrich and Robert D. Danielson, and to my grandparents, Dorothy Danielson, Robert W. Danielson, Jeanette Haubrich, and Harold Haubrich, for forming the foundation of my life. To my beloved friends, for teaching me what life is about. To my great aunt and uncle June Drake and Jim Drake, for teaching me what science is about.

No elementary phenomenon is a phenomenon until it is an observed phenomenon.

—John Archibald Wheeler, “Law Without Law.”

TABLE OF CONTENTS

LIST OF FIGURES

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CHAPTER 1

INTRODUCTION

General relativity and quantum field theory are the two fundamental pillars of modern physics. Their union in the form of a theory of quantum gravity remains the most significant open issue in theoretical physics. Although one can formulate an essentially satisfactory theory of linearized quantum gravity perturbed off of some fixed background spacetime, severe difficulties arise in formulating a nonperturbative theory of quantum gravity. While strong arguments can be given that gravity should be quantized [Bronstein, 2012, Page and Geilker, 1981, Eppley and Hannah, 1977, Mattingly, 2006, Carlip, 2008, Giampaolo and Macrì, 2019], these difficulties have led some to suggest that gravity may be fundamentally classical, that the description of gravity with quantum mechanics requires a radical modification of quantization [Hossenfelder, 2010, Penrose, 2014, Diósi, 1987], or that the question of quantization is ill-posed [Dyson, 2013]. Of central importance to this debate is the prediction that it should be possible to entangle two initially uncorrelated bodies purely by virtue of their gravitational interaction. Intimately related with this is the prediction of quantized gravitational radiation in the form of gravitons, the existence of which has not yet been verified experimentally.

As already noted by Feynman in the 1950s [DeWitt-Morette and Rickles, 2011, Zeh, 2011], some key issues regarding the quantization of gravity can be explored by considering the gravitational field sourced by a quantum superposition of a massive body. Due to recent advances in maintaining coherent spatial superpositions,[1] many actual experiments involving such superpositions have recently been proposed [Ford, 1982, **?**, Bahrami et al., 2015]. Given the rapid progress toward proposed "low-energy" tabletop experiments [Bose et al., 2017, Marletto and Vedral, 2017, Carney et al., 2019, Haine, 2021, Qvarfort et al., 2020, Carlesso et al., 2019, Howl et al., 2021, Matsumura and Yamamoto, 2020, Pedernales et al., 2022, Liu

1. Spatial superpositions of masses on the scale of 10^5 amu over distances of order microns have been achieved [Gerlich et al., 2011, Eibenberger et al., 2013, Romero-Isart, 2017, Fein et al., 2019] and recent proposals have suggested up to nanogram scale superpositions [Pino et al., 2018, Brand et al., 2017].

et al., 2021, Datta and Miao, 2021, Gonzalez-Ballestero et al., 2021, Krisnanda et al., 2020, Margalit et al., 2021, Christodoulou and Rovelli, 2019, Bose et al., 2022], it is of interest to understand what such low-energy phenomena might teach us about the fundamental nature of quantum gravity.

The analysis by Belenchia *et al.* [Belenchia et al., 2018, 2019] of a gedankenexperiment originally proposed by [Mari et al., 2016] provides strong evidence that low-energy experiments can probe quantum field theoretic aspects of gravity. In this gedankenexperiment, an experimenter, Alice, puts a massive body (hereinafter referred to as a "particle") into a quantum superposition at different spatial locations. At a later time, she recombines the particle and determines its quantum coherence. In the meantime—at a spacelike separation from the recombination portion of Alice's experiment—another experimenter, Bob, measures the Newtonian gravitational field of Alice's particle to try to determine its position. If Bob succeeds, then by complementarity, Alice's particle must be decohered. But, if Bob influences the state of Alice's particle, then causality would be violated. The analysis by Belenchia *et al.* [Belenchia et al., 2018, 2019] showed that, in order to avoid contradictions with complementarity or causality, quantum gravity must have fundamental features of a quantum field theory at low energies, specifically the quantization of gravitational radiation (which decoheres Alice's particle without the presence of Bob) and local vacuum fluctuations (which limits Bob's ability to measure the position of Alice's particle). However, the analysis of [Belenchia et al., 2018, 2019] made only back-of-the-envelope estimates for the decoherence effects associated with Alice's recombination and Bob's measurement. Furthermore, it considered only a particular type of measurement that Bob might make. An important purpose of Chapter 2 (based on [Danielson et al., 2022a]) is to reanalyze this gedankenexperiment, allowing Bob to make any measurement whatsoever in the region spacelike separated from Alice's recombination region. We provide a precise analysis of the decoherence associated with radiation emitted by Alice's particle and the decoherence associated with Bob's measurement.

We thereby confirm in a rigorous way the conclusions that had been drawn in [Belenchia et al., 2018, 2019] from their back-of-the-envelope estimates.

Proposed experimental probes of gravitationally mediated entanglement [Ford, 1982, **?**, Bahrami et al., 2015, Bose et al., 2017, Marletto and Vedral, 2017, Carney et al., 2019, 2022, Feng and Vedral, 2022] will search for entanglement mediated by the superposed Newtonian field of a body. Thus the implications of such a discovery are of significant practical interest. The crux of our rigorous reformulation of the gedankenexperiment is to show that, under the protocols of the thought experiment, no meaningful distinction can be made between quantized gravitational radiation—gravitons, and the Newtonian field of a body. In fact, we show the causal consistency of the theory dictates that the existence of entanglement mediated by the constraints of gravity, i.e., the "Newtonian field," implies the existence of entanglement mediated by gravitons. It follows therefore that the experimental discovery of entanglement mediated by a quantum state of the Newtonian field may be viewed as implying the existence of the graviton as a fundamental constituent of Nature.

Entanglement and decoherence are two sides of the same coin, insofar it is impossible for a system to become entangled with another without accruing a commensurate degree of decoherence. In the gedankenexperiment involving Alice and Bob, we establish that Alice's particle must be decohered by gravitational radiation whenever it is possible for Bob to measure the Newtonian field of her particle. In flat spacetime, the requirement that Bob measures the field at spacelike separation from the recombination portion of Alice's experiment implies that if Alice recombines very slowly and thus minimizes the energy radiated into the gravitational field, then Bob must perform his measurement a great distance away to remain spacelike. Therefore, Bob must resolve a very small distinction between the superposed Newtonian fields, which of course fall off with distance. The precision of Bob's field measurement is, however, limited by the local vacuum fluctuations of the metric, which force Bob to measure the field over a longer and longer time to make a measurement at farther and farther distances. In flat spacetime this implies that if Alice performs her experiment adiabatically and radiates an arbitrarily small amount of energy during her recombination process then Bob will only gain an arbitrarily small amount of which-path information as he is forced to measure Alice's Newtonian field at an arbitrarily far distance from its source. This entire situation changes dramatically in the spacetime of a black hole, which is what we demonstrate in Chapter 3.

Chapter 3 describes a new gedankenexperiment, in which Alice uses rocket thrusters to hold her laboratory at a fixed distance from a black hole. There she adiabatically moves a massive body into a quantum superposition of positions, and later adiabatically recombines the spatial branches of her superposition and determines its quantum coherence. Another experimenter, Bob, will again attempt to decohere Alice's particle by measuring its Newtonian field, and as before he must perform his measurement entirely at spacelike separations from the recombination portion of Alice's experiment. If Bob performs his measurement outside the black hole, then the conclusion is no different than in flat spacetime: Alice has performed her experiment adiabatically, so Bob is forced to remain extremely far from Alice and gains almost no which-path information. Now, however, Bob has another option available to him: he can "hide" his experiment in the black hole interior. By doing this he can remain spacelike separated from Alice's recombination process at a fixed distance, regardless of how slowly Alice performs her experiment (we assume the black hole is sufficiently massive that it does not evaporate a significant fraction of its mass in this time, and so that Bob can successfully measure the Newtonian field before reaching the singularity). Now nothing prevents Bob from performing a successful which-path measurement while remaining spacelike from Alice's recombination process, in spite of the fact that Alice can minimize the energy she radiates into the black hole to an arbitrarily small amount. Complementarity dictates that if Bob is able to obtain which-path information, then Alice's particle must be decohered. On the other hand, causality dictates that Alice cannot learn about a measurement performed by

Bob inside the black hole. This paradox suggests one resolution: there would be no paradox if the mere presence of a black hole decoheres all superpositions in its exterior *as if* Bob were performing a measurement of the Newtonian field in the interior.

Motivated by this gedankenexperiment, we proceed along the lines of [Danielson et al., 2022b] to show that if a massive body is put in a quantum superposition of spatially separated states, the mere presence of a black hole in the vicinity of the body will eventually destroy the coherence of the superposition. This occurs because, in effect, the gravitational field of the body radiates soft gravitons into the black hole, allowing the black hole to acquire “which path” information about the superposition. A similar effect occurs for quantum superpositions of electrically charged bodies, and we provide estimates of the decoherence time for such quantum superpositions.

Chapter 4 (based on [Danielson et al., 2023]) generalizes the results of Chapter 3 to spacetimes with Killing horizons, i.e., spacetimes with a Killing vector field such that there is a null surface to which the Killing field is normal. The event horizon of a stationary black hole is a Killing horizon [Hawking and Ellis, 1973, Hawking, 1972, Alexakis et al., 2010], so spacetimes with Killing horizons encompass the case of stationary spacetimes that contain black holes. However, there are many cases of interest where Killing horizons are present without the presence of black holes. One such case is that of Minkowski spacetime, where the Rindler horizon is a Killing horizon with respect to the Lorentz boost Killing field. Another such case is de Sitter spacetime, where the cosmological horizon is a Killing horizon. We will show that in these cases, a spatial superposition that is kept stationary (with respect to the symmetry generating the Killing horizon) will decohere in a manner similar to the black hole case. We will also provide an estimate of the maximum amount of time during which coherence can be maintained.

The main purpose of Chapter 5 (based on [Danielson et al., 2025b]) is to show that one can give a purely local description of the decoherence in terms of the behavior of the quantum field within Alice's lab. From this viewpoint, the decoherence arises from the behavior of the unperturbed two-point function of the quantum field in the region where the superposition was created. In particular, the decoherence in the presence of a black hole can be understood as resulting from the extremely low frequency Hawking radiation that partially penetrates into Alice's lab before being reflected back into the black hole by the effective potential of the black hole. This local viewpoint will enable us to gain insights into various aspects of the decoherence process, such as the differences in decoherence that occur in different vacuum states and in different spacetimes. We will also gain insight into the requirements on a material body to mimic the decoherence effects of a black hole.

CHAPTER 2

NEWTONIAN FIELD VERSUS GRAVITONS

The analysis of this chapter sheds additional light on the issue of whether tabletop experiments probe only quantum properties of the Newtonian gravitational field [Anastopoulos and Hu, 2018]. In the gedankenexperiment of Chapter 1, since Bob sees only the Newtonian gravitational field of Alice's superposition during the time of his measurement, it is natural to view this Newtonian field as mediating entanglement between Bob and Alice. Indeed, if Alice decides to recombine her body at a much later time, the resulting correlations between the state of Bob's measuring apparatus and the state of Alice's particle must be viewed as having been mediated by the Newtonian field of Alice's particle. However, we will show that if Alice follows her protocol and recombines her particle in a region spacelike separated from Bob's measurements, then it is much more natural to view Bob as having measured on-shell gravitons that were emitted by Alice's particle; i.e., although Bob may believe that he is measuring a Newtonian gravitational field, he is actually measuring long wavelength gravitons. This viewpoint makes it clear that if the protocols of the gedankenexperiment are followed, then Bob is merely a "bystander" and his measurements have no relevance to the decoherence of Alice's particle.

Thus, in the circumstances of our gedankenexperiment, there is no clear distinction between entanglement of Alice's particle with Bob's apparatus that is mediated by a Newtonian field and entanglement of Alice's particle with gravitons that then interact with Bob's apparatus. This suggests that, in more general circumstances, entanglement mediated by a Newtonian field is not fully distinguishable from entanglement with gravitons and, hence, that the experimental discovery of entanglement by a Newtonian field may be viewed as evidence for existence of the graviton as a fundamental particle of nature.[1] Furthermore, our analysis provides support for the conclusions of [Belenchia et al., 2019] that the Newtonian

1. Additional arguments for this conclusion have been given in [Carney, 2022].

field itself can store and transmit quantum information.

In Sec. 2.1, we review the gedankenexperiment of [Mari et al., 2016] and its analysis by [Belenchia et al., 2018]. In Sec. 2.2 we analyze the decoherence effects associated with the emission of quantized radiation by Alice's particle and the decoherence effects associated with measurements made by Bob. In Sec. 2.3 we reanalyze the gedankenexperiment in a more precise way and provide a proof that no violations of causality or complementarity occur. Some further remarks and conclusions are given in Sec. 2.4.

Throughout the chapter, we will work in Planck units where $G = c = \hbar = 1$.

2.1 The Gedankenexperiment of Mari *et al.* and its Resolution by Belenchia *et al.*

In this section we review the gedankenexperiment initially proposed by Mari *et al.* [Mari et al., 2016] and its resolution given by Belenchia *et al.* [Belenchia et al., 2018]. There are electromagnetic and gravitational versions of this gedankenexperiment. For simplicity and definiteness, we shall first focus on the electromagnetic version and then discuss the modifications to the analysis needed for the gravitational case.

The gedankenexperiment is illustrated in Fig. 2.1. At some time in the distant past, Alice sent a charged particle with spin in the positive x direction through a Stern-Gerlach apparatus that is oriented in the z direction. We assume that this process was done sufficiently slowly so as to produce negligible radiation and that Alice's particle can be described by ordinary, nonrelativistic quantum mechanics. After going through the Stern-Gerlach apparatus, her particle is then in a superposition state of the form

$$\frac{1}{\sqrt{2}}\big(\,|{\uparrow}; A_1\rangle + |{\downarrow}; A_2\rangle\,\big) \tag{2.1.1}$$

where $|A_1\rangle$ and $|A_2\rangle$ describe spatially separated wave packets and $|{\uparrow}\rangle$ and $|{\downarrow}\rangle$ represent

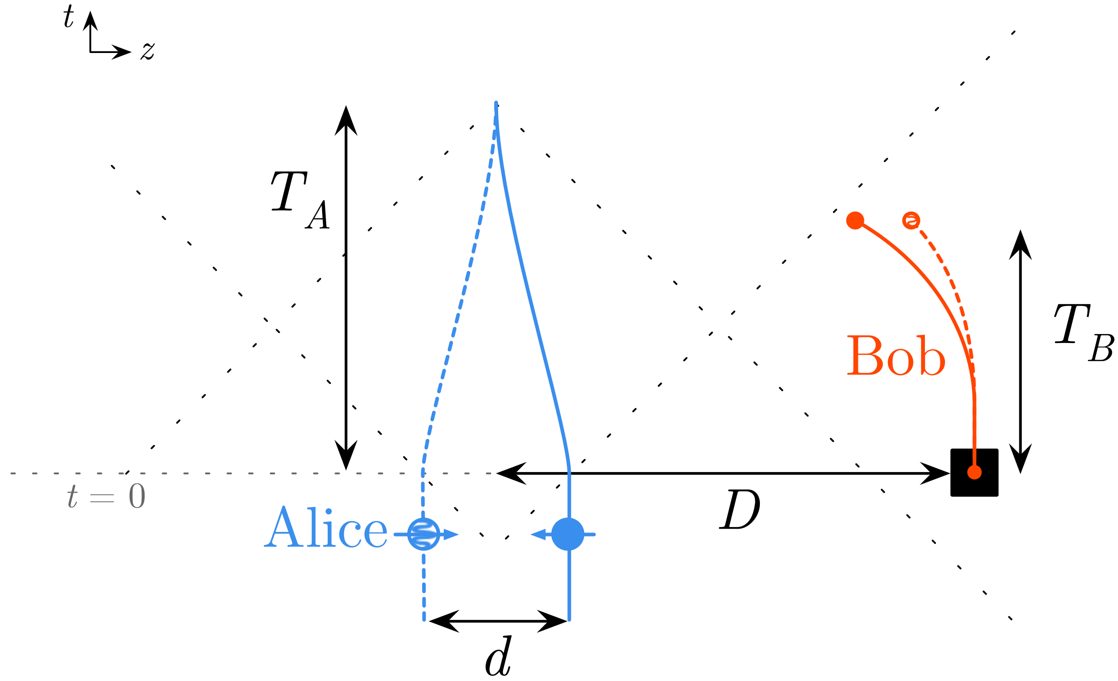

Figure 2.1: The setup for the gedankenexperiment of [Mari et al., 2016], as analyzed in [Belenchia et al., 2018]. Alice's particle (in blue) is originally in the superposition state Eq. (2.1.1) with the two wave packets separated by distance d. Bob is at a distance $D \gg d$ from Alice and, at a prearranged time, he releases a particle (in orange) from a trap and attempts to gain information about which path Alice's particle took by determining the strength of the Coulomb/Newtonian field of Alice's particle. Meanwhile, at a corresponding prearranged time, Alice recombines her particle and determines its coherence as described in the text. Bob does his measurement within time $T_B < D$ and Alice recombines her particle in time $T_A < D$, so their actions are performed in spacelike separated regions.

eigenstates of z spin. At a prearranged time, Bob attempts to determine which path Alice's particle followed by measuring the Coulomb field of Alice's particle. One way that Bob could do this is to release a charged particle from a trap at the prearranged time; if Alice's particle takes the right path in Fig. 2.1, the Coulomb field near Bob will be stronger and the motion of Bob's particle will be influenced more, so by measuring the position of his particle at a later time, Bob can obtain some "which-path" information about Alice's particle. At a corresponding, prearranged time, Alice recombines her particle by putting it through a "reversing Stern-Gerlach apparatus" [Mari et al., 2016, Bose et al., 2017]. Alice then determines the coherence of her recombined particle by measuring its spin in the x direction.[2] If her had particle maintained perfect coherence, it would evolve back to an eigenstate of spin in the positive x direction. By contrast, if the components of the original superposition Eq. (2.1.1) had completely decohered, Alice would find that the spin is in the positive x direction only 50% of the time. By repeating the gedankenexperiment as many times as necessary, Alice can build up good statistics on the x spin and thereby determine the degree of decoherence of her particle. By the prearranged protocol, the spacetime region in which Alice does the recombination and spin measurement is spacelike separated from the region in which Bob does his measurements, as illustrated in Fig. 2.1.

This gedankenexperiment appears to lead to a contradiction with complementarity or causality. If Bob acquires any which-path information from his measurement, the state of Bob's particle must be correlated with Alice's to some degree. In that case, by complementarity, Alice's particle cannot be in a perfectly coherent superposition and she will find her particle to have spin in the negative x direction some of the time. On the other hand, since Bob and Alice perform their actions in spacelike separated regions, by causality, it is impossible for Bob's measurements to have any effect on Alice's results, so the fact that he obtained some which-path information cannot degrade the coherence of Alice's particle. So, if Bob's measurement does not influence Alice's spin measurement, we would appear to have a violation of complementarity, whereas if Bob's measurement does influence Alice's spin measurement, we have a clear violation of causality.

A resolution of this apparent paradox was given in [Belenchia et al., 2018]. This resolution is based upon Bob's limitations in acquiring which-path information due to vacuum fluctuations and Alice's limitations in maintaining coherence due to the emission of entangling

2. In the version of the gedankenexperiment discussed in [Belenchia et al., 2018], Alice determines the coherence of her particle by performing an interference experiment on the particle wave packets. An alternative resolution of that version of the gedankenexperiment was proposed in [Rydving et al., 2021], based upon postulating fundamental limits to the ability to resolve interference fringes as originally proposed by [Baym and Ozawa, 2009]. This alternative resolution would not be applicable to the version of the gedankenexperiment being considered here.

radiation. Bob's limitations due to vacuum fluctuations can be estimated as follows. In the electromagnetic case, the difference of the Coulomb electric fields associated with the different paths of Alice's particle is given by

$$E \sim \frac{\mathcal{D}_A}{D^3} \tag{2.1.2}$$

where D is the distance between Alice and Bob and $\mathcal{D}_A = q_A d$, where q_A is the charge of Alice's particle and $d \ll D$ is the distance between the two paths of Alice's particle. If Bob must perform his measurement in time T_B, the difference in the final position of his particle due to the difference in the Coulomb fields of Alice's particle is

$$\delta x \sim \frac{q_B}{m_B} \frac{\mathcal{D}_A}{D^3} T_B^2 \tag{2.1.3}$$

where q_B is the charge of Bob's particle and m_B is its mass. On the other hand, vacuum fluctuations of the electromagnetic field produce fluctuations in the position of Bob's particle of order

$$\Delta x \sim \frac{q_B}{m_B}. \tag{2.1.4}$$

Thus, on account of the "noise" due to vacuum fluctuations, Bob can acquire significant which-path information only if

$$\frac{\mathcal{D}_A}{D} > \left(\frac{D}{T_B}\right)^2. \tag{2.1.5}$$

In particular, if Bob abides by his protocol $T_B < D$, he can acquire significant which-path information only when $\mathcal{D}_A > D$.

Alice's limitations on maintaining coherence due to radiation can be estimated as follows. When Alice recombines her particle over a time T_A, she reduces the initial effective dipole $\mathcal{D}_A$ to zero. By the Larmor formula, this should result in emission of entangling radiation corresponding to an average energy flux $\sim (\mathcal{D}_A/T_A^2)^2$. Thus the total energy radiated should be $\sim \mathcal{D}_A^2/T_A^3$. This radiation should be composed of photons of frequency $\sim 1/T_A$. Thus the total number of entangling photons emitted when Alice recombines her particle should be

$$N \sim \frac{\mathcal{D}_A^2}{T_A^2}. \tag{2.1.6}$$

If $N > 1$, then Alice's particle will undergo significant decoherence due to entanglement with radiation, independent of what Bob does. In particular, if Alice abides by her protocol $T_A < D$, she can maintain coherence only when $\mathcal{D}_A < D$.

The above estimates allow one to provide the resolution given in [Belenchia et al., 2018]. If $\mathcal{D}_A > D$, then Bob can acquire significant which-path information, so by complementarity, Alice's particle must correspondingly be significantly decohered. However, in this case the radiation emitted when Alice does her recombination will decohere her particle independently of what Bob does, so there is no reason to believe that Bob's measurement "caused" the decoherence, i.e., there is no obvious violation of causality. On the other hand, if $\mathcal{D}_A < D$, then Alice should be able to largely maintain the coherence of her particle during the recombination. But in this case, Bob cannot acquire significant which-path information, so complementarity does not imply decoherence of Alice's particle and, again, there is no obvious violation of causality.

The analysis of the gravitational version of the gedankenexperiment within the context of linearized quantum gravity is very similar, with the main difference being the replacement of "dipole" by "quadrupole." Alice's original separation of the particle into a superposition of two paths does not produce an effective dipole on account of conservation of center of mass—her laboratory must produce an equal and opposite compensating mass dipole. Thus, Eq. (2.1.3) gets replaced by

$$\delta x \sim \frac{\mathcal{Q}_A}{D^4} T_B^2 \tag{2.1.7}$$

where $\mathcal{Q}_A = m_A d^2$, where m_A is the mass of Alice's particle. The replacement of Eq. (2.1.4) is the Planck length which, in our units, is given by

$$\Delta x \sim 1. \tag{2.1.8}$$

Since Alice now dominantly would emit quadrupole radiation during her recombination, the replacement of Eq. (2.1.6) is

$$N \sim \frac{\mathcal{Q}_A^2}{T_A^4}. \tag{2.1.9}$$

Suppose that Bob and Alice follow their protocols, so that $T_B < D$ and $T_A < D$. Then if $\mathcal{Q}_A > D^2$, Bob can acquire significant which-path information but Alice decoheres her particle with gravitational radiation independent of what Bob does. Conversely, if $\mathcal{Q}_A < D^2$, then Alice should be able to largely maintain the coherence of her particle during the recombination, but Bob cannot acquire significant which-path information. Thus, as in the electromagnetic case, there is no obvious contradiction with complementarity or causality.

The above analysis of [Belenchia et al., 2018] resolves the apparent paradox posed by the gedankenexperiment. Interestingly, this analysis shows that both quantized radiation and vacuum fluctuations are essential for resolving the paradox. Nevertheless, there are some unsatisfactory aspects of this analysis. In particular, only back-of-the-envelope estimates of the various effects were made, so only a rough, order of magnitude relation was obtained between the decoherence due to radiation during Alice's recombination and the decoherence associated with Bob's measurement. Furthermore, one might consider ways in which Bob might improve his ability to obtain which-path information. For example, suppose that Bob, together with $n-1$ assistants, sets up n separate experiments like the one pictured in Fig. 2.1 to measure the Coulomb/Newtonian field of Alice's particle. Suppose that each of these n experiments are done in regions that are spacelike separated from Alice's recombination region and spacelike separated from each other. If each of these experiments could be treated as independent, one would obtain an improvement of $1/\sqrt{n}$ in Bob's ability to overcome the noise due to vacuum fluctuations. Bob would then be able to obtain a corresponding improvement in his acquisition of which-path information, so if n could be taken to be sufficiently large, we would again get a contradiction with complementarity or causality. In fact, vacuum fluctuations over spacelike separated regions are correlated, so it is not obvious that the n experiments can be treated as independent. But it also is not obvious that a scheme of this sort would not work. Thus, while the analysis of [Belenchia et al., 2018] is satisfactory for indicating that there are no obvious contradictions with complementarity or causality, it is not adequate for conclusively showing that no such contradictions can ever occur in this type of gedankenexperiment.

As already stated in the Introduction, an important purpose of this chapter is to improve the analysis of [Belenchia et al., 2018] by giving much more precise versions of the above estimates. We will thereby show in a much more rigorous way that no contradictions with complementarity or causality can occur in this type of gedankenexperiment. As a very important by-product, we will also obtain additional insights into how the state of Alice's particle and the state of Bob's apparatus become correlated. Should this correlation be viewed as being mediated by the Coulomb/Newtonian field of Alice's particle or by on-shell photons/gravitons emitted during the recombination process? We will show that both viewpoints are correct, i.e., they are equivalent descriptions of the same phenomena. We begin in the next section by giving precise descriptions of the decoherence due to Alice and the decoherence due to Bob.

2.2 Decoherence due to Alice and Decoherence due to Bob

In this section, we give a more precise characterization of the decoherence of Alice's particle due to radiation emitted when she recombines her particle and the decoherence associated with Bob's measurements. These characterizations will be used in the next section to reanalyze the gedankenexperiment. In this section we will explicitly discuss the electromagnetic version of the gedankenexperiment, since the language and concepts are more familiar in this context. However, exactly the same discussion applies to the gravitational case, with appropriate substitutions of "graviton" for "photon," "Newtonian" for "Coulomb," etc.

2.2.1 Decoherence due to Alice

We first consider the decoherence of Alice's particle that would occur in the absence of Bob or any other external influence.

Previously, we stated that after Alice sends her particle through a Stern-Gerlach apparatus at an early time, the particle is in the superposition state Eq. (2.1.1). However, this expression ignores the electromagnetic field, which is in a different state depending upon the state of Alice's particle. Heuristically, the state of the total system should be of the form

$$\frac{1}{\sqrt{2}}\big(\,|{\uparrow}; A_1\rangle \otimes |\psi_1\rangle + |{\downarrow}; A_2\rangle \otimes |\psi_2\rangle\,\big) \tag{2.2.1}$$

where states $|\psi_1\rangle$ and $|\psi_2\rangle$ formally correspond to coherent states of the Coulomb field of Alice's particle in states $|{\uparrow}; A_1\rangle$ and $|{\downarrow}; A_2\rangle$ respectively. However, this is only a formal expression because the "Coulomb states" $|\psi_1\rangle$, $|\psi_2\rangle$ are not well defined—we would need to define the state space of the full interacting quantum field theory to define them. Nevertheless, formally, one could argue that these formal Coulomb states should be orthogonal and that therefore Alice's particle is already decohered at the earliest time depicted in Fig. 2.1. However this decoherence is a "false decoherence" in the sense of [Unruh, 2000]. If Alice recombines her particle slowly enough and if there are no external influences, she will be able to fully restore the coherence of her particle.

As Alice recombines her particle and moves its components along noninertial paths, formally the total state should continue to be of the form Eq. (2.2.1). However, while the recombination process is occurring, there is no way to meaningfully separate $|\psi_1\rangle$ or $|\psi_2\rangle$

into a "Coulomb part" (which is not an independent degree of freedom and should cause only a false decoherence of Alice's particle) and a "radiation part" (which is a state of the free electromagnetic field that should be responsible for a true decoherence). Since we do not have a well-defined inner product between $|\psi_1\rangle$ and $|\psi_2\rangle$, we cannot, in general, meaningfully say how much true decoherence has occurred at any finite time during this process.

However, the situation improves considerably if we go to asymptotically late times. At asymptotically late times, the electromagnetic field naturally decomposes into a radiation field that propagates to null infinity and a Coulomb field that follows Alice's particle to timelike infinity. The asymptotic Coulomb field is completely determined by the asymptotic state of Alice's particle and does not represent an independent degree of freedom (see e.g. [Prabhu et al., 2022]). Thus, at asymptotically late times, the state of the total system is of the form

$$\frac{1}{\sqrt{2}}\big(\,|{\uparrow}; A_1\rangle_{i^+} \otimes |\Psi_1\rangle_{\mathscr{I}^+} + |{\downarrow}; A_2\rangle_{i^+} \otimes |\Psi_2\rangle_{\mathscr{I}^+}\,\big). \tag{2.2.2}$$

Here $|{\uparrow}; A_1\rangle_{i^+}$ and $|{\downarrow}; A_2\rangle_{i^+}$ represent the asymptotically late time states of the components of Alice's recombined particle and $|\Psi_1\rangle_{\mathscr{I}^+}$ and $|\Psi_2\rangle_{\mathscr{I}^+}$ represent the states of the radiation field at null infinity that would arise if, over all time, the states of Alice's particle were $|{\uparrow}; A_1(t)\rangle$ and $|{\downarrow}; A_2(t)\rangle$, respectively. Note that after recombination, the spatial wave packets describing the "1" and "2" states coincide, so, in particular, we have $|A_1\rangle_{i^+} = |A_2\rangle_{i^+}$, but we keep the 1 and 2 subscripts for notational clarity.

