



Magnetized rotational neutron stars and mass-radius relation

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We calculate radii and masses of neutron stars for various EoSs using a perturbative approach. We also calculate increased masses due to magnetic fields. Moreover, we calculate the radius of a neutron star as a function of its increased total mass due to rotation. As for the EoSs, we adopt various kinds of models in relativistic mean field (RMF) theory. We calculate mass-radius relations (MR relations) using 5 hadronic EoSs. We have obtained the result that the total masses are over twice the solar mass for all 5 hadronic EoSs in the presence of magnetic fields together with rotation.

KEYWORDS: Neutron star, Equation of State, MR relation, rapid rotation, strong magnetic fields

1. Introduction

Rotating neutron star, called pulsar, was observed in 1967. Since then, many pulsars and magnetars have been observed as neutron stars. A neutron star is an object of radius about 10 km, and most have masses of approximately 1.4 times the solar mass. However, the neutron star PSR J1614-2230 forming a binary system with a white dwarf was discovered in 2010, whose observed mass was $1.94 \pm 0.04 M_{\odot}$ [1]. In 2013, the neutron star PSR J0348+0432 with mass of $2.01 \pm 0.04 M_{\odot}$ was observed, and the existence of neutron stars which have masses twice the solar mass is established beyond doubt [2]. Recently an analysis of the gravitational wave suggests a massive neutron star with mass between $2.01 \pm 0.04 M_{\odot}$ and $2.16 \pm 0.17 M_{\odot}$ [3]. Such a very heavy neutron star gives a strong limit on equations of state, for which typical mass about $1.4 M_{\odot}$ is standard.

The presence of such massive neutron stars can be easily described if the neutron star matter consists of only nucleons and some varieties of leptons. However, hyperons should appear naturally in the high-density region of the neutron star where its density is a few times higher than the nuclear saturation density. The appearance of hyperons softens the equation of state (EoS) of neutron star matter, and makes it difficult to explain the presence of the massive ($2M_{\odot}$) neutron stars. This problem is referred to as the hyperon puzzle.

In our previous study [5], the mass-radius (MR) relation of deformed neutron stars in the axially symmetric poloidal magnetic field is calculated. The MR relation is obtained by solving the Hartle equations, whereas the one for spherical stars is obtained by the Tolman-Oppenheimer-Volkoff equations. The anisotropic effects of the poloidal magnetic fields are found to be non-negligible for a strong magnetic field more than $3 \times 10^{18} G$ at the center of a neutron star. In this study, we consider magnetic fields or/and rotation. From the observation, there exist strong magnetized neutron stars, called magnetars, having a magnetic field of about $2 \times 10^{15} G$ on the surface [4]. Also a rotational neutron star with 716 Hz frequency has been observed [6]. Moreover, observed radii of neutron stars also give a good indication for giving NS a strong limit. If we write the radius of a neutron star as R_{NS} ,



the neutron star radius is $R_{NS} \leq 13.6$ km, which is derived from the observation of the GW170817 gravitational wave event [7]. We calculate radii and masses for 5 hadronic EoSs. We compare the mass-radius relation of various EoSs for magnetized neutron stars and rotational neutron stars.

2. Formulations

2.1 Equation of state

In this paper the neutron star matter is supposed to be static and uniform in the high-density region, which is described in the relativistic mean field (RMF) theory based on the nonlinear Walecka model. We use the Lagrangian given in Ref. [8–12, 14–16]. This Lagrangian includes the baryon octet $\{p, n, \Lambda, \Sigma^0, \Sigma^\pm, \Xi^0, \Xi^\pm\}$, electron, and muon are taken into account among fermions. In this model, the scalar meson σ , the vector meson ω , and the vector-isovector meson ρ are introduced. Occasionally, σ^* and ϕ^μ mesons are included. We determine the parameters in the model so that the basic properties at nuclear saturation density are reproduced. Occasionally, we add terms of the magnetic field in the Lagrangian.

