Quantum field theory with boundary conditions at the horizons*

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A procedure to derive a unitary evolution law for a quantised black hole has been proposed by the author. The proposal implies several assumptions, which seem almost unavoidable to this author. We start off with the question how to describe the energy eigenstates of a black hole. The background metric required for this cannot be the Vaidya metric let alone the metric proposed by Hawking, who included the effect of the final evaporation of the black hole. This however leads to the formation if firewalls at both the future and the past event horizon, unless one anticipates the effects that the firewalls have. These effects can be handled as new boundary conditions at the horizons, describing the flow of the participating particles. It is subsequently explained how these boundary conditions must involve the antipodes of the outside world. Imposing unitarity and continuity then automatically leads to a unique, unitary evolution operator. We exhibit the resulting, quite coherent picture.

Keywords: Quantum black hole; past and future event horizon; firewall; Penrose diagram; eternal black hole; time slices; entanglement; vector representation; Shapiro shift; Cauchy surface; boundary condition; antipodal identification.

1. Introduction

There is a general agreement that a theoretical study of black holes in a regime, where quantum mechanical effects play a role, is important for a more complete understanding of general relativity and/or some modifications of this theme, in its relation to quantum mechanics.[1,2] There are several problems and disagreements, however. What is the classical limit from where one should start adding quantum corrections? The first thing many investigators assume is that there is a classical metric describing a black hole from beginning to end, and after that, the dynamical variables should be subject to some quantum mechanical replacement.[3] This should give the "quantum black hole".

However, there is a difficulty concerning the way these quantum variables would evolve. First, in accordance with the no-hair theorem, the evolution of these

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"quantum corrections" would quickly fade away, together with all other quantum field theoretical degrees of freedom. There seems to be just a single state that survives: the Hartle–Hawking state.[4] We do have evolution in this Hartle–Hawking state in a quantum mechanical manner, which is why investigators did continue along such directions, but, entanglement or not, only a single state seems to survive, and this appears to be incompatible with unitarity.

Second, if there is a quantum mechanical evolution law, it should also apply both during the initial implosion and the final puff of the evaporation process. These depend on quantum processes, and demanding their tie evolution to be fixed destroys unitarity.

Third, the quantum states will be entangled with particles entering or leaving the scene far from the black hole.

There is a way out of these difficulties, which is to consider more carefully how ingoing particles assemble near the past event horizon, and how the emitted Hawking particles are prepared near the future event horizon. They form what has become to be known as *firewalls* .[5–7] This is the subject of the present investigation. Already years ago, the author reported about what seems to be considerable progress along these lines ,[8] but there continue to be communication barriers that should be overcome.

Particle states emerging from the past event horizon will be shown here to generate a rich spectrum of different possible quantum states of out-going particles, and, considering the equally rich spectrum of in-going particles, the quantum mechanical interactions between the two horizons can be seen to restore unitarity.

There will be limitations to what we can do presently, as will be briefly explained later.

2. The Background Metric

The Penrose diagram[9,10] for the eternal black hole is depicted in Fig. 1(a). This diagram is the unique analytic extension of the Schwarzschild metric that solves Einstein's equations without matter effects added. In line of arguments just sketched above, and clarified further below, this is the only metric that we believe can be used for the quantum theory.

The metric often considered in quantum speculations for black holes, is the Vaidya metric of the kind sketched in Fig. 1(b). It was suspected to be superior since it includes the black hole formation process, so it may seem to contain more freely adjustable parameters that can be subject to quantisation.

Adding to that the gravitational effects of the Hawking evaporation, Hawking proposed to substitute this by Fig. 1(c). This diagram,[11] like the others, has the singularity at $r = 0$ absorbing in-going material, which is problematic since unitarity would demand information to be returned in the entire process. Anyway, one should

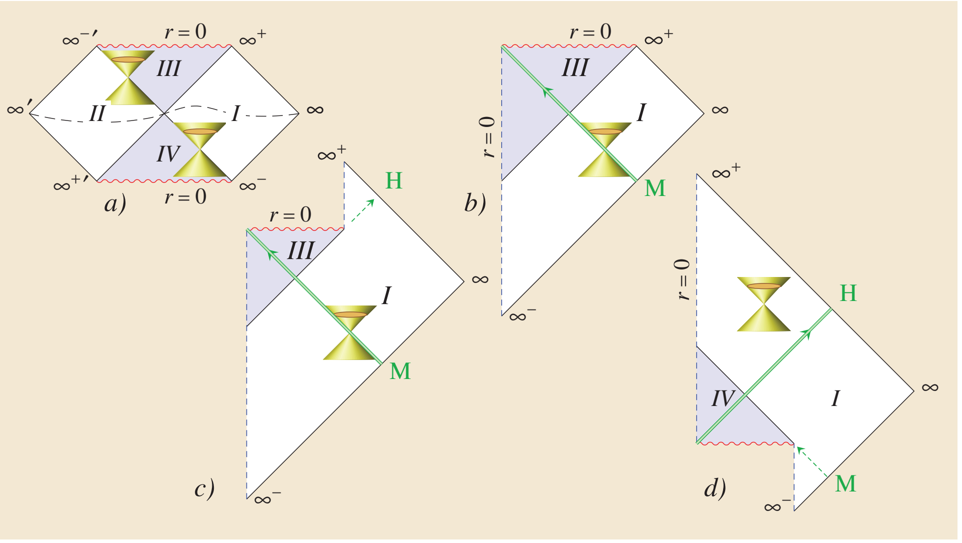

Fig. 1. Different Penrose diagrams for the Schwarzschild black hole. M stands for matter moving in, H for Hawking radiation going out. See text.

realise that the underlying equations, both for general relativity and for quantum mechanics, are invariant under time reversal, so that Fig. 1(d) should at least be considered as possibility as well.

We shall later conclude that only Fig. 1(a) can be used for the quantum process, but the arguments will be more convincing after we have treated the details of our theory. At this point, let us emphasise here that Fig. 1(a) for the eternal black hole is the most suitable one to use for the description of the stationary modes of a black hole, in particular its energy eigenstates, which make use of beams of in-going and out-going particles asymptotically far away, so as to make up for the evaporation.

To set up our program, we momentarily return to Schwarzschild coordinates, we divide the Schwarzschild time coordinate t in equal slices, each lasting for at most one period $\Delta t \approx M_{\rm BH}$ in Planck units, see Fig. 2. Within each of these time slices, the physical variables are defined to consist of all standard model particles and the slightly hidden degrees of freedom such as dark matter and gravitons. These particles are following their (quantum) trajectories, but, and this is essential for our procedure, particles that venture too close to either the future or the past horizon, will *not* be included. The idea is that such particles will become more manifestly observable in future or past slices (where they belong).

This paper is about stitching these slices back together in order to get the complete picture. Particularly at the horizons, this is a nontrivial procedure.

There may be some misunderstanding as to the quantum state of the Hawking particles. The Hawking particles are seen to be entangled, and as such it may appear

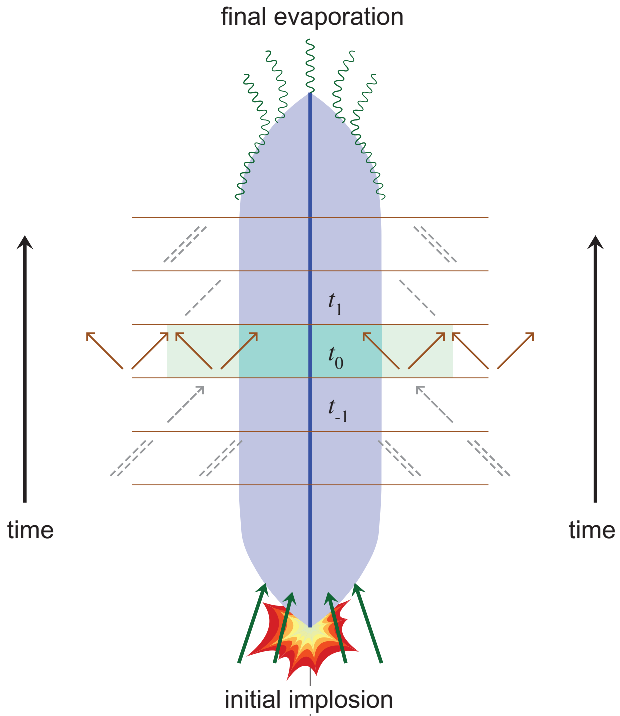

Fig. 2. Slicing the Schwarzschild coordinates in sequences of short periods. The quantum variables can be defined on each slice, lasting not longer than $\mathcal{O}(M_{\rm BH})$ in Planck units.

that they can only form a single quantum state. This happens only, however, if none of the Hawking particles are subject to observation, and if nothing is thrown into the black hole for large stretches in time. This will become more clear below.[a]

Quantum field theory (QFT), projected on this background of time-slices, will reveal how particles propagate and interact. At any safe distance (of the order of a couple of Planck lengths), the outcomes of QFT calculations will be decisive as to what happens. Up to this point, it should be clear that the metric of an eternal black hole serves just fine as a background.

However, particles inevitably will approach the horizons more closely. There are particles entering the future event horizon, and particles emerging from the past event horizon. A theory is needed that *dictates* how particles appear out of the past horizon, and how, if at all, this is determined by the in-going ones.

At this point, it seems that Hawking's original calculation[1] cannot be the entire story since it puts all out-particles into just one state, the Hartle–Hawking state.

[a]For a better understanding of these aspects of quantum mechanics, it may be helpful to view quantum mechanics itself as a *vector representation* of physically observable objects, see the author's ideas about this, in Refs. 12 and 13.

In contrast, we expect that different states of particles entering the future horizon should lead to different states of out-particles.

It suffices to produce a prescription that holds just for a few Planck lengths away from the intersection point of future and past event horizons. There, we expect "something" that causes in-particles to bounce against, or turn into, out-particles. So-far, in this treatment, we have only used the physically accessible or observable part of the Schwarzschild metric. Quantum gravity has been used only in its perturbative regime, so it only enters in the form of some stray gravitons. But now, this is going to change.

What has been found is that *interactions* between in- and out-particles can do the job. We need interactions that become sufficiently strong only at tiny distances. Which force acts more strongly at close distances than the renormalizable interactions in the Standard Model? Of course, the answer is: the gravitational force.

3. The In–Out Interaction Caused by Gravity

Indeed, the gravitational effect of an in-going particle on an out-going one is that the wave function of the out-particle is modified. This is illustrated in Fig. 3: the dashed arrowline marked "in", going through the future event horizon very early in time, carries a large momentum δp^-. Coming from region (IV) in the diagram, a Hawking particle was on its way out (wiggled line marked "out"). Due to δp^-, a shock wave[14] from the in-particle drags the out-particle along, displacing it by a distance δu^-, which is proportional to δp^- and a Green function $F(\theta)$ depending on the transverse, angular, distance θ between the particles. This function $F(\theta)$ has been calculated.[15]

This effect is closely related to the Shapiro effect,[16] the fact that a light or radio signal from an astrophysical source or a spacecraft, grazing the Sun, experiences a delay due the solar gravitational field. All Hawking particles are delayed in a different way, depending on their transverse positions, and so it happens that the entire quantum state describing the out-particles is modified.

The modification operator is easy to calculate, in principle

$$U = e^{i\delta p^- \sum_{\rm out} p^+_{\rm out} G F(\theta_{\rm out})} = e^{i \sum_{\rm in,out} p^+_{\rm out} p^-_{\rm in} G F(\theta_{\rm out}, \varphi_{\rm out}, \theta_{\rm in}, \varphi_{\rm in})}, \tag{3.1}$$

where G is Newton's constant, and furthermore, in the second part we replaced δp^- by $\sum_{\rm in} p^-_{\rm in}$ since *all* in-particles have this effect on *all* out-particles.

It must be emphasised that the dependence of U on the in-particle momenta increases exponentially with Schwarzschild time, because, the later the time t where we study the particles in the background metric, the closer the older in-particles squeeze against the past horizon, so the larger their influence will be. The very first in-particles in fact were the particles that formed the black hole by imploding. Since they came in very early, their exponentiated in-momenta p^- are gigantic, in terms

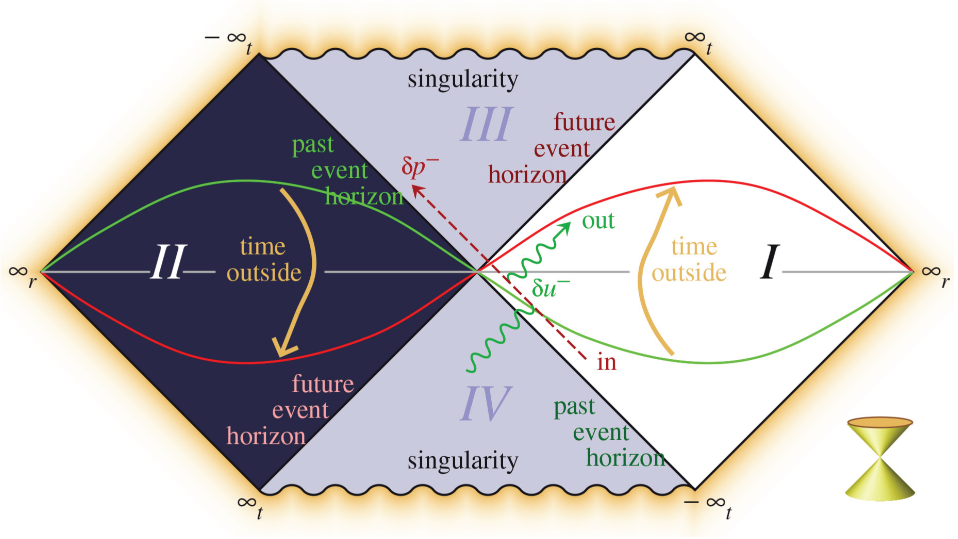

Fig. 3. The Penrose diagram for the eternal black hole, see text. Light cones everywhere in the diagram are oriented the same way as the one shown.

of the Penrose coordinates for the particles later in time, *but this is nothing to worry about*: they caused the wave functions of the early out particles to be separated by equally large separation variables $u^-_{\rm out}$.

The increase of u^- changes from exponential to linear in time as soon as the out-particles move out of the direct gravitational influence of the black hole. They are joining the other distant out-particles that no longer play any role for us. Our complete quantum states of course include the early out-going particles, and our calculations show that they are entangled with the later particles, but from this moment onwards we can just re-arrange the basis of quantum states for the out-particles, and describe their wave functions any way we please.

The reader may have felt something strange in the emerging picture: the time dependence of all particle states is such that the outgoing particles move away exponentially from the past and future event horizons, but there are *two* regions in which they can escape. The Penrose diagram used so-far is twice the size of the universe that we usually consider as the home universe of this black hole. We labelled these two regions (I) and (II) in Fig. 3. What happens to a particle that makes its way out in region (II)? If we ignore it, we make a mistake. The operator U derived in Eq. (3.1) is the operator for the Fourier transform (apart from the Green function F that will be treated later). *The Fourier transformation process is unitary*, which means that we can use it to describe a unitary quantum evolution of the black hole, *but it is only unitary if we consider in- and out-wave functions to cover the entire line* $[-\infty, +\infty]$, and not just the half-lines that form the boundaries of region (I) or region (II) of a black hole.

Indeed, the Shapiro effect can easily transport the geodesic of a particle across the other horizon. This is the real reason why we are destined to use the entire, double, Penrose diagram for our considerations. Before disclosing the *physical* role of region (II), we sketch the picture obtained so-far.

A time boost for an outside observer, $t \Rightarrow t+\Delta t$, corresponds to a *Lorentz boost*, $(u^+,\, u^-) \Rightarrow (u^+ e^{-t/4GM_{\rm BH}},\; u^- e^{t/4GM_{\rm BH}})$ for an observer close to the origin of this diagram. This implies that Cauchy surfaces for the particles in the regions (I) and (II) take the shape of the green curve in Fig. 3, that evolves into and beyond the red curve as time continues.

The Cauchy surfaces never enter into the regions (*III*) *or* (*IV*) *in the Penrose diagram.* They pivot around the origin. This is where our procedure contrasts with other work.[3]

However, we do have to pay for this — in an interesting way. Our general finding is that, when particles go into and out of the black hole, they cause shifts along the horizons. During black hole formation, these shifts are large. Effectively, we are using the eternal Penrose diagram, but the *boundary conditions*, by which the horizons connect different domains of the spacetime metric, are strongly affected by the gravitational force: the Shapiro effect.

Now, we have to address the question what region (II) stands for physically. It cannot just be ignored. It also cannot just be there to describe another black hole. That other black hole would not only be strongly entangled with the first one, as some investigators argue, but the two black holes exchange information. Entangled particles do *not* exchange information without violating locality.

Mathematically, it is easy to "cure" our theory. We have to find an isometry transformation in spacetime, whose square is the identity. We can write it as $\mathbb{Z}_2$. One such transformation is the identity itself, or, regions (I) and (II) are then assumed to represent the same spacetime. Our problem is then that this does not work since the Shapiro shift would not commute with such a mapping, or, it would generate cusp singularities near the origin. However, such cusp singularities do not arise if we accompany the $\mathbb{Z}_2$ mapping with an antipodal mapping:

$$(\theta,\, \varphi) \leftrightarrow (\pi - \theta,\; \varphi + \pi). \tag{3.2}$$

In flat Minkowski spacetime, this would also generate a cusp singularity at the origin, but for Schwarzschild (and its rotating and charged relatives), it does not. This is because in the entire spacetime that we are using, $r \geq 2\mathrm{GM}_{\rm BH}$ so that angular positions never come close to their antipodes. Only deep inside the regions (III) and (IV), $r \to 0$, and points coincide with their antipodes. We never venture more than infinitesimal distances into these unphysical regions.

Nevertheless, at the beginning of this paragraph, we put the word "cure" in quotation marks. The fact that a solution looks mathematically pleasing does not automatically mean that it embodies the physical solution.

This antipodal mapping procedure may help us to avoid the "quantum no-cloning" condition, but does it? We do see a potential danger. In region (II), compared with the time variable for a local observer, the arrow of time for the distant observer will flip over. *Time runs backwards there!* This means that, if, near the antipodes, an observer sees a particle, the same particle will appear to move backwards in time, or, have negative energy, when viewed by an observer in region (II).

Our resolution to that problem is to redefine energy by a constant. A particle in region (II) has energy $E_{\mathrm{II}} = E^{\max} - E_{\mathrm{I}}$, where $E^{\max}$ is the *energy of the "anti-vacuum" state.* The antivacuum is the state completely filled with particles, having the largest possible amount of energy.

Better understanding is obtained by adhering to expansions in partial harmonics, item 9 in the next section, but this we leave for future considerations.

4. On the Complete Theory

We hope the above can serve as a more elaborate explanation as to the various ingredients for a consistent theory of quantum black holes. It can be viewed as a "package deal": there are several postulates that by now sound natural and unavoidable by this author, but seem to meet with considerable skepticism from other researchers:

(1) Up to a few Planck lengths from the horizon, matter particles may be handled using conventional quantum field theory, only during small time steps, each lasting for the "scrambling time", $T_{\mathrm{scrambl}} = \mathcal{O}(M_{\mathrm{BH}})$. This is far too short to include collapsing matter or late evaporation. These would act as projection operators that ruin unitarity. In practice, we can only handle modest amounts of matter going in and/or out during any of these short time spans.
(2) One has to use the metric of the eternal black hole as background.
(3) The metric must be divided by $\mathbb{Z}_2$, to restore a single asymptotic region.
(4) The only way to do that is by *antipodal identification.*
(5) Time outside runs *backwards* in region (II). This implies that the local states to be compared with the black hole states must approach the *antivacuum* in region (II). Creating a particle near the black hole corresponds to creating a particle in (I) or annihilating a particle in (II).
(6) During the time stretch around time $t \approx t_0$ for the distant observer, in-particles at $t \ll t_0$ are seen to generate a Shapiro shift along the past event horizon; out-particles at $t \gg t_0$ generate Shapiro shift along future event horizon.
These form *firewalls* that can be removed by re-arranging the corrections at the horizons (cut-and-paste procedures that effectively modify the boundary conditions there).
(7) This links the *positions* of the out particles to the momenta on the in-particles, and *vice versa.* That opens the way to define a unitary evolution law.
(8) Initial imploding matter and the final evaporating matter have decisive effects at the horizons; rather than merely shifting the geodesics, these may *initiate*

the links between the antipodes there. Initial and final black holes are mere *seeds*, to be described as gravitational *instantons*. This is as yet a suspicion; it is not known how to substantiate that with calculations.

(9) To do explicit calculations, it is advisable to expand the in-going momenta and the out-going momenta or positions at the horizons in *spherical harmonics* for optimal use of linearity of the equations. The Green function $F(\theta,\, \varphi,\ \theta',\, \varphi')$ then diagonalises, much like Schrödinger's equation for the hydrogen atom.
Note that these spherical harmonics are not to be confused; in our case, we are dealing with *operators*. Since these operators involve all of the in-going and out-going particles, it is not appropriate to try to "second quantise" them. At every $(\ell,\, m)$, we have only one position operator and one momentum operator.

The mathematics works correctly only when these views are all combined. In particular, we may *only* compare states in a background metric to states as seen from the outside, if the mapping on the Cauchy surfaces is 1 to 1.

This enforced the antipodal mapping (it cannot be avoided).

Note that this mapping keeps all information visible on the Cauchy surfaces; we have no information loss.

The procedure described above can lead to an accurately defined scattering matrix linking every in-state to an out state in a unitary manner. Unitarity may follow because, at every (ℓ, m) mode, where ℓ must be odd because of the antipodal identification, the Fourier transformation is a unitary operation.

It is important to emphasise that the Shapiro effect only acts on energy and momentum, not on other quantum numbers. This means that further analysis is needed to establish how data other than positions and momenta are transported across horizons.

Thus, we are not yet ready to consider the complete Standard Model to describe the interactions in the immediate vicinity of the black hole. This is somewhat reminiscent to the situation in string theory. This theory also requires first to characterise all its excited states in terms of momentum and/or position operators, while its relation with the Standard Model can only be established by extensive studies of the relevant symmetry groups and algebras.

A new energetic generation of young students and ambitious postdocs is asked for to continue research along these lines.

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