It is very important to recognize that—unlike Eq. (2.2.1)—Eq. (2.2.2) is not merely a formal expression. The states $|\Psi_1\rangle_{\mathscr{I}^+}$ and $|\Psi_2\rangle_{\mathscr{I}^+}$ are well-defined Fock space states of the "out" Hilbert space of the electromagnetic field and have a well-defined description in terms of photons.[3] The failure of $|\Psi_1\rangle_{\mathscr{I}^+}$ and $|\Psi_2\rangle_{\mathscr{I}^+}$ to coincide implies a decoherence of Alice's

3. In a general scattering process, there will be a nontrivial electromagnetic "memory effect," resulting in infrared divergences in the description of the quantum state (see e.g. [Prabhu et al., 2022, Kulish and Faddeev, 1970, Bloch and Nordsieck, 1937]). In that case, the electromagnetic "out" state cannot be expressed as a state in the standard Fock space and cannot be given a proper description in terms of photons. However, such infrared divergences do not occur in cases where the charges are not relatively boosted at

particle. The degree of decoherence of the asymptotic state of Alice's particle is given by

$$\mathscr{D}_{\text{Alice}} = 1 - |\langle \Psi_1 | \Psi_2 \rangle_{\mathscr{I}^+}| \tag{2.2.3}$$

where $\langle \Psi_1 | \Psi_2 \rangle_{\mathscr{I}^+}$ denotes the inner product of the states $|\Psi_1\rangle_{\mathscr{I}^+}$ and $|\Psi_2\rangle_{\mathscr{I}^+}$ on $\mathscr{I}^+$. This equation is a precise and general version of the decoherence estimate given in Sec. 2.1 based on the number of "entangling photons" that are emitted. If $|\Psi_1\rangle_{\mathscr{I}^+}$ and $|\Psi_2\rangle_{\mathscr{I}^+}$ differ by more than one photon, they should be nearly orthogonal, and the decoherence will be nearly complete.

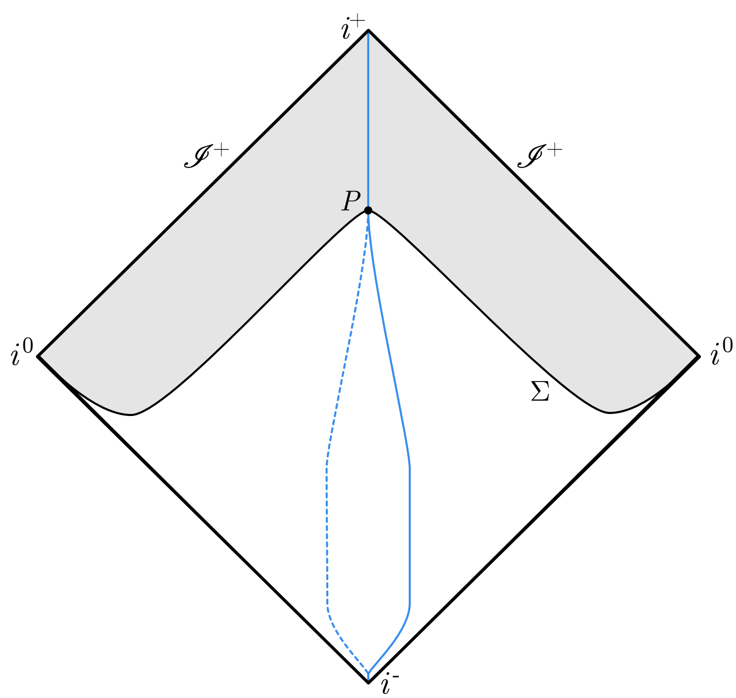

Figure 2.2: Alice recombines her particle at event P and subsequently keeps her recombined particle in inertial motion. Σ is an arbitrary Cauchy surface passing through P.

We are interested in the case depicted in Fig. 2.2 where Alice recombines her particle as in the gedankenexperiment—but without the presence of Bob—and after recombination, she

asymptotically early and late times as we consider here, so such infrared issues play no role in the analysis of this gedankenexperiment. Similar divergences which arise due to the gravitational memory effect also play no role in the (linearized) gravitational version of the gedankenexperiment.

keeps her combined particle in inertial motion at all future times. Then, to the causal future of the recombination event P, the electromagnetic field will correspond to the Coulomb field of the recombined particle. Let Σ be an arbitrary Cauchy surface passing through P. Extend the Coulomb field of the recombined particle to the entire region to the future, $I^+(\Sigma)$, of Σ (i.e., not just the causal future of P). Subtract this Coulomb field from the electromagnetic field in this region. The electromagnetic field associated with $|\uparrow; A_1\rangle$ with the final Coulomb field subtracted will thus correspond to a well-defined state $|\Psi_1\rangle_\Sigma$ of the source-free electromagnetic field on Σ. Similarly, the electromagnetic field associated with $|\downarrow; A_2\rangle$ with the final Coulomb field subtracted will correspond to a well-defined state $|\Psi_2\rangle_\Sigma$ on Σ. At "time" Σ, the joint state of Alice's particle and the electromagnetic field is described by

$$\frac{1}{\sqrt{2}}\big(|\uparrow; A_1\rangle_\Sigma \otimes |\Psi_1\rangle_\Sigma + |\downarrow; A_2\rangle_\Sigma \otimes |\Psi_2\rangle_\Sigma \big). \tag{2.2.4}$$

In contrast to Eq. (2.2.1), this is a completely meaningful expression; $|\Psi_1\rangle_\Sigma$ and $|\Psi_2\rangle_\Sigma$ are well-defined states of the source-free electromagnetic field. Under time evolution, $|\Psi_1\rangle_\Sigma$ and $|\Psi_2\rangle_\Sigma$ evolve to $|\Psi_1\rangle_{\mathscr{I}^+}$ and $|\Psi_2\rangle_{\mathscr{I}^+}$, respectively. Since time evolution is unitary, we may express the decoherence Eq. (2.2.3) of Alice's particle as

$$\mathscr{D}_{\text{Alice}} = 1 - |\,\langle \Psi_1|\Psi_2\rangle_\Sigma\,|. \tag{2.2.5}$$

This is our desired expression for the decoherence due to Alice. It is clear that if there are no time constraints on Alice's recombination, then by doing the recombination adiabatically—so that negligible radiation is emitted to infinity—she can make the decoherence arbitrarily small.

2.2.2 Decoherence due to Bob

We now consider the decoherence that would occur if Bob makes a measurement that obtains some which-path information about Alice's particle. We assume that Alice recombines her particle adiabatically in the distant future—after Bob has completed his measurements—in such a way that had Bob not been present, no decoherence would have occurred. Thus, any decoherence in this situation can be attributed to Bob. This situation corresponds to experimental proposals such as [Bose et al., 2017].

Since Bob is now part of the system, heuristically, the state of the total system after Alice has put her particle through the initial Stern-Gerlach apparatus but before Bob has begun his measurements is now

$$\frac{1}{\sqrt{2}}\big(\,|{\uparrow}; A_1\rangle \otimes |\psi_1\rangle + |{\downarrow}; A_2\rangle \otimes |\psi_2\rangle\,\big) \otimes |B_0\rangle \tag{2.2.6}$$

where $|B_0\rangle$ is the initial state of Bob's apparatus and again $|\psi_1\rangle$ and $|\psi_2\rangle$ are the formal Coulomb states of Alice's particle. We wish to consider a situation wherein Bob turns on his apparatus for a time T_B and makes a measurement of the Coulomb field of Alice's particle in order to try to obtain which-path information. We assume that Bob carries out his measurement in such a way that he emits negligible radiation to infinity. For example if Bob measures the motion of a charged particle released from a trap as described in the previous section, the sensitivity of his experiment will depend on q_B/m_B but the emitted radiation will vary as q_B^2, so by taking q_B and m_B sufficiently small, he should be able to carry out his measurements with negligible emitted radiation.[4] We allow Bob to make any field measurement whatsoever, i.e., we do not restrict him to measuring the trajectory of a particle released from a trap. For the analysis of this subsection, we do not place any limits on T_B, i.e., we do not require $T_B < D$.

4. The assumption that Bob emits negligible radiation is being made so as to make our discussion simpler and cleaner, but it is not essential for the analysis.

Since no radiation is emitted by Bob or Alice, at asymptotically late times, the state of the electromagnetic field at null infinity will be $|0\rangle_{\mathscr{I}^+}$ for either state of Alice's superposition. Thus, the final state of the electromagnetic field plays no role in entanglement and we need only be concerned with the Alice-Bob system. The final state of the Alice-Bob system will be of the form

$$\frac{1}{\sqrt{2}}\big(\,|\uparrow; A_1\rangle_{i^+} \otimes |B_1\rangle_{i^+} + |\downarrow; A_2\rangle_{i^+} \otimes |B_2\rangle_{i^+}\,\big) \tag{2.2.7}$$

where $|B_1\rangle_{i^+}$ and $|B_2\rangle_{i^+}$ are the final states of Bob's apparatus for Alice's states $|\uparrow; A_1\rangle$ and $|\downarrow; A_2\rangle$, respectively. The failure of $|B_1\rangle_{i^+}$ and $|B_2\rangle_{i^+}$ to coincide corresponds to Bob having acquired which-path information about Alice's particle. The corresponding decoherence of Alice's particle is

$$\mathscr{D}_{\text{Bob}} = 1 - |\,\langle B_1|B_2\rangle_{i^+}\,|. \tag{2.2.8}$$

However, since Bob stops interacting at time T_B, we can equivalently calculate the inner product at time T_B

$$\mathscr{D}_{\text{Bob}} = 1 - |\,\langle B_1|B_2\rangle_{T_B}\,|. \tag{2.2.9}$$

This gives the decoherence associated with Bob's measurement. In the circumstance considered here where Alice emits no radiation, it is clear that this decoherence can be viewed as being caused by Bob. It also is clear that in this circumstance, the decoherence should be viewed as being mediated by the Coulomb field of Alice's particle.

Equation (2.2.9) is a precise and general version of the decoherence estimate given in Sec. 2.1 based upon Bob's ability to get which-path information. The amount of which-path information Bob can obtain is determined by the extent to which Bob can design a measurement so that $|B_1\rangle_{T_B}$ is nearly orthogonal to $|B_2\rangle_{T_B}$. The degree to which $|B_1\rangle_{T_B}$ is orthogonal to $|B_2\rangle_{T_B}$ determines how much decoherence of Alice's particle must occur.

2.3 Reanalysis of the Gedankenexperiment

We now are in a position to reanalyze the gedankenexperiment of Sec. 2.1. We will again explicitly consider the electromagnetic version of the gedankenexperiment, but the exactly same discussion applies to the gravitational case with the appropriate word substitutions. The spacetime diagram of the gedankenexperiment is redrawn in Fig. 2.3 in order to show three Cauchy surfaces, Σ_1, Σ_2, and Σ_3, that will play an important role in our reanalysis.

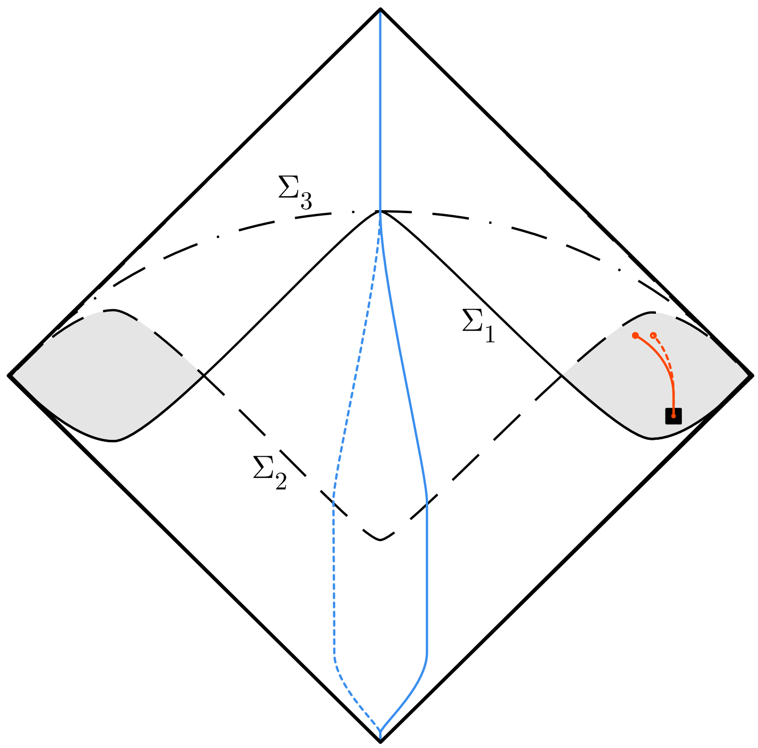

Figure 2.3: A spacetime diagram of the gedankenexperiment of Fig. 2.1 showing the three Cauchy surfaces, Σ_1, Σ_2, and Σ_3. The Cauchy surface Σ_1 passes through Alice's region after recombination but is such that the region in which Bob performs his measurements (shaded in gray) lies to the future of Σ_1. (We have depicted Bob as releasing a particle from a trap, but Bob is allowed to perform any measurement whatsoever in the gray region.) The Cauchy surface Σ_2 is such that it passes through Alice's region before she starts the recombination process but is such that Bob's measurement lies to the past of Σ_2. The Cauchy surface Σ_3 passes through Alice's region after recombination and is such that Bob's measurement lies to the past of Σ_3.

We reanalyze the decoherence of Alice's particle using the results of the previous section as follows. First, consider the portion of the spacetime of Fig. 2.3 that lies to the past of

Cauchy surface Σ_1. At the time represented by Σ_1, Alice has completed her recombination but Bob has not yet begun performing his measurements. The portion of the spacetime lying to the past of Σ_1 is identical to the portion of the spacetime of Fig. 2.2 lying to the past of a corresponding Cauchy surface Σ. Thus, we may apply the results of Sec. 2.2.1 to conclude that the decoherence of Alice's particle is given by

$$\mathscr{D}_{\text{Alice}} = 1 - \left| \langle \Psi_1 | \Psi_2 \rangle_{\Sigma_1} \right| , \tag{2.3.1}$$

where $|\Psi_1\rangle_{\Sigma_1}$ and $|\Psi_2\rangle_{\Sigma_1}$ are the radiation states on Σ_1 obtained by subtracting the common Coulomb field from the states of the electromagnetic field corresponding to Alice's particle being in states $|{\uparrow}; A_1\rangle$ and $|{\downarrow}; A_2\rangle$, respectively. Since Alice's recombination is complete at time Σ_1, Eq. (2.3.1) should yield the exact expression for the decoherence of Alice's particle.

However, we also can analyze the decoherence of Alice's particle by considering the portion of the spacetime that lies to the past of the Cauchy surface Σ_2. At time Σ_2, Alice has not yet started her recombination, but Bob has completed his measurements. Thus, the situation here is identical to the setup considered in Sec. 2.2.2. Hence, we may apply the results of Sec. 2.2.2 to conclude that a decoherence of Alice's particle given by

$$\mathscr{D}_{\text{Bob}} = 1 - |\, \langle B_1 | B_2 \rangle \,| \tag{2.3.2}$$

must occur as a result of Bob's measurements, where $|B_1\rangle$ and $|B_2\rangle$ represent the states of Bob's apparatus after completion of his measurement. It is possible that more decoherence of Alice's particle could occur as Alice performs her recombination. However, since Bob has completed his measurement and stops interacting after time Σ_2, it is impossible for the decoherence of Alice's particle to be less than this.

It follows that there would be a paradox if it were possible for Bob to do a measurement in such a way that

$$| \langle B_1 | B_2 \rangle | < \left| \langle \Psi_1 | \Psi_2 \rangle_{\Sigma_1} \right| , \tag{2.3.3}$$

i.e., such that the decoherence associated with Bob's measurement is greater than the decoherence due to Alice. If Eq. (2.3.3) held, then Bob's measurement either would result in a violation of causality [if it induced an additional decoherence of Alice's particle beyond that given by Eq. (2.3.1)], or it would result in a violation of complementarity (if it did not induce such an additional decoherence). Eq. (2.3.3) is a precise statement of the potential paradox posed by the gedankenexperiment of Sec. 2.1.

However, it is now easy to see that no such paradox can ever arise. At time Σ_1, the state of the joint Alice-field-Bob system is described by

$$\frac{1}{\sqrt{2}} \big(|{\uparrow}; A_1\rangle \otimes |\Psi_1\rangle_{\Sigma_1} + |{\downarrow}; A_2\rangle \otimes |\Psi_2\rangle_{\Sigma_1} \big) \otimes |B_0\rangle \tag{2.3.4}$$

where $|\Psi_1\rangle_{\Sigma_1}$ and $|\Psi_2\rangle_{\Sigma_1}$ are the radiation states on Σ_1 (with the common Coulomb field subtracted), and $|B_0\rangle$ is the initial state of Bob's detector. We now consider the evolution of this state to the Cauchy surface Σ_3. There is no evolution of Alice's state, since Σ_3 is the same time as Σ_1 as far as Alice's state is concerned. However, the radiation interacts with Bob's measuring apparatus. In the case where Alice's state is $|{\uparrow}; A_1\rangle$, Bob's state evolves to $|B_1\rangle$, whereas if Alice's state is $|{\downarrow}; A_2\rangle$, Bob's state evolves to $|B_2\rangle$. It follows that the state Eq. (2.3.4) on Σ_1 must evolve to the state on Σ_3 described by

$$\frac{1}{\sqrt{2}} \big(|{\uparrow}; A_1\rangle \otimes |\Psi_1'\rangle_{\Sigma_3} \otimes |B_1\rangle + |{\downarrow}; A_2\rangle \otimes |\Psi_2'\rangle_{\Sigma_3} \otimes |B_2\rangle \big). \tag{2.3.5}$$

Here $|\Psi_1'\rangle_{\Sigma_3}$ and $|\Psi_2'\rangle_{\Sigma_3}$ are the radiation states that arise from $|\Psi_1\rangle_{\Sigma_1}$ and $|\Psi_2\rangle_{\Sigma_1}$, respectively, after interaction with Bob. The states $|\Psi_1'\rangle_{\Sigma_3}$ and $|\Psi_2'\rangle_{\Sigma_3}$ depend on the interaction with Bob, so they cannot be calculated without knowing exactly what Bob is measuring. However, no matter what Bob does, the joint evolution from Σ_1 to Σ_3 must be unitary. It follows that the norms of states are preserved and that

$$\begin{aligned}\langle \Psi'_1|\Psi'_2\rangle_{\Sigma_3}\langle B_1|B_2\rangle &= \langle \Psi_1|\Psi_2\rangle_{\Sigma_1}\langle B_0|B_0\rangle \\ &= \langle \Psi_1|\Psi_2\rangle_{\Sigma_1}. \end{aligned} \tag{2.3.6}$$

It then follows immediately that

$$|\langle B_1|B_2\rangle| \geq \left|\langle \Psi_1|\Psi_2\rangle_{\Sigma_1}\right| \tag{2.3.7}$$

so the inequality Eq. (2.3.3) can never be satisfied. This is precisely what we wished to show.

Although the above argument completes our proof that no contradiction with causality or complementarity can ever arise in this gedankenexperiment—no matter what Bob chooses to measure—it remains to give a more intuitive explanation of our new resolution of the gedankenexperiment and connect it with the discussion of Sec. 2.1.

The main new ingredient that we have added to the analysis is that we may view Bob as measuring aspects of the radiation emitted by Alice's particle. It may seem strange to talk about "emitted radiation" that is present in a region that is spacelike separated from the region where the emission is taking place. Indeed, this may, by itself, appear to be a violation of causality! However, this kind of phenomenon is a basic feature of quantum field theory, with no violation of causality involved. The mode function of a particle in quantum field theory is a positive frequency solution and cannot be sharply localized. If a photon is emitted by a source in some localized region $\mathcal{O}$, there always will be some amplitude for the photon to be present in a region spacelike separated from $\mathcal{O}$. Indeed, as discussed in detail in [Unruh and Wald, 1984], there are cases where the emitted photon is *mostly* localized in a spacelike separated region. This does not lead to a violation of causality because an observer in the spacelike separated region will not be able to tell whether she is observing a photon or a vacuum fluctuation—she can tell the difference between these possibilities only when she enters the causal future of $\mathcal{O}$. In the present case, the electromagnetic field in Bob's region can be viewed either as corresponding to the superposition of the Coulomb fields of Alice's particle with no radiation—as would be natural to do if we view Bob's region as lying to the past of time Σ_2—or as the single Coulomb field of Alice's combined particle together with free radiation—as would be natural to do if we view Bob's region as lying to the future of time Σ_1. These viewpoints are indistinguishable in Bob's region.

The radiation viewpoint allows us to understand why Bob cannot produce any additional decoherence beyond what Alice produces during her recombination. Bob can obtain which-path information only by measuring (i.e., scattering and/or absorbing) the entangling photons that "previously" were emitted by Alice. Therefore, the state of his apparatus cannot become more correlated with Alice's particle than the radiation emitted by Alice, as we have proven above in Eq. (2.3.7).

Note that, as we have just argued, in the gedankenexperiment, Bob is merely an "innocent bystander" with regard to the decoherence of Alice's particle, since he is merely measuring the entangling radiation emitted by the particle that was the true cause of the decoherence. However, suppose that Alice does not follow the protocol assigned to her in the gedankenexperiment and instead recombines her particle very slowly at a later time, so as not to produce any radiation. Then, despite her attempts to keep perfect coherence, she will find that her particle has decohered by the amount Eq. (2.3.2). In this case, Bob's measurement is the true cause of her particle's decoherence [Belenchia et al., 2019]. Interestingly, when Bob performs his measurements, he has no way of knowing whether he will turn out to be an "innocent bystander" or the cause of decoherence of Alice's particle.

Finally, we note that the analysis of the gedankenexperiment summarized in Sec. 2.1 was based upon the limitations on Alice's ability to maintain coherence due to radiation and the limitations on Bob's ability to get which-path information due to vacuum fluctuations. The reanalysis of the gedankenexperiment given above gave a more precise version of Alice's limitations on maintaining coherence due to radiation. However, we did not mention "vacuum fluctuations" in the discussion of the decoherence associated with Bob's measurements, so it might appear that the reanalysis differs in this respect. However, this is not the case: The radiation fields $|\Psi_1\rangle_{\Sigma_1}$ and $|\Psi_2\rangle_{\Sigma_1}$ have different expected values of the electromagnetic field. Their failure to be orthogonal can be viewed as a manifestation of the same type of fluctuations in these states as occurs in the vacuum state; if these states did not have such fluctuations, they would be fully distinguishable and hence orthogonal. But, as is evident from 2.3.7, it is the failure of $|\Psi_1\rangle_{\Sigma_1}$ and $|\Psi_2\rangle_{\Sigma_1}$ to be orthogonal that limits Bob's ability to make $|B_1\rangle$ and $|B_2\rangle$ orthogonal. Thus, there is a direct connection between vacuum fluctuations and the limitations on Bob's ability to obtain which-path information.

2.4 Summary and Conclusions

In this chapter, we have reanalyzed the gedankenexperiment discussed in [Belenchia et al., 2018]. Our reanalysis validates the arguments that had been made in [Belenchia et al., 2018] using only back-of-the-envelope estimates, and it shows in a much more precise way—and under completely general assumptions about the measurements that Bob makes—that no violations of causality or complementarity can occur.

Perhaps the most interesting aspect of our reanalysis is the equivalence of two viewpoints on how the state of Bob's measuring apparatus becomes correlated with the state of Alice's particle. In the gravitational version of the gedankenexperiment, one can say either that (i) Alice's particle became entangled with on-shell gravitons emitted during the recombination process and Bob's apparatus then interacted with these gravitons—thereby transferring some of the entanglement present in these gravitons to his apparatus—or that (ii) the Newtonian gravitational field of Alice's particle mediated an entanglement of Bob's apparatus with Alice's particle. If Alice follows her protocol but Bob fails to make any measurement, then it is essential to take viewpoint (i) to understand why Bob's inaction has no effect whatsoever on the decoherence of Alice's particle. Conversely, if Bob follows his protocol but Alice recombines her particle adiabatically at a later time, one must take viewpoint (ii) to understand how Bob's measuring apparatus becomes correlated with the Alice's particle [Belenchia et al., 2019]. But if Alice and Bob each follow the protocols of the gedankenexperiment, then both (i) and (ii) provide a valid description of the process that occurs.

Indeed, it is essential that both (i) and (ii)—or, alternatively, neither (i) nor (ii)—be valid descriptions of the process. To see this, suppose that (i) fails, i.e., Alice's particle does not emit entangling gravitons, but suppose that (ii) holds, i.e., Bob's apparatus is able to entangle with Alice's particle via its Newtonian gravitational field. Then Alice's particle would not decohere in the absence of Bob. It follows that if it decohered in the presence of Bob we would have a violation of causality, whereas if it did not decohere in the presence of Bob we would have a violation of complementarity. Thus, it is not consistent for (i) to fail but (ii) to hold. Conversely, suppose (i) holds, i.e., Alice's particle emits quantized entangling gravitational radiation, but suppose that (ii) fails, i.e., Bob's apparatus is unable to entangle with Alice's particle via its Newtonian gravitational field. Then, since, as we have seen, under the protocol of the gedankenexperiment, the difference of the Newtonian fields of Alice's particle can be equivalently viewed as quantized radiation emitted by Alice's particle, this would imply that Bob is unable to interact with quantized gravitational radiation in any way that results in entanglement. This would not make sense in any theory where quantized gravitational radiation can be produced.[5]

These considerations show that there is a direct relationship between Newtonian entanglement and the existence of gravitons. Our argument for such a relationship is strictly valid only within the protocol of the gedankenexperiment, where the measurement of the Newtonian field/gravitons is carried out within a time span no longer than the light travel

5. It has been argued that it may be impossible, in principle, to measure the energy of a single graviton [Dyson, 2013]. Bob is not required here to resolve an individual graviton but merely to become entangled, at least to some degree, with gravitons.

time to the source. Nevertheless, these considerations yield strong support for the view that any observation of entanglement mediated by a Newtonian field provides evidence for the existence of the graviton.

CHAPTER 3

BLACK HOLES DECOHERE QUANTUM SUPERPOSITIONS

The gedankenexperiment of the previous chapter suggests an intimate connection between the causal structure of a theory, and the amount of entanglement that exists between spacetime regions. This chapter introduces a variation of the gedankenexperiment that illustrates this point in dramatic fashion, by asking Alice to perform her coherence experiment in the exterior of a black hole.

Black holes have long been known to be destroyers of quantum coherence. If one member of an entangled pair of particles falls into a black hole, all that will remain is the particle that stayed outside the black hole, which will be in a mixed state. Much more generally, if matter of any kind falls into a black hole, it will, in effect, eventually emerge as Hawking radiation and be in a highly mixed state. While it may be debated as to whether the quantum coherence is lost forever in this process (see, e.g., [Unruh and Wald, 2017, Marolf, 2017]), there is a general consensus that the state outside the black hole is highly mixed at least up to the "Page time" in black hole evaporation.

The purpose of this chapter is to show that black holes are even more insidious destroyers of quantum coherence than has been previously known. If one puts any quantum matter in a spatial superposition, the mere presence of a black hole in the vicinity of the matter will eventually destroy the coherence of this superposition. This happens because the long-range (i.e., electromagnetic and gravitational) fields associated with the quantum matter affect the quantum state of these fields on the black hole horizon. In effect, the black hole thereby acquires "which path" information about the quantum superposition. As we shall show, this inflicts a fundamental rate of decoherence even on stationary superpositions outside its event horizon. This is sufficient to decohere any quantum superposition over a sufficiently long period of time.

To understand how this works, it is useful to first consider a quantum superposition in flat spacetime and see how decoherence can be avoided, following the analysis given in Chapter 2. For simplicity and definiteness, we consider an electrically charged body and the decoherence effects of the electromagnetic field, but an exactly similar analysis will apply for a massive body in the gravitational case. Below, we will refer to the charged body as a "particle" although it need not be an elementary particle, e.g. it could be an atom or a nanoparticle. Suppose an experimenter, Alice, sends a particle of charge q with spin initially in the positive x-direction through a Stern-Gerlach apparatus oriented in the z-direction, so that the state of her particle after the process is in a superposition state of the following form:

$$\frac{1}{\sqrt{2}}\left(|\uparrow; \mathcal{A}_1\rangle + |\downarrow; \mathcal{A}_2\rangle \right). \tag{3.0.1}$$

Here $|\mathcal{A}_1\rangle$ and $|\mathcal{A}_2\rangle$ are spatially separated wavepackets with separation d, with $|\uparrow\rangle$ and $|\downarrow\rangle$ being eigenstates of the z-spin. We wish to know whether the coherence of this superposition is preserved at a later time. In order to make this into a well defined experimental/observational question, Alice can put the particle through a reversing Stern-Gerlach apparatus at some later time and measure the x-spin. If the coherence of the superposition Eq. (3.0.1) has been maintained, the spin will always be found to be in the positive x-direction, whereas if any coherence has been lost the spin will sometimes be found to be in the negative x-direction.

