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PHYSICAL EFFECTS OF HADRONIC BREMSSTRAHLUNG — REACTIONS AT LARGE AND SMALL MOMENTUM TRANSFERS*

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ABSTRACT

The synthesis of Regge behavior and other hadronic features with the parton model is discussed. Regge behavior of electromagnetic and weak amplitudes is shown to be a consequence of hadronic bremsstrahlung and the Regge behavior of hadronic amplitudes. A characterization of the hadronic wavefunction in terms of two components is emphasized. One component, which arises from the constituents of the hadronic bremsstrahlung, yields the wee parton spectrum and Regge behavior and falls rapidly as any one constituent is constrained to take most of the momentum of the incident hadron. The second component, which is hadron irreducible, and most probably involves the minimum number of parton constituents, must have approximately power law fall off in invariant variables. Only this second component contributes to large angle exclusive scattering, the form factors at large t, and the scaling structure function for x near 1. This component is responsible for the Drell-Yan-West relation and yields simple crossing to the annihilation channel at $x \sim 1$. Our approach and analysis clarifies the nature of the impulse approximation in hadronic scattering. For example, the dramatic fixed pole nature of the Compton amplitude at large t is discussed and its importance emphasized.

I. INTRODUCTION

In view of the success of the parton model in its description of scaling behavior of deep inelastic electron scattering and large transverse momentum processes, it is an intriguing question whether there exists a fundamental dynamical description of hadronic interactions at the microscopic level. In our previous papers,[1,2,3] we have argued that both electromagnetic and hadronic scattering processes at large transverse momentum probe the fundamental properties and interactions of hadrons at short distances. Further, we have shown that a description of large transverse momentum interactions based on a simple, basic-scattering mechanism, will, by iteration (in the t- or u-channels), join on smoothly to the usual Regge (or multiperipheral) desription of low momentum transfer events. In this way, the parton or quark field-theoretic model of composite hadrons can provide an elegant, unified description of hadron dynamics based on a very few degrees of freedom.

In this paper we will discuss the physical synthesis of Regge behavior and other essential features of hadronic interactions within a broad class of parton models.[4] Our discussion will generally be independent of the exact nature of the microscopic theory; i.e., the basic interactions of the parton constituents. We shall argue, based on the detailed work of Ref. 3, that for large angle exclusive processes ($s \to \infty$, t/s fixed) only the most elementary parton processes contribute. But then, as we have shown, the t-channel iteration of any such elementary amplitude automatically leads to Regge behavior in the low momentum transfer region. Because of this iteration of hadronic states in the t-channel, the parton model becomes consistent with the coherent nature of hadronic interactions at small momentum transfers.

Physically, the inclusion of multiple interactions in the t-channel can be thought of as allowing one of the incoming hadrons to bremsstrahlung a secondary hadron which, in turn, undergoes the basic interaction with the other incoming hadron at a lower effective energy. In a frame in which an incident hadron B has momentum P with $P \to \infty$, it is easy to show that the (virtual) bremsstrahlung spectrum of hadrons H with longitudinal momentum zP ($0 < z \leq 1$) has the behavior (for $z \to 0$)

$$G_{H/B}(z)\, dz \;\propto\; dz\, z^{-\alpha} \tag{1}$$

where α (~ 1 for Pomeron behavior) is determined by the leading energy behavior $\sigma_{HB} \sim s^{\alpha-1}$ of the total $\bar{H}B$ cross section. By momentum conservation

$$\sum_H \int_0^1 dz\, z\, G_{H/B}(z) = 1 \quad .$$

If the basic interactions between particles fall with energy (as required, for example, by the parton model without elementary vector gluon interactions between constituents of different hadrons) then, unless there are kinematic or dynamical restrictions, the basic collision tends to occur between the light and wee hadronic components of the incident particles. Because of this effect, a typical inclusive reaction actually involves only low energy collisions, with most of the beam energy expected along the beam line (in the center-of-mass) as pionization and perhaps fragmentation products. The slow, perhaps logarithmic dependence in energy of such reactions is thus not unexpected.

The key determinant of how far an inclusive process probes short distance effects thus has little to do with the incident energy; a more relevant measure is the production energy

$$s_p = 4\left(p_T^{\text{total}}\right)^2$$

as calculated from the total transverse momentum of the particle or particles produced on one side of the collision.

Similarly, in the case of exclusive processes, the high energy projectile will prefer to emit (virtual) hadrons with less longitudinal momentum — provided that the momentum transfer is sufficiently small that the required coherent reabsorption processes are not suppressed. The resulting theory of exclusive scattering has all of the complexities of normal Regge behavior in the forward and backward regions, but joins smoothly onto a simple elementary impulse approximation result at large t and u.[3]

It should be emphasized that the parton model is incomplete until the physical effects of hadronic bremsstrahlung are taken into account. Regge behavior and peripheral reactions demand the propagation of the hadronic interactions over large longitudinal distances. Since free quark-parton states have not been observed, the light-mass virtual hadronic states are evidently required to propagate the strong interactions beyond a very short range. In terms of a pure set of quark bases states, these intermediate hadronic states are simply those multi-quark systems which have hadronic quantum numbers and whose members travel close together in phase space to provide maximum binding.

The validity of the impulse approximation at large transverse momenta in exclusive and inclusive processes thus emerges naturally as a result of the control or suppression of the intermediate hadron states. In the asymptotic large angle region of exclusive scattering, the interaction time is so short that only the simplest "hadron irreducible" interactions can take place and the use of the impulse approximation is justified. In this region the coherent Regge effects are suppressed, exposing the basic mechanisms which underly the interactions between hadrons. As a byproduct of this analysis we can give the regions of validity of the impulse approximation; i.e., the kinematic domain of various processes, which despite the strong force, probe the short distance structure of elementary processes within the hadrons. The list of such processes turns out to be surprisingly broad; they include

(1) Fixed (center-of-mass) angle scattering: exclusive scattering at $s \to \infty$, all invariant ratios $p_i \cdot p_j/s$ fixed,

(2) Large transverse momentum inclusive processes,

(3) Electroproduction of exclusive channels at large q^2, as well as

(4) The Bjorken scaling region of deep inelastic scattering and annihilation.

At sufficiently high momentum transfers, the hadronic bremsstrahlung processes can be controlled and/or suppressed and these processes then involve only the minimum number of interactions along a parton propagator.

The key to unravelling the basic parton processes from the complicated hadronic effects to which they give rise is to recognize the <u>two component</u> nature of the composite hadron' wavefunction. Consider for instance electron-proton inelastic scattering in a frame in which the proton has large momentum P in the z direction. We can distinguish two different contributions to deep inelastic processes: (see Fig. 1)

(i) The parton with which the photon interacts is actually the constituent of a secondary hadron H bremsstrahlunged from the proton,

(ii) The struck parton originates directly from the proton.

Processes (i) and (ii) are associated with the <u>reducible</u> and <u>irreducible</u> components of the hadron wavefunction, respectively. We can then make the following identification: The irreducible part of the wavefunction is responsible for the

Drell-Yan relation, simple crossing relations to the annihilation channel in the threshold region ($\omega \sim 1$), and the inverse power-law behavior of form factors and large transverse momentum amplitudes. On the other hand, the reducible part of the wavefunction reflects the Regge behavior of the hadronic bremsstrahlung component, and leads to Regge or "wee parton" behavior in weak and electromagnetic processes in the appropriate kinematic domain. Further, it allows a self-consistent treatment of Reggeization in hadronic processes. The reducible terms do not contribute to deep inelastic scattering near threshold ($\omega \sim 1$), to the form factor at high t, or to exclusive processes at large momentum transfers. However, they do play an indirect role in inclusive processes at moderately high p_T away from the edge of phase space.

In Table I we summarize the properties of these two distinct wavefunction components and their consequences. In the following section we will expand upon the table entries and further clarify the two component wavefunction picture.

II. THE EFFECTS OF HADRONIC BREMSSTRAHLUNG

A. Deep Inelastic Scattering

The physical distinction between the reducible and irreducible components of the hadron wavefunction is most clearly illustrated by considering deep inelastic electron scattering in a frame in which the target proton has large momentum P in the z direction as in Fig. 2. The irreducible contribution is defined by the requirement that there is no two (or higher) hadron state in the t channel. Thus only the simplest Bethe-Salpeter wavefunction (or vertex amplitude) with the minimum number of constituent quark or parton fields contributes, and we obtain a scaling contribution to νW_2 of the form

$$2q\cdot p_H \,\mathrm{Im}\, K_{\gamma H}(\omega_H) = \pi \sum_{a=1}^{n} e_a^2 \int_0^1 dx\, f_{aH}(x)\, x\,\delta(x-1/\omega_H) \quad , \tag{2}$$

where $\omega_H = -q^2/2q\cdot p_H$ and $f_{aH}(x)$ is the normalized fractional longitudinal probability distribution obtained from the n-parton wavefunction. The amplitude $K_{\gamma H}$ corresponds to virtual Compton scattering on the t-channel hadron-irreducible component of the hadron wavefunction. Since this hadron-irreducible wavefunction obeys some type of elementary bound state equation such as the Bethe-Salpeter equation (as in the Drell-Lee model[5]) or the corresponding equation in terms of time-ordered perturbation theory as described in Ref. (1), the basic inverse power-law dependence of such wavefunctions in the off-shell variables implies that $f_{aH}(x)$ vanishes as a power at both the $x \sim 1$ and $x \sim 0$ limits. The latter behavior is equivalent to the absence of Regge behavior in this amplitude.

The reducible contribution to the deep inelastic amplitude (illustrated in Fig. 2a) is given by[6]

$$q\cdot p \; \mathrm{Im}\, T^{\mu\nu}_{\gamma p}(\omega) = \sum_H \int \frac{d^2k_\perp}{2(2\pi)^3} \int_0^1 \frac{dx}{(1-x)} \int d\sigma^2 \; q\cdot p$$

$$\cdot \; \mathrm{Im}\, T^{\mu\nu}_{\gamma H}(\omega_H, \lambda^2)\, W_{\bar{H}p}(\sigma^2, \lambda^2) \left(\lambda^2 - M_H^2\right)^{-2} \quad ,$$

with

$$\lambda^2 - M_H^2 = x\left[M_p^2 - \left(k_\perp^2 + x\sigma^2 + (1-x)\, M_H^2\right)/x(1-x)\right]$$

with $\omega_H = x\omega$ in the Bjorken limit. Here $W_{\bar{H}p}(\sigma^2, \lambda^2)$ is the imaginary part of the $\bar{H}$-p forward scattering amplitude with center-of-mass energy σ and $\bar{H}$ mass λ. Since both $\omega = -2q\cdot p/q^2$ and $\omega_H = -2q\cdot p_H/q^2$ are greater than 1, the x integral is restricted to $\omega^{-1} < x < 1$. We now note the asymptotic definitions

$$q\cdot p \; \mathrm{Im}\, T^{\mu\nu}_{\gamma p} \sim p^\mu p^\nu F_2^p(\omega)$$

$$q\cdot p \; \mathrm{Im}\, T^{\mu\nu}_{\gamma H} \sim \frac{1}{x}\, q\cdot p_H \, p_H^\mu p_H^\nu W_2^H = x p^\mu p^\nu F_2^H(\omega_H) \quad .$$

Thus

$$F_2^{p\text{-red.}}(\omega) = \sum_H \int \frac{d^2k_\perp}{2(2\pi)^3} \int_0^1 \frac{dx}{(1-x)} \int d\sigma^2 \, \frac{W_{\bar{H}p}(\sigma^2, \lambda^2)}{\left(\lambda^2 - M_H^2\right)^2}\, x F_2^{H\text{-irred.}}(x\omega)$$

$$= \sum_H \int_{1/\omega}^1 dx \; G_{\bar{H}/p}(x)\, F_2^{H\text{-irred.}}(x\omega) \qquad (3)$$

These relations provide the connection between the reducible and the irreducible structure functions. This result may intuitively be understood as describing the ability of the proton (at $P \to \infty$) to emit a hadron H (with longitudinal momentum xP) which then undergoes deep inelastic scattering at a lower energy: $\omega_H = x\omega$. The Regge behavior of the forward hadronic amplitude

$$W_{\bar{H}p}(\sigma^2, \lambda^2) = \sum_i (\sigma^2)^{\alpha_i} \beta_i(\lambda^2)$$

for $\sigma^2 \to \infty$, plus the variable scaling $x\sigma^2 = \bar{\sigma}^2$ required to keep λ^2 finite, immediately implies a Feynman scaling form

$$G_{H/p} \sim x^{-\alpha}$$

for $x \sim 0$, where α gives the leading behavior in $W_{\bar{H}p}$. Further, because of the strong off-shell damping in λ^2 of the hadron amplitude $W_{\bar{H}p}(\sigma^2, \lambda^2)$, the probability function $G_{H/p}$ vanishes rapidly at $x \sim 1$, $\lambda^2 \propto -(1-x)^{-1} \to -\infty$. Thus as $\omega \to 1$, x is also constrained to be close to 1 in Eq. (3) and the reducible contribution to $F_2^p(\omega)$ vanishes extremely rapidly due to the vanishing of both F_2^H and $G_{H/p}$. By comparison, the irreducible component vanishes only as $(\omega-1)^3$ for the proton.

This result enables us to understand the origin of the Drell-Yan relation connecting the asymptotic t-dependence of the elastic form factor and the threshold dependence of F_2^p at $\omega \sim 1$. At large momentum transfer, the reducible hadronic bremsstrahlung component to the elastic form factor (see Fig. 2b) vanishes much more rapidly than the contribution from the hadron irreducible wavefunctions due to the strong damping of the nonforward hadronic amplitude $W_{\bar{H}p}(\sigma^2, \lambda^2, t)$ for t large. Thus the threshold dependence of νW_2 and the asymptotic t-dependence of the form factors are both controlled by the off-shell power-law damping of the simple Bethe-Salpeter quark-field component wavefunction, and the Drell-Yan relation holds (modulo logarithmic and possible spin complications). Furthermore, this makes it apparent that the simple continuation between timelike and spacelike asymptotic form factors also implies a simple continuation between the annihilation process $e^+e^- \to p+X$

and deep inelastic scattering $ep \to eX$ in the threshold region. This continuation is always possible in the case of simple Bethe-Salpeter wavefunctions without anomalous thresholds in the off-shell variables.

Equation (3) for the reducible contribution to F_2^p also enables us to understand how Regge behavior results from the bremsstrahlung contribution despite the fact that the irreducible structure functions $F_2^{H\text{-irred}}(\omega_H)$ vanish as $\omega_H \to \infty$. Taking large ω in Eq. (3) we scale $x = \bar{x}/\omega$ and obtain the leading contribution for $\omega \to \infty$,

$$F_2^p \propto \omega^{\alpha-1} \quad ,$$

where α is the leading Regge contribution to the $\bar{H}$-p forward scattering amplitudes. This result can also be immediately re-expressed in terms of the quark-proton scattering amplitude, and one obtains the original Landshoff-Polkinghorne-Short formula.[7] In their language, the Regge behavior of F_2^p follows from the Regge behavior of this amplitude. Here we have shown that the Regge behavior of the quark-proton amplitude is an inevitable consequence of the Regge behavior of the (hadronic) $\bar{H}$-proton scattering amplitude. Thus the "wee component" of the proton wavefunction measured in deep inelastic scattering actually consists of constituents with a finite fraction of the longitudinal momentum of "wee" hadrons emitted by the proton in the $P \to \infty$ frame. In the target rest frame the wee contribution corresponds to constituents of hadrons in the cloud describing the periphery of the "dressed" proton.

It is interesting to re-express the above features of the reducible and irreducible contributions to deep inelastic scattering in terms of configuration space variables. Let us recall that the deep inelastic structure functions are obtained as

$$W_{\mu\nu} \sim \int d^4y \, e^{iq\cdot y} < p \left| \left[J_\mu(0), J_\nu(y) \right] \right| p > \quad .$$

The Regge behavior of $W_{\mu\nu}$ implies that the commutator has contributions for large longitudinal distances (along the light cone), $y\cdot p$, between the current interactions. In the hadronic bremsstrahlung picture, the fundamental irreducible kernel amplitude has contributions only for a finite limited range of $y\cdot p_H$, with $p_H = xp$ in the $P \to \infty$ frame. However, the Regge behavior of $W_{\bar{H}p}$ emphasizes small x, for large ω, thus giving rise to a contribution at large longitudinal distances $y\cdot p \sim \langle \frac{1}{x} \rangle \, y\cdot p_H$. Hence, although the basic Compton process on the hadron H has finite $y\cdot p_H$, the time $y_0 = y\cdot p/m_p$ measured in the proton rest frame appears dilated.

An important question is whether we are actually "double-counting" by including both contributions (a) and (b) of Fig. 1 in the proton wavefunction. It might be argued that summation over all hadron states H in Fig. 1a is complete and thus already includes the irreducible component (1b). This, however, could not be possible in a literal quark-parton model — just by quantum number considerations — since the system of propagating hadrons H can only carry the triality number zero. The state H in Fig. 1a more closely resembles a multiquark intermediate state than a single quark or parton state. Thus in the models considered here the physical particle spectra include both composite states of triality zero plus elementary single particle states with quark quantum numbers, which — at least over short distances — propagate as free particles and are the basic source of the electromagnetic and weak currents.

B. Exclusive Processes: Compton Scattering and Photoproduction

The hadronic bremsstrahlung picture greatly clarifies some heretofore confusing aspects of the parton model as applied to real photon processes.

Using this concept, we can unravel the pointlike versus vector dominance or hadronlike aspects of photon interactions. Further, we can understand the criteria for the validity of impulse approximation and fixed-pole type predictions.

Let us start with the consideration of the full Compton amplitude $\gamma+p \to \gamma+p$. Now the photons can be real or virtual depending upon the experiment. As before, we must distinguish the two contributions: t-channel hadron reducible and irreducible (depicted in Fig. 3), which exhibit quite different characteristics.

At large t, the irreducible contributions are clearly dominant due to the strong damping in momentum transfer of the hadronic bremsstrahlung amplitudes ($M_{\overline{HB}}$ in Fig. 3b). Because the irreducible amplitude does not have a Regge spectrum, the large energy of the initial photon must pass through the parton propagator. In fact, for $\nu \to \infty$, $\langle x\nu \rangle \to \infty$, and the impulse approximation must apply: the two photons are connected on to the same parton line without hadronic intercession. In fact, the leading $s \to \infty$, fixed t behavior of the nonspin flip $\gamma+q \to \gamma+q$ T_1 amplitude is a fixed pole (or more correctly a Kronecker δ_{J0}) singularity in the amplitude at J=0. (This contribution arises from an explicit seagull diagram in the case of spin 0 partons; in the spin 1/2 case it is part of the normal Feynman propagator and can be associated with a time-ordered perturbation theory Z-graph in the infinite momentum frame). Thus the Compton amplitude $\gamma+p \to \gamma+p$ always contains a J=0 fixed pole arising from the pointlike (local) 2 photon coupling of the parton which is dominant for $s \to \infty$ at fixed but large t. Thus for $|t| \gg M^2$, $s \to \infty$,

$$\frac{d\sigma}{dt} \propto \frac{\alpha^2 F^2(t)}{s^2} \tag{4}$$

with

$$F(t) = \sum_b e_b^2 F_b(t) \tag{5}$$

and $F_b(t)$ is the $C=+1$ vertex function for parton b appropriate to the seagull diagram; it has the same large t dependence as the elastic form factor. Furthermore, this contribution does not depend on the photon masses k_1^2 or k_2^2. Since this $J=0$ fixed pole behavior is predicted to be absent in vector meson, photo- and electroproduction, a dramatic breakdown of vector meson dominance in Compton amplitudes is expected. These predictions for a leading energy-independent k^2-independent real part to the Compton amplitude can be directly tested by measuring the interference of the Bethe-Heitler and Compton amplitudes in the difference of the cross sections for electron and positron wide angle bremsstrahlung $e^{\pm}p \to e^{\pm}p\,\gamma$.

In contrast, the hadron-reducible contribution to the Compton amplitude generally does not allow the application of the impulse approximation — no matter how large the energy s is. As long as t is sufficiently small, so that at least one normal Regge trajectory lies above zero, the secondary hadron H will prefer (in order to implement the leading s^{α} Regge behavior of the Compton amplitude) to carry only a small fraction $x \sim O(1/s)$ of the initial hadron's momentum. Thus the irreducible γH scattering actually occurs at a low energy of order $xs \sim m^2$. The impulse approximation is clearly not valid here: interactions along the parton line which absorbed the photon are to be expected before the photon is re-emitted. In fact, it is natural to expect that these hadronic vertex corrections such as illustrated in Fig. 4, may even bind the partons into the usual $J^P = 1^-$ hadronic spectrum and will then produce the familiar vector-dominance aspects of the forward Compton amplitude.

But again, when t becomes sufficiently large such that either the bremsstrahlung amplitude $M_{\overline{H}B}$ is vanishingly small or all of its trajectories lie below zero, the impulse approximation will set in and the Compton amplitude will again be energy-independent.

We may also make a similar analysis of meson photo- and electroproduction processes, $\gamma+N \rightarrow M+N$. As before, at large t the hadron-reducible component of the hadronic wavefunction is suppressed, exposing the more fundamental irreducible contribution, which depends on the simple properties of the parton amplitude $\gamma+q \rightarrow M+q$. We expect at large s and $|t|$, that

$$\frac{d\sigma}{dt}(\gamma+N \rightarrow M+N) \sim F_N^2(t)\,\frac{d\sigma}{dt}(\gamma+q \rightarrow M+q) \quad . \tag{6}$$

Assuming the meson form factor has a monopole behavior, the resultant asymptotic fall off is predicted to be $(s)^{-7}$ at fixed t/s, $s \rightarrow \infty$. This behavior is independent of the photon mass, q^2 and is in excellent agreement with experiment.[8] In the case of the electroproduction limit $s \rightarrow \infty$ with t/s and q^2/s fixed, the effective Compton amplitude has the asymptotic behavior $d\sigma/dt \sim s^{-7} f(t/s,\ q^2/s)$. In each case the elementary amplitude for $\gamma+q \rightarrow M+q$ is required at large t, xs, and xu, with one leg of the Bethe-Salpeter amplitude for M far off-shell. The dominant contribution is obtained by iterating the binding interaction once in the bound state equation in order to display the routing of the large momentum transfer. In a sense this corresponds to a generalized type of impulse approximation in the wavefunction.[9]

At small t, the reducible hadronic bremsstrahlung component of the nucleon wavefunction leads to the dominant, leading Regge-behaved amplitude; the underlying irreducible process occurs at low energy, and the impulse approximation is in general inapplicable. However, it is interesting to note the following: if q^2 is large, and x is small, then the parton leg is far off-shell: $(xp \pm q)^2 \sim q^2$, and again the underlying irreducible process involves an off-shell meson leg. Let us consider the limit $s \to \infty$ with $\omega = 1-s/q^2$ fixed and $t \sim 0$. After integrating over transverse loop momenta [see Eq. (6) of Ref. 1], the matrix element for forward meson electroproduction has the form

$$M(\text{"}\gamma\text{"}+N \to M+N) = \int_0^1 dx\, g(x)\, M(\text{"}\gamma\text{"}+q \to M+q) \quad , \tag{7}$$

where $M(\text{"}\gamma\text{"}+q \to M+q)$ is the on-shell parton subprocess matrix element evaluated at

$$\bar{s} = xs + (1-x)\, q^2 = q^2(1-x\omega)$$

$$\bar{u} = -q^2(x\omega)$$

and $\bar{t} = t \sim 0$. The function g(x) has damping at $x \sim 1$ and behaves as $x^{-1-\alpha(t)}$ for $x \sim 0$. This equation must thus be evaluated for $\alpha < 0$, and then continued to positive α after integration. The matrix element thus factorizes as a function of q^2 times a function of ω, and we obtain in the limit $s \to \infty$, ω fixed, $t \sim 0$,

$$\frac{d\sigma}{dt}(\text{"}\gamma\text{"}+N \to M+N) \propto \frac{1}{-q^2}\, F_M^2(q^2)\, f(\omega) \quad , \tag{8}$$

where $f(\omega) \to \omega^{2\alpha(t)-2}$ for $\omega \to \infty$. This result agrees with the q^{-6} prediction of Bjorken and Kogut[10] and Preparata.[11] The quantum numbers of the trajectory $\alpha(t)$ depend on the particular meson that is produced. Thus despite the contribution of the Regge region $x \sim \omega^{-1}$, a modified impulse approximation involving a minimum number of binding interactions holds.

C. Exclusive Processes: Hadron Scattering

It is clear from the above discussion that the simplest features of hadronic structure and scattering will be evident at large t and u where the reducible components of the hadron wavefunctions can be suppressed. That is, we expect exclusive scattering, at fixed center-of-mass angle and large s, to involve only the simplest components of the wavefunctions of the scattering hadrons and to probe the fundamental parton interactions at large energy and small distances. A number of approaches based on these physical ideas yield the result for $s \to \infty$, at fixed t/s, [1,9]

$$\frac{d\sigma}{dt} \sim s^{-n} f(t/s) \quad , \tag{9}$$

which is in striking agreement with experiment.

The possible fundamental parton interactions can be classified into two distinct types:

(1) Interactions involving the direct elementary interactions of quarks at large momentum transfer (presumably via gluon exchange) and

(2) Parton interchange or rearrangement between the interacting hadrons.

There is, however, already strong evidence that the first mechanism is not important in the hadronic processes observed so far. First, elementary vector gluon exchange implies a scale-invariant inclusive cross section for the production of large transverse momentum hadrons, contrary to recent measurements. Second, such interactions if present would lead to an effective Regge trajectory, at $s \to \infty$ for fixed but large t: $M \sim (s)^{\alpha(t)} \beta(t)$ or $(u)^{\alpha(t)} \beta(t)$ which does not fall below $\alpha \sim 1$ for vector gluon exchange (single or multiple) or $\alpha = 0$ for spin zero exchange, again contrary to experiment.

Third, particle-particle and particle-antiparticle cross sections should be equal at large t, which is also contrary to experiment. The interchange interaction, which is analogous to rearrangement collisions in the atomic and nuclear domain, is an inevitable force in all composite theories. If it dominates, then we can write the cross section for exclusive scattering at large t and u via the interchange process in a simple covariant form:

$$\frac{d\sigma}{dt}(A+B \to C+D) \sim F^2_{BD}(t)\, \frac{d\sigma}{dt}(A+q \to C+q)\Big|_{\substack{s'=\langle x\rangle s \\ t'=t}} \tag{10}$$

This formula assumes that B and D have the most convergent wavefunctions. The value $\langle x \rangle$ is the effective fractional longitudinal momentum (obtained via the mean value theorem) of the interacting quark constituent expressed in hadron B's infinite momentum frame. The function $F_{BD}(t)$ falls at large t at the same rate as the form factor of particle B or D, depending upon which one has the slowest fall off.

If we assume $F_N(t) \sim t^{-2}$ for the nucleon form factors, and $F_M(t) \sim t^{-1}$ for meson form factors, then this result predicts N=8 for meson-baryon scattering, N=7 for meson-photoproduction, and N=10 (or 12) for baryon-baryon scattering — assuming either a 3-component quark-field wavefunction[9] or a 2-component quark + core baryon state.[1] Further, as $t \to -\infty$ the effective trajectory $\alpha(t)$ approaches -1 for M+B → M+B scattering, and (-2)[9] (or -3)[1] for the leading effective B+B → B+B trajectory. These and other predictions of the interchange theory are consistent with experiment and are discussed dynamically in more detail in Ref. 3.

As the momentum transfer t (or u) becomes progressively smaller, the reducible hadron-bremsstrahlung components of the wavefunctions — which certainly contain all manner of iterations of the irreducible hadronic interchange process — will become progressively more important. The t-channel iterations thus eventually serve to completely Reggeize the basic interchange amplitude as $|t| \to 0$ as discussed in detail in Ref. 3. This then provides a specific realization of the microscopic dynamical mechanism behind the Regge behavior of hadronic amplitudes. If a trajectory moves towards unity as $t \to 0$, then s-channel iterations must of course also be included to insure that unitarity in the s-channel is respected.

D. The Interchange Mechanism and Duality Diagrams

The above description of hadronic scattering leads to an interpretation of the Harari-Rosner duality diagrams[12]: If one accepts the interchange picture of deep scattering, and allows for the possibility of a three-body baryon wavefunction, then, deep hadronic scattering is describable by interchange diagrams which have the same topologies as duality diagrams. However, unlike ordinary Harari-Rosner duality diagrams which apply only to the nondiffractive part of the amplitude, the interchange diagrams give the whole amplitude at large $|t|$, including the Pomeron. The t-channel interations of an irreducible kernel will build up moving j-plane poles at small $|t|$. Iterations of the interchange graphs may also be interpreted as duality diagrams with internal loops, a simple example of which is shown in Fig. 6 and a more complicated example in Fig. 7. Since these iterations are expected to be important at smaller momentum transfers, this procedure yields a dynamical interpretation of duality diagrams at small $|t|$: The extent to which the original topology is washed out by the insertion of bubbles (i.e., by intermediate t-channel states which are different from the external t-channel states) is a measure of the extent to which the Pomeron is more important than the lower lying ordinary Regge trajectories. (See Ref. 3 and references therein for a further discussion.)

E. Inclusive Reaction at Large Transverse Momentum

The predictions of the interchange model for large transverse momentum processes of the form $A+B \to C+X$ and the important role of hadronic bremsstrahlung have already been discussed in detail in Refs. 2 and 3. The cross section involving only the irreducible part of hadron A's wavefunction can be written as

$$\frac{d\sigma}{dt dx}(A+B \to C+X) = \frac{F_{2B}(x)}{x} \frac{d\sigma}{dt}(A+q \to C+q)\Big|_{\substack{s'=xs \\ t'=t}} \tag{11}$$

where

$$t = (p_A - p_C)^2, \qquad \mathscr{m}^2 = p_X^2 \quad ,$$

and

$$x = -t/(\mathscr{m}^2 - t) \quad .$$

The quantity x is the fractional longitudinal momentum of the quark in the infinite momentum frame of B. This formula is of course directly applicable to deep inelastic electron or Compton scattering. The basic process occurring at short distances is thus $A+q \to C+q$.

Unless we are close to the kinematic boundary $|t|, |u| \sim O(s)$, where the hadronic bremsstrahlung contribution is suppressed relative to the irreducible contribution by phase space considerations (longitudinal momentum fractions near 1), the Reggeization effects of the reducible contributions become important. In this case, one finds

$$\frac{d\sigma}{dt dx}(A+B \to C+X) = \sum_H \int_0^1 dz\, G_{H/A}(z) \frac{d\sigma}{dt dx}(H+B \to C+X)\Big|_{\substack{s'=zs \\ u'=u \\ t'=zt}} \tag{12}$$

corresponding to contributions of the form illustrated in Fig. 8. In fact for $s \to \infty$ at fixed u (or fixed $p_T^2 = tu/s$) we obtain Feynman scaling (s-independence) since $G(z) \sim z^{-1}$ at $z \sim 0$. The hadronic bremsstrahlung process allows the basic process $H_I + q \to C + q$, which is falling sharply with energy, to take place at the minimum possible energy consistent with the observed p_T. Since G is scale invariant and $d\sigma/dt\ (M+q \to M+q) \sim s^{-4}$ at fixed t/s, where M is any pseudoscalar meson, we can predict the asymptotic form[2]

$$
\begin{aligned}
E \frac{d\sigma}{d^3p} (p+p \to M+X) &\sim s^{-4}\, g(t/s,\ M^2/s) \\
&\sim p_T^{-8}\, f(2p_T/\sqrt{s},\ 2p_L/\sqrt{s}) \quad ,
\end{aligned}
\tag{13}
$$

where p_L is the longitudinal momenta. This result, as well as the specific prediction for the function $f(x_T, 0)$, are in excellent agreement with the recent ISR[12] and NAL data,[14] (see Figs. 9 and 10) if reasonable choices for $G_{\pi/p}(z)$ and $F_2^p(x)$ are made. We also note that Eq. (12) and (13) imply a smooth connection between the exclusive and inclusive cross sections in accordance with the correspondence principle or generalized Drell-Yan relation discussed by Bjorken and Kogut.[9]

The detailed picture of Reggeization provided by the hadron bremsstrahlung model allows one to contrast exclusive and inclusive amplitudes and to understand the almost kinematic origin of the slow approach to Feynman scaling at fixed p_T in the latter case.[15] The model demands that the leading trajectories in the two cases be the same. However, the secondary trajectories are very different. The effective secondary trajectories in the inclusive case enter with a negative sign and have a very large coupling relative to the leading term. The correct Regge variable is s/p_T^2 for large p_T and the approach to scaling from below is easily understood in terms of the build up of the bremsstrahlung as the energy increases at fixed p_T. This kinematic origin of effective daughter trajectories is to be compared with the dynamical origin of secondary trajectories in the exclusive case where unitarity in the t channel can place strong constraints on the residues of such terms. Unless one fully extracts these kinematic effects in the approach to scaling, simple exoticity rules based on dual models should fail.

III. CONCLUSION

If, as we have assumed, hadrons are composites of elementary fields and their wavefunctions have the inverse power-law dependence suggested by bound state equations, then large transverse momentum, electromagnetic and hadronic processes will probe the basic interactions and the simplest states of hadronic matter at short distances.

In sharp contrast to this behavior, small momentum transfer processes involve the most complex and coherent aspects of strong hadronic interactions. For these processes, the basic interaction occurs usually at the lowest possible energy just as in the multiperipheral model. Stated another way, low momentum transfer processes require the propagation of the hadronic interactions over long distances. Since the basic constituents have not been observed, this requires that hadronic matter be arranged in a highly coherent state of the lightest possible mass. Such states are in a sense very "fragile," and hence can only contribute for small momentum transfers.

In this paper we have emphasized the importance of understanding the role of intermediate virtual hadronic states in low and intermediate momentum transfer exclusive processes and in inclusive processes away from thresholds and phase space boundaries. Because of the failure to observe physical quark states far from the interior of hadrons, the hadronic bremsstrahlung components are necessary to propagate the strong interactions beyond this short range. As we have shown, this mechanism then allows for a simple interpretation of Regge behavior in deep inelastic scattering and other electromagnetic processes, the multipherial nature of inclusive reactions, and the wee spectrum of the hadronic wavefunctions.

In a previous paper,[3] detailed calculations were performed by taking into account in the irreducible kernel only the two-hadron intermediate states in the t-channel. There is no real necessity for restricting the analysis to only two particles; higher numbers of hadrons could be included in principle. However, there are two different limits in which the two hadron state should dominate. For large momentum transfers, a dimensional argument shows that the minimum number of hadron lines will dominate the irreducible kernel. At small momentum transfers on the other hand, the state with the longest range dominate and this is again the state with the fewest hadrons. Finally we note that the basic interaction always occurs at the lowest possible (virtual) energy and this suggests that the four point irreducible kernel will be more important than higher point functions corresponding to production.

At large momentum transfers or at large fractional momentum $x \sim 1$, the intermediate hadron states are not important and we expect that the simplest quark-field components of the nucleon and meson wavefunctions should dominate, i.e.,

$$|p\rangle \sim |uud\rangle , \qquad |\pi^+\rangle = |u\bar{d}\rangle \quad ,$$

modulo possible color symmetries. Therefore, any antiquark or strange quark component of the proton must arise from a meson bremsstrahlung state which can only be important at smaller x.

Perhaps one of the most exciting aspects of the hadronic bremsstrahlung picture is that it makes clear in detail how exclusive and inclusive processes probe physics at short distances in the large $|t|$, large $|u|$, or deep scattering, domain, yet convert smoothly on to normal Regge-type behavior in the forward and backward direction. The predicted breakdown of vector-dominance relations and the predicted fixed pole nature of electromagnetic processes are only one aspect of a very important and exciting new arena of physics.

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TABLE I

The Two Components of the Hadron Wavefunction

Irreducible Component	Reducible Component
Properties	
I. The parton is a direct constituent of the primary hadron A.	The parton is a constituent of a secondary hadron H emitted by the primary hadron A.
II. It contains the minimum number of partons or parton fields. The wavefunction is derived from a Bethe-Salpeter equation and has power-law fall-off in the off shell parton variables.	It has multiparticle states of high multiplicity, including a quark-antiquark component arising from leading Pomeron and Regge components of σ_{HA}. The wavefunction is a convolution of the hadronic HA amplitude and the irreducible wavefunction of H.
III. The probability of finding parton with fraction x of the longitudinal momentum (in a frame in which $p_A \to \infty$) vanishes at $x \to 0$.	The fractional longitudinal distribution function of parton momentum diverges at $x \to 0$; $f(x) \sim x^{-\alpha}$ where α is leading trajectory in σ_{HA}.
IV. The associated irreducible parton-hadron scattering amplitude falls as a power in t, the momentum transfer.	The associated reducible parton-hadron scattering amplitude has exponential fall-off in t characteristic of the H-A hadronic scattering amplitude.
Consequences	
1. Provides the dominant contribution to the deep inelastic structure functions at $\omega \to 1$, $F_2(\omega) \sim (\omega-1)^n$.	Provides the dominant contribution to deep inelastic scattering at $\omega \to 0$. $F_2(\omega) \sim \omega^{\alpha-1}$.
2. Gives the dominant contribution to the form factor and vertex functions at large t: $F(t) \sim t^{-p}$.	Gives a negligible exponentially-damped contribution to form factors at large t.
3. Provides relations of the Drell-Yan type between the form factors and structure functions at threshold: $n=2p-1$ (modulo logarithms, spin complications).	Does not contribute to the Drell-Yan relations.
4. Allows continuation of form factors at large t and structure functions at $\omega \sim 1$ from spacelike to timelike (annihilation) domain.	Negligible in regions where continuation is possible.

5. Allows application of impulse approximation, short distance concepts where dominant; e.g., dominates Compton amplitude at large t; gives simple J=0 fixed pole behavior.	Impulse approximation generally fails except for high q^2 electroproduction. Can produce normal Regge behavior (including VMD-type features) in electromagnetic and weak amplitudes.
6. Dominant contribution to large angle scattering: yields power-law $d\sigma/dt \sim s^{-N} f(t/s)$ for $s \rightarrow \infty$ at fixed t/s.	Negligible contribution to processes at large t or u.
7. Gives effective Regge trajectories which move asymptotically to negative values: $\alpha(t) \rightarrow -n$ $(n \geq 0)$ for $\|t\| \rightarrow \infty$.	Moves trajectories away from their asymptotic limit as $\|t\|$ decreases from infinity.
8. Contributions to large angle scattering associated with skeleton duality-type diagrams (i.e., the parton interchange contributions).	Contributions to exclusive scattering associated with loop corrections to the skeleton diagrams, and the Regge behavior of hadronic amplitudes.
9. Contributes only at limited longitudinal distances.	Associated with processes involving propagation over large longitudinal distances.
10. Dominates inclusive reactions at high $p_\perp$ near the phase-space boundaries; $2p_\perp/\sqrt{s} \sim O(1)$.	Results in Feynman scaling at fixed $p_\perp$, $s \rightarrow \infty$. Also dominates the transition region of inclusive reactions at $p_\perp >> m$, $p_\perp/\sqrt{s}$ away from phase-space boundaries.
11. Contributes negligibly to forward high-energy processes.	Causes high energy collision at small t to occur predominantly via the interaction of slow-moving bremsstrahlung components as in the multiperipheral model.

FIGURE CAPTIONS

1. The hadron-reducible and hadron-irreducible components of the hadron wavefunction. In the reducible component, the parton is a constituent of a secondary hadron H emitted by the proton. (See Table I.)
2. The t-channel hadron-reducible contributions to: (a) The scaling part of the virtual Compton amplitude; (b) The elastic form factor. The amplitude denoted by K is itself hadron irreducible; i.e., contains no two-hadron intermediate state in the t-channel.
3. The hadron irreducible and hadron-reducible contributