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Photon propagation in a magnetized electron plasma

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Abstract. We have given a general decomposition of the photon self-energy in a magnetized medium for any magnetic field strength. In the long wave length limit this can be expressed interms of only three independent form factors. We consider the finite temperature magnetized electron gas as the background in which the photon is propagating and derived the dispersion relations in the non-relativistic limit.

1. Introduction

The propagation of photons in a material medium in the presence of a magnetic field gives rise to many observable effects. For example, in astrophysical contexts, the fact that the photons with different polarizations have different dispersion relations (birefringence), leads to the Faraday rotation effects that have been observed for various astrophysical objects[1]. The subject of the propagation of photons in magnetized plasmas is well studied[2]. The field-theoretical methods are generally useful in this kind of problem, since they are applicable to a wider range of physical situations for which the semiclassical methods breakdown[3].

The existence of super critical fields (4.4×10^{13} G) in nature have grown stronger recently with the observations of few Soft Gamma Rays Repeaters (SGR) and Anomalous X-Rays Pulsars (AXP), which are very likely magnetars, that is, isolated neutron stars with surface magnetic fields of order $10^{14} - 10^{15}$ G. A model for extragalactic gamma rays burst in terms of merger of massive binary stars suggests also that magnetic fields up to the order of 10^{17} G may exist. In the context of the Early Universe, very large magnetic fields (10^{23} G) may be generated during the electroweak phase transition due to gradients in the Higgs field. Thus, there are physical environments of interest - that involve, in addition to matter in extreme relativistic and/or degenerate conditions, strong magnetic fields - for which neither the semiclassical methods nor the weak field approximation are directly applicable. We derive the general decomposition of the photon self-energy in such a medium, and calculate the dispersion relations for the different modes and calculate the Faraday rotation for the plane polarized light in an electron gas at finite temperature[4].

2. Photon polarization tensor

In the absence of the magnetic field, the photon self-energy $\pi_{\mu\nu}$ depends in general on the photon momentum q^μ, and on the velocity four-vector of the medium u^μ. In the frame of reference in which the medium is at rest, u^μ has components given by $u^\mu = (1, \vec{0})$ and in that frame we write $q^\mu = (\omega, \vec{Q})$. In the presence of a magnetic field, but otherwise an isotropic medium, $\pi_{\mu\nu}$ depends in addition on the vector b^μ that is determined by the magnetic field. The vector b^μ is defined such that, in the frame in which the medium is at rest, $b^\mu = (0, \hat{b})$ where we denote the magnetic field vector by $\vec{B} = B\hat{b}$.

In the most general case, the photon self-energy can be expressed as a linear combination of the bilinears $\epsilon_i^\mu \epsilon_j^\nu$ in the form

$$\pi_{\mu\nu}^{(eff)}(\omega, \vec{Q}) = -\sum_{i,j} \pi^{(ij)}(\omega, \vec{Q}) \epsilon_{\mu i}(\vec{Q}) \epsilon_{\nu j}(\vec{Q}) \,, \tag{1}$$

involving the nine independent coefficient functions $\pi^{(ij)}(\omega, \vec{Q})$. In the long wavelength limit all the $\pi^{(ij)}(\omega, \vec{Q})$ can be determined in terms of just three independent functions, $\epsilon_1^\mu(\vec{Q}) = (0, \hat{e}_1)$, $\epsilon_2^\mu(\vec{Q}) = (0, \hat{e}_2)$ and $\epsilon_3^\mu(\vec{Q}) = \frac{1}{\sqrt{q^2}}(Q, \omega\hat{e}_3)$. The long wavelength limit is valid under the condition $\omega \gg v_e Q$, where v_e stands for the average velocity of an electron in the gas. In the limit $\vec{Q} \to 0$, we can write $q^\mu = \omega u^\mu$ and therefore only u^μ and b^μ are independent vectors. The most general form of $\pi_{\mu\nu}^{(eff)}$ consistent with the transversality condition in this case is then

$$\pi_{\mu\nu}^{(eff)}(\omega, \vec{Q} \to 0) = \pi_T(\omega, \vec{Q} \to 0) R_{\mu\nu} + \pi_L(\omega, \vec{Q} \to 0) Q_{\mu\nu} + \pi_P(\omega, \vec{Q} \to 0) P_{\mu\nu} \,, \tag{2}$$

where $R_{\mu\nu}, P_{\mu\nu}$ and $Q_{\mu\nu}$ are

$$\begin{aligned} Q_{\mu\nu} &= -b_\mu b_\nu \\ R_{\mu\nu} &= g_{\mu\nu} - u_\mu u_\nu - Q_{\mu\nu} \\ P_{\mu\nu} &= i\epsilon_{\mu\nu\alpha\beta} b^\alpha u^\beta \end{aligned} \tag{3}$$

and the functions $\pi_{T,L,P}$ are determined from the one-loop expression for $\pi_{\mu\nu}^{(eff)}$ by means of the projection formulas

$$\begin{aligned} \pi_T(\omega, \vec{Q}) &= \frac{1}{2} R^{\mu\nu} \pi_{\mu\nu}^{(eff)}(\omega, \vec{Q}) \\ \pi_L(\omega, \vec{Q}) &= Q^{\mu\nu} \pi_{\mu\nu}^{(eff)}(\omega, \vec{Q}) \\ \pi_P(\omega, \vec{Q}) &= -\frac{1}{2} P^{\mu\nu} \pi_{\mu\nu}^{(eff)}(\omega, \vec{Q}) \,. \end{aligned} \tag{4}$$

3. Photon self-energy

In one-loop, the 11 element of the photon thermal self-energy is given by

$$i\pi_{11\mu\nu}(\omega, \vec{Q}) = -(-ie)^2 \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\,\left[\gamma_\mu i S_e(p+q) \gamma_\nu i S_e(p)\right] , \tag{5}$$

where S_e stands for the 11 component of the electron thermal propagator and $S_e(p) = S_{Fe}(p) + S_{Te}(p)$ with

$$iS_F(p) = \int_0^\infty d\tau G(p,s) e^{i\tau\Phi(p,s) - \epsilon\tau} , \tag{6}$$

where

$$\Phi(p,s) = p^2 - m_e^2 + \left(s^{-1}\tan(s) - 1\right) R_{\mu\nu}p^\mu p^\nu \,,$$
$$G(p,s) = \not{V}_1(p,s) + i\gamma_5 \not{V}_2(p,s) + m_e + m_e \tan(s) i\gamma_5 \not{u}\not{b} \,, \tag{7}$$

with

$$V_{1\mu}(p,s) = sec^2(s)p_\mu + tan^2(s)\left[(p\cdot b)b_\mu - (p\cdot u)u_\mu\right]$$
$$V_{2\mu}(p,s) = tan(s)\left[(p\cdot b)u_\mu - (p\cdot u)b_\mu\right] \,, \tag{8}$$

and $s = |e|B\tau$. The thermal part is given by

$$iS_{Te}(p) = -\eta_e(p)\int_{-\infty}^{\infty} d\tau G(p,s) e^{i\tau\Phi(p,s)-\epsilon|\tau|}. \tag{9}$$

The term which will contribute to the real part is given by

$$\pi_{\mu\nu}^{(eff)}(\omega,\vec{Q}) = ie^2\int\frac{d^4p}{(2\pi)^4}\Bigg\{\eta_e(p)\int_{-\infty}^{\infty} d\tau e^{\lambda(p,\tau)} \int_0^{\infty} d\tau' e^{\lambda(p',\tau')}\mathrm{Tr}\left[\gamma_\mu G(p',\tau')\gamma_\nu G(p,\tau)\right]$$
$$+ \eta_e(p')\int_0^{\infty} d\tau e^{\lambda(p,\tau)}\int_{-\infty}^{\infty} d\tau' e^{\lambda(p',\tau')}\mathrm{Tr}\left[\gamma_\mu G(p',\tau')\gamma_\nu G(p,\tau)\right]\Bigg\} \,, \tag{10}$$