2.2 Magnetic fields

In this work we adopt a baryon density dependent (ρ) magnetic field strength given by [16, 21]

$$B(\rho) = B_s + B_0 \left[1 - \exp \left\{ -\alpha \left(\frac{\rho}{\rho_0} \right)^\gamma \right\} \right], \quad (1)$$

where B_s and B_0 indicate the strength on the surface and that for a much denser region than that with the saturation density $\rho_0 = 0.15 \text{ fm}^{-3}$, respectively. We adopt the parameters $\alpha = 0.05$ and $\gamma = 2$ [16]. Here, $B_s = 10^{15} \text{ G}$ is employed.

2.3 Hartle Equations

A theoretical method to calculate masses and eccentricities of axially deformed objects due to slow rotation was first introduced by J. B. Hartle and others in Ref. [24–27]. The metric for such an object can be written as

$$ds^2 = -e^\nu [1 + 2 \{h_0 + h_2 P_2(\cos \theta)\}] dt^2 + e^\lambda \left[1 + \frac{2e^\lambda}{r} \{m_0 + m_2 P_2(\cos \theta)\} \right] dr^2 + r^2 [1 + 2k_2 P_2(\cos \theta)] \times [d\theta^2 + \sin^2 \theta (d\phi - \omega dt)^2], \quad (2)$$

where $\omega(r, \theta)$ represents the local angular velocity of a rotating star, and $h_0(r)$, $h_2(r)$, $m_0(r)$, $m_2(r)$, and $k_2(r)$ are the second order perturbative terms with respect to the angular velocity Ω . The second order Legendre polynomial is given as $P_2(\cos \theta) = \frac{1}{2}(3 \cos^2 \theta - 1)$.

3. Results

In this study, we employ various kinds of EoSs. We choose 5 hadronic EoSs, namely, GM1, TM1-a, TM1-b, TM2- $\omega\rho$ -a, and TM2- $\omega\rho$ -b. Figure 1 shows the mass versus radius relation of a neutron star. These figures show the total mass of a neutron star as a function of its radius. These are so-called the MR relations. The basic properties reproduced by various EoSs are shown in Ref. [10–13]. The left upper panel shows those cases without rotation and magnetic fields, the right upper panel is that with rotation, the left bottom panel is that with magnetic fields, and the right bottom panel is that with both rotation and magnetic fields. Here, in the rotational case, Ω is assumed to be 0.03 km^{-1} . In the magnetized case, B_0 is assumed to be $2 \times 10^{18} \text{ G}$. The yellow solid line and orange solid line indicate the pulsars, PSR J1614-2230 and PSR J0348+0432, respectively. The colors of lines indicate EoS as

follows, dark blue; GM1 EoS, green; TM1-a EoS, light blue; TM1-b EoS, purple; TM2- $\omega\rho$ -a EoS, red; TM2- $\omega\rho$ -b EoS. Table I shows the compatibility for various EoSs. If mass is over twice the solar mass, it is written as \circ . If not, as \times . We obtain neutron star masses more than twice the solar mass ($>2M_{\odot}$) either in the strong magnetic field of 2×10^{18} G in the center or in the rapid rotation of $\Omega = 0.03 \text{ km}^{-1}$ in the TM2- $\omega\rho$ -b EoS. Furthermore, we obtain neutron stars with masses more than $2 M_{\odot}$ both with a strong magnetic field and in a rapid rotation for 5 hadronic EoSs. However, this rotation ($\Omega = 0.03 \text{ km}^{-1}$) corresponds to a neutron star revolving at about 13,700 Hz at maximum mass, so it is not realistic. We also assume Ω to be 0.01 km^{-1} and the mass in rotation slightly increases. In this case, it does not give masses over twice the solar mass. It is apparent, however, that if one makes the frequency larger, the mass increases more.

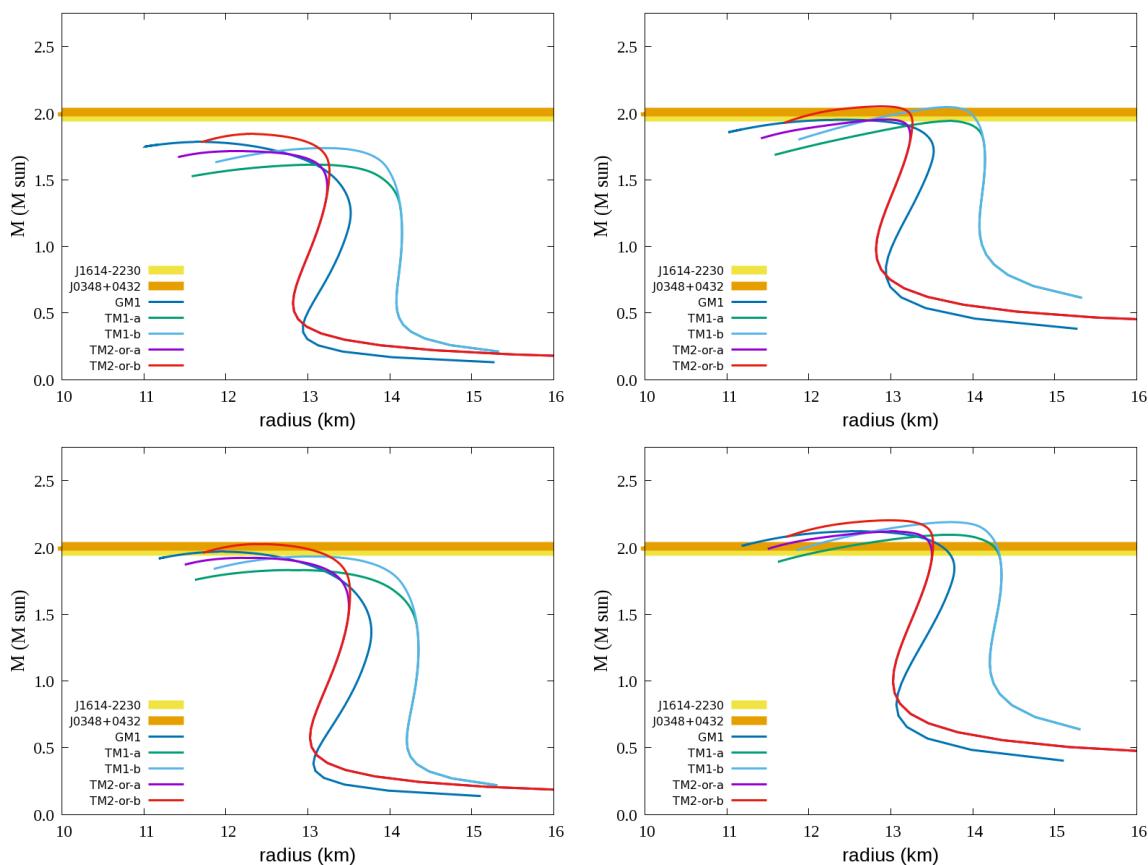


Fig. 1. Various MR relations. The left upper panel shows those cases without rotation and magnetic fields, the right upper panel is that with rotation ($\Omega = 0.03 \text{ km}^{-1}$), the left bottom panel is that with magnetic fields ($B_0 = 2 \times 10^{18}$ G), and the right bottom panel is that with both rotation ($\Omega = 0.03 \text{ km}^{-1}$) and magnetic fields ($B_0 = 2 \times 10^{18}$ G).

4. Summary

We calculated the mass-radius relations for magnetized and rotating neutron stars using various kinds of EoSs. There exist two EoSs (TM1-a and TM2- $\omega\rho$ -b) with rotation, one EoS (TM2- $\omega\rho$ -b) with magnetic fields, which give the masses over twice the solar mass. All five EoSs with both rotation and magnetic fields that reach beyond the masses over twice the solar mass.

Table I. Two-solar-mass compatibility.

EoS	original	rotation	magnetic fields	rotation & magnetic fields
GM1	×	×	×	○
TM1-a	×	×	×	○
TM1-b	×	○	×	○
TM2- $\omega\rho$ -a	×	×	×	○
TM2- $\omega\rho$ -b	×	○	○	○

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