We assume that there are no external influences whatsoever on Alice's particle. It might then seem obvious that coherence must be maintained. However, this is not necessarily the case because, since the particle is charged, an electromagnetic field is present and it is part of the system. Heuristically, the state of the total system after passage through the initial Stern-Gerlach apparatus is actually of the following form:

$$\frac{1}{\sqrt{2}}\left(|\uparrow; \mathcal{A}_1\rangle \otimes |\psi_1\rangle + |\downarrow; \mathcal{A}_2\rangle \otimes |\psi_2\rangle \right) \tag{3.0.2}$$

where $|\psi_1\rangle$ and $|\psi_2\rangle$ formally correspond to the states of the electromagnetic field for the charge-current sources determined by $|\mathcal{A}_1\rangle$ and $|\mathcal{A}_2\rangle$, respectively. Since $|\psi_1\rangle$ and $|\psi_2\rangle$ clearly are distinguishable electromagnetic fields, it might seem that Alice's particle is already decohered at the outset. However, this decoherence is a "false decoherence" in the sense of Ref. [Unruh, 2000]. If Alice recombines her particle slowly enough so as to avoid radiating, she will be able to fully restore the coherence of her particle.

In order to give a precise description of the true decoherence of Alice's particle associated with the electromagnetic field, it is necessary to separate the electromagnetic field into a "Coulomb part" (which is not an independent degree of freedom and should cause only a false decoherence of Alice's particle) and a "radiation part" (which corresponds to the true degrees of freedom of the electromagnetic field that should be responsible for a true decoherence, observable by Alice). In general, this distinction is not possible to make in a meaningful way at any finite time. However, the situation improves considerably if we go to asymptotically late times. At asymptotically late times, the electromagnetic field naturally decomposes into a radiation field that propagates to null infinity and a Coulomb field that follows Alice's particle to timelike infinity. The asymptotic Coulomb field is completely determined by the asymptotic state of Alice's particle and does not represent an independent degree of freedom. Thus, at asymptotically late times, the state of the total system is of the following form:

$$\frac{1}{\sqrt{2}}\big(\,|\uparrow;\mathcal{A}_1\rangle_{i^+}\otimes|\Psi_1\rangle_{\mathscr{I}^+}+|\downarrow;\mathcal{A}_2\rangle_{i^+}\otimes|\Psi_2\rangle_{\mathscr{I}^+}\,\big). \tag{3.0.3}$$

Here $|\uparrow;\mathcal{A}_1\rangle_{i^+}$ and $|\downarrow;\mathcal{A}_2\rangle_{i^+}$ represent the asymptotically late-time states of the components of Alice's particle and $|\Psi_1\rangle_{\mathscr{I}^+}$ and $|\Psi_2\rangle_{\mathscr{I}^+}$ represent the quantum states of the electromagnetic radiation at future null infinity $\mathscr{I}^+$. If Alice has recombined her particle at some finite time, then $|\mathcal{A}_1\rangle = |\mathcal{A}_2\rangle$. Thus, the decoherence of Alice's superposition will be determined by the orthogonality of the radiation states

$$\mathscr{D} = 1 - |\langle\Psi_1|\Psi_2\rangle_{\mathscr{I}^+}|\,. \tag{3.0.4}$$

In the absence of any external influences, Alice can ensure that the coherence of her particle is maintained (i.e. $\mathscr{D} \approx 0$) if she recombines her particle in such a way that negligible entangling radiation is emitted. As estimated in Ref. [Belenchia et al., 2018] and in the previous chapter, this will be possible if the recombination is done over a time span T such that

$$T \gg \frac{qd}{\sqrt{\epsilon_0 c^3 \hbar}}. \tag{3.0.5}$$

In other words, if Eq. (3.0.5) holds, Alice can ensure that $|\Psi_1\rangle_{\mathscr{I}^+} \approx |\Psi_2\rangle_{\mathscr{I}^+} \approx |0\rangle_{\mathscr{I}^+}$, so $\mathscr{D} \approx 0$. Thus, in Minkowski spacetime, Alice can, in principle, maintain the quantum coherence of her spatial superposition by recombining the components of the superposition slowly enough.

The previous chapter also analyzed the complementary point of view of an experimenter, Bob, who attempts to measure the superposed Newtonian field of Alice's particle while remaining in the causal complement of the recombination portion of Alice's experiment. We found that the vacuum fluctuations of the electromagnetic field reduced the precision of Bob's field measurement, thereby obscuring from him any significant amount of which-path information so long as he remained spacelike separated from Alice's recombination process. Bob might attempt to overcome this vacuum fluctuation noise by averaging his field measurement over a longer time. This, however does him no good in flat spacetime: to perform a longer experiment while remaining at spacelike separation from Alice's recombination process, Bob must measure the field at a further distance from Alice's source. Although Bob can increase his precision in this way, the falloff of Alice's sourced field with distance eliminates any benefit Bob might have hoped to obtain, by shrinking the difference between the two Coulomb fields he is attempting to distinguish.

We now consider how this situation changes if there is a black hole in the vicinity of Alice. First, Alice must ensure that her lab does not fall into the black hole. One way of doing this would be for Alice to orbit the black hole. However, this may result in some unwanted emission of radiation. Therefore it would be better to equip Alice with a rocket engine that keeps her lab stationary. She must then also apply some force to her particle (e.g., via a uniform electric field) to keep it stationary. There also may be other effects in her lab due to the spacetime curvature associated with the black hole. However, Alice can take the effects of the gravitational field of the black hole on her lab into account in such a way that they will not result in the decoherence of her particle. Therefore, we shall ignore these effects.

Again, Bob will attempt to decohere Alice's particle by measuring its Coulomb field. If he attempts this from outside the black hole, he will find the analysis of the previous chapter goes through essentially unchanged. If, however, Bob is willing to perform his measurement in the region of the black hole interior that is spacelike separated from Alice's recombination process, the situation changes dramatically. Of course, Bob will eventually reach the singularity. Given a sufficiently massive black hole, however, he can sample the Coulomb field for a sufficiently long time that the effects of vacuum fluctuations will average out. Unlike in flat spacetime, Bob can do this while remaining at an essentially fixed proper distance from Alice. In fact, the longer the time T over which Alice recombines her superposition, the more which-path information Bob can obtain while remaining at spacelike separations. As always, causality requires that it is impossible for Bob to effect the coherence of Alice's particle. The fact that Bob could in principle measure Alice's Coulomb field from inside the black hole, then, suggests that the mere presence of a black hole should decohere Alice's superposition regardless of the presence of Bob. Furthermore, this gedankenexperiment suggests that the decoherence inflicted by the black hole should in fact grow with T, so that if Alice performs her experiment adiabatically to minimize the energy radiated to null infinity, she should in fact suffer even more decoherence due to the black hole.

As we shall now explain, the black hole itself does indeed acquire "which path" information about Alice's particle, which will result in decoherence. With regard to the decoherence of Alice's particle, the key difference arising when a black hole is present is that electromagnetic radiation can now propagate through the black hole horizon as well as to null infinity. Thus, when a black hole is present, the asymptotically late-time state of Alice's particle and the electromagnetic field is now

$$\frac{1}{\sqrt{2}}\big(|{\uparrow};\mathcal{A}_1\rangle_{i^+}\otimes|\Psi_1\rangle_{\mathscr{I}^+}|\Phi_1\rangle_{\mathscr{H}^+}+|{\downarrow};\mathcal{A}_2\rangle_{i^+}\otimes|\Psi_2\rangle_{\mathscr{I}^+}|\Phi_2\rangle_{\mathscr{H}^+}\big) \tag{3.0.6}$$

where $|\Psi_1\rangle_{\mathscr{I}^+}$ and $|\Psi_2\rangle_{\mathscr{I}^+}$ are as before and $|\Phi_1\rangle_{\mathscr{H}^+}$ and $|\Phi_2\rangle_{\mathscr{H}^+}$ are the corresponding states of the electromagnetic field on the event horizon, $\mathscr{H}^+$, of the black hole. The decoherence of Alice's particle in the presence of a black hole is now given by

$$\mathscr{D}=1-|\langle\Psi_1|\Psi_2\rangle_{\mathscr{I}^+}\langle\Phi_1|\Phi_2\rangle_{\mathscr{H}^+}|\,. \tag{3.0.7}$$

As in Minkowski spacetime, if Alice recombines her particle adiabatically, she can ensure that there is negligible radiation to infinity, so $|\Psi_1\rangle_{\mathscr{I}^+}\approx|\Psi_2\rangle_{\mathscr{I}^+}\approx|0\rangle_{\mathscr{I}^+}$, in which case any decoherence will be entirely due to radiation propagating into the black hole

$$\mathscr{D}_{\text{BH}}=1-|\langle\Phi_1|\Phi_2\rangle_{\mathscr{H}^+}|\,. \tag{3.0.8}$$

It might be thought that, by performing her recombination adiabatically, Alice also can ensure that no radiation enters the black hole. However, this is not the case.

To see this, we first consider a classical point charge outside of a Schwarzschild black hole. The explicit solution for a static point charge outside of a Schwarzschild black hole has long been known [Copson and Whittaker, 1928, Cohen and Wald, 1971, Linet, 1976]. On the horizon, the electric field of a static point charge is purely radial, i.e. the only nonvanishing component of the electric field on the horizon is $E_r=cF_{ab}\ell^a n^b$, where n^a denotes the affinely parametrized null normal to the horizon and ℓ^a is the unique past-directed radial null vector satisfying $\ell^a n_a=1$. Electromagnetic radiation on the horizon is described by the pullback, E_A, of the electric field $E_a = cF_{ab}n^b$ to the horizon, where capital Latin indices denote angular components on the horizon. Since $E_A = 0$ for a static point charge, there is no radiation through the horizon, as would be expected. However, suppose we now quasi-statically move the point charge to a new location. After the charge has reached its new location, the electric field will again be radial, but E_r on the horizon will be different from what it was initially. However, it follows from Maxwell's equations at the horizon that

$$\mathcal{D}^A E_A = -\partial_V E_r \tag{3.0.9}$$

where $\mathcal{D}_A$ denotes the covariant derivative on the 2-sphere cross-sections of the horizon, angular indices are raised and lowered with the metric, q_{AB}, on the cross-section, and V is an affine parameter such that $n^a = (\partial/\partial V)^a$. Therefore, we must have $E_A \neq 0$ on the horizon as the charge is being moved and, indeed, $\int E_A dV$ is constrained by initial and final values of E_r, independently of how the charge is moved between its initial and final positions. Thus, there is necessarily some radiation that crosses the horizon of the black hole due to the displacement of the charge. We can make the total energy flux of this radiation through the horizon arbitrarily small by moving the charge very slowly, but, as we will now show, we cannot make the "total photon flux" of this radiation small by moving the charge quasi-statically.

In order to analyze quantum aspects of the radiation, we need to give a precise specification of the quantum state of electromagnetic radiation on the horizon of a black hole. For an unperturbed Schwarzschild black hole formed by gravitational collapse, the state of the electromagnetic field on the horizon of the black hole is described by the Unruh vacuum. However, we will be concerned here only with low frequency phenomena ($\omega \ll 1$), in which case the Unruh and Hartle-Hawking vacua near the horizon are essentially indistinguishable. For the electromagnetic field in a gauge where $A_a n^a = 0$ on the horizon, the "free data" of the electromagnetic field on the horizon is the pull-back, A_A, of the vector potential. In the Fock space associated with the Hartle-Hawking vacuum, a “particle” corresponds to a solution that is purely positive frequency with respect to affine parameter on the horizon. The inner product on the one-particle Hilbert space is given by [Kay and Wald, 1991]

$$\langle A_{1,B}|A_{2,C}\rangle_{\mathscr{H}^+} \equiv \frac{2\epsilon_0 c}{\hbar}\int_{\mathbb{S}^2} r_{\rm S}^2 d\Omega \int_0^\infty \frac{\omega d\omega}{2\pi}\ q^{BC}\overline{\hat{A}_{1,B}(\omega, x^A)}\hat{A}_{2,C}(\omega, x^A) \tag{3.0.10}$$

where $r_{\rm S}$ is the Schwarzschild radius of the black hole and $\hat{A}_A$ is the Fourier transform of A_A with respect to affine parameter V. Equation (3.0.10) corresponds to a Klein-Gordon type of inner product on the positive frequency part of A_A. Now suppose that the black hole is perturbed by a classical charge-current source of the quantum electromagnetic field. The quantum state of the electromagnetic field will then be described by the coherent state (relative to the unperturbed vacuum) associated with the classical retarded solution. The expected number of “horizon photons” in this electromagnetic state at the horizon is given by

$$\langle N\rangle = \|A_A\|^2_{\mathscr{H}^+}, \tag{3.0.11}$$

where A_A is the classical retarded solution and the norm of A_A is defined by the inner product Eq. (3.0.10).

Let us apply this result to the electromagnetic field of a point charge that starts at a point x outside the black hole, is moved to another point x' outside of the black hole and remains at x' forever. We have already seen in this case that $\int E_A dV \neq 0$. Since $E_A = -c\partial_V A_A$, this means that A_A does not return to its initial value at the end of the process. This is closely analogous to the memory effect that occurs at null infinity [Bieri and Garfinkle, 2013, Satishchandran and Wald, 2019]. The fact that A_A does not return to its initial value implies that its Fourier transform diverges as $1/\omega$ as $\omega \to 0$. It then follows immediately from Eq. (3.0.10) that $\|A_A\|^2_{\mathscr{H}^+} = \infty$. Thus, if one moves a point charge from x to x' and leaves the particle at x' forever, no matter how quickly or slowly the charge is moved, an infinite number of "soft horizon photons" will be radiated into the black hole. This is closely analogous to the infrared divergences at null infinity that arise in scattering theory in quantum electrodynamics [Prabhu et al., 2022]. Note that the infinite number of "soft photons" carry negligible energy, and by moving the charge quasi-statically, the total electromagnetic energy radiated into the black hole can be made to be arbitrarily small.

The case of more relevance for us is one in which the point charge is moved from x to x', is held at x' for a long proper time T, and then is returned to x. In that case, A_A returns to its initial value at late times, so there is no infrared divergence in the sense that $\langle N \rangle$ is finite. Nevertheless, the following estimates show that $\langle N \rangle$ is very large when T is very large. The radial electric field of a point charge located a distance b from the black hole is roughly $E_r \sim q/\epsilon_0 b^2$ [Copson and Whittaker, 1928, Cohen and Wald, 1971, Linet, 1976]. The change in the radial electric field when the charge is moved from x to x' is therefore roughly $\Delta E_r \sim qd/\epsilon_0 b^3$, where d is the distance between x and x' and we have assumed that $d \ll b$. Taking account of the fact that the 2-spheres on the horizon are of radius $2GM/c^2$, it then follows from Eq. (3.0.9) that the change in the vector potential, A_A, on the horizon when the particle is moved from x to x' is

$$\Delta A_A \sim \frac{G^2 M^2}{c^5} \frac{qd}{\epsilon_0 b^3}. \tag{3.0.12}$$

Eventually, when the particle is moved back to x, the change in A_A will be equal and opposite to Eq. (3.0.12). But if the charge is held at point x' for a very long time T, the contribution of A_A to the norm defined by Eq. (3.0.10) will be dominated by the low-frequency contribution arising from the time interval over which Eq. (3.0.12) holds. We obtain

$$\langle N \rangle = \|A_A\|^2_{\mathscr{H}^+} \sim \frac{G^4 M^4 q^2 d^2}{\hbar c^9 \epsilon_0 b^6} \ln V \tag{3.0.13}$$

where V is the affine time on the horizon corresponding to the proper time T along the particle trajectory. However, the relation between affine time, V, and Killing time, v, on the horizon of a black hole is given by $V = \exp(\kappa v/c)$, where $\kappa = c^4/4GM$ is the surface gravity of the black hole. Furthermore, the Killing time is related to the proper time of the particle by the redshift factor. We shall assume that Alice's lab is not extremely close to the black hole and neglect the departure of the redshift factor from 1. We then obtain

$$\langle N \rangle = \|A_A\|^2_{\mathscr{H}^+} \sim \frac{G^3 M^3 q^2 d^2}{\hbar c^6 \epsilon_0 b^6} T. \tag{3.0.14}$$

Thus, the number of "soft photons" radiated into the black hole in the above process grows linearly with the time, T, that the point charge spends at point x'.

We now have all of the ingredients needed to analyze Alice's coherence experiment, under the assumption that Alice splits and recombines her particle slowly enough that negligible radiation is emitted to infinity. Although our results hold much more generally, it is easiest to consider the case where, after passing through the Stern-Gerlach apparatus, the first component of Alice's particle remains at position x and the second component of her particle moves to position x'. After these components stay at x and x', respectively, for a time T, they are recombined in such a way that the first component continues to remain at x and the second component moves from x' to x. In that case, no radiation is emitted by the first component, so in Eq. (3.0.6), we have $|\Phi_1\rangle_{\mathscr{H}^+} = |0\rangle_{\mathscr{H}^+}$. However, our above analysis applies to the second component, which moves from x to x', stays at x' for a time T, and then returns to x. Thus, $|\Phi_2\rangle_{\mathscr{H}^+}$ will be a state with expected number of photons given by Eq. (3.0.14). If $\langle N \rangle \gtrsim 1$, then $|\Phi_2\rangle_{\mathscr{H}^+}$ will be nearly orthogonal to $|\Phi_1\rangle_{\mathscr{H}^+} = |0\rangle_{\mathscr{H}^+}$. This means that—due entirely to the presence of a black hole—Alice's particle will decohere in a time

$$T_D \sim \frac{\epsilon_0 \hbar c^6 b^6}{G^3 M^3 q^2 d^2} \tag{3.0.15}$$

$$\sim 10^{43} \text{ years} \left(\frac{b}{\text{a.u.}}\right)^6 \cdot \left(\frac{M_\odot}{M}\right)^3 \cdot \left(\frac{e}{q}\right)^2 \cdot \left(\frac{\text{m}}{d}\right)^2. \tag{3.0.16}$$

Thus, if our Sun were a black hole and if one separated an electron into two components one meter apart in a laboratory experiment on Earth, it would not be possible to maintain the coherence of the electron for more than 10^{43} years. On the other hand, if this experiment were done at $b = 6GM/c^2$, then $T_D \sim 5$ minutes.

A closely parallel analysis can be given for the case of a gravitating particle. In the gravitational case, the electric part of the Weyl tensor $E_{ab} = c^2 C_{acbd} n^c n^d$ plays a role closely analogous to the role played by the electric field E_a in the electromagnetic case. For a static point mass outside a Schwarzschild black hole the only non-vanishing component of the electric part of the Weyl tensor on the horizon is $E_{rr} = c^2 C_{acbd} \ell^a n^c \ell^b n^d$. Gravitational radiation on the horizon is described by the pullback, E_{AB}, of E_{ab}, which vanishes for a static point mass. However, the process of moving the particle quasi-statically to a new location will involve a change in E_{rr}. The (once-contracted) Bianchi identity on the horizon yields

$$\mathcal{D}^A E_{AB} = -\partial_V E_{rB}, \qquad \mathcal{D}^A E_{rA} = -\partial_V E_{rr} \tag{3.0.17}$$

which implies

$$\mathcal{D}^A \mathcal{D}^B E_{AB} = \partial_V^2 E_{rr} \tag{3.0.18}$$

in close analogy with Eq. (3.0.9). Thus, if a point mass is moved quasi-statically, there necessarily will be radiation through the horizon. To determine the number of gravitons emitted, we treat the quantum gravitational field at the level of linearized perturbation theory about the black hole background. For a metric perturbation h_{ab} in a gauge where

$h_{ab}n^a = 0 = q^{AB}h_{AB}$ on the horizon, the "free data" on the horizon is h_{AB}. As in the electromagnetic case, a "particle" in the Fock space associated to the Hartle-Hawking vacuum is a solution with positive frequency with respect to affine parameter V. The inner product on the one-particle Hilbert space is given by a direct analog of Eq. (3.0.10) with A_A replaced by h_{AB}. Finally, E_{AB} is given in terms of h_{AB} by $E_{AB} = -\frac{1}{2}c^2\partial_V^2 h_{AB}$.

The analysis of the decoherence of a quantum superposition of a body of mass m in the presence of a black hole now proceeds in exact parallel with the electromagnetic case. The only significant difference is that, for the same reason as in the analysis of Ref. [Belenchia et al., 2018], it is now the effective mass quadrupole md^2 of the superposition that enters, rather than the effective electrostatic dipole qd that entered the electromagnetic analysis. We find that a black hole will decohere a quantum superposition of a massive body in a time

$$T_D^{\mathrm{GR}} \sim \frac{\hbar c^{10} b^{10}}{G^6 M^5 m^2 d^4} \tag{3.0.19}$$

$$\sim 10\ \mu\mathrm{s} \left(\frac{b}{\mathrm{a.u.}}\right)^{10} \cdot \left(\frac{\mathrm{M}_\odot}{M}\right)^{5} \cdot \left(\frac{\mathrm{M}_{\mathrm{Earth}}}{m}\right)^{2} \cdot \left(\frac{\mathrm{R}_{\mathrm{Earth}}}{d}\right)^{4}. \tag{3.0.20}$$

Thus, if the Sun were a black hole and the Earth occupied a quantum state with its center of mass spatially superposed by a separation on the order of its own radius, this superposition would decohere due to the presence of the black hole in about 10 μs. Of course, it would not be easy to put the Earth into such a quantum superposition.

In summary, we have found that black holes, in effect, gather information about quantum superpositions of spatially separated components by means of the long range fields sourced by the matter comprising these components. Eventually, a black hole will decohere any quantum superposition. Although this may not be of practical importance for any presently contemplated experiments, it may be of fundamental significance for our understanding of the nature of black holes in a quantum theory of gravity.

CHAPTER 4

KILLING HORIZONS DECOHERE QUANTUM SUPERPOSITIONS

Consider a stationary spacetime in which an experimentalist, Alice, is present. Alice's lab is stationary, and she has control of a charged or massive body (hereinafter referred to as a "particle"). She sends her particle through a Stern-Gerlach apparatus or other device that puts her particle in a quantum superposition of two spatially separated states.[1] She keeps these spatially separated components stationary for a time T and then recombines them. Will Alice be able to maintain the coherence of these components, so that, when recombined, the final state of her particle will be pure — or will decoherence have occurred, so that the final state of her particle will be mixed?

Ordinarily, any decoherence effects will be dominated by "environmental influences," i.e., additional degrees of freedom present in Alice's lab that interact with her particle. We assume that Alice has perfect control of her laboratory and its environment so that there is no decoherence from ordinary environmental effects. However, for a charged or massive particle, Alice cannot perfectly control the electromagnetic or gravitational field, since her particle acts as a source for these fields and some radiation will be emitted during the portions of her experiment where she separates and recombines her particle. Nevertheless, in Minkowski spacetime, if her lab is stationary in the ordinary, inertial sense, she can perform her experiment in a sufficiently adiabatic manner that negligible decohering radiation is emitted. In principle, she can keep the particle separated for an arbitrarily long time T and still maintain coherence when the components are recombined.

1. Quantum spatial superpositions of massive bodies have been of recent interest in both theoretical as well as experimental probes of fundamental properties of quantum gravity, e.g., [Bose et al., 2017, Marletto and Vedral, 2017, Belenchia et al., 2018, Christodoulou and Rovelli, 2019, Giacomini et al., 2019, Gonzalez-Ballestero et al., 2021, Danielson et al., 2022a, Carney, 2022, Christodoulou et al., 2023, Carney et al., 2022, Feng and Vedral, 2022, Zhou et al., 2022, Overstreet et al., 2023].

In the preceding chapter we showed that the above situation changes dramatically if a black hole is present in the spacetime — even though the experiment is carried out entirely in the black hole's exterior. In effect, a black hole horizon harvests "which path" information about any quantum superposition in its exterior, via the long-range fields sourced by the superposed matter. We showed that this results in the unavoidable radiation of entangling "soft photons or gravitons" through the horizon that carry the "which path" information into the black hole. Consequently, the mere presence of the black hole implies a fundamental rate of decoherence on the quantum superposition.[2] Although the rate of decoherence will be small if the black hole is far away, the coherence decays exponentially in the time, T, that the spatial superposition is maintained. Thus, in any spacetime with a black hole, there will be essentially complete decoherence within a finite time.[3]

The purpose of this chapter is to generalize the results of Chapter 3 to spacetimes with Killing horizons, i.e., spacetimes with a Killing vector field such that there is a null surface to which the Killing field is normal (see, e.g., [Kay and Wald, 1991] for a discussion of properties of Killing horizons). The event horizon of a stationary black hole is a Killing horizon [Hawking and Ellis, 1973, Hawking, 1972, Alexakis et al., 2010], so spacetimes with Killing horizons encompass the case of stationary spacetimes that contain black holes. However, there are many cases of interest where Killing horizons are present without the presence of black holes. One such case is that of Minkowski spacetime, where the Rindler horizon is a Killing horizon with respect to the Lorentz boost Killing field. Another such case is de Sitter spacetime, where the cosmological horizon is a Killing horizon. We will show that in these cases, a spatial superposition that is kept stationary (with respect to the symmetry generating the Killing horizon) will decohere in a manner similar to the black hole case. We will also provide

2. In QED, this effect applies only to superpositions of charged particles. However, since all matter sources gravity, the quantum gravitational decoherence applies to all superpositions.

3. This maximal coherence time for superpositions in the exterior can be much smaller than the evaporation time of the black hole.

an estimate of the maximum amount of time during which coherence can be maintained.

The case of the Rindler horizon is particularly instructive. The relevant symmetry here is that of Lorentz boosts, so Alice's lab will be "stationary" if it is following orbits of Lorentz boosts, which are uniformly accelerating worldlines. Our analysis based upon radiation through the Rindler horizon shows that decoherence of a uniformly accelerating spatially separated superposition occurs because of the emission of "soft" (i.e., very low frequency) gravitons or photons, where the frequency is defined relative to an affine parameter on the Rindler horizon. As we shall show, the decoherence effect of this radiation of soft gravitons or photons is distinct from the (smaller) decoherence effect resulting from the presence of Unruh radiation. To gain further insight, we also analyze the decohering radiation in the electromagnetic case from the inertial point of view, using the Liénard-Wiechert solution to determine the radiation at future null infinity. As we shall show, the decohering photons are of high frequency at null infinity.

In Sec. 4.1 we provide a general discussion of the decoherence of a quantum superposition due to radiation in a stationary spacetime. In Sec. 4.2 we consider the decoherence of a uniformly accelerating superposition, analyzing it from both the Rindler and Minkowski viewpoints. We also show that this decoherence is distinct from (and larger than) the decoherence effects due to the presence of Unruh radiation. In Sec. 4.3 we analyze the decoherence in de Sitter spacetime associated with the cosmological horizon. We will work in Planck units where $G = c = \hbar = k_{\mathrm{B}} = 1$ and, in electromagnetic formulas, we also put $\epsilon_0 = 1$, but we will restore these constants in our formulas that give estimates for decoherence times. Lower case Latin indices represent abstract spacetime indices. Upper case Latin indices from the early alphabet correspond to spatial indices on horizons or null infinity.

4.1 Decoherence due to Radiation in a Stationary Spacetime

In this section, we will give a general analysis of the decoherence of a spatial superposition in a stationary spacetime due to emission of radiation by the body. Our analysis applies both to the decoherence of a charged body due to emission of electromagnetic radiation and to the decoherence of a gravitating body due to emission of linearized gravitational radiation. The analyses of these two cases are very closely parallel. In order to avoid repetition, we will analyze only the electromagnetic case in detail, but near the end of this section, we will state the corresponding results in the linearized gravitational case, which can be obtained straightforwardly by replacing the vector potential A_a with the perturbed metric h_{ab}, the charge-current j_a with the stress-energy T_{ab}, etc.

Consider a charged particle[4] in a stationary spacetime. We assume that the particle is initially in a stationary state. The particle is then put through a Stern-Gerlach (or other) apparatus, resulting in it being in a superposition state[5]

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|\psi_1\rangle + |\psi_2\rangle\right) \tag{4.1.1}$$

where $|\psi_1\rangle$ and $|\psi_2\rangle$ are normalized states that are spatially separated after passing through the apparatus. The particle is then recombined via a reversing Stern-Gerlach (or other) apparatus and returns to a stationary state. We are particularly interested in the case where, between separation and recombination, $|\psi_1\rangle$ and $|\psi_2\rangle$ are kept stationary for a long period of time, T, but we do not make any such assumption in this section. We wish to estimate how much decoherence due to emission of electromagnetic radiation will have occurred by the time of recombination.[6]

4. As already indicated above, the "particle" need not be an elementary particle but could be a "nanoparticle" or any other body whose only relevant degree of freedom for our analysis is its center of mass.

5. For simplicity, we have assumed that we have a 50-50 superposition of $|\psi_1\rangle$ and $|\psi_2\rangle$, but this assumption is not necessary.

6. The decoherence of Alice's particle can be experimentally determined as follows. We assume that

A key assumption that we shall make is that the fluctuations in the charge-current operator $\boldsymbol{j}^a$ in the states $|\psi_1\rangle$ and $|\psi_2\rangle$ are negligibly small over the scales of interest so that we can treat the charge current in each of these states as c-number sources in Maxwell's equations, given by $j_1^a = \langle\psi_1|\boldsymbol{j}^a|\psi_1\rangle$ and $j_2^a = \langle\psi_2|\boldsymbol{j}^a|\psi_2\rangle$, respectively. In the initial and final stationary eras, $|\psi_1\rangle$ and $|\psi_2\rangle$ are assumed to coincide spatially (though they may differ in other characteristics, such as spin) so that $j_1^a = j_2^a$ at very early and very late times.

In order to proceed further, we must specify the initial state of the electromagnetic field. Since, prior to going through the Stern-Gerlach apparatus, the charge is assumed to be stationary, at early times we may subtract the "Coulomb field" C_a^{in} of the charge, i.e., at early times we may consider the electromagnetic field observable

$$\boldsymbol{A}_a^{\text{in}} = \boldsymbol{A}_a - C_a^{\text{in}}\mathbf{1} \tag{4.1.2}$$

where C_a^{in} is the (assumed to be unique) stationary classical solution to Maxwell's equations with the early-time stationary charged particle source $j_1^a = j_2^a$ and $\boldsymbol{A}_a$ is the vector potential operator. We need not assume any specific choice of gauge for $\boldsymbol{A}_a^{\text{in}}$. Then $\boldsymbol{A}_a^{\text{in}}$ satisfies the source-free Maxwell's equations at early times, and we may extend its definition to all times by requiring it to satisfy the source-free Maxwell equations everywhere.

The initial state of the electromagnetic field may be specified by giving the "radiation state" of $\boldsymbol{A}_a^{\text{in}}$. The choice of this state depends on the physical situation being considered. If the spacetime were globally stationary — i.e., if the stationary Killing field were everywhere timelike, so, in particular, there are no Killing horizons — it would be natural to assume that

Alice's particle initially has spin in the positive x-direction and thus is in a 50-50 superposition of z-spin after passing through the initial Stern-Gerlach apparatus. After recombination, Alice measures the x-spin of her particle. If coherence of the superposition eq. (4.1.1) has been maintained, then (assuming that Alice has made appropriate corrections if there are any phase differences between the paths) the spin will always be found to be in the positive x-direction. On the other hand, if any coherence has been lost, the particle will not be in a state of definite spin, and the spin will sometimes be found to be in the negative x-direction. By repeating the experiment many times, Alice can, in principle, determine the decoherence to any desired accuracy.

the initial state of the radiation is the stationary vacuum state, i.e., the ground state relative to the time translations. For the case of a black hole spacetime, it would be correspondingly natural to assume that the initial state of the radiation is that of the Unruh vacuum, since for a black hole formed by gravitational collapse, the state of a quantum field is expected to approach the Unruh vacuum after the black hole has "settled down" to a stationary state. For the case of Minkowski spacetime, we take the initial state of the radiation to be the ordinary (inertial) Minkowski vacuum. For de Sitter spacetime, we take the initial state of the radiation to be the de Sitter invariant vacuum[7] for the electromagnetic field [Allen and Jacobson, 1986]. We denote the initial state of the radiation in all of the above cases by $|\Psi_0\rangle$.

In each of the above cases, $|\Psi_0\rangle$ is a pure, quasi-free (i.e., Gaussian) state. It follows (see, e.g., [Wald, 1995] or Appendix A of [Kay and Wald, 1991]) that we can construct a one-particle Hilbert space $\mathcal{H}_{\rm in}$ and corresponding Fock space $\mathcal{F}(\mathcal{H}_{\rm in})$ wherein $|\Psi_0\rangle$ plays the role of the vacuum state and the field operator $\boldsymbol{A}^{\rm in}_a$ is represented on $\mathcal{F}(\mathcal{H}_{\rm in})$ by

$$\boldsymbol{A}^{\rm in}_a(f^a) = i\boldsymbol{a}(\overline{K\sigma_f}) - i\boldsymbol{a}^\dagger(K\sigma_f). \tag{4.1.3}$$

Here f^a is a divergence-free[8] test function, σ_f denotes the advanced minus retarded solution to Maxwell's equations with source f^a, and $K : S \to \mathcal{H}_{\rm in}$ denotes the map taking the space S of classical solutions to their representatives in the one-particle Hilbert space $\mathcal{H}_{\rm in}$. The commutator of the creation and annihilation operators in Eq. (4.1.3) is given by

$$[\boldsymbol{a}(\overline{K\sigma_f}), \boldsymbol{a}^\dagger(K\sigma_g)] = \langle K\sigma_f|K\sigma_g\rangle\, \mathbf{1}, \tag{4.1.4}$$

where $\langle K\sigma_f|K\sigma_g\rangle$ is the inner product on $\mathcal{H}_{\rm in}$, which is given by a natural generalization of

7. A de Sitter invariant vacuum state does not exist for the massless scalar field [Allen, 1985] but such a state does exist for the electromagnetic field [Allen and Jacobson, 1986] and linearized gravitational field [Allen, 1986].

8. Restriction of the smearing to divergence-free test functions is necessary and sufficient to eliminate the gauge dependence of $\boldsymbol{A}^{\rm in}_a$ (see, e.g., P.101 of [Wald, 1995]).

the Klein-Gordon inner product to electromagnetic fields.

For the case of a globally stationary spacetime in the stationary vacuum state, $K\sigma_f$ corresponds to taking the positive frequency part of σ_f with respect to the time translations generating the stationary symmetry. For the case of a stationary black hole in the Unruh vacuum state, $K\sigma_f$ corresponds to taking the positive frequency part of σ_f with respect to affine time on the past horizon and with respect to Killing time at past null infinity. For Minkowski spacetime in the inertial Minkowski vacuum, $K\sigma_f$ corresponds to taking the positive frequency part of σ_f with respect to inertial time translations. Equivalently, $K\sigma_f$, in this case, corresponds to the solution obtained by taking the positive frequency part of the restriction of σ_f to any null hyperplane $\mathscr{N}$ (i.e., any Rindler horizon) with respect to an affine parametrization of the null geodesics generating $\mathscr{N}$. For de Sitter spacetime in the de Sitter invariant vacuum, $K\sigma_f$ corresponds to the solution obtained by taking the positive frequency part of the restriction of σ_f to any cosmological horizon with respect to an affine parametrization of the null geodesics generating that horizon.

Under the above assumption that the charge-currents of $|\psi_1\rangle$ and $|\psi_2\rangle$ can be treated as c-number sources, the electromagnetic field $\boldsymbol{A}_{i,a}$ in the presence of the charge in state $|\psi_i\rangle$ for $i = 1, 2$ is given in terms of the source-free field $\boldsymbol{A}^{\text{in}}_a$ by [Yang and Feldman, 1950]

$$\boldsymbol{A}_{i,a} = \boldsymbol{A}^{\text{in}}_a + G^{\text{ret}}_a(j^b_i)\mathbf{1} \tag{4.1.5}$$

where $G^{\text{ret}}_a(j^b_i)$ denotes the classical retarded solution for source j^b_i. In particular, since the field $\boldsymbol{A}^{\text{in}}_a$ is in state $|\Psi_0\rangle$, the correlation functions of the electromagnetic field $\boldsymbol{A}_{i,a}$ for $|\psi_i\rangle$

are given by[9]

$$\begin{aligned}&\langle \boldsymbol{A}_{i,a_1}(x_1)\dots \boldsymbol{A}_{i,a_n}(x_n)\rangle\\&\quad=\langle\Psi_0|\left[\boldsymbol{A}^{\rm in}_{a_1}(x_1)+G^{\rm ret}_{a_1}(j_i^b)(x_1)\mathbf{1})\right]\\&\qquad\qquad\dots\left[\boldsymbol{A}^{\rm in}_{a_n}(x_n)+G^{\rm ret}_{a_n}(j_i^b)(x_n)\mathbf{1})\right]|\Psi_0\rangle.\end{aligned}\tag{4.1.6}$$

Equation (4.1.6) is valid at all times. However, at late times — i.e., to the future of any Cauchy surface Σ corresponding to the time at which recombination has occurred — we can again subtract off the common stationary Coulomb field, $C_a^{\rm out}$, of $j_1^a = j_2^a$ to obtain the source-free field[10] $\boldsymbol{A}^{\rm out}_{i,a}$ that describes the radiation at late times for the states $|\psi_i\rangle$,

$$\boldsymbol{A}^{\rm out}_{i,a}=\boldsymbol{A}_{i,a}-C^{\rm out}_a\mathbf{1}\,. \tag{4.1.7}$$

By Eq. (4.1.6), at late times, the correlation functions of $\boldsymbol{A}^{\rm out}_a$ are given by

$$\begin{aligned}&\langle \boldsymbol{A}^{\rm out}_{i,a_1}(x_1)\dots \boldsymbol{A}^{\rm out}_{i,a_n}(x_n)\rangle\\&\quad=\langle\Psi_0|\left[\boldsymbol{A}^{\rm in}_{a_1}(x_1)+\mathcal{A}_{i,a_1}(x_1)\mathbf{1})\right]\\&\qquad\qquad\dots\left[\boldsymbol{A}^{\rm in}_{a_n}(x_n)+\mathcal{A}_{i,a_n}(x_n)\mathbf{1})\right]|\Psi_0\rangle\end{aligned}\tag{4.1.8}$$

where

$$\mathcal{A}_{i,a}=G^{\rm ret}_a(j_i^b)-C^{\rm out}_a. \tag{4.1.9}$$

Note that $\mathcal{A}_{i,a}$ is a classical solution of the source-free Maxwell equations in the late-time region.

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9. It is understood that each of the x_k variables should be smeared with a divergence-free test vector field f_k^a.

10. Note that $\boldsymbol{A}^{\rm in}_a$ did not have a subscript "i" whereas $\boldsymbol{A}_{i,a}$ and $\boldsymbol{A}^{\rm out}_{i,a}$ do carry such subscripts. This is a consequence of the fact that we are working in the "in" representation — i.e., the Heisenberg representation on the Hilbert space $\mathcal{F}(\mathcal{H}_{\rm in})$ — so $\boldsymbol{A}^{\rm in}_a$ does not depend on the sources, but the other fields do.

The correlation functions Eq. (4.1.8) on any late-time Cauchy surface are precisely those of the coherent state

$$|\Psi_i\rangle = e^{-\frac{1}{2}\|K\mathcal{A}_i\|^2} \exp\left[\boldsymbol{a}^\dagger(K\mathcal{A}_i)\right]|\Psi_0\rangle\,, \tag{4.1.10}$$

where the norm is that of the one-particle inner product of Eq. (4.1.4). Thus, the coherent state $|\Psi_1\rangle$ describes the "out" radiation state corresponding to charged particle state $|\psi_1\rangle$ and the coherent state $|\Psi_2\rangle$ describes the "out" radiation state corresponding to charged particle state $|\psi_2\rangle$. The joint "out" state, $|\Upsilon\rangle$, of the particle-radiation system is given by

$$|\Upsilon\rangle = \frac{1}{\sqrt{2}}\left(|\psi_1\rangle \otimes |\Psi_1\rangle + |\psi_2\rangle \otimes |\Psi_2\rangle\right). \tag{4.1.11}$$

Therefore, the decoherence of $|\psi_1\rangle$ and $|\psi_2\rangle$ due to emission of electromagnetic radiation is given by

$$\mathscr{D} = 1 - |\langle\Psi_1|\Psi_2\rangle|. \tag{4.1.12}$$

We wish to evaluate $\mathscr{D}$.

By the general formula for the inner product of coherent states, we have

$$|\langle\Psi_1|\Psi_2\rangle| = \exp\left[-\frac{1}{2}\|K(\mathcal{A}_1 - \mathcal{A}_2)\|^2\right]. \tag{4.1.13}$$

Now, in the late-time era, $\mathcal{A}_{1,a} - \mathcal{A}_{2,a}$ is just the difference between the classical retarded solutions with sources j_1^a and j_2^a,

$$\mathcal{A}_{1,a} - \mathcal{A}_{2,a} = G_a^{\text{ret}}(j_1^b) - G_a^{\text{ret}}(j_2^b) = G_a^{\text{ret}}(j_1^b - j_2^b). \tag{4.1.14}$$

Consider the coherent state associated with $G_a^{\text{ret}}(j_1^b - j_2^b)$ in the late-time era. We refer to photons in this state as *entangling photons*. By the general properties of coherent states, the expected number, $\langle N \rangle$, of entangling photons is given by

$$\langle N \rangle \equiv \left\| K \left[G^{\mathrm{ret}}(j_1 - j_2) \right] \right\|^2 . \tag{4.1.15}$$

Thus, we have

$$| \langle \Psi_1 | \Psi_2 \rangle | = \exp \left[-\frac{1}{2} \langle N \rangle \right] \tag{4.1.16}$$

so

$$\mathscr{D} = 1 - | \langle \Psi_1 | \Psi_2 \rangle | = 1 - \exp \left[-\frac{1}{2} \langle N \rangle \right] \tag{4.1.17}$$

and we see that the necessary and sufficient condition for significant decoherence ($\mathscr{D} \sim 1$) is $\langle N \rangle \gtrsim 1$.

We summarize the results that we have obtained above as follows. Under the assumptions we have made above, in order to calculate the decoherence, $\mathscr{D}$, of the particle due to radiation, we carry out the following steps:

1. We obtain the expected charge current, j_1^a and j_2^a, for the particle in states $|\psi_1\rangle$ and $|\psi_2\rangle$ of the superposition.

2. We calculate the classical retarded solution, $G_a^{\mathrm{ret}}(j_1^b - j_2^b)$ for the difference of these charge currents, which is a source-free solution at late times, since $j_1^a = j_2^a$ at late times.

3. We calculate the one-particle state $KG^{\mathrm{ret}}(j_1 - j_2)$ corresponding to $G_a^{\mathrm{ret}}(j_1^b - j_2^b)$ at late times. In the various cases, this corresponds to the following: (i) For a globally stationary spacetime initially in the stationary vacuum state, this one-particle state is the positive frequency part of the solution with respect to the time translations generating the stationary symmetry. (ii) For the case of a stationary black hole initially in the Unruh vacuum, the one-particle state is the positive frequency part of the solution with respect to affine time on the past horizon and with respect to Killing time at past

null infinity. (iii) For Minkowski spacetime initially in the Minkowski vacuum, the one-particle state is the positive frequency part of the solution with respect to inertial time or, equivalently, the positive frequency part with respect to affine time on any Rindler horizon. (iv) For de Sitter spacetime initially in the de Sitter invariant vacuum, the one-particle state is the positive frequency part of the solution with respect to affine time on any cosmological horizon.

4. We compute the squared norm, $\|K[G^{\rm ret}(j_1 - j_2)]\|^2$, of this one-particle state at late times. This quantity is equal to the expected number of entangling photons, $\langle N \rangle$. The decoherence due to radiation is then given by

$$\mathscr{D} = 1 - \exp\left[-\frac{1}{2}\|K\left[G^{\rm ret}(j_1 - j_2)\right]\|^2\right]. \tag{4.1.18}$$

As previously stated, the above analysis extends straightforwardly to the linearized gravitational case, where the perturbed metric, h_{ab}, is treated as a linear quantum field propagating in the background classical stationary spacetime. To compute the decoherence due to gravitational radiation in this case, we carry out the above steps, replacing A_a by h_{ab} and the charge-current j^a by the stress-energy tensor T_{ab}. The retarded solution $G^{\rm ret}_a(j^b)$ for Maxwell's equations is replaced by the retarded solution $G^{\rm ret}_{ab}(T_{cd})$ for the linearized Einstein equation. The map $K : S \to \mathcal{H}_{\rm in}$ is again obtained as in item 3 above and the inner product on $\mathcal{H}_{\rm in}$ is again given by a natural generalization of the Klein-Gordon inner product to linearized gravitational fields. The decoherence due to gravitational radiation is then given by the analog of Eq. (4.1.18).

The above analysis applies for any motion of the components of Alice's superposition. We are primarily interested in the case where, during a time interval T_1, Alice puts a particle of charge q (or mass m) into a spatial superposition, where the distance between the components of the particle wavefunction is d. She then keeps this superposition stationary in her lab for a time T. Finally, she recombines her particle over a time interval T_2.

In Minkowski spacetime in the case where Alice's lab is inertial, $G_a^{\rm ret}(j_1^b - j_2^b)$ will be nonzero at null infinity only at the retarded times corresponding to the time intervals T_1 and T_2. A rough estimate of the number of entangling photons was obtained in [Belenchia et al., 2018] using the Larmor formula for radiation in these eras, which, in natural units, yields

$$\langle N \rangle \sim \frac{q^2 d^2}{[\min(T_1, T_2)]^2} \quad \text{(Minkowski, EM)}. \tag{4.1.19}$$

The corresponding result in the linearized gravitational case is [Belenchia et al., 2018]

$$\langle N \rangle \sim \frac{m^2 d^4}{[\min(T_1, T_2)]^4} \quad \text{(Minkowski, GR)}. \tag{4.1.20}$$

Therefore, if Alice recombines her particle sufficiently slowly that $T_1, T_2 \gg qd$ in the electromagnetic case or $T_1, T_2 \gg md^2$ in the gravitational case, then she can maintain the quantum coherence of her particle. In particular, Alice can keep the components of her particle separated for as long a time T as she likes without destruction of the coherence.

As shown in Chapter 3, the situation is quite different if a black hole is present. In the electromagnetic case, even if $T_1, T_2 \gg qd$ so that a negligible number of entangling photons is emitted to infinity, there will be entangling radiation emitted into the black hole. For large T, the number of entangling photons increases with T as[11]

$$\langle N \rangle \sim \frac{M^3 q^2 d^2}{D^6} T \qquad \text{(black hole, EM)} \tag{4.1.21}$$

where M is the mass of the black hole, D is the proper distance of Alice's lab from the horizon of the black hole, and we assume that $D \gtrsim M$. The corresponding result in the linearized

11. In the analysis of Chapter 3, we used the fact that the Unruh vacuum is well approximated by the Hartle-Hawking vacuum at low frequencies near the horizon of the black hole.

gravitational case is

$$\langle N \rangle \sim \frac{M^5 m^2 d^4}{D^{10}} T \qquad \text{(black hole, GR).} \tag{4.1.22}$$

Thus, the coherence of Alice's particle will always be destroyed within a finite time.

In the next two sections, we will apply the above analysis to the cases of Rindler spacetime and de Sitter spacetime. Although we will explicitly analyze only the Rindler and de Sitter cases, it will be clear from our analysis of the next two sections — as well as our analysis in Chapter 3 — that it can be applied to any Killing horizon, provided only that the initial "vacuum state" $|\Psi_0\rangle$ of the electromagnetic and/or linearized gravitational field corresponds to one-particle states that are positive frequency with respect to affine time on the future Killing horizon.

4.2 Rindler Horizons Decohere Quantum Superpositions

We now consider the case of Minkowski spacetime[12] with Alice's lab uniformly accelerating with acceleration a. Specifically, we take Alice's lab to follow the orbit

$$t = \frac{1}{a}\sinh(a\tau), \qquad z = \frac{1}{a}\cosh(a\tau) \tag{4.2.1}$$

of the boost Killing field

$$b^a = a\left[z\left(\frac{\partial}{\partial t}\right)^a + t\left(\frac{\partial}{\partial z}\right)^a\right]. \tag{4.2.2}$$

Here we have normalized b^a such that $b^a b_a = -1$ on the worldline of Alice's laboratory. Thus, b^a is the four-velocity of Alice's laboratory and τ is the proper time in her lab. We introduce the null coordinates

$$U \equiv t - z, \qquad V \equiv t + z \tag{4.2.3}$$

12. We explicitly treat the case of 4 spacetime dimensions, but our analysis generalizes straightforwardly to all higher dimensions.

and the corresponding vector fields

$$n^a \equiv (\partial/\partial V)^a, \qquad \ell^a \equiv (\partial/\partial U)^a, \tag{4.2.4}$$

which are globally defined, future-directed null vector fields that satisfy $\ell^a n_a = -1$. In terms of these coordinates, the Minkowski spacetime metric is

$$\eta = -dUdV + dx^2 + dy^2 \tag{4.2.5}$$

and the boost vector field is given by

$$b^a = a\big[-U\ell^a + Vn^a\big]. \tag{4.2.6}$$

The boost Killing field is null on the two "Rindler horizons," i.e., the two null planes $U = 0$ and $V = 0$, which divide Minkowski spacetime into four wedges. The orbits of the boost Killing field are future-directed and timelike within the "right Rindler wedge" $\mathscr{W}_{\rm R}$ which is the region $U < 0$ and $V > 0$. Thus, the "right Rindler wedge" $\mathscr{W}_{\rm R}$ — where Alice performs her experiment — is a static, globally hyperbolic spacetime where the notion of "time translations" is defined by Lorentz boosts.

We refer to the null surface $U = 0$ as the future Rindler horizon and denote it as $\mathscr{H}_{\rm R}^+$. On the region $V > 0$ of $\mathscr{H}_{\rm R}^+$, it is useful to introduce the coordinate v by

$$V = V_0 e^{av} \tag{4.2.7}$$

where V_0 is an arbitrary constant. Then, for $V > 0$ on $\mathscr{H}_{\rm R}^+$, we have

$$b^a\big|_{\mathscr{H}_{\rm R+}} = aV\left(\frac{\partial}{\partial V}\right)^a\bigg|_{\mathscr{H}_{\rm R+}} = \left(\frac{\partial}{\partial v}\right)^a\bigg|_{\mathscr{H}_{\rm R+}}. \tag{4.2.8}$$

Since $(\partial/\partial V)^a$ on the horizon is tangent to the affinely parametrized null geodesic generators of $\mathscr{H}_{\rm R}^+$, we refer to V as the "affine time" on $\mathscr{H}_{\rm R}^+$, whereas we refer to v as the "boost Killing time" on $\mathscr{H}_{\rm R}^+$.

4.2.1 Decoherence due to Radiation of Soft Photons/Gravitons through the Rindler Horizon

We are now in position to apply the results of Sec. 4.1 to the Rindler case. We will first analyze the electromagnetic case and then give the corresponding results in the gravitational case.

We assume that the electromagnetic field is initially in the Minkowski vacuum state. We assume that Alice possesses a charged particle that is initially stationary (with respect to the boost Killing field) in her (uniformly accelerating) lab. She then creates a quantum spatial superposition which is held stationary (with respect to the boost Killing field) for a proper time T and is then recombined. We wish to know the degree of decoherence of Alice's particle due to emission of radiation. We may directly apply the analysis of Sec. 4.1 to answer this question.

The future Rindler horizon $\mathscr{H}_{\rm R}^+$ $(U = 0)$ does not meet the technical requirements of being a Cauchy surface for Minkowski spacetime, since there are inextendible timelike curves that remain in the past of $\mathscr{H}_{\rm R}^+$ as well as inextendible timelike curves that lie in the future of $\mathscr{H}_{\rm R}^+$. However, as argued in [Unruh and Wald, 1984], it is effectively a Cauchy surface for determining evolution of solutions to the wave equation. This is most easily seen in the conformally completed spacetime, where $\mathscr{H}_{\rm R}^+$ is the past light cone of a point $p \in \mathscr{I}^+$ except for the single generator that lies on $\mathscr{I}^+$ and it also is the future light cone of a point on $p' \in \mathscr{I}^-$ except for the single generator that lies on $\mathscr{I}^-$. Data on the full past light cone of p would determine a solution to the past of $\mathscr{H}_{\rm R}^+$ and data on the full future light cone of p' would determine a solution to the future of $\mathscr{H}_{\rm R}^+$, thereby determining a solution everywhere in Minkowski spacetime. However, for solutions with appropriate decay, the data on the missing null geodesic generators of $\mathscr{I}^+$ and $\mathscr{I}^-$ can be determined by continuity from the data on $\mathscr{H}_R^+$. Consequently, data on $\mathscr{H}_R^+$ suffices to uniquely characterize solutions with appropriate decay. Consequently, the "out" states $|\Psi_1\rangle$ and $|\Psi_2\rangle$ of the radiation are completely determined by data on $\mathscr{H}_R^+$. Note that this contrasts sharply with the black hole case, where one would need data on both the future event horizon and future null infinity to characterize the "out" state of radiation.

The decoherence of Alice's particle due to radiation is given by Eq. (4.1.17). In order to evaluate this, we first consider a classical point charge of charge q in the "right Rindler wedge" $\mathcal{W}_R$ that is stationary with respect to the boost Killing field and lies at proper distance D from the bifurcation surface of the Rindler horizon. Such a charge will be uniformly accelerating with acceleration a given by

$$a = \frac{1}{D}\,, \tag{4.2.9}$$

as depicted in Fig. 4.1.

The explicit solution for such a stationary charge in the Rindler wedge has long been known [Whittaker, 1927, Bondi and Gold, 1955, Rohrlich, 1961, Boulware, 1980, Padmanabhan and Padmanabhan, 2010, Eriksen and Grøn, 2004]. The only nonvanishing component of the electromagnetic field in the region $V > 0$ of $\mathscr{H}_R^+$ is

$$E_U \equiv F_{ab}\ell^a n^b = \frac{2a^2 q}{\pi(1 + a^2\rho^2)^2} \tag{4.2.10}$$

where $\rho^2 \equiv x^2 + y^2$. Electromagnetic radiation through the Rindler horizon is described by the pullback, E_A, of the electric field[13] $E_a = F_{ab}n^b$ to $\mathscr{H}_R^+$, where the capital Latin indices

13. The electric field as measured by an observer with 4-velocity u^b is $F_{ab}u^b$. Although n^b is null rather than timelike, it is natural (and standard) to use the terminology "electric field" for $F_{ab}n^b$ on the horizon.

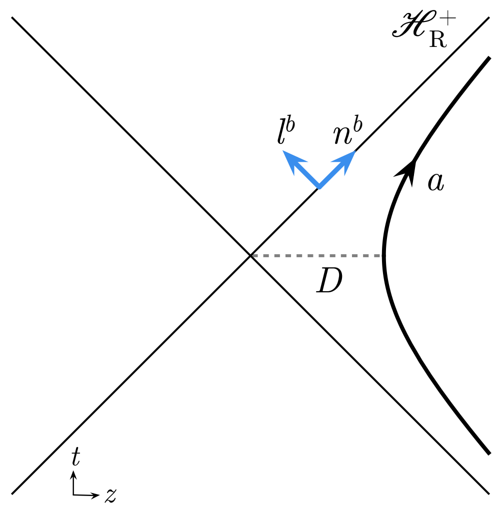

Figure 4.1: Alice's laboratory undergoes uniform acceleration a in the z-direction in Minkowski spacetime and thus follows an orbit of a boost Killing field. The future Rindler horizon $\mathscr{H}_R^+$ is a Killing horizon for this boost Killing field. The future-directed null vector $n^b = (\partial/\partial V)^b$ points along the horizon, while $l^b = (\partial/\partial U)^b$ is transverse to it. D is the proper distance from Alice's lab to the horizon.

from the early alphabet denote spatial components in the x and y directions. Since $E_A = 0$ on the horizon for a uniformly accelerated charge, one may say that a charge held stationary in Alice's lab does not produce any radiation as determined on $\mathscr{H}_R^+$ — even though a uniformly accelerated charge radiates (inertial) energy to future null infinity.[14]

Now consider the case where the point charge is initially uniformly accelerating with acceleration a at a proper distance $D = 1/a$ from the bifurcation surface of the Rindler horizon. The charge is then moved in the z-direction[15] to a different orbit of the same boost Killing field, so that it has uniform acceleration a' and lies at proper distance $D' = 1/a'$ from the Rindler horizon. After the charge has reached its new location, the electric field on $\mathscr{H}_R^+$ is again given by Eq. (4.2.10), but its value, E'_U, will be different from its value at early

14. A uniformly accelerating charge has a nonvanishing inertial energy current flux $T_{ab}t^a$ through both $\mathscr{H}_R^+$ and $\mathscr{I}^+$, where t^a denotes a Minkowski time translation. However, the flux of "boost energy" $T_{ab}b^a$ vanishes at both $\mathscr{H}_R^+$ and $\mathscr{I}^+$.

15. We consider a z-displacement for simplicity. Similar results would hold if the charge were displaced in the x or y directions.

times. Maxwell's equations on $\mathscr{H}_{\rm R}^+$ imply that

$$\mathcal{D}^A E_A = \partial_V E_U \tag{4.2.11}$$

where $\mathcal{D}_A$ is the derivative operator on the $\mathbb{R}^2$ cross sections of the horizon and capital Latin indices from the early alphabet are raised and lowered with the metric, δ_{AB}, on the cross sections. Eq. (4.2.11) implies that $E_A \neq 0$ whenever $\partial_V E_U \neq 0$, so there will be radiation through the horizon as the charge is being moved. Most importantly, it implies that

$$\mathcal{D}^A \left(\int_{-\infty}^{\infty} dV E_A \right) = \Delta E_U \tag{4.2.12}$$

where $\Delta E_U = E'_U - E_U$ is the change in the radial electric field between the charge at positions D' and D. Now, in a gauge where $A_a n^a = 0$ on the horizon, the transverse (i.e., x-y) components of the electric field are related to the corresponding components of the vector potential by

$$E_A = -\partial_V A_A. \tag{4.2.13}$$

Since the transverse components of the Coulomb field of a static charge vanish, we may replace the vector potential A_A by the "Coulomb subtracted" vector potential $\mathcal{A}_A$ defined by Eq.(4.1.9), so we have

$$E_A = -\partial_V \mathcal{A}_A. \tag{4.2.14}$$

It then follows immediately from Eq. (4.2.12) that the difference, $\Delta\mathcal{A}_A$, between the final and initial values of $\mathcal{A}_A$ is given by

$$\mathcal{D}^A(\Delta\mathcal{A}_A) = -\Delta E_U \tag{4.2.15}$$

independently of the manner in which the charge is moved from D to D'. Equation (4.2.15) is an exact mathematical analog of the electromagnetic memory effect at null infinity [Bieri and Garfinkle, 2013]. For the explicit solution Eq. (4.2.10), we have

$$\Delta E_U \approx \frac{-4qda^3(1-a^2\rho^2)}{\pi(1+a^2\rho^2)^3}. \tag{4.2.16}$$

where $d = D' - D$ and we have assumed that

$$d \ll D = \frac{1}{a}\,. \tag{4.2.17}$$

From Eq. (4.2.15), we find that $\Delta\mathcal{A}_A$ points in the $\hat{\rho}$-direction and has magnitude

$$|\Delta\mathcal{A}_A| = \Delta\mathcal{A}_\rho \sim \frac{qda^3\rho}{(1+a^2\rho^2)^2}. \tag{4.2.18}$$

The key point is that even though $E_A = 0$ at both late and early times, $\mathcal{A}_A$ does not return to its original value at late times, and the change, $\Delta\mathcal{A}_A$, in the vector potential between late and early times is determined only by the initial and final positions of the charge.

We now consider the quantized radiation through the horizon resulting from the displacement of the charge, assuming that, after the displacement, the charge is held at its new position, D', forever. For the Fock space associated with the Minkowski vacuum state, the map $K : S \to \mathcal{H}_{\text{in}}$ that associates one-particle states to classical solutions is given by taking the positive frequency part of the classical solution with respect to inertial time, with the inner product on $\mathcal{H}_{\text{in}}$ given by the Klein-Gordon product. For the electromagnetic field on $\mathscr{H}_{\text{R}}^+$ in a gauge where $\mathcal{A}_a n^a = 0$ on $\mathscr{H}_{\text{R}}^+$, the "free data" on $\mathscr{H}_{\text{R}}^+$ is the pullback, $\mathcal{A}_A$, of the vector potential. For two classical solutions with data $\mathcal{A}_{1,A}$ and $\mathcal{A}_{2,A}$ on $\mathscr{H}_{\text{R}}^+$, the inner product of their corresponding one-particle states is given by [Kay and Wald, 1991, Dappiaggi et al., 2017]

$$\langle K\mathcal{A}_1 |\, K\mathcal{A}_2\rangle_{\mathscr{H}_{\text{R}}^+} = 2\int\limits_{\mathbb{R}^2} dxdy \int\limits_0^\infty \frac{\omega d\omega}{2\pi}\delta^{AB}\overline{\hat{\mathcal{A}}_{1,A}}\hat{\mathcal{A}}_{2,B} \tag{4.2.19}$$

where $\hat{\mathcal{A}}_A(\omega, x^B)$ is the Fourier transform of $\mathcal{A}_A(V, x^B)$ with respect to the affine parameter V. By the same reasoning as led to Eq. (4.1.15), the expected number of photons on $\mathscr{H}_{\rm R}^+$ in the coherent state associated to any classical solution $\mathcal{A}_A$ is simply

$$\langle N \rangle = \|K\mathcal{A}\|^2_{\mathscr{H}_{\rm R}^+} \tag{4.2.20}$$

where the norm is defined by the inner product Eq. (4.2.19). However, since $\Delta\mathcal{A}_A \neq 0$, the Fourier transform, $\hat{\mathcal{A}}_A(\omega, x^B)$, of $\mathcal{A}_A$ diverges as $1/\omega$ as $\omega \to 0$. It follows that the integrand of the expression for the norm given by the right side of Eq. (4.2.19) also diverges as $1/\omega$ as $\omega \to 0$, so the integral is logarithmically divergent. Thus, $\|K\mathcal{A}\|^2_{\mathscr{H}_{\rm R}^+} = \infty$. Therefore, if Alice displaces a charged particle to a different orbit of the boost Killing field and the particle remains on this new uniformly accelerated trajectory forever, an infinite number of "soft horizon photons" will be radiated through the Rindler horizon[16] regardless of how quickly or slowly this process is done.

The above infrared divergence is an exact mathematical analog of the infrared divergences that occur at null infinity in QED for processes with nonzero memory (see e.g., [Ashtekar, 1987, Satishchandran and Wald, 2019, Carney et al., 2017, 2018, Asorey et al., 2018]). Note that infrared divergences at null infinity arise only in $d = 4$ spacetime dimensions. The reason for this is that in d dimensions, radiation falls off at infinity in null directions as $1/r^{d/2-1}$, whereas Coulomb fields and associated memory effects fall off as $1/r^{d-3}$, so it is only in $d = 4$ dimensions that memory effects occur at radiative order [Pate et al., 2018, Satishchandran and Wald, 2019]. By contrast, radial Coulomb fields will penetrate a Killing horizon in all spacetime dimensions (see [Garfinkle, 2021] for the case of a Schwarzschild black hole) and a

16. These "soft horizon photons" are closely related to the "soft hair" discussed by Hawking, Perry, and Strominger [Hawking et al., 2016] in the case of black hole horizons (see also [Hotta et al., 2001]). However, while Hawking, Perry, and Strominger considered effects of matter falling into a black hole, our "soft radiation" arises from the displacement of matter sourcing a long range field outside of a horizon. Note that in the case of a black hole, the "soft radiation" of Alice's experiment increases the entanglement of the black hole with its exterior.

displacement of a charge will result in a change in the radial Coulomb field in all dimensions. As analyzed above, this will result in radiation through the horizon in all dimensions high enough for the field in question to admit radiation (i.e., $d \geq 3$ for electromagnetism and $d \geq 4$ for gravity). Consequently, the logarithmic divergence in Eq. (4.2.20) occurs in all spacetime dimensions that admit radiation.[17]

Now suppose that Alice displaces the particle a z-distance $d \ll D = 1/a$ from D to $D' = D + d$ as above, but instead of leaving the particle at D' forever, she leaves it there for proper time[18] T and then returns it to D. In this case, the transverse components of the vector potential, $\mathcal{A}_A$, return to their initial values at late times, so there is no "memory effect" at the horizon. Correspondingly, there are no infrared divergences in the expected number of photons that propagate through $\mathscr{H}_{\rm R}^{+}$. Nevertheless, if T is very large then the expected number of photons $\langle N \rangle$ will be correspondingly large. To see this, we note that if, for convenience, we work in a gauge where $\mathcal{A}_A = 0$ initially, then during the era at which the particle is at D', $\mathcal{A}_A$ will be given by the right side of Eq. (4.2.18). If we keep the manner in which the particle is moved from D to D' as well as from D' to D fixed but take T to be very large, the asymptotic behavior of the norm Eq. (4.2.19) will be dominated by the low-frequency contribution from the era of time T that the particle is displaced. The logarithmic divergence at $\omega = 0$ that would occur if the particle remained at D' forever is now effectively cut off at frequency $\omega \sim 1/V$, where V denotes the affine time duration on the horizon $\mathscr{H}_{\rm R}^{+}$ over which the particle remains at D'. We obtain

$$\langle N \rangle = \| K \mathcal{A} \|^2_{\mathscr{H}_{\rm R}} \sim q^2 d^2 a^2 \ln \left(\frac{V}{\min[V_1, V_2]} \right) \tag{4.2.21}$$

17. Indeed, there would also be infrared divergences for a particle that sources a massive field, since the Yukawa field of the particle will also penetrate the horizon.

18. We have normalized the boost Killing field b^a so that Killing time equals proper time on the orbit at D with acceleration a. Since we assume $d = D' - D \ll D$, Killing time and proper time are also (nearly) equal on the orbit at D'. Thus, T is also the elapsed Killing time that Alice keeps the particle at D'.

where $V_1, V_2 \ll V$ are the durations of affine time over which the particle is displaced from D to D' and from D' back to D, so that $1/\min[V_1, V_2]$ provides an effective high-frequency cutoff. However, the affine time V on the horizon is related to boost Killing time on the horizon by

$$V = V_0 \exp(av) \tag{4.2.22}$$

and the boost Killing time v corresponds to the proper time T in Alice's lab. Thus, we obtain

$$\langle N \rangle \sim q^2 d^2 a^3 T \qquad \text{(Rindler, EM)}\,. \tag{4.2.23}$$

Therefore, no matter how slowly the particle is displaced, it is forced to radiate a number of "soft Rindler horizon photons" through the Rindler horizon that is proportional to the time T that the particle remains on the displaced trajectory.

We are now in a position to fully analyze Alice's experiment. Alice's lab is uniformly accelerating with acceleration a in Minkowski spacetime. She puts her particle of charge q into a superposition of states separated by z-distance $d \ll 1/a$ and keeps these components stationary in her lab for a proper time T. She then recombines the components and determines their coherence.[19] By the analysis of Sec. 4.1, the decoherence is given by Eq. (4.1.18). However, for large T, the calculation of $\|K\left[G^{\text{ret}}(j_1 - j_2)\right]\|^2$ corresponds precisely to the calculation we have given above of the number of photons radiated through the Rindler horizon when a charge is displaced for a time T. Thus, we obtain

$$\|K\left[G^{\text{ret}}(j_1 - j_2)\right]\|^2 \sim q^2 d^2 a^3 T. \tag{4.2.24}$$

In other words, for large T, Alice's superposition will decohere due to radiation of "soft

19. The coherence can be determined as described in footnote 6.

Rindler horizon photons," as

$$\mathscr{D} = 1 - \exp(-\Gamma_{\text{rad}} T) \tag{4.2.25}$$

where the "decoherence rate" Γ_{rad}, is given by

$$\Gamma_{\text{rad}} = q^2 d^2 a^3. \tag{4.2.26}$$

Thus, restoring the constants c, $\hbar$, and ϵ_0, Alice's particle will decohere within a time

$$T_{\text{D}} \sim \frac{\epsilon_0 \hbar c^6}{a^3 q^2 d^2} \qquad \text{(Rindler, EM)} \tag{4.2.27}$$

$$\sim 10^{33} \text{ years} \left(\frac{\text{g}}{a}\right)^3 \cdot \left(\frac{\text{e}}{q}\right)^2 \cdot \left(\frac{\text{m}}{d}\right)^2. \tag{4.2.28}$$

Thus, if Alice's lab uniformly accelerates at one g in flat spacetime and she separates an electron into two components one meter apart, she would not be able to maintain coherence of the electron for more than 10^{33} years.

A similar analysis holds in the gravitational case[20] where Alice separates a massive body with mass m across a distance d and maintains this superposition for a time T. In the gravitational case, the "electric part" of the perturbed Weyl tensor $E_{ab} = C_{acbd} n^c n^d$ plays an analogous role to the electric field E_a in the electromagnetic version of the gedankenexperiment. For a uniformly accelerating point mass, the only nonvanishing component of the electric part of the Weyl tensor on $\mathscr{H}_{\text{R}}^+$ is $E_{UU} = C_{acbd} \ell^a n^c \ell^b n^d$.

Gravitational radiation on the horizon is described by the pullback, E_{AB}, of E_{ab}, which vanishes for the static point mass. However, the process of quasistatically moving the static point mass involves a change in E_{UU} on $\mathscr{H}_{\text{R}}^+$. The (once-contracted) Bianchi identity on the horizon yields

$$\mathcal{D}^A E_{AB} = \partial_V E_{UB}, \qquad \mathcal{D}^A E_{UA} = \partial_V E_{UU} \tag{4.2.29}$$

20. In the gravitational case, additional stress-energy will be needed to keep Alice's particle in uniform acceleration. We will ignore the gravitational effects of this additional stress-energy.

which implies that

$$\mathcal{D}^A \mathcal{D}^B E_{AB} = \partial_V^2 E_{UU} \tag{4.2.30}$$

which is closely analogous to Eq. (4.2.11). As in the electromagnetic case, if a uniformly accelerating point mass is quasistatically moved there is necessarily gravitational radiation through $\mathscr{H}_{\mathrm{R}}^+$.

To determine the number of "Rindler horizon gravitons" emitted we quantize the linearized gravitational field. For a metric perturbation h_{ab} in a gauge where $h_{ab}n^a = 0$ and $\delta^{AB} h_{AB} = 0$, the "free data" on $\mathscr{H}_{\mathrm{R}}^+$ is h_{AB}. A "particle" in the standard Fock space associated to the Poincaré invariant vacuum is then a positive frequency solution with respect to affine parameter V and the inner product on the one-particle Hilbert space is given by a direct analog of Eq. (4.2.19) with the vector potential $\mathcal{A}_A$ replaced with the metric perturbation h_{AB}, namely

$$\langle K h_1 | K h_2 \rangle_{\mathscr{H}_{\mathrm{R}}^+} = \frac{1}{8} \int_{\mathbb{R}^2} dx dy \int_0^\infty \frac{\omega d\omega}{2\pi} \delta^{AB} \delta^{CD} \overline{\hat{h}_{1,AC}} \hat{h}_{2,BD}. \tag{4.2.31}$$

Finally, E_{AB} is related to the metric perturbation h_{AB} by

$$E_{AB} = -\frac{1}{2} \partial_V^2 h_{AB} \,. \tag{4.2.32}$$

Equations (4.2.30) and (4.2.32) directly imply that a permanent change, $\Delta E_{UU} \neq 0$, in the U-U component of the electric part of the Weyl tensor on $\mathscr{H}_{\mathrm{R}}^+$ implies a permanent change, $\Delta h_{AB} \neq 0$, in the perturbed metric on $\mathscr{H}_{\mathrm{R}}^+$ between early and late times. In the quantum theory, as in the electromagnetic case, this implies a logarithmic infrared divergence in the number of gravitons emitted through $\mathscr{H}_{\mathrm{R}}^+$ in the process where a uniformly accelerating charge is moved to a new orbit of the same boost Killing field and then remains at the new position forever.

The analysis of Alice's experiment proceeds in a similar manner to the electromagnetic case. Alice does not maintain the relative separation of her wavefunction forever but closes her superposition after a proper time T. As before, the number of entangling gravitons emitted to the Rindler horizon is logarithmically growing in affine time and therefore linearly growing in the proper time duration T of Alice's experiment. We obtain

$$\langle N \rangle \sim m^2 d^4 a^5 T \qquad \text{(Rindler, GR)}\,. \tag{4.2.33}$$

Thus, restoring constants, we find that the Rindler horizon decoheres the quantum superposition of a uniformly accelerating massive body in a time

$$T_D^{\text{GR}} \sim \frac{\hbar c^{10}}{G m^2 d^4 a^5} \qquad \text{(Rindler, GR)} \tag{4.2.34}$$

$$\sim 2 \text{ fs} \left(\frac{\mathrm{M}_{\mathrm{Moon}}}{m}\right)^2 \cdot \left(\frac{\mathrm{R}_{\mathrm{Moon}}}{d}\right)^4 \cdot \left(\frac{\mathrm{g}}{a}\right)^5 . \tag{4.2.35}$$

Therefore, if the Moon were accelerating at one g and occupied a quantum state with its center of mass superposed by a spatial separation of the order of its own radius then it would decohere within about 2 femtoseconds. Of course, it would not be easy to put the moon in such a coherent quantum superposition.

Note the acceleration of a stationary observer outside of a black hole who is reasonably far[21] ($D \gtrsim M$) from the event horizon is $a \sim M/D^2$. If we substitute $a = M/D^2$ in Eqs. (4.2.27) and (4.2.34), we obtain Eqs. (4.1.21) and (4.1.22), respectively. Therefore, it might be tempting to believe that what is important in all cases is the acceleration of Alice's lab. However, this is not the case. In particular, if we replace the black hole by an ordinary star (and if there are no dissipative effects in the star), then there will not be any analogous

21. It should be emphasized that the estimates made in Chapter 3 that yielded Eqs.(4.1.21) and (4.1.22) assumed that Alice's lab is reasonably far from the black hole. If Alice's lab is extremely close to the black hole (i.e., at a distance $D \ll M$ from the horizon), then the black hole analysis would reduce to the Rindler case analyzed here.

decoherence effect, even though the acceleration of Alice's lab is the same as in the case of a black hole. Furthermore, as we shall see in Sec. 4.3, decoherence effects associated with the cosmological horizon occur in de Sitter spacetime even for nonaccelerating observers. It is the presence of a Killing horizon that is the essential ingredient for the fundamental rate of decoherence of quantum superpositions as described in this dissertation.

We now consider another potential cause of decoherence, namely Unruh radiation.

4.2.2 Decoherence due to Scattering of Unruh Radiation

The Minkowski vacuum state restricted to a Rindler wedge is a thermal state at the Unruh temperature

$$\mathcal{T} = \frac{a}{2\pi} \tag{4.2.36}$$

relative to the notion of time translations defined by the Lorentz boost Killing field b^a, Eq. (4.2.2). Thus, the superposition state of Alice's particle will be buffeted by this thermal bath of Unruh radiation. Scattering of this radiation will cause some decoherence of Alice's particle. Indeed, since this decoherence should occur at a steady rate while the superposition is kept stationary (and thus the decoherence will be proportional to T), one might even suspect that scattering of Unruh radiation could be the same effect as found in the previous section but expressed in a different language. The purpose of this subsection is to show that this is not the case, i.e., decoherence due to scattering of Unruh radiation and decoherence due to radiation of "soft" photons/gravitons through the horizon are distinct effects. Furthermore, we shall show that, for reasonable parameter choices, the decoherence rate due to the scattering of Unruh radiation is smaller than the decoherence rate due to emitted radiation as obtained in the previous section. We will consider only the electromagnetic case in this subsection.

The decoherence rate of a spatial superposition due to collisions with particles in an environment has been analyzed in [Joos and Zeh, 1985, Gallis and Fleming, 1990, Diósi, 1995, Hornberger and Sipe, 2003], and we will adapt this analysis to obtain a rough estimate of the decoherence caused by the scattering of Unruh radiation. As in Eq. (4.1.1), Alice has a particle of charge q in a state $|\psi\rangle = (|\psi_1\rangle + |\psi_2\rangle)/\sqrt{2}$, where $|\psi_1\rangle$ and $|\psi_2\rangle$ are spatially separated by a distance d. Since we require $d \ll 1/a$ [see eq. (4.2.17)] and since the typical wavelength of Unruh photons at temperature Eq. (4.2.36) is $\lambda \sim 1/a$, we are in the scattering regime where $\lambda \gg d$. In an elastic scattering event between Alice's particle and a photon in the Unruh radiation, the final outgoing state of the photon will depend upon which branch of the superposition the photon scattered off of. Let $|\chi_1\rangle$ denote the outgoing state of the Unruh photon for scattering off of $|\psi_1\rangle$ and let $|\chi_2\rangle$ denote the outgoing state for scattering off of $|\psi_2\rangle$. Decoherence will occur to the extent to which these outgoing states of the scattered Unruh photon are distinguishable, i.e., $\mathscr{D} = 1 - |\langle\chi_1|\chi_2\rangle|$.

In order to obtain a rough estimate of the decoherence resulting from a single scattering event, we consider the corresponding Minkowski process of the scattering of a photon of momentum p off of an inertial superposition separated by d, with $d \ll 1/p$. Assuming that the charged particle states $|\psi_1\rangle$ and $|\psi_2\rangle$ are identical except for their location, the scattered photon states $|\chi_1\rangle$ and $|\chi_2\rangle$ should differ only by the action of the translation operator $e^{-i\vec{\mathcal{P}}\cdot\vec{d}}$, i.e.,

$$|\chi_2\rangle \approx e^{-i\vec{\mathcal{P}}\cdot\vec{d}}|\chi_1\rangle \tag{4.2.37}$$

where $\vec{\mathcal{P}}$ denotes the photon momentum operator. Expanding the exponential, we obtain the following rough estimate of the decoherence resulting from a single scattering event involving a photon of momentum p

$$1 - |\langle\chi_1|\chi_2\rangle| \sim p^2 d^2 \tag{4.2.38}$$

where we have ignored any dependence on the angle between the incoming momentum $\vec{p}$ and the separation $\vec{d}$. We will take Eq. (4.2.38) as our estimate of the decoherence of Alice's particle resulting from the scattering of a single Unruh photon of "Rindler momentum" p (i.e., of energy $\epsilon = p$ with respect to the boost Killing field b^a).

The total decoherence rate due to scattering of Unruh radiation is then given by

$$\Gamma_{\text{scatt}} \sim d^2 \int_0^\infty dp \; p^2 \varrho(p) \sigma(p) \tag{4.2.39}$$

where $\varrho(p)$ is the number density of photons at momentum p [so $\varrho(p)$ is also the incoming flux of photons] and $\sigma(p)$ is the scattering cross section. For a thermal distribution of photons.[22] we have

$$\varrho(p) \sim \frac{p^2}{e^{p/\mathcal{T}} - 1}. \tag{4.2.40}$$

We take σ to be given by the Thomson cross section

$$\sigma = \frac{8\pi}{3} \frac{q^4}{(4\pi m)^2}, \tag{4.2.41}$$

where m is the mass of Alice's particle. Putting this all together, our estimate of the decoherence rate due to scattering of Unruh photons is

$$\Gamma_{\text{scatt}} \sim \frac{q^4 d^2 a^5}{m^2} \qquad \text{(Rindler, EM)}\,. \tag{4.2.42}$$

Comparing Eq. (4.2.42) to the rate of decoherence, Γ_{rad} due to the emission of soft photons given by Eq. (4.2.26), one can immediately see that the effects are distinct. In particular, Γ_{rad} has no dependence on the mass, m, of Alice's particle, whereas Γ_{scatt} does depend on m on account of the mass dependence of the scattering cross section. The ratio of these decoherence rates is given by

$$\frac{\Gamma_{\text{scatt}}}{\Gamma_{\text{rad}}} \sim \frac{q^2 a^2}{m^2} = \left(\frac{q/m}{D} \right)^2 \tag{4.2.43}$$

22. The factor of p^2 in the numerator of Eq. (4.2.40) arises from the density of states in Minkowski spacetime. We ignore here any differences between the Minkowski and Rindler densities of states

Now, q/m is the "charge radius" of Alice's particle and, as argued in [Belenchia et al., 2018], it represents a fundamental lower bound to the spread of a charged particle due to vacuum fluctuations of the electromagnetic field. Therefore, in order that $|\psi_1\rangle$ and $|\psi_2\rangle$ not overlap, we must have $d > q/m$. Since $d \ll D$, we conclude that

$$\frac{\Gamma_{\text{scatt}}}{\Gamma_{\text{rad}}} \ll 1 \tag{4.2.44}$$

i.e., the contribution to decoherence from the scattering of Unruh radiation is negligible compared with the decoherence due to emission of soft photons through the Rindler horizon.

A similar analysis holds for a charged particle superposition outside of a black hole. It is worth noting, that the decoherence effects due to scattering of Hawking radiation will decrease with distance, D, from the black hole only as $1/D^2$ for large D, giving

$$\Gamma_{\text{scatt}} \sim \frac{q^4 d^2}{m^2 M^3} \frac{1}{D^2} \quad \text{(black hole, EM)}. \tag{4.2.45}$$

On the other hand, by Eq. (4.1.21) the decoherence effects of radiation of soft photons through the horizon decreases with D as $1/D^6$. Thus at sufficiently large D, the decoherence effects due to scattering of Hawking radiation will dominate. However, in this regime, both effects are extremely small.

4.2.3 Decoherence from the Inertial Perspective

In our analysis of the decoherence of a spatial superposition in the presence of a black hole [Danielson et al., 2022b] as well as in our analysis of the decoherence of a spatial superposition in Rindler spacetime given above in Sec. 4.2.1, it may appear that we have introduced a radical new mechanism for decoherence, namely radiation of soft photons and gravitons through a horizon. The main purpose of this subsection is to show that, in fact, the decoherence we derived in the Rindler case can also be obtained by entirely conventional means. In the

Rindler case, we are simply considering a uniformly accelerating superposition in Minkowski spacetime. The radiation of entangling photons to infinity from such a superposition can be calculated in the inertial viewpoint by standard methods, without introducing concepts such as a Rindler horizon. It is instructive to calculate the decoherence from the inertial viewpoint both in order to validate the results of Sec. 4.2.1 as well as to gain insight into how the emitted "soft photons" would be interpreted by an inertial observer. As we shall see, the entangling photons as seen by a faraway inertial observer along the forward axis of acceleration will be "hard" even though, from her point of view, Alice has performed the experiment adiabatically. We will restrict our analysis in this subsection to the electromagnetic case.

The Liénard-Wiechert solution for the potential of a point charge in Minkowski spacetime following an arbitrary worldline $X^\mu(\tau)$ is, in Lorenz gauge,

$$A^\mu(x) = \frac{1}{4\pi}\frac{1}{\alpha}\frac{q}{|\vec{x}-\vec{X}(t_{\rm ret})|}\frac{dX^\mu}{dt}(t_{\rm ret}) \tag{4.2.46}$$

where

$$\alpha \equiv 1 - \hat{n}\cdot\frac{d\vec{X}}{dt}(t_{\rm ret}) \quad \text{and } \hat{n} = \frac{\vec{x}-\vec{X}(t_{\rm ret})}{|\vec{x}-\vec{X}(t_{\rm ret})|}. \tag{4.2.47}$$

For a uniformly accelerated trajectory with acceleration a, we have

$$X^\mu(\tau) = \left(\frac{1}{a}\sinh(a\tau), 0, 0, \frac{1}{a}\cosh(a\tau)\right). \tag{4.2.48}$$

In Bondi coordinates (u, r, θ, ϕ) with

$$u \equiv t - r \tag{4.2.49}$$

the future light cone of an event at proper time τ on the worldline Eq. (4.2.48) reaches null infinity at

$$au = \sinh(a\tau) - \cos\theta\cosh(a\tau). \tag{4.2.50}$$

Electromagnetic radiation is described by the pullback of the electromagnetic field, Eq. (4.2.46), to null infinity. Taking the limit as $r \to \infty$ at fixed u, we obtain[23]

$$A_A(u,\theta,\phi) = \frac{-q}{4\pi} \frac{\sinh(a\tau)\sin\theta}{\cosh(a\tau) - \cos\theta \sinh(a\tau)} (d\theta)_A \tag{4.2.51}$$

where, in this subsection, capital indices from the early alphabet denote angular components on the 2-sphere cross-sections of $\mathscr{I}^+$. We will be concerned with the difference, at fixed (u,θ,ϕ), between the electromagnetic radiation of a particle following the trajectory Eq. (4.2.48) and a particle following a similar trajectory that is displaced in the z-direction by a proper distance $d \ll 1/a$ and thus has

$$\delta a = a^2 d. \tag{4.2.52}$$

We denote this difference by

$$A^{\mathrm{d}}_A(u,\theta,\phi) \equiv A_A(a+\delta a) - A_A(a) \approx \delta a \left(\frac{\partial A_A}{\partial a}\right)_{u,\theta} \tag{4.2.53}$$

From Eq. (4.2.51), we obtain

$$A^{\mathrm{d}}_A = -\frac{a^2 q d}{4\pi} \frac{u \sin\theta}{(\cosh(a\tau) - \cos\theta \sinh(a\tau))^3} (d\theta)_A \tag{4.2.54}$$

where Eq. (4.2.50) was used to compute $(\partial\tau/\partial a)_{(u,\theta)}$.

In her experiment, Alice starts with her particle in a uniformly accelerating state. Over a proper time T_1, she separates it into two uniformly accelerating components separated by a distance d as above. She keeps these components separated for a proper time T, and she then recombines them over a proper time T_2. The difference between the radiation fields of

23. The vector potential is not smooth at $\mathscr{I}^+$ in Lorenz gauge but one can do an asymptotic gauge transformation such that A_a is smooth at $\mathscr{I}^+$. Such a gauge transformation does not affect the angular components A_A at $\mathscr{I}^+$ [Satishchandran and Wald, 2019], so we can calculate A_A using our Lorenz gauge expression.

these components is given by

$$\mathcal{A}_A \equiv \mathcal{A}_{1,A} - \mathcal{A}_{2,A} = F(\tau) A^{\mathrm{d}}_A \tag{4.2.55}$$

where the smooth function F is such that $F(\tau) = 0$ for $\tau < -T_1$ and $\tau > T + T_2$, whereas $F(\tau) = 1$ for $0 < \tau < T$. The entangling photon content is then given by

$$\langle N \rangle = \|K\mathcal{A}\|^2 = 2 \int_{\mathbb{S}^2} d\Omega \int_0^\infty \frac{\omega d\omega}{2\pi} \, \overline{\hat{\mathcal{A}}_A} \hat{\mathcal{A}}^A \tag{4.2.56}$$

where $\hat{\mathcal{A}}_A(\omega, \theta, \phi)$ denotes the Fourier transform of $\mathcal{A}_A(u, \theta, \phi)$ with respect to u, i.e.,

$$\hat{\mathcal{A}}_A(\omega, \theta, \phi) = \int_{-\infty}^{\infty} du \; e^{i\omega u} \mathcal{A}_A(u, \theta, \phi). \tag{4.2.57}$$

We are interested in estimating $\langle N \rangle$ for large T.

In order to evaluate the Fourier transform integral, it is useful to note that, at fixed a, we have

$$\frac{du}{d\tau} = \cosh(a\tau) - \cos\theta \sinh(a\tau) \tag{4.2.58}$$

and

$$\frac{d^2u}{d\tau^2} = a^2 u. \tag{4.2.59}$$

It follows that

$$\begin{aligned} \frac{d}{du}\left(\frac{1}{du/d\tau}\right) &= \frac{1}{du/d\tau} \frac{d}{d\tau}\left(\frac{1}{du/d\tau}\right) \\ &= \frac{-a^2 u}{(\cosh(a\tau) - \cos\theta \sinh(a\tau))^3} \end{aligned} \tag{4.2.60}$$

Thus, we have

$$A^{\mathrm{d}}_A = \frac{qd\sin\theta}{4\pi}(d\theta)_A \frac{d}{du}\left(\frac{1}{du/d\tau}\right) \tag{4.2.61}$$

and

$$\hat{\mathcal{A}}_A = \frac{qd\sin\theta}{4\pi}(d\theta)_A \int_{-\infty}^{\infty} du\ e^{i\omega u} F(\tau)\frac{d}{du}\left(\frac{1}{du/d\tau}\right). \tag{4.2.62}$$

Integrating by parts, we obtain

$$\hat{\mathcal{A}}_A(\omega, x^A) = -\frac{qd\sin\theta}{4\pi}(d\theta)_A\left[i\omega \int_{-\infty}^{\infty} du\ e^{i\omega u}\frac{F(\tau)}{du/d\tau} + \int_{-\infty}^{\infty} du\ e^{i\omega u}\frac{F'(\tau)}{(du/d\tau)^2}\right]. \tag{4.2.63}$$

The second term in this equation contributes only during the time intervals $(-T_1, 0)$ and $(T, T+T_2)$ when Alice opens and closes the superposition. For large T, its contribution can be shown to be negligible compared with the first term. Therefore, we have

$$\hat{\mathcal{A}}_A(\omega, x^A) \approx -(d\theta)_A \frac{i\omega qd\sin\theta}{4\pi} I \tag{4.2.64}$$

where

$$I \equiv \int_{-\infty}^{\infty} du\ e^{i\omega u}\frac{F(\tau)}{du/d\tau}. \tag{4.2.65}$$

To evaluate I, we approximate F by a step function in the τ-interval $[0, T]$. The corresponding interval, $[u_0, u_T]$, in u is

$$\begin{aligned} u_0 &= -\frac{1}{a}\cos\theta \\ u_T &= \frac{1}{2a}\left[e^{aT}(1-\cos\theta) - e^{-aT}(1+\cos\theta)\right]. \end{aligned} \tag{4.2.66}$$

Noting that

$$\frac{du}{d\tau} = \sqrt{a^2u^2 + \sin^2\theta} \tag{4.2.67}$$

we obtain

$$I \approx \int_{u_0}^{u_T} du \, \frac{e^{i\omega u}}{\sqrt{a^2u^2 + \sin^2\theta}}. \tag{4.2.68}$$

It can be seen that for large T, the dominant contribution to I will come from small angles, $\theta \ll 1$. For $aT \gg 1$, the upper limit of the integral may then be approximated as

$$\begin{aligned} u_T &\approx \frac{1}{4a}e^{aT}\theta^2 - \frac{1}{a}e^{-aT} \quad \text{for } \theta \ll 1 \\ &\sim \begin{cases} 0 & \text{for } \theta^2/4 < e^{-aT} \\ \frac{1}{4a}\theta^2 e^{aT} & \text{for } \theta^2/4 \geq e^{-aT} \end{cases}. \end{aligned} \tag{4.2.69}$$

For $aT \gg 1$, the contribution to I from $\theta^2/4 < e^{-aT}$ can be shown to make a negligible contribution to $\langle N \rangle$, Eq. (4.2.56). Therefore, we may approximate I as

$$I \sim \Theta(\theta^2 - 4e^{-aT}) \int_{-1/a}^{\exp(aT)\theta^2/(4a)} du \, \frac{e^{i\omega u}}{\sqrt{a^2u^2 + \sin^2\theta}} \tag{4.2.70}$$

where

$$\Theta(x) \equiv \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } x \geq 0. \end{cases} \tag{4.2.71}$$

For $0 < \omega < 4ae^{-aT}/\theta^2$, we may bound I by replacing $e^{i\omega u}$ by 1. The integral can then be evaluated explicitly, and it can be shown that for $aT \gg 1$, the contribution to $\langle N \rangle$ from this frequency range is negligible. For $\omega > 4ae^{-aT}/\theta^2$, the integrand is oscillatory for $u > \exp(aT)\theta^2/(4a)$, and, for $aT \gg 1$, we will make negligible error in our estimate of $\langle N \rangle$ if we replace the upper limit of Eq. (4.2.70) by ∞. We will also make a negligible error by replacing the lower limit by 0. Thus, for $aT \gg 1$, we may approximate I as

$$I \sim \Theta(\theta^2 - 4e^{-aT})\Theta(\omega - 4ae^{-aT}/\theta^2) \int_0^\infty du \, \frac{e^{i\omega u}}{\sqrt{a^2u^2 + \sin^2\theta}}. \tag{4.2.72}$$

Evaluating the integral we obtain

$$I \sim \frac{1}{a}\Theta(\theta^2 - 4e^{-aT})\Theta(\omega - 4ae^{-aT}/\theta^2)\left(\frac{1}{2}i\pi I_0(\sin\theta\omega/a) + K_0(\sin\theta\omega/a) - \frac{1}{2}i\pi \boldsymbol{L}_0(\sin\theta\omega/a)\right) \tag{4.2.73}$$

where I_0, K_0 are Bessel functions and $\boldsymbol{L}_0$ is a Struve function. This expression is highly suppressed for $\omega > a/\theta$, so we can expand in $\theta\omega/a$ and truncate the function above $\omega = a/\theta$ to obtain,

$$I \sim -\frac{1}{a}\Theta(1 - \theta\omega/a)\Theta(\theta^2 - 4e^{-aT})\Theta(\omega - 4ae^{-aT}/\theta^2)\ln(\theta\omega/a). \tag{4.2.74}$$

Note that the restrictions $\omega < a/\theta$, and $\theta > 2e^{-aT/2}$ imply a frequency cutoff at $\omega \sim ae^{aT/2}/2$. By Eqs.(4.2.74) and (4.2.64), the frequency spectrum of $\hat{\mathcal{A}}_A$ goes as $\omega\ln(\omega/a)$ up to this cutoff, i.e., the spectrum is "hard" and becomes increasingly so for large T. This contrasts with the increasingly "soft" spectrum on the Rindler horizon, which goes as $1/\omega$ down to a low frequency cutoff $\sim 1/V \propto e^{-aT}$. Thus, the "soft horizon photons" from the Rindler perspective are "hard" photons from the inertial perspective.

From Eq. (4.2.56) for $\langle N\rangle$ together with our expression Eq. (4.2.64) for $\hat{\mathcal{A}}_A$ and the expression Eq. (4.2.74) that we have just derived for I, we obtain

$$\langle N\rangle \sim \left(\frac{qd}{a}\right)^2 \int d\omega d\theta \, \theta^3\omega^3 \left(\ln\frac{\omega\theta}{a}\right)^2 \tag{4.2.75}$$

where the region of ω-θ integration is determined by the Θ-functions appearing in Eq. (4.2.74) as well as the geometrical restriction $\theta \lesssim 1$. We can break up this region into the portion with $\omega \leq a$ and the portion with $\omega > a$. Since the region with $\omega \leq a$ and $\theta \lesssim 1$ is bounded and the integrand of Eq. (4.2.75) is bounded in this region, the contribution to $\langle N \rangle$ from $\omega \lesssim a$ is bounded by a constant that is independent of T. We may therefore discard this contribution. In the region $\omega > a$, the third Θ-function in Eq. (4.2.74) is redundant, and the integration region is

$$a \leq \omega \leq a e^{aT/2}/2 \tag{4.2.76}$$

$$2e^{-aT/2} \leq \theta \leq \frac{a}{\omega}. \tag{4.2.77}$$

For $aT \gg 1$, we will make negligible error by replacing the lower limit of θ by 0. We thereby obtain

$$\langle N \rangle \sim \left(\frac{qd}{a}\right)^2 \int\limits_a^{a\exp(aT/2)/2} d\omega \int\limits_0^{a/\omega} d\theta \; \theta^3 \omega^3 \left(\ln \frac{\omega\theta}{a}\right)^2. \tag{4.2.78}$$

Making the change of variables from θ to

$$x = \frac{\omega}{a}\theta \tag{4.2.79}$$

we find that the θ-integral becomes

$$\int\limits_0^{a/\omega} d\theta \; \theta^3 \omega^3 \left(\ln \frac{\omega\theta}{a}\right)^2 = \frac{a}{\omega} a^3 \int\limits_0^1 dx \; x^3 (\ln x)^2 \sim \frac{a^4}{\omega}. \tag{4.2.80}$$

Thus, we obtain

$$\begin{aligned}\langle N\rangle &\sim \left(\frac{qd}{a}\right)^2 a^4 \int\limits_a^{a\exp(aT/2)/2} \frac{d\omega}{\omega} \\ &\sim a^2q^2d^2\ln[\exp(aT/2)] \\ &\sim a^3q^2d^2T. \end{aligned} \tag{4.2.81}$$

This estimate agrees with Eq. (4.2.23).

Thus, we have succeeded — with considerable effort! — in our goal of deriving the decoherence of Alice's superposition by entirely conventional means. It is notable how much simpler the calculation of Sec. 4.2.1 was compared to the calculation that we have just completed.

4.3 Cosmological Horizons Decohere Quantum Superpositions

In this section, we apply our analysis to de Sitter spacetime. The de Sitter metric in a static patch is given by

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 q_{AB}dx^A dx^B \tag{4.3.1}$$

where, in this section, x^A are angular coordinates on the 2-sphere, q_{AB} is the unit round metric on the 2-sphere, and

$$f(r) = 1 - r^2/R_H^2 \tag{4.3.2}$$

where R_H (the "Hubble radius") is a constant. The coordinate singularity at $r = R_H$ corresponds to the "cosmological horizon," which is a Killing horizon of the static Killing field $(\partial/\partial t)^a$. The relation between "affine time," V, and "Killing time," v, on the future cosmological horizon is

$$V = e^{v/R_H}. \tag{4.3.3}$$

The general analysis of Sec. 4.1 applies to the decoherence of a static superposition in de Sitter spacetime. The estimates of the decoherence due to emission of soft photons and gravitons through the cosmological horizon when Alice keeps the superposition present for a time T can be made in exact parallel with the analysis of Sec. 4.2 in the Rindler case and Chapter 3 in the black hole case. The only noteworthy new ingredient in de Sitter spacetime is that the worldline $r = 0$ is an orbit of the static Killing field that is inertial, i.e., nonaccelerating. We now estimate the decoherence of a spatial superposition created in Alice's lab at $r = 0$ and thereby show that decoherence will occur even though Alice's lab is not accelerating.

By Gauss' law, a point charge placed at $r = 0$ will give rise to a radial electric field E_U on the future cosmological horizon given by

$$E_U \sim \frac{q}{R_H^2} \tag{4.3.4}$$

where $E_U = F_{ab}\ell^a n^b$ on the horizon with $n^a = (\partial/\partial V)^a$ tangent to the affinely parametrized null generators of the horizon and $\ell^a = (\partial/\partial U)^a$ a radial null vector with $n^a \ell_a = -1$. The change in the electric field on the horizon resulting from a displacement of the charge to $r = d \ll R_H$ is

$$\Delta E_U \sim \frac{qd}{R_H^3}. \tag{4.3.5}$$

By paralleling the steps that led to Eq. (4.2.18) above, we find that the change in the tangential components of the vector potential at the horizon is

$$|\Delta \mathcal{A}_A| \equiv \left(R_H^{-2} q^{AB} \Delta \mathcal{A}_A \Delta \mathcal{A}_B\right)^{1/2} \sim \frac{qd}{R_H^2}. \tag{4.3.6}$$

By paralleling the steps that led to Eq. (4.2.23) — assuming that the electromagnetic field is initially in the de Sitter invariant vacuum (see footnote 7) — we obtain the estimate

$$\langle N \rangle \sim \frac{q^2 d^2}{R_H^3} T \qquad \text{(de Sitter, EM)}\,. \tag{4.3.7}$$

Thus, restoring constants, the decoherence time due to the presence of the cosmological horizon is

$$T_{\rm D} \sim \frac{\hbar \epsilon_0 R_H^3}{q^2 d^2} \qquad \text{(de Sitter, EM)}\,. \tag{4.3.8}$$

Since $d \ll R_H$, the decoherence time will be much larger than the Hubble time R_H/c unless q is extremely large relative to the Planck charge $q_P \equiv \sqrt{\epsilon_0 \hbar c}$. Nevertheless, we see that decoherence does occur despite the fact that Alice's lab is inertial.

A similar analysis applies in the gravitational case for a spatial superposition of a massive particle in Alice's lab at $r = 0$. In parallel with the derivation given in Sec. 4.2.1 above, we find

$$\langle N \rangle \sim \frac{m^2 d^4}{R_H^5} T \qquad \text{(de Sitter, GR)} \tag{4.3.9}$$

which leads to a decoherence time

$$T_{\rm D}^{\rm GR} \sim \frac{\hbar R_H^5}{G m^2 d^4} \qquad \text{(de Sitter, GR)}\,. \tag{4.3.10}$$

CHAPTER 5

LOCAL DESCRIPTION OF DECOHERENCE DUE TO BLACK HOLES AND OTHER BODIES

The analysis of Chapters 3 and 4 strongly suggests that global aspects of the structure of the spacetime—specifically, the presence of a horizon—are essential for the decoherence effect. The main purpose of the present chapter is to show that one can give an alternative, purely local description of the decoherence in terms of the behavior of the quantum field within Alice's lab. From this viewpoint, the decoherence arises from the behavior of the unperturbed two-point function of the quantum field in the region where the superposition was created. In particular, the decoherence in the presence of a black hole can be understood as resulting from the extremely low frequency Hawking radiation that partially penetrates into Alice's lab before being reflected back into the black hole by the effective potential of the black hole. This local viewpoint will enable us to gain insights into various aspects of the decoherence process, such as the differences in decoherence that occur in different vacuum states and in different spacetimes. We will also gain insight into the requirements on a material body to mimic the decoherence effects of a black hole.

We note that, very recently, Wilson-Gerow, Dugad, and Chen [Wilson-Gerow et al., 2024] also have given a local formulation of our decoherence results, focusing particularly on the Rindler case, i.e., an accelerating observer in Minkowski spacetime. The methods and arguments used in [Wilson-Gerow et al., 2024] are quite different from the ones we shall give in our analysis below. Nevertheless, there are a number of significant points of overlap in the results. In particular, our result Eq. (5.3.8) relating the decoherence to the local two-point function of the electric field corresponds to Eq. (103) of [Wilson-Gerow et al., 2024].

We also note that in a previous chapter we analyzed the decohering effects of the scattering of Unruh radiation on a charged superposition in an accelerating laboratory in Minkowski spacetime. We concluded that this decoherence was distinct from (and smaller than) the decohering effects of emission of entangling radiation through the Rindler horizon. However, in Chapter 4 we considered only incoherent scattering effects of individual Unruh photons. We did not consider the coherent effects of the presence of a large number of Unruh photons of frequency $\omega \sim 1/T \ll 1/a$, where a denotes the acceleration of the laboratory. As we shall see in the present chapter, the presence of these very low frequency photons can be viewed as stimulating the emission of entangling radiation from the superposition. Thus, the decoherence effect in Rindler spacetime is, in fact, intimately related to the presence of very low frequency Unruh radiation in the Minkowski vacuum. Similarly, the decoherence effect in a black hole spacetime is intimately related to the presence of very low frequency Hawking radiation in the Unruh vacuum.

Our local reformulation of the decoherence makes manifest that one can interpret the decoherence of Alice's superposition in terms of the interaction of Alice's particle with the local state of the quantum field in her lab. It should be emphasized that the thermal nature of the state is, by itself, insufficient to account for this effect [Danielson et al., 2023, Wilson-Gerow et al., 2024]. In particular, for the decoherence in the Unruh vacuum in the presence of a black hole, it is essential that there is an extremely large reservoir of "soft" Hawking quanta in the Unruh vacuum as compared with an ordinary inertial thermal bath in Minkowski spacetime at the same temperature. Furthermore, in the Boulware vacuum in a black hole spacetime—which is the ground state with respect to the timelike Killing field and thus has no particles—Alice's superposition still spontaneously emits entangling soft photons/gravitons into the black hole, but the number of entangling particles grows only logarithmically with time. The Unruh vacuum corresponds to a thermal population whose density of states diverges at low frequencies. The presence of these low-frequency quanta stimulate the emission of entangling soft radiation into the horizon, so that the number of entangling soft photons/gravitons grows linearly in time.

Our local reformulation of Alice's decoherence also allows one to also consider what happens when one replaces the black hole by a body without a horizon. It is instructive to consider the case where Alice's lab is in the spacetime outside of a static, spherical star rather than a black hole but we do not consider any internal degrees of freedom of the matter composing the star, i.e., we consider only the effect of replacing the spacetime geometry of a black hole with the spacetime geometry of a star. If the quantum field is in its stationary ground state in the spacetime of the star, then the two-point function of the quantum field in Alice's lab should look very much like the Boulware vacuum in Schwarzschild spacetime with respect to the incoming modes from infinity. However, the "white hole incoming modes" of Schwarzschild will be entirely absent for the star. These white hole modes are responsible for the decoherence effects that grow with T in Schwarzschild, so a similar decoherence will not occur for the star. Even if the quantum field is in a thermal state in the spacetime of the static star, there will be no decoherence effects that grow with T. Thus, the presence of a horizon is essential for the kind of decoherence obtained for a Schwarzschild black hole.

Nevertheless, one can get decoherence without a horizon if one has a material body with internal degrees of freedom that interact electromagnetically and/or gravitationally with the particle in Alice's lab. In this situation, the interaction is now mediated by the long-range Couloumbic/Newtonian field of the superposition without any emission of radiation, analogous to the gedankenexperiment [Belenchia et al., 2018, Danielson et al., 2022a] in flat spacetime where Alice and Bob both perform their experiments adiabatically and in causal contact with one another. As we shall show, the material body will mimic the decoherence effects of the black hole if, at very low frequencies, the thermal fluctuations of its electric dipole moment and/or mass quadrupole moment agree with black hole case [see Eqs. (5.3.48) and (5.3.49) below]. This issue has recently been investigated by Biggs and Maldacena [Biggs and Maldacena, 2024]. In order for a body of size comparable to that of a black hole to be able to absorb and emit low frequency electromagnetic or gravitational waves as efficiently as the black hole, a conducting or gravitating body must have a very large resistance or viscosity. There does not appear to be any difficulty, in principle, in achieving this in the electromagnetic case [Biggs and Maldacena, 2024]. However, some extraordinary physical properties of matter would be required to mimic the quantum gravitational decoherence effect [Biggs and Maldacena, 2024].

In Sec. 5.1, we review our previous derivation of decoherence in the presence of a horizon. In Sec. 5.2, we provide a local reformulation of this decoherence in terms of the two-point function of the quantum field in Alice's laboratory over the duration of her experiment. In Sec. 5.3, we compute the decoherence in the Unruh vacuum in Schwarzschild using our local formulation, which requires the computation of the two-point function of the electric field along the worldline of Alice's lab. Finally, in Sec. 5.4, we compute the decoherence for different vacua in Schwarzschild and in different spacetimes, including a brief discussion of the decoherence due to entanglement with an ordinary material body.

Unless otherwise stated, we will work in Planck units where $G = c = \hbar = k_{\mathrm{B}} = 1$ and, in electromagnetic formulas, we also put $\epsilon_0 = 1$. We will generally follow the notational conventions of [Wald, 1984]. In particular, abstract spacetime indices will be denoted with lowercase Latin indices from the early alphabet $(a, b, c\dots)$. Spacetime coordinate components will be denoted with Greek indices. Spatial coordinates and components will be denoted with Latin indices from the middle alphabet $(i, j, k, \dots)$.

5.1 Decoherence of a Quantum Superposition due to Radiation

In this section we briefly review the analysis of decoherence due to radiation through a Killing horizon previously given in Chapters 3 and 4. We will focus on the electromagnetic case and merely state the corresponding results in the gravitational case.

An experimenter, Alice, in a stationary lab in a stationary spacetime $(\mathscr{M}, g_{ab})$ controls a charged particle[1] which is initially held stationary in her lab. The particle is put through a

1. The "particle" need not be "elementary," e.g., it could be a nanoparticle. All that is required is that the

Stern-Gerlach apparatus over a time T_1 so that at the end of this process its quantum state is of the form

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\psi_1\rangle + |\psi_2\rangle) \tag{5.1.1}$$

where $|\psi_1\rangle$ and $|\psi_2\rangle$ are the spatially separated, normalized states of the particle after passing through the Stern-Gerlach apparatus. Alice maintains this stationary superposition for a (proper) time T, and she subsequently recombines her particle over a time T_2 where we assume that $T \gg T_1, T_2$. The recombined particle is then kept stationary. We now analyze the decoherence of Alice's particle due to emission of entangling electromagnetic radiation sourced by Alice's superposition.

We assume that $|\psi_1\rangle$ and $|\psi_2\rangle$ are sufficiently spatially separated that $\langle\psi_1|\boldsymbol{j}^a|\psi_2\rangle = 0$ and we further assume that the fluctuations of the charge current $\boldsymbol{j}^a$ in states $|\psi_1\rangle$ and $|\psi_2\rangle$ are negligible compared with their expected values. We may then treat the charge-current of each component of the superposition as a c-number source in Maxwell's equations. Thus, if Alice's particle is in state $|\psi_n\rangle$ for $n = 1, 2$, then the electromagnetic field operator is given by [Yang and Feldman, 1950]

$$\boldsymbol{A}_{n,a} = \boldsymbol{A}_a^{\text{in}} + G_a^{\text{ret}}(j_n)\mathbf{1} \tag{5.1.2}$$

where $\boldsymbol{A}_a^{\text{in}}$ is the unperturbed ("in") field operator and $G_a^{\text{ret}}(j_n)$ is the retarded solution associated to the classical charge-current $j_n^a = \langle\psi_n|\boldsymbol{j}^a|\psi_n\rangle$. The "out" radiative field at late times is obtained by subtracting the final Coulomb field C_a of the recombined particle from $\boldsymbol{A}_{n,a}$

$$\begin{aligned} \boldsymbol{A}_{n,a}^{\text{out}} &= \boldsymbol{A}_{n,a} - C_a\mathbf{1} \\ &= \boldsymbol{A}_{n,a}^{\text{in}} + \mathcal{A}_{n,a}\mathbf{1} \end{aligned} \tag{5.1.3}$$

degrees of freedom of the particle apart from its center of mass may be neglected.

where

$$\mathcal{A}_{n,a} \equiv G^{\text{ret}}_a(j_n) - C_a \,. \tag{5.1.4}$$

We assume that the initial state of the quantum electromagnetic field is some "vacuum state" (i.e., a pure, quasifree state) $|\Omega\rangle$ that is invariant under the time translation symmetries of the spacetime. The unperturbed field operator $\boldsymbol{A}^{\text{in}}$ on the Fock space, $\mathscr{F}(\mathscr{H}_{\text{in}})$, associated with $|\Omega\rangle$ can be expressed in terms of annihilation and creation operators on $\mathscr{F}(\mathscr{H}_{\text{in}})$ as

$$\boldsymbol{A}^{\text{in}}_a(f^a) = i\boldsymbol{a}(\overline{K\Delta(f)}) - i\boldsymbol{a}^\dagger(K\Delta(f)) \tag{5.1.5}$$

where f^a is a divergence-free[2] test vector field and $\Delta(f)$ is the advanced minus retarded solution to Maxwell's equation with source f^a

$$[\Delta(f)]_a(x) = \int_{\mathscr{M}} \sqrt{-g} d^4x' \Delta_{aa'}(x,x') f^{a'}(x') \tag{5.1.6}$$

where $\Delta_{aa'}(x,x')$ is the advanced minus retarded Greens function. Here K is the map that takes classical solutions into the corresponding one-particle states in the Fock space defined by $|\Omega\rangle$.

As can be seen from Eq. (5.1.3), the "out" state corresponding to the "in" vacuum $|\Omega\rangle$ has field correlation functions at late times that are obtained from the vacuum correlation functions by shifting the field operator by a multiple of the identity operator. It follows that if Alice's particle is in state $|\psi_n\rangle$, then the "out" state of the electromagnetic field will be given by the coherent state

$$|\Psi_n\rangle = e^{-\frac{1}{2}\|K\mathcal{A}_n\|^2} \exp\left[\boldsymbol{a}^\dagger(K\mathcal{A}_n)\right] |\Omega\rangle \tag{5.1.7}$$

2. Restriction of the smearing to divergence-free test functions is necessary and sufficient to eliminate the gauge dependence of $\boldsymbol{A}_{\text{in},a}$ (see, e.g., p. 101 of [Wald, 1995]).

where, for notational simplicity, we drop the spacetime index "a" from $\mathcal{A}_{n,a}$, Eq. (5.1.4), here and elsewhere in the remainder of this section. The norm $||K\mathcal{A}_n||$ appearing in Eq. (5.1.7) is taken in the one-particle Hilbert space of the Fock space of $|\Omega\rangle$.

The joint quantum state of Alice's particle together with the emitted electromagnetic radiation at late times is given by

$$\frac{1}{\sqrt{2}}\big(\,|\psi_1\rangle \otimes |\Psi_1\rangle + |\psi_2\rangle \otimes |\Psi_2\rangle\,\big). \tag{5.1.8}$$

Thus, the decoherence of Alice's particle due to the emission of electromagnetic radiation is then given by

$$\mathscr{D}_{\text{Alice}} = 1 - |\,\langle\Psi_1|\Psi_2\rangle\,|\,. \tag{5.1.9}$$

The magnitude of the inner product of the coherent states $|\Psi_1\rangle$ and $|\Psi_2\rangle$ is computed to be

$$|\,\langle\Psi_1|\Psi_2\rangle\,| = \exp\left(-\frac{1}{2}||K(\mathcal{A}_1 - \mathcal{A}_2)||^2\right) \tag{5.1.10}$$

where $K(\mathcal{A}_1 - \mathcal{A}_2)$ denotes the one-particle state associated with late time classical solution

$$\mathcal{A}_1 - \mathcal{A}_2 = G^{\text{ret}}(j_1 - j_2). \tag{5.1.11}$$

But $||K(\mathcal{A}_1 - \mathcal{A}_2)||^2$ is equal to the expected number of photons, $\langle N\rangle$, in the coherent state associated with the late time classical solution $\mathcal{A}_1 - \mathcal{A}_2$ sourced by $j_1 - j_2$

$$\langle N\rangle \equiv ||K(\mathcal{A}_1 - \mathcal{A}_2)||^2 = ||KG^{\text{ret}}(j_1 - j_2)||^2. \tag{5.1.12}$$

Thus, we have

$$\mathscr{D} = 1 - \exp\left(-\frac{1}{2}\langle N\rangle\right). \tag{5.1.13}$$

We shall refer to $\langle N\rangle$ as the expected number of *entangling photons*. If the expected number of entangling photons is significantly bigger than 1, then Alice's superposition will be completely decohered.

Thus, we see that to compute the decoherence of a superposition created by Alice under the assumptions stated above, we proceed as follows:

1. We compute the expected currents j_1 and j_2 of the components of Alice's superposition.
2. We compute the classical retarded solution $G^{\text{ret}}(j_1 - j_2)$ sourced by the difference of these currents.
3. We compute the one-particle state $KG^{\text{ret}}(j_1 - j_2)$ of this classical solution at late times and its squared norm $\|KG^{\text{ret}}(j_1 - j_2)\|^2$. This yields the expected number of entangling photons, $\langle N \rangle$, and thereby the decoherence, Eq. (5.1.13). Note that the one-particle map K depends on the choice of vacuum state $|\Omega\rangle$.

The above analysis extends directly to the linearized quantum gravitational case, where the linearized metric perturbation h_{ab} is treated as a field propagating on a fixed spacetime background. In the above formulas, we simply replace A_a with h_{ab} and we replace the current j_a with the linearized stress tensor T_{ab}. The expected number of entangling gravitons is then given by the analog of Eq. (5.1.12) and the decoherence is given by Eq. (5.1.13).

In Minkowski spacetime, we may take the notion of stationarity to be given by ordinary, inertial time translations and we may take $|\Omega\rangle$ to be the Poincaré invariant vacuum. If a particle of charge q is put in a superposition separated by a distance d, then we may estimate $G^{\text{ret}}(j_1 - j_2)$ near null infinity using the Larmor formula. The one-particle state $KG^{\text{ret}}(j_1 - j_2)$ is the positive frequency part of this solution with respect to inertial time translations. The norm of this one-particle state is given by the Klein-Gordon norm. The expected number of entangling photons is thereby estimated to be [Belenchia et al., 2018, Danielson et al., 2022a]

$$\langle N\rangle \sim \frac{q^2 d^2}{\min[T_1, T_2]^2} \quad \text{(Minkowski, EM).} \tag{5.1.14}$$

Thus, the decoherence does not depend upon T and can be made arbitrarily small by performing the separation and recombination of the superposition sufficiently slowly, so that $T_1, T_2 \gg qd$.

In the analysis of the corresponding gravitational case we must take into account the fact that conservation of total stress-energy implies that the center of mass cannot change. Thus if the component $|\psi_1\rangle$ of the superposition corresponds to the particle moving to the right, then Alice's lab must move a tiny bit to the left to keep the center of mass unchanged. The upshot is that the leading order contribution to the retarded solution with source $T_1^{ab} - T_2^{ab}$ arises from quadrupole radiation rather than dipole radiation. The estimate corresponding to Eq. (5.1.14) for the number of entangling gravitons is [Belenchia et al., 2018, Danielson et al., 2022a]

$$\langle N\rangle \sim \frac{m^2 d^4}{\min[T_1, T_2]^4} \quad \text{(Minkowski, GR).} \tag{5.1.15}$$

Again, the decoherence does not depend upon T and can be made arbitrarily small by performing the separation and recombination of the superposition sufficiently slowly, so that $T_1, T_2 \gg \sqrt{md^2}$.

However, it was shown in Chapter 3 that the situation is drastically different in the presence of a black hole or, more generally, any Killing horizon [Danielson et al., 2023]. In the case of a black hole, the relevant vacuum is the "Unruh vacuum" $|\Omega_{\mathrm{U}}\rangle$. If T_1, T_2 are sufficiently large—i.e., if Alice separates and recombines the particle sufficiently slowly—then the number of entangling photons/gravitons emitted to infinity will again be negligible. However, if an initially stationary source is moved to a new position and held there forever, the retarded solution will exhibit a "memory effect" on the horizon [Hawking et al., 2016].

Consequently, it can be seen that if Alice were to keep her superposition open forever, an infinite number of soft entangling photons/gravitons would be emitted through the horizon, in close analogy with the infrared divergences at infinity that arise in scattering theory (see, e.g., [Ashtekar, 1987, Ashtekar et al., 2018, Prabhu et al., 2022, Prabhu and Satishchandran, 2024]). If Alice closes her superposition after time T, then the number of entangling photons radiated through the horizon will be finite but will grow linearly with T. In the electromagnetic case the number of photons grows as [Danielson et al., 2022b]

$$\langle N \rangle \sim \frac{M^3 q^2 d^2}{D^6} T \quad \text{(black hole, EM)} \tag{5.1.16}$$

where M is the mass of the black hole and D is the proper distance of Alice's lab from the horizon (and, for simplicity, we have assumed that $D \gtrsim M$ so that, e.g., the redshift factor at Alice's lab is of order unity and can be absorbed in the "$\sim$"). The analogous computation in the gravitational case[3] yields [Danielson et al., 2022b]

$$\langle N \rangle \sim \frac{M^5 m^2 d^4}{D^{10}} T \quad \text{(black hole, GR).} \tag{5.1.17}$$

More generally, it was shown that in the presence of any Killing horizon (e.g., a Rindler or cosmological horizon) the number of entangling soft photons and gravitons grows linearly in the time T that the superposition is maintained [Danielson et al., 2023].

The above results were obtained by calculating the quantum state of the electromagnetic and linearized gravitational fields on the horizon associated with the retarded solution sourced by the components of Alice's superposition. The decoherence of Alice's particle was attributed to the emission of entangling photons/gravitons through the horizon. Thus, it might appear that the global properties of the spacetime—specifically, the presence of a horizon—are

3. In the gravitational case, it will be necessary to have some additional stress-energy present to hold Alice's lab stationary and keep her particle components stationary. We neglect any effects of such additional stress-energy.

essential for the description of the decoherence phenomenon we have just given. However, we will now show that the decoherence can alternatively be described purely in terms of the local properties of the unperturbed quantum field within Alice's laboratory. This alternative viewpoint will enable us to compare decoherence phenomena in the presence of a black hole with decoherence phenomena occurring when no horizon is present.

5.2 Local Reformulation of the Decoherence

As in the previous section, we first consider the electromagnetic case and then state the corresponding results in the gravitational case.

A local reformulation of the electromagnetic decoherence results of the previous section is obtained from the following simple observations: First, since $j_1 = j_2$ at late times, the retarded solution $G^{\text{ret}}(j_1 - j_2)$ is equal to $-\Delta(j_1 - j_2)$ at late times, where $\Delta = G^{\text{adv}} - G^{\text{ret}}$. Thus, we may replace G^{ret} by $-\Delta$ in Eqs. (5.1.11) and (5.1.12), and we no longer have to evaluate these quantities at late times. Second, we note that it follows immediately from Eq. (5.1.5) that for any (divergence-free) test vector field f^a, we have

$$\langle\Omega| \left[\boldsymbol{A}^{\text{in}}_a(f^a)\right]^2 |\Omega\rangle = ||K\Delta(f)||^2 \tag{5.2.1}$$

where $\boldsymbol{A}^{\text{in}}$ denotes the unperturbed electromagnetic field. Combining Eq. (5.2.1) with Eq. (5.1.12) (with G^{ret} replaced by $-\Delta$), we obtain

$$\langle N\rangle = \langle\Omega| \left[\boldsymbol{A}^{\text{in}}_a(j_1^a - j_2^a)\right]^2 |\Omega\rangle\,. \tag{5.2.2}$$

Thus, we see that the prescription for computing the decoherence of Alice's superposition outlined in the bullet points given in the previous section can be equivalently reformulated as follows:

- We compute the expected currents j_1^a and j_2^a of the components of Alice's superposition.
- We compute the two-point function $\langle\Omega|\, \boldsymbol{A}_a^{\text{in}}(x)\boldsymbol{A}_{a'}^{\text{in}}(x')|\, |\Omega\rangle$ of the unperturbed field in the vacuum state $|\Omega\rangle$.
- We smear this two-point function in both variables with the test vector field $f^a = j_1^a - j_2^a$. This yields the expected number of entangling photons, $\langle N\rangle$, and thereby the decoherence, Eq. (5.1.13).

The remarkable feature of this reformulation is that it requires only knowledge of the expected currents and the unperturbed two-point function of the quantum field in Alice's lab, i.e., unlike the previous prescription, we do not need to calculate anything about the particle content of the perturbed field at late times. In particular, this explicitly demonstrates that the decoherence can be viewed as a purely local phenomenon occurring entirely in Alice's lab.

The corresponding result in the linearized gravitational case is

$$\langle N\rangle = \langle\Omega| \left[\boldsymbol{h}_{ab}^{\text{in}}(T_1^{ab} - T_2^{ab})\right]^2 |\Omega\rangle \tag{5.2.3}$$

where $T_1^{ab} - T_2^{ab}$ is the difference in the stress-energy of the components of Alice's particle (also taking into account the tiny correlated motion of Alice's lab that keeps the center of mass fixed). Again, the calculation of decoherence is seen to require only a knowledge of the expected stress-energy of the components of Alice's particle as well as the unperturbed two-point function of the quantum field in Alice's lab, so the decoherence can be viewed as a purely local phenomenon occurring entirely in Alice's lab.

Note that Eqs. (5.2.2) and (5.2.3) show that the quantity $\langle N\rangle$—and hence the corresponding decoherence, $\mathscr{D}$, given by Eq. (5.1.13)—are determined by the *vacuum fluctuations* of the quantum field smeared into the difference of the sources in Alice's lab.

In the next section, we recompute the black hole decoherence Eq. (5.1.16) using our local reformulation. This will enable us to gain further insights into the nature of the decoherence in the presence of a black hole and to compare it with cases where no horizon is present.

5.3 Local Calculation of the Decoherence in the Unruh Vacuum around a Schwarzschild Black Hole

We now compute the decoherence of Alice's particle in the presence of a Schwarzschild black hole by the methods of the previous section. We will focus upon the electromagnetic case and merely comment briefly on the linearized gravitational case near the end of this section.

If we neglect the spatial extent of the particle components, then we have

$$j_1^a(t, x^i) \approx \frac{q}{\sqrt{-g}} \delta^{(3)}[x^i - X_1^i(t)] u_1^a \frac{d\tau_1}{dt} \tag{5.3.1}$$

and similarly for j_2^a. Here t is the Killing time coordinate, x^i are spatial coordinates on the hypersurfaces Σ_t orthogonal to the timelike Killing field t^a, $X_1^i(t)$ is the path taken by the center of mass of the first component of the particle, u_1^a is the 4-velocity of that path, τ_1 is the proper time along the path, and $\delta^{(3)}$ is the "coordinate delta function" defined so that $\int \delta^{(3)}[x^i - X_1^i(t)] d^3x = 1$. For nonrelativistic motion relative to the rest frame of t^a, we have $d\tau_1/dt \approx \sqrt{-g_{tt}}$ and

$$j_1^a(t, x^i) \approx \frac{q}{\sqrt{-g}} \delta^{(3)}[x^i - X_1^i(t)](t^a + v_1^a) \tag{5.3.2}$$

where v^a is the coordinate velocity of the component, i.e., $v_1^i = dX_1^i/dt$ and $v_1^t = 0$. We represent the displacement of the two components of Alice's particle at time t by the tangent vector $S^a(t)$ to the geodesic segment in Σ_t of unit affine parameter that connects the centers of mass of the two components. We write $S^a(t) = d(t)s^a(t)$, where s^a is a unit vector. Then $d(t)$ represents the proper distance between the components. We assume that s^a is Lie transported along t^a (i.e., the direction of separation does not change with time) and that

$d(t)$ is smoothly varying and is such that

$$d(t) = \begin{cases} d \text{ for } |t| < T/2 \\ 0 \text{ for } t < -T/2 - T_1 \text{ and } t > T/2 + T_2. \end{cases} \tag{5.3.3}$$

The difference between the current densities of the two components is given by

$$(j_1^a - j_2^a) \;\approx\; \frac{qd(t)}{\sqrt{-g}} t^a s^b \nabla_b \delta^{(3)}(x^i - X^i) - \frac{q}{\sqrt{-g}} \delta^{(3)}(x^i - X^i) s^a t^b \nabla_b d(t) \tag{5.3.4}$$

where X^i is the position of Alice's lab. Here, the first term arises from the difference in charge densities and the second term arises from the difference in spatial currents. We may rewrite this as

$$(j_1^a - j_2^a) \approx \frac{2q}{\sqrt{-g}} t^{[a} s^{b]} \nabla_b \left[d(t) \delta^{(3)}(x^i - X^i) \right]. \tag{5.3.5}$$

We define the electric field E_a on the static slices by[4]

$$E_a = F_{ab} t^b = (\nabla_a A_b - \nabla_b A_a) t^b . \tag{5.3.6}$$

It follows immediately from Eq. (5.3.5) and the definition of E that the unperturbed field $\boldsymbol{A}^{\text{in}}$ smeared in with $j_1^a - j_2^a$ (with the volume element $\sqrt{-g} d^4x$ understood in the smearing) is given by

$$\boldsymbol{A}_a^{\text{in}}(j_1^a - j_2^a) \approx -q \int dt d(t) s^a \boldsymbol{E}_a^{\text{in}}(t, X^i). \tag{5.3.7}$$

Thus, from Eq. (5.2.2), we have

$$\langle N \rangle = q^2 \int dt dt' d(t) d(t') \left\langle s^a \boldsymbol{E}_a^{\text{in}}(t, X^i) s^{a'} \boldsymbol{E}_{a'}^{\text{in}}(t', X^i) \right\rangle_\Omega . \tag{5.3.8}$$

4. Note that this differs from the notion of the "electric field on the horizon" used in Chapters 3 and 4, which was defined as $F_{ab}k^b$, where k^b is the null normal to the horizon.

Thus, to calculate $\langle N \rangle$ and thereby the decoherence Eq. (5.1.13) of Alice's particle, we simply evaluate the two-point function of the component, $s^a \boldsymbol{E}_a^{\rm in}$ of the electric field in the direction of the separation, s^a, of the components of Alice's particle evaluated at Alice's lab, $x^i = X^i$, and smeared in time via the separation $d(t)$.

Thus, the remaining task is to obtain the two-point function of the unperturbed electric field, which we will do via a mode expansion. We shall simplify this task by restricting consideration to the case of radial separation of the components of Alice's particle, so that we need only calculate the two-point function of the radial component of $\boldsymbol{E}_a^{\rm in}$. The magnetic parity modes do not contribute to the radial component of the electric field so we need only consider the electric parity modes [Wald, 2022]. The two-point function of the radial coordinate component $\boldsymbol{E}_r^{\rm in}$ has been calculated for the Boulware, Unruh and Hartle-Hawking vacuum states by Zhou and Yu [Zhou and Yu, 2012] and Menezes [Menezes, 2016], who obtained[5]

$$\langle \boldsymbol{E}_r(x)\boldsymbol{E}_r(x')\rangle_\Omega = \sum_{\ell=1}^{\infty} \frac{C_\ell P_\ell(\hat{r}\cdot\hat{r}')}{r^2 r'^2} \int_{-\infty}^{\infty} \frac{d\omega}{\omega} e^{-i\omega(t-t')} \times \\ \times \left[\vec{G}(\omega)\vec{R}_{\omega\ell}(r)\vec{R}^*_{\omega\ell}(r') + \overleftarrow{G}(\omega)\overleftarrow{R}_{\omega\ell}(r)\overleftarrow{R}^*_{\omega\ell}(r') \right]. \tag{5.3.9}$$

Here,

$$C_\ell \equiv \frac{1}{16\pi^2}\ell(\ell+1)(2\ell+1) \tag{5.3.10}$$

and P_ℓ is the ℓth Legendre polynomial (so $P_\ell(\hat{r}\cdot\hat{r}') = 1$ for the case of interest below where $x^i = x'^i$). The mode functions $\vec{R}_{\omega\ell}(r)$ and $\overleftarrow{R}_{\omega\ell}(r)$ satisfy the differential equation

$$\frac{d^2 R_{\omega\ell}}{dr^{*2}} + \left[\omega^2 - V(r)\right] R_{\omega\ell} = 0 \tag{5.3.11}$$

5. These results are given in Eqs. (51)-(53) of [Zhou and Yu, 2012] and Eqs. (A13)-(A16) of [Menezes, 2016]. We have used the addition theorem for spherical harmonics to rewrite their sum of spherical harmonics over azimuthal number m in terms of $P_\ell(\hat{r}\cdot\hat{r}')$.

where

$$V(r) = \left(1 - \frac{2M}{r}\right) \frac{\ell(\ell+1)}{r^2} \tag{5.3.12}$$

and r^* is the radial "tortoise coordinate"

$$r^* = r + 2M \ln\left(\frac{r}{2M} - 1\right) , \tag{5.3.13}$$

which satisfies $dr^*/dr = (1-2M/r)^{-1}$ and ranges from $r^* \to -\infty$ at the horizon to $r^* \to +\infty$ at infinity. The modes $\vec{R}_{\omega\ell}$ correspond to waves that are incoming from the white hole and are defined by the asymptotic conditions

$$\vec{R}_{\omega\ell}(r) \to \begin{cases} e^{i\omega r^*} + \vec{A}_{\omega\ell} e^{-i\omega r^*} \text{ as } r \to 2M \\ \vec{B}_{\omega\ell} e^{i\omega r^*} \text{ as } r \to \infty \end{cases} \tag{5.3.14}$$

whereas the modes $\overleftarrow{R}_{\omega\ell}$ correspond to waves that are incoming from infinity and are defined by the asymptotic conditions

$$\overleftarrow{R}_{\omega\ell}(r) \to \begin{cases} \overleftarrow{B}_{\omega\ell} e^{-i\omega r^*} \text{ as } r \to 2M \\ e^{-i\omega r^*} + \overleftarrow{A}_{\omega\ell} e^{i\omega r^*} \text{ as } r \to \infty. \end{cases} \tag{5.3.15}$$

Finally, the coefficients $\vec{G}(\omega)$ and $\overleftarrow{G}(\omega)$ appearing in Eq. (5.3.9) depend on the choice of vacuum state $|\Omega\rangle$. For the Boulware vacuum [Boulware, 1975], $|\Omega_{\rm B}\rangle$, we have

$$\vec{G}_{\rm B}(\omega) = \overleftarrow{G}_{\rm B}(\omega) = \Theta(\omega) \tag{5.3.16}$$

corresponding to the fact that Boulware vacuum is positive frequency with respect to Killing time at both the white hole horizon and past infinity. For the Unruh vacuum [Unruh, 1976],

$|\Omega_{\rm U}\rangle$, we have

$$\vec{G}_{\rm U}(\omega) = \frac{1}{1 - e^{-2\pi\omega/\kappa}} \quad \text{and} \quad \overleftarrow{G}_{\rm U}(\omega) = \Theta(\omega) \tag{5.3.17}$$

where κ is the surface gravity of the black hole, corresponding to the fact that the Unruh vacuum is positive frequency with respect to Killing time at past null infinity but is positive frequency with respect to affine time (and thus is thermally populated with respect to Killing time at temperature $\kappa/2\pi$) on the white hole horizon. Finally, for the Hartle-Hawking vacuum [Hartle and Hawking, 1976], $|\Omega_{\rm HH}\rangle$, we have

$$\vec{G}_{\rm HH}(\omega) = \overleftarrow{G}_{\rm HH}(\omega) = \frac{1}{1 - e^{-2\pi\omega/\kappa}} \tag{5.3.18}$$

corresponding to the fact that the Hartle-Hawking vacuum is a thermal state at both the white hole horizon and past null infinity.

We now plug our expression Eq. (5.3.9) for the two-point function into our formula Eq. (5.3.8) for $\langle N\rangle$. We obtain

$$\langle N\rangle = q^2 \sum_{\ell=1}^{\infty} \frac{C_\ell(1-2M/r)}{r^4} \int\limits_{-\infty}^{\infty} \frac{d\omega}{\omega} |\hat{d}(\omega)|^2 \left[\vec{G}(\omega)|\vec{R}_{\omega\ell}(r)|^2 + \overleftarrow{G}(\omega)|\overleftarrow{R}_{\omega\ell}(r)|^2\right]. \tag{5.3.19}$$

Here r is the radial coordinate of Alice's lab and $\hat{d}(\omega)$ is the Fourier transform of $d(t)$

$$\hat{d}(\omega) = \int\limits_{-\infty}^{\infty} dt\; e^{i\omega t} d(t)\,. \tag{5.3.20}$$

The factor of $(1-2M/r)$ arises from converting the proper distance component $s^a E_a$ appearing in Eq. (5.3.8) to the coordinate component E_r appearing in Eq. (5.3.9), and we used the fact that $P_\ell(1) = 1$.

For $d(t)$ of the form Eq. (5.3.3) with T large, the magnitude of the Fourier transform $|\hat{d}(\omega)|$ behaves like $d/|\omega|$ as $\omega \to 0$ until this divergent behavior levels off below $|\omega| \sim 1/T$.

There will also be a high frequency cutoff at $|\omega| \sim 1/\min[T_1, T_2]$. Thus, we may approximate the contribution of $|\hat{d}(\omega)|$ to the integral in Eq. (5.3.19) using

$$|\hat{d}(\omega)| \sim \begin{cases} \frac{d}{\omega} & \frac{1}{T} < |\omega| < \frac{1}{\min[T_1,T_2]} \\ 0 & |\omega| < \frac{1}{T} \text{ or } |\omega| > \frac{1}{\min[T_1,T_2]}. \end{cases} \tag{5.3.21}$$

Thus, the behavior of $\langle N \rangle$ at large T will be determined by the behavior of the integrand of Eq. (5.3.19) near the low frequency end, $|\omega| \sim 1/T$, of the range of integration. In order to determine this behavior, we need to obtain expressions for the mode functions $\vec{R}_{\omega\ell}(r)$ and $\overleftarrow{R}_{\omega\ell}(r)$ at very low frequencies.

In order to determine these mode functions at low frequencies, we divide the exterior into three regions (see Fig. 5.1):

$$\text{Region I} \quad 2M < r \leq r_1 \tag{5.3.22}$$

$$\text{Region II} \quad r_1 < r \ll r_2 \tag{5.3.23}$$

$$\text{Region III} \quad 3M \ll r < \infty \tag{5.3.24}$$

where [Fabbri, 1975]

$$r_1 = 2M + \frac{8\omega^2 M^3}{\ell(\ell+1)} \tag{5.3.25}$$

$$r_2 = \frac{[\ell(\ell+1)]^{1/2}}{\omega}. \tag{5.3.26}$$

Note that for $\omega M \ll 1$, there will be a large overlap of regions II and III. In region I, we may neglect the potential, $V(r)$, in Eq. (5.3.12) compared with ω^2 and the solutions take the form

$$R^{\mathrm{I}}_{\omega\ell}(r) \approx \alpha^{\mathrm{I}}_{\ell}(\omega) e^{i\omega r^*} + \beta^{\mathrm{I}}_{\ell}(\omega) e^{-i\omega r^*}. \tag{5.3.27}$$

In region II, the potential, $V(r)$, dominates over ω^2 and the solutions are well approximated

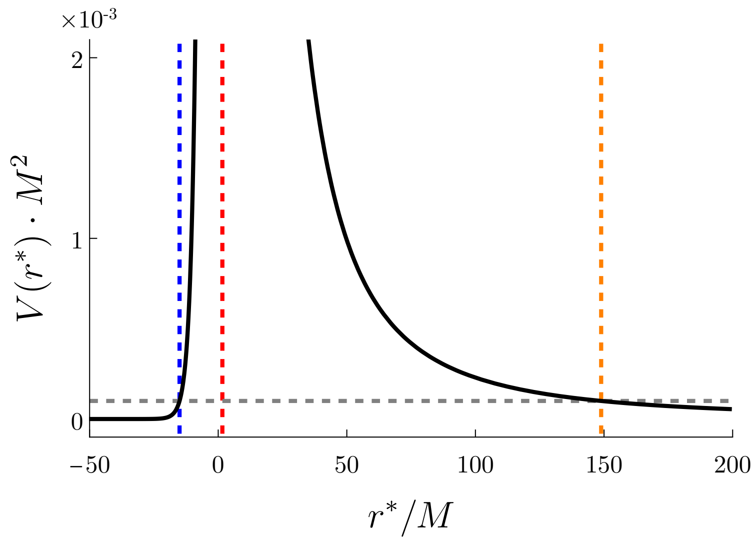

Figure 5.1: The potential $V(r^*)$ plotted as a function of r^* for $\ell = 1$. The horizontal, grey dashed line corresponds to square of the frequency $\omega = 0.01/M$. The vertical blue and orange dashed lines correspond to the turning points r_1^* and r_2^* respectively. The vertical, red dashed line is the peak of the potential at $r = 3M$. The radial mode solutions in regions II and III are matched in the regions where they overlap. The solutions in regions I and II are both good approximations in a neighborhood of $r^* = r_1^*$ and so can be matched there.

by the static (zero frequency) solutions [Cohen and Wald, 1971, Fabbri, 1975]

$$\begin{aligned} R^{\rm II}_{\omega\ell}(r) \approx &\alpha^{\rm II}_\ell(\omega)\left[\frac{y}{2}P_\ell(y-1) - \frac{P_{\ell+1}(y-1) - P_{\ell-1}(y-1)}{2(2\ell+1)}\right] \\ &+\beta^{\rm II}_\ell(\omega)\left[\frac{y}{2}Q_\ell(y-1) - \frac{Q_{\ell+1}(y-1) - Q_{\ell-1}(y-1)}{2(2\ell+1)}\right] \end{aligned} \tag{5.3.28}$$

where $y \equiv r/M$.

Finally, in region III, we may approximate the potential as $V(r) \approx \ell(\ell+1)/r^{*2}$ and we may then approximate the solutions by the flat spacetime solutions with r^* replacing r

$$R^{\rm III}_{\omega\ell}(r) \approx \alpha^{\rm III}_\ell(\omega) r^* j_\ell(\omega r^*) + \beta^{\rm III}_\ell(\omega) r^* n_\ell(\omega r^*) \tag{5.3.29}$$

where j_ℓ and n_ℓ denote the spherical Bessel and Neumann functions. Note that in the overlap between regions II and III, we may neglect[6] the difference between r and r^* and the solutions take the form

$$R^{\rm II,III}_{\omega\ell}(r) \approx \alpha_\ell(\omega) r^{\ell+1} + \frac{\beta_\ell(\omega)}{r^\ell}. \tag{5.3.30}$$

In order to determine $\vec{R}_{\omega\ell}(r)$, we start with the solution $\vec{B}_{\omega\ell} e^{-i\omega r^*}$ in region III [see Eq. (5.3.14)], with initially unknown coefficient $\vec{B}_{\omega\ell}$. We match this solution to the general solution Eq. (5.3.28) in region II and then match the resulting solution to the general solution Eq. (5.3.27) in region I. Finally, we adjust $\vec{B}_{\omega\ell}$ so as to give a coefficient of 1 to the term $e^{i\omega r^*}$ as $r \to 2M$ in Eq. (5.3.14). Similarly, to obtain $\overleftarrow{R}_{\omega\ell}(r)$, we start with the solution $\overleftarrow{B}_{\omega\ell} e^{-i\omega r^*}$ in region I [see Eq. (5.3.15)], with initially unknown coefficient $\overleftarrow{B}_{\omega\ell}$. We match this solution to the general solution Eq. (5.3.28) in region II, match the resulting solution to the general solution Eq. (5.3.29) in region III, and adjust $\overleftarrow{B}_{\omega\ell}$ so as to give a coefficient of 1 to the term $e^{-i\omega r^*}$ as $r \to \infty$ in Eq. (5.3.15).

For simplicity, we shall assume that Alice's lab is located in the region $M \ll r \ll 1/\omega$ for the relevant range of frequencies $\omega \sim 1/T$, so that it lies in the overlap of regions II and III. This is the regime in which the estimates of Chapters 3 and 4 reviewed in Sec. 5.1 apply, so we will be able to make a direct comparison of our results with the results of the previous calculation. The mode functions $\overleftarrow{R}_{\omega\ell}(r)$ were previously obtained by Fabbri [Fabbri, 1975], since they are needed to analyze scattering of classical waves by a black hole. In region III, we find that $\beta^{\rm III}_\ell = O([\omega M]^{2\ell+2})$ and thus the Neumann term in Eq. (5.3.29) may be neglected. The solution with the correct normalization in region III is

$$\overleftarrow{R}_{\omega\ell}(r) \approx -2 i^{3\ell+1} \omega r^* j_\ell(\omega r^*). \tag{5.3.31}$$

6. Replacement of r^* by r in Eq. (5.3.29) would give rise to an arbitrarily large phase error in the solutions as $r \to \infty$, so the difference between r and r^* cannot be neglected throughout region III. However, the difference between r and r^* makes only a small correction, which we neglect, in the overlap of regions II and III.

If, in addition, we have $\omega r \ll 1$, then

$$\overleftarrow{R}_{\omega\ell}(r) \approx -\frac{i^{3\ell+1}2^{\ell+1}\ell!}{(2\ell+1)!}(\omega r)^{\ell+1} \qquad (M \ll r \ll \omega^{-1}). \tag{5.3.32}$$

Thus, as might be expected, if we assume that Alice's lab is not close to the black hole ($r \gg M$), the modes in Alice's lab corresponding to low frequency incoming waves from infinity are essentially unaffected by the black hole. As in flat spacetime, they are suppressed by the factor $(\omega r)^{\ell+1}$ due to the angular momentum barrier. Since $\omega r \ll 1$, the dominant contribution to the two-point function in Alice's lab from modes that are incoming from infinity arises from the $\ell = 1$ mode.

Performing the similar analysis for $\vec{R}_{\omega\ell}(r)$, we obtain

$$\vec{R}_{\omega\ell}(r) \approx a_\ell \left(\frac{M}{r}\right)^{\ell} (M\omega) \qquad (M \ll r \ll \omega^{-1}), \tag{5.3.33}$$

where

$$a_\ell = \frac{-i2^{l+2}(\ell-1)!(\ell+1)!}{(2\ell+1)(2\ell)!}. \tag{5.3.34}$$

Note that, although at low frequencies the white hole modes are essentially entirely reflected back into the black hole by the potential barrier $V(r)$, these modes fall off in r only as the power law $1/r^\ell$ and, thus, they penetrate far beyond the peak of the potential barrier at $r = 3M$ and can have a nontrivial effect in Alice's lab. Note also that, as opposed to the incoming modes from infinity, the frequency dependence of the white hole modes is ℓ independent. Since $r \gg M$, the dominant contribution to the two-point function in Alice's lab from the modes emerging from the white hole arises from the $\ell = 1$ modes.

We now estimate $\langle N \rangle^{\mathrm{U}}$, Eq. (5.3.19), for the case of the Unruh vacuum, $|\Omega_{\mathrm{U}}\rangle$. (The cases of the Boulware and Hartle-Hawking vacua will be treated in the next section.) We first consider the contribution, $\langle N \rangle^{\mathrm{U}}_{\leftarrow}$, of the incoming modes from infinity. We keep only the $\ell = 1$ contribution and use Eq. (5.3.32) to evaluate $\overleftarrow{R}_{\omega 1}$. We use Eq. (5.3.21) to evaluate $\hat{d}$

and we also use $\overleftarrow{G}_{\mathrm{U}}(\omega) = \Theta(\omega)$. Ignoring all subleading terms and all factors of order unity, we obtain the following expression for the contribution of the incoming modes from infinity in the Unruh vacuum

$$\langle N \rangle^{\mathrm{U}}_{\leftarrow} \sim \frac{q^2}{r^4} \int_{1/T}^{1/\min[T_1,T_2]} \frac{d\omega}{\omega} \frac{d^2}{\omega^2} (\omega r)^4 \sim \frac{q^2 d^2}{\min[T_1, T_2]^2} \,. \tag{5.3.35}$$

This agrees with the estimate Eq. (5.1.14) for Minkowski spacetime obtained by considering radiation of entangling photons to infinity. Note that the contribution from the incoming modes from infinity does not grow with T.

Next, we estimate the contribution, $\langle N \rangle^{\mathrm{U}}_{\rightarrow}$ of the incoming modes from the white hole to $\langle N \rangle^{\mathrm{U}}$. We keep only the $\ell = 1$ contribution and use Eq. (5.3.33). In the Unruh vacuum, we have

$$\vec{G}_{\mathrm{U}}(\omega) = \frac{1}{1 - e^{-2\pi\omega/\kappa}} \approx \frac{\kappa}{2\pi\omega}. \tag{5.3.36}$$

Ignoring all subleading terms and all factors of order unity and setting $r = D$, we obtain the following expression for the contribution of the incoming modes from the white hole in the Unruh vacuum

$$\langle N \rangle^{\mathrm{U}}_{\rightarrow} \sim \frac{q^2 d^2 \kappa M^4}{D^6} \int_{1/T}^{1/\min[T_1,T_2]} \frac{d\omega}{\omega^2} \sim \frac{q^2 d^2 M^3}{D^6} T. \tag{5.3.37}$$

For large T, this contribution dominates over Eq. (5.3.35), so we have

$$\langle N \rangle^{\mathrm{U}} = \langle N \rangle^{\mathrm{U}}_{\leftarrow} + \langle N \rangle^{\mathrm{U}}_{\rightarrow} \approx \langle N \rangle^{\mathrm{U}}_{\rightarrow} \sim \frac{q^2 d^2 M^3}{D^6} T. \tag{5.3.38}$$

This agrees with the estimate Eq. (5.1.16) for the decoherence resulting from the emission of entangling photons through the black hole horizon. Thus, our purely local analysis reproduces the results previously obtained in Chapters 3 and 4.

We now briefly comment on the analogous computation in the linearized quantum gravitational case. If we approximate the stress-energy tensor of the first component of Alice's particle as being essentially a point particle, then its stress-energy tensor would take the form

$$T_1^{ab}(t) \approx \frac{m}{\sqrt{-g}} \delta^{(3)}[x^i - X_1^i(t)] u_1^a u_1^b \frac{d\tau_1}{dt} \tag{5.3.39}$$

in analogy with Eq. (5.3.1). If this component was not interacting with any other matter, then conservation of stress-energy would imply that it must move on a geodesic. However, since we want the component to follow a nongeodesic trajectory, Alice must apply some "external force" to it. The external forces on the different components act oppositely on the different components during separation and recombination and will have a backreaction effect on Alice's lab. In Minkowski spacetime, conservation of total stress-energy implies that Alice's lab would have to move oppositely to the particle components so as to keep the center of mass of the total system fixed. In the case of a black hole spacetime, the situation is more complicated, since a further external system would be needed to keep Alice's lab stationary. Nevertheless, the analog of the dipole contribution Eq. (5.3.5) to the difference in stress-energy of the components should be canceled by the stress-energy effects of Alice's lab, and the leading order contribution should be given by

$$(T_1^{ab} - T_2^{ab}) \approx \frac{2m}{\sqrt{-g}} \frac{dt}{d\tau} t^{[a} s^{c]} t^{[b} s^{d]} \nabla_c \nabla_d \left[d^2(t) \delta^{(3)}(x^i - X^i) \right]. \tag{5.3.40}$$

The analog of Eq. (5.3.7) is then

$$\boldsymbol{h}_{ab}^{\text{in}}(T_1^{ab} - T_2^{ab}) \approx -m \int dt d^2(t) s^a s^b \boldsymbol{E}_{ab}^{\text{in}}(t, X^i). \tag{5.3.41}$$

where $\boldsymbol{E}_{ab}^{\text{in}}$ is the quantum field observable corresponding to the electric part of the Weyl tensor $E_{ab} = C_{acbd} t^c t^d$. Thus, the computation of $\langle N \rangle$, Eq. (5.2.3), reduces to obtaining the two-point function of the Weyl tensor. Again, we can simplify calculations by restricting to the case of radial separation. The upshot is that the order of magnitude estimates that we obtained above for the electromagnetic case apply with the substitutions $q \to m$, $d \to d^2$ and the mode sum now running over $\ell \geq 2$, so that the dominant contribution arises from $\ell = 2$. For the Unruh vacuum, this yields the estimate

$$\langle N \rangle_{\leftarrow}^{\mathrm{U,GR}} \sim \frac{m^2 d^4}{\min[T_1, T_2]^4} \tag{5.3.42}$$

in agreement with Eq. (5.1.15), and the estimate

$$\langle N \rangle_{\rightarrow}^{\mathrm{U,GR}} \sim \frac{M^5 m^2 d^4}{D^{10}} T \tag{5.3.43}$$

in agreement with Eq. (5.1.17).

Finally, we note that Eq. (5.3.8) shows that in the electromagnetic case, we have

$$\langle N \rangle = q^2 \left\langle \left(\int dt d(t) s^a \boldsymbol{E}_a^{\mathrm{in}} \right)^2 \right\rangle_{\Omega} \sim q^2 d^2 T^2 \left[\Delta(s^a \boldsymbol{E}_a^{\mathrm{in}}) \right]^2 \tag{5.3.44}$$

where $\Delta(s^a \boldsymbol{E}_a^{\mathrm{in}})$ is defined by

$$[\Delta(s^a \boldsymbol{E}_a^{\mathrm{in}})]^2 = \left\langle \left(\frac{1}{T} \int dt \frac{d(t)}{d} s^a \boldsymbol{E}_a^{\mathrm{in}} \right)^2 \right\rangle_{\Omega} \tag{5.3.45}$$

and thus can be interpreted as the root mean square of the time average of the s^a component of the electric field fluctuations in state $|\Omega\rangle$ on Alice's worldline during the duration of her experiment.

The fluctuations of the electric field are most usefully characterized by its power spectrum. The power spectrum of the radial component of the electric $S_r^{\mathrm{U}}(\omega)$ is given by

$$S_r^{\mathrm{U}}(\omega) = \int_{-\infty}^{\infty} dt\ e^{i\omega(t-t')} \langle \boldsymbol{E}_r(t, X^i) \boldsymbol{E}_r(t', X^i) \rangle_{\Omega_{\mathrm{U}}} . \tag{5.3.46}$$

The modes that dominantly contribute to this power spectrum in Alice's lab are the white hole modes $\vec{R}_{\omega\ell}$ with $\ell = 1$ and $\omega \sim 1/T$. By Eqs. (5.3.9) and (5.3.33), in the Unruh vacuum these modes contribute[7]

$$\begin{aligned} S_r^{\mathrm{U}}(\omega) &\sim \frac{1}{r^4}\frac{1}{\omega}\vec{G}_U(\omega)|\vec{R}_{\omega 1}(r)|^2 \\ &\sim \frac{\kappa}{r^4\omega^2}\left(\frac{M^2\omega}{r}\right)^2 \\ &\sim \frac{M^3}{r^6}. \end{aligned} \tag{5.3.47}$$

This corresponds to the black hole in the Unruh vacuum acting as though it were an ordinary body with a randomly fluctuating electric dipole moment, $\vec{P}_U$ with constant power spectrum

$$\Delta|\vec{P}_{\mathrm{U}}|(\omega) \sim \frac{\sqrt{\epsilon_0\hbar}G^{3/2}M^{3/2}}{c^3} \sim 10\ \frac{\mathrm{e{\cdot}m}}{\sqrt{\mathrm{Hz}}}\left(\frac{M}{M_\odot}\right)^{3/2}, \tag{5.3.48}$$

where we have restored fundamental constants to emphasize that this is an $O(\sqrt{\hbar})$ effect.

Similarly, in the gravitational case, the black hole acts as though it were an ordinary body with a fluctuating mass quadrupole moment of magnitude

$$\Delta|Q_{\mathrm{U}}|(\omega) \sim \frac{\sqrt{\hbar}G^2M^{5/2}}{c^5} \sim 10^{-1}\ \frac{\mathrm{g{\cdot}m^2}}{\sqrt{\mathrm{Hz}}}\left(\frac{M}{M_\odot}\right)^{5/2}. \tag{5.3.49}$$

More generally, the power spectra of the higher electric multipole fluctuations and mass multipole fluctuations of the black hole go as

$$\Delta|\mathscr{Q}_\ell^{\mathrm{EM}}|(\omega) \sim M^{\ell+1/2}, \quad \Delta|\mathscr{Q}_\ell^{\mathrm{GR}}|(\omega) \sim M^{\ell+1/2}. \tag{5.3.50}$$

7. In Rindler spacetime, the analogous horizon modes similarly make a contribution to the power spectrum of the electric field that is nonvanishing as $\omega \to 0$ [Wilson-Gerow et al., 2024]. This fact is undoubtedly intimately related to the phenomena analyzed in [Higuchi et al., 1992b,a, Matsas et al., 1996, Higuchi et al., 1997].

There also are similar fluctuations of the magnetic parity multipole moments. The dominant contribution to the decoherence in Alice's experiment, however, comes from the lowest electric parity multipole moment.

In conclusion, we have successfully reproduced the main results of Chapters 3 and 4 using our purely local reformulation. In the next section, we will use our local reformulation to compare the results for the decoherence in the Unruh vacuum around a black hole to other cases.

5.4 Comparison with Decoherence Arising in Other Cases

The results we have obtained in the previous section will now enable us to analyze the decoherence arising in other situations. Specifically, we will analyze the cases of (i) a Schwarzschild black hole in the Boulware or Hartle-Hawking vacuum, (ii) Minkowski spacetime in the Minkowski vacuum or filled with a thermal bath of radiation, (iii) a spacetime corresponding to the gravitational field of a star with no internal degrees of freedom assigned to the star, and (iv) a material body with internal degrees of freedom in a thermal state.

5.4.1 Decoherence in the Boulware and Hartle-Hawking Vacua

The Boulware vacuum, $|\Omega_{\rm B}\rangle$, is the ground state for the exterior region ($r > 2M$) of Schwarzschild with respect to the timelike Killing field. The Boulware vacuum is singular on the past and future event horizons of Schwarzschild. Since it is singular on the future horizon, it does not correspond to a physically reasonable state for a black hole formed by gravitational collapse. Nevertheless, the Boulware vacuum is a well-defined state in Alice's lab, and it is instructive to compute the decoherence of her particle in the Boulware vacuum using the results of the previous section.

The Boulware vacuum differs from the Unruh vacuum only in that $\vec{G}$ and $\overleftarrow{G}$ are now given by Eq. (5.3.16) rather than Eq. (5.3.17). Since $\overleftarrow{G}_{\rm B} = \overleftarrow{G}_{\rm U}$, it follows immediately that

$\langle N \rangle^{\mathrm{B}}_{\leftarrow}$ is again given by Eq. (5.3.35), i.e.,

$$\langle N \rangle^{\mathrm{B}}_{\leftarrow} = \langle N \rangle^{\mathrm{U}}_{\leftarrow} \sim \frac{q^2 d^2}{\min[T_1, T_2]^2} \,. \tag{5.4.1}$$

On the other hand, in the Boulware vacuum, we have $\vec{G}_{\mathrm{B}} = \Theta(\omega)$ rather than being given by Eq. (5.3.36). Consequently, the integrand of the formula for $\langle N \rangle^{\mathrm{B}}_{\rightarrow}$ will differ from the integrand appearing on the right side of Eq. (5.3.37) by a factor of $\sim \omega/\kappa$. We obtain

$$\begin{aligned} \langle N \rangle^{\mathrm{B}}_{\rightarrow} &\sim \frac{q^2 d^2 M^4}{D^6} \int\limits_{1/T}^{1/\min[T_1,T_2]} \frac{d\omega}{\omega} \\ &= \frac{q^2 d^2 M^4}{D^6} \ln\left(\frac{T}{\min[T_1, T_2]}\right). \end{aligned} \tag{5.4.2}$$

Additionally, we note that the Boulware vacuum at $M\omega \ll 1$ has a randomly fluctuating electric dipole $\Delta|\vec{P}_{\mathrm{B}}|$ and mass quadrupole $\Delta|Q_{\mathrm{B}}|$ of magnitude

$$\Delta|\vec{P}_{\mathrm{B}}|(\omega) \sim M^2\sqrt{\omega}, \quad \Delta|Q_{\mathrm{B}}|(\omega) \sim M^3\sqrt{\omega} \tag{5.4.3}$$

which are much smaller than the corresponding fluctuations in the Unruh vacuum given by Eqs. (5.3.48) and (5.3.49).

Equation (5.4.2) could also be derived by the methods used in Chapters 3 and 4. Indeed, the only change that needs to be made to the calculations done in those chapters is that when we compute the one-particle norm corresponding to the retarded solution with source $j_1^a - j_2^a$ on the horizon, we now have to take the positive frequency part with respect to Killing time rather than affine time. The same calculation as led to Eq. (13) of Chapter 3—which yielded $\langle N \rangle$ varying as $\ln V$, where V denotes the affine time duration of the separation—now yields the $\ln T$ dependence[8] given in Eq. (5.4.2).

8. Affine time V is related to Killing time T by $V \propto \exp(\kappa T)$, so, for the Unruh vacuum, the logarithmic

Next, we consider decoherence in the Hartle-Hawking vacuum, $|\Omega_{\rm HH}\rangle$. In the exterior region ($r > 2M$) of Schwarzschild, the Hartle-Hawking vacuum is a thermal (KMS) state with respect to all modes at temperature $\mathscr{T} = \kappa/2\pi$. Since $\vec{G}_{\rm HH} = \vec{G}_{\rm U}$, it follows immediately that $\langle N\rangle^{\rm HH}_{\rightarrow}$ is again given by Eq. (5.3.37), i.e.,

$$\langle N\rangle^{\rm HH}_{\rightarrow} = \langle N\rangle^{\rm U}_{\rightarrow} \sim \frac{q^2 d^2 M^3}{D^6} T. \tag{5.4.4}$$

On the other hand, in the Hartle-Hawking vacuum we have

$$\overleftarrow{G}_{\rm HH}(\omega) = \frac{1}{1 - e^{-\omega/\mathscr{T}}} \tag{5.4.5}$$

with $\mathscr{T} = \kappa/2\pi = 1/8\pi M$ rather than $\overleftarrow{G} = \Theta(\omega)$ as for the Boulware and Unruh vacua. At low frequencies, we have $\overleftarrow{G}_{\rm HH}(\omega) \approx \mathscr{T}/\omega$. Consequently, the integrand (of the formula for $\langle N\rangle^{\rm HH}_{\leftarrow}$ will differ from the integrand appearing on the right side of Eq. (5.3.35) by a factor of $\mathscr{T}/\omega$ at low frequencies. We obtain

$$\langle N\rangle^{\rm HH}_{\leftarrow} \sim \frac{q^2 d^2 \mathscr{T}}{\min[T_1, T_2]} \sim \frac{q^2 d^2}{M \min[T_1, T_2]}\,, \tag{5.4.6}$$

which differs from Eq. (5.3.35) in that a factor of M has replaced a factor of $\min[T_1, T_2]$ in the denominator. Nevertheless, the thermal population of incoming modes from infinity does not lead to a decoherence that grows with T. The key point is that although the radiation incoming from infinity is thermal, it does not have the necessary population of "soft modes" to provide a decoherence effect similar to the white hole modes [Wilson-Gerow et al., 2024]. For sufficiently large T the contribution of the incoming modes from infinity will be negligible

dependence on V is converted to the linear dependence on T obtained above. However, for an extremal black hole ($\kappa = 0$), the relation between V and T is linear, so one would expect only logarithmic growth of $\langle N\rangle$ with T in the extremal case. In fact, in the electromagnetic case, the coefficient of this logarithmic term also vanishes in extremal Kerr [Gralla and Wei, 2024] (the "black hole Meisner effect") but a $\ln T$ dependence occurs for a scalar field [Gralla and Wei, 2024].

compared with the contribution from the white hole modes, Eq. (5.4.4), and the decoherence in the Hartle-Hawking vacuum will be the same as in the Unruh vacuum.

It should be noted that there can be additional decoherence effects resulting from thermal populations of modes emerging from the white hole and/or infinity that have not been taken into account in our analysis above. In particular, we have implicitly assumed in our analysis that the components of Alice's particle move on fixed trajectories that are not affected by the incoming radiation. This would be the case if, e.g., the components of Alice's particle are rigidly held in traps.[9] However, if these components are free to move in response to the incoming electromagnetic radiation, there will be Thompson scattering of the radiation. Since the Thompson scattering will be slightly different for the different components, this will result in decoherence that will grow with time for a steady influx of radiation. The decoherence arising from Thompson scattering of low frequency thermal radiation was estimated in Chapter 4, based upon previous analyses of collisional decoherence given in [Diósi, 1995, Gallis and Fleming, 1990, Hornberger and Sipe, 2003, Joos and Zeh, 1985]. It was shown in Chapter 4 that in the Rindler case, this collisional decoherence can be neglected compared with the decoherence due to emission of soft radiation. For the case of a black hole in the Unruh or Hartle-Hawking states, the same would be true if Alice's lab is sufficiently near the black hole. However, the decoherence rate due to emission of soft radiation falls off rapidly with distance, D, from the black hole, whereas the collisional decoherence rate falls off more slowly in the Unruh vacuum and does not fall off at all in the Hartle-Hawking vacuum. Thus, if the particle components are free to respond to the incoming radiation, the collisional decoherence effects will dominate at sufficiently large distances from the black hole.

Finally, we briefly mention the corresponding results for the gravitational case. In the

9. It would be best to use nonelectromagnetic traps, so that the traps do not produce any shielding or other electromagnetic effects that could interfere with Alice's experiment.

gravitational case, a calculation analogous to that which led to Eq. (5.4.2) now yields

$$\langle N\rangle^{\mathrm{B,GR}}_{\rightarrow} \sim \frac{m^2 d^4 M^6}{D^{10}} \ln\left(\frac{T}{\min[T_1, T_2]}\right) \tag{5.4.7}$$

whereas $\langle N\rangle^{\mathrm{B,GR}}_{\leftarrow}$ is the same as for the Unruh vacuum, Eq. (5.3.42). A calculation analogous to that which led to Eq. (5.4.6) now yields

$$\langle N\rangle^{\mathrm{HH,GR}}_{\leftarrow} \sim \frac{m^2 d^4 \mathscr{T}}{\min[T_1, T_2]^3} \sim \frac{m^2 d^4}{M \min[T_1, T_2]^3} \tag{5.4.8}$$

whereas $\langle N\rangle^{\mathrm{HH,GR}}_{\rightarrow}$ is the same as for the Unruh vacuum, Eq. (5.3.43).

5.4.2 Decoherence in Minkowski Spacetime

In Minkowski spacetime, there are no "white hole modes," $\vec{R}_{\omega\ell}(r)$, of the quantum field. The incoming modes from infinity, $\overleftarrow{R}_{\omega\ell}(r)$, are given by

$$\overleftarrow{R}_{\omega\ell}(r) = -2i^{3l+1}\omega r j_\ell(\omega r)\,, \tag{5.4.9}$$

corresponding to taking the limit as $M \to 0$ of the Schwarzschild modes. The two point function of the radial component of the electric field can be obtained from Eq. (5.3.9) by deleting the white hole modes and using Eq. (5.4.9) for the incoming modes from infinity. The Minkowski vacuum, $|\Omega_M\rangle$, corresponds to $\overleftarrow{G}(\omega) = \Theta(\omega)$. It follows immediately that the decoherence of Alice's particle in the Minkowski vacuum will be given by the same estimate as we previously obtained for the decoherence effects of the incoming modes from infinity in Schwarzschild for the Boulware or Unruh vacua [see Eqs. (5.3.35) and (5.4.1)], namely

$$\langle N\rangle^{\mathrm{M}} \sim \frac{q^2 d^2}{\min[T_1, T_2]^2}\,. \tag{5.4.10}$$

This agrees with the estimate originally given in [Belenchia et al., 2018]. In particular, the decoherence effects do not grow with T.

If we thermally populate the modes $\overleftarrow{R}_{\omega\ell}(r)$ in Minkowski spacetime at temperature $\mathscr{T}$, then the decoherence will be given by the same estimate as we previously obtained in Eq. (5.4.6) for the decoherence effects of the incoming modes from infinity in Schwarzschild for the Hartle-Hawking vacuum, namely

$$\langle N \rangle^{\rm M}_{\rm th.} \sim \frac{q^2 d^2 \mathscr{T}}{\min[T_1, T_2]} \,. \tag{5.4.11}$$

In particular, the decoherence effects do not grow with T, despite the presence of the thermal bath.

In a similar manner, in the gravitational case, for the Minkowski vacuum, we obtain

$$\langle N \rangle^{\rm M,GR} \sim \frac{m^2 d^4}{\min[T_1, T_2]^4} \tag{5.4.12}$$

in agreement with the original estimate of [Belenchia et al., 2018]. If Minkowski spacetime is populated with a thermal bath of gravitons at temperature $\mathscr{T}$, then we obtain the same estimate as in Eq. (5.4.8), namely

$$\langle N \rangle^{\rm M,GR}_{\rm th.} \sim \frac{m^2 d^4 \mathscr{T}}{\min[T_1, T_2]^3}. \tag{5.4.13}$$

Again, the decoherence effects do not grow with T, despite the presence of a thermal bath of gravitons.

Finally, we point out that for a scalar field it is possible, in principle, to get decoherence in an inertial laboratory in Minkowski spacetime from "soft radiation" despite the absence of a horizon. In Minkowski spacetime, a memory effect and associated infrared divergences occur at null infinity for a massless field as a result of a permanent change in the field at order

$1/r$. Since charge is conserved in electromagnetism, such $O(1/r)$ changes can occur in the electromagnetic case only via Lorentz boosting of the Coulomb fields of the charged particles. This generically occurs in scattering, since the outgoing charged particles generically have different momenta from the incoming particles. However, the protocol of Alice's experiment requires her to keep the components of her particle confined to her lab, which precludes changes in particle momenta lasting a long enough time T to produce significant decoherence via "soft radiation." This is in accord with what we have found above. Similarly, since mass is conserved in linearized gravity, there also are no significant "soft radiation" decoherence effects. However, for a scalar field, scalar charge need not be conserved, and a change in the scalar field at order $1/r$ can be achieved by simply changing the monopole moment of the source. Consequently, a source with a permanent change of scalar charge will radiate an infinite number of "soft" massless scalar particles in $\ell = 0$ modes. We can use this fact to obtain decoherence via soft radiation to null infinity in Minkowski spacetime in a manner previously suggested in [Gralla and Wei, 2024] as follows.

Suppose that a massless scalar field $\boldsymbol{\phi}$ exists in nature and Alice performs her experiment in an inertial laboratory in Minkowski spacetime with a particle with scalar charge. Suppose, further, that her protocol includes changing the charge of one of the components during separation and then restoring the charge during the recombination.[10] The scalar analog of Eqs. (5.2.2) and (5.2.3) is

$$\langle N \rangle = \langle \Omega | \left[\boldsymbol{\phi}^{\text{in}}(j_1 - j_2) \right]^2 | \Omega \rangle . \tag{5.4.14}$$

The mode expansion of the two-point function of a scalar field in Schwarzschild is given in [Candelas, 1980]. It takes a form very similar to Eq. (5.3.9) except that (i) the factor of $1/r^2 r'^2$ is replaced by $1/rr'$ for the definition of scalar mode functions analogous to our definition of electromagnetic mode functions used in Eq. (5.3.9) and (ii) the mode sum

10. If the experiment is performed in the presence of a black hole or other gravitating body, such a change in scalar charge as determined at infinity automatically occurs from redshift effects if the components are separated in the radial direction [Gralla and Wei, 2024].

begins at $\ell = 0$ rather than $\ell = 1$. Only the incoming modes from infinity are relevant for Minkowski spacetime, and they again take the form Eq. (5.4.9). The $\ell = 0$ modes contribute to Eq. (5.4.14) an extra factor of $1/\omega^2$ relative to the $\ell = 1$ modes. For the case where the scalar field initially is in the Minkowski vacuum state $|\Omega_{\rm M}\rangle$, a calculation in direct parallel to Eq. (5.3.35) yields

$$\langle N\rangle^{\rm M,S} \sim (\Delta q_{\rm S})^2 \ln\left(\frac{T}{\min[T_1, T_2]}\right) \tag{5.4.15}$$

where $\Delta q_{\rm S}$ denotes the scalar charge difference of the two components during their separation. This behavior is analogous to the decoherence occurring in the presence of a black hole for the Boulware vacuum [see Eqs. (5.4.2) and (5.4.8)]. If Minkowski spacetime is initially filled with a thermal bath of scalar particles at temperature $\mathscr{T}$, we obtain

$$\langle N\rangle^{\rm M,S}_{\rm th.} \sim (\Delta q_{\rm S})^2 \mathscr{T} T \tag{5.4.16}$$

which is analogous to the decoherence in the presence of a black hole in the Unruh or Hartle-Hawking vacua.[11] In both cases, the decoherence grows with T due to the emission of soft radiation to infinity, and we thus see that such decoherence is possible, in principle, without the presence of a horizon.

5.4.3 Decoherence in the Spacetime of a Static Star

We now consider the decoherence effects arising in Alice's lab when we place it outside of a star rather than a black hole. In this subsection, we do not consider the decoherence effects that may arise from interactions with degrees of freedom of the matter composing the star,

11. For a scalar field the similarity of the decoherence rate in a global thermal state in Minkowski spacetime, as compared to the decoherence due to a Killing horizon is related to the fact that the restriction of the two-point function of the Minkowski vacuum to a uniformly accelerating world line is identical to the restriction of the two-point function of the global Minkowski thermal state at the Unruh temperature to an inertial world line. However, for the electromagnetic and gravitational fields, no such equivalence holds [Boyer, 1980], and as we have seen, these fields do not exhibit the analogous decoherence in a global thermal state.

i.e., we are concerned only with the effects of replacing the black hole spacetime with a spacetime without a horizon. Decoherence effects due to interactions with matter will be considered in the next subsection.

The metric outside of a static, spherical star is identical to the metric of a Schwarzschild black hole. If the electromagnetic field in the spacetime of a static star is initially in its ground state, then one might expect that if Alice performs her experiment outside of the star, she would get essentially the same results as she would have obtained by performing her experiment at the same radius in Schwarzschild spacetime with the electromagnetic field initially in the Boulware vacuum state.[12] Similarly, if the electromagnetic field in the spacetime of the star is initially in a thermal state at temperature $\mathscr{T} = 1/8\pi M$, one might expect that Alice would get essentially the same results as for a Schwarzschild black hole with the electromagnetic field initially in the Hartle-Hawking vacuum state. The purpose of this subsection is to explain why these expectations are not correct.

The key point is that the behavior of a quantum field in the spacetime of a star differs significantly from that of a quantum field around a black hole in that the white hole modes, $\vec{R}_{\omega\ell}(r)$, are absent. The complete absence of the white hole modes in the case of a star is very different from the modes being present but in their ground state, as occurs for the Boulware vacuum in Schwarzschild. The white hole modes in Schwarzschild represent additional degrees of freedom of the quantum field that are not present in the case of the star. It is these additional degrees of freedom—associated with the presence of a horizon—that are responsible for the decoherence effects that grow with T in Alice's experiment.

To see this explicitly, we note that in the spacetime of the star, the two-point function of the radial component of the electric field is modified from Eq. (5.3.9) in that (i) the white hole modes, $\vec{R}_{\omega\ell}(r)$, are absent and (ii) the incoming modes from infinity, $\overleftarrow{R}_{\omega\ell}(r)$, are modified by the presence of the star. However, at very low frequencies, $\omega\mathcal{R} \ll 1$, where $\mathcal{R}$ denotes the

12. In contrast to a static star, a body that collapses to a black hole produces the Unruh vacuum in its exterior, so that $\langle N\rangle$ grows linearly in time, as we have shown.

radius of the star, the corrections to $\overleftarrow{R}_{\omega\ell}(r)$ are negligibly small. The ground state of the star satisfies $\overleftarrow{G}(\omega) = \Theta(\omega)$. It follows immediately that the decoherence in the spacetime of a star with the electromagnetic field initially in its ground state is the same as the decoherence in Schwarzschild due to the incoming modes from infinity in the Boulware or Unruh vacua [see Eq. (5.4.1)], which, in turn, is the same as the decoherence in Minkowski spacetime in the Minkowski vacuum [see Eq. (5.4.10)]. Thus, we obtain

$$\langle N \rangle^{\text{star}} \sim \frac{q^2 d^2}{\min[T_1, T_2]^2} \,. \tag{5.4.17}$$

Similarly, if the electromagnetic field around the star is in a thermal state at temperature $\mathscr{T}$, we obtain the same result as in Eq. (5.4.8), namely

$$\langle N \rangle^{\text{star}}_{\text{th.}} \sim \frac{q^2 d^2 \mathscr{T}}{\min[T_1, T_2]} \,. \tag{5.4.18}$$

In the gravitational case, we obtain results in agreement with Eqs. (5.4.12) and (5.4.13), respectively.

In summary, the presence of a horizon is essential for the black hole decoherence effects. Similar effects do not occur in the spacetime of a static star.

5.4.4 Decoherence due to the Presence of a Body with Internal Degrees of Freedom

As we have just seen, in the electromagnetic and gravitational cases, decoherence due to emission of "soft radiation" does not occur in a static asymptotically flat spacetime without a horizon.[13] This can be understood as resulting from the absence of any "white hole mode" degrees of freedom associated with the horizon. However, if an actual material body is

13. However, as discussed at the end of Sec. 5.4.2, in the scalar case one can get decoherence due to emission of soft radiation to null infinity.

present, there will be additional degrees of freedom associated with the material body. These degrees of freedom can couple to the components of Alice's particle via ordinary Coulombic (or, in the gravitational case, Newtonian) interactions. If there is suitable dissipation in the material body system, this can result in the decoherence of Alice's particle. Indeed, ordinary environmental decoherence is exactly of this nature. In this subsection, we will consider whether the decoherence of Alice's particle resulting from Coulombic/Newtonian interactions with a material body can mimic the decoherence obtained for the case of a black hole.

As we have seen in Sec. 5.3 above, in the electromagnetic case the dominant contribution to decoherence of Alice's particle near a Schwarzschild black hole in the Unruh vacuum comes from the $\ell = 1$ white hole modes at very low frequencies. Very near the horizon of the black hole, these modes correspond to radiation and they represent genuine additional degrees of freedom of the electromagnetic field. Nevertheless, we saw at the end of Sec. 5.3 that in Alice's lab, these modes look just like the exterior dipole field of an ordinary body, with a fluctuating electric dipole moment given by Eq. (5.3.48). Thus, if we have a material body with the property that its ordinary thermal fluctuations cause its electric dipole moment at very low frequencies ω to fluctuate in accord with Eq. (5.3.48), then that material body should mimic the decoherence effects of a black hole. Similarly, in the gravitational case, a material body will mimic the decoherence effects of a black hole if ordinary thermal fluctuations cause its mass quadrupole moment at very low frequencies ω to fluctuate in accord with Eq. (5.3.49).

The issue of whether an ordinary material body can mimic a black hole of the same temperature in this manner has very recently been investigated by Biggs and Maldacena [Biggs and Maldacena, 2024]. They have shown that in the electromagnetic case, there are no difficulties in constructing a physically reasonable matter model that mimics the "soft radiation" decoherence effects of a black hole. However, in the gravitational case, the mimicking of black hole decoherence effects by an ordinary body of the same physical size and temperature as the black hole appears to require extraordinary properties of the matter.

The underlying difficulty is the weakness of the coupling of matter to gravity. In order to produce a fluctuating quadrupole moment of the required size Eq. (5.3.49), it seems possible that the body would need to have a mass comparable to that of a black hole as well as extremely large dissipation. This issue appears worthy of further investigation.

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