where we have defined $\lambda(p,\tau) = i\tau\Phi(p,s) - \epsilon|\tau|$ and $p' = p + q$. Using this we can calculate the scalar form factors π_L, π_T and π_P. Here we consider the case $\omega \ll \sqrt{|e|B}$ and $\omega \ll 2\langle E_e\rangle$, where $\langle E_e\rangle$ stands for an average value of the energy of an electron in the gas. Thus, if the gas is non-relativistic, the condition holds for $\omega \ll m_e$. If the gas is extremely relativistic, it also holds for $\omega > m_e$. Together with the condition $\omega \gg v_e Q$ this implies that we are considering a regime in which $v_e Q \ll \omega \ll \sqrt{|e|B}\,,\ 2\langle E_e\rangle$, which we refer as the low frequency limit.

4. Nonrelativistic electron gas

In this case the temperature and chemical potential of the electron gas are $\beta m_e \gg 1$, $\frac{m_e}{\mu} \gg 1$ which gives $f_{\bar{e}} \approx 0$ and $E_n \simeq m_e$. Then the dispersion relations to the leading order are given by

$$\pi_T(\omega) = \frac{\omega^2\Omega_0^2}{\omega^2-\omega_B^2} \,,$$
$$\pi_L(\omega) = \Omega_0^2 \,,$$
$$\pi_P(\omega) = \frac{\omega\omega_B\Omega_0^2}{\omega^2-\omega_B^2} \,, \tag{11}$$

where $\omega_B \equiv \frac{|e|B}{m_e}$ and $\Omega_0^2 \equiv \frac{e^2 n_e}{m_e}$ with

$$n_e = \frac{|e|B}{4\pi^2}\left[\sum_{n=0}^{\infty}(2-\delta_{n,0})\int_{-\infty}^{\infty} dp_\parallel f_e(E_n)\right] \,. \tag{12}$$

The ω_B, Ω_0 and n_e are the standard expressions for the cyclotron frequency, the plasma frequency and total electron number density, respectively. For example, if the photon propagates parallel to the magnetic field, the dispersion relations are

$$\begin{aligned} &\omega^2 - Q^2 - \frac{\omega\Omega_0^2}{(\omega - \omega_B)} = 0, \\ &\omega^2 - Q^2 - \frac{\omega\Omega_0^2}{(\omega + \omega_B)} = 0, \\ &\omega = \Omega_0. \end{aligned} \tag{13}$$

The formulas in Eq. (11), and whence those in Eq. (13), represent the leading term in powers of $1/m_e$, and they neglect entirely the momentum-dependent terms

The two transverse modes having different dispersion relations leads to the Faraday rotation effect. After traveling a distance L, the direction of polarization of the wave has rotated by an angle $\theta = \frac{1}{2}\omega\Delta nL$, where $\Delta n = (n_-(\omega) - n_+(\omega))$. with $n_\mp(\omega)$ being the refractive indices of the left and right polarized modes, respectively. These can be computed from the dispersion relations given above by using $n(\omega) = Q/\omega$. For non-relativistic limit we obtain For values of ω such that $\omega \gg \Omega_0$,we have $n_- \simeq n_+ \simeq 1$ and therefore

$$\frac{\theta}{L} \simeq \frac{\Omega_0^2\omega_B}{2(\omega^2 - \omega_B^2)}. \tag{14}$$

This result coincides with the one given in Ref.[5].

We have given a general decomposition of the photon self-energy in a matter background that contains a magnetic field, in terms of the minimal set of tensors consistent with isotropy and the transversality condition. From this result, we have shown that the self-energy can be expressed in terms of nine independent form factors, that in the long wavelength limit reduce to three. In this limit, by applying the (real-time) finite temperature field theory method, we have calculated the one-loop formulas for the form factors and in the non-relativistic limit recover the Faraday rotation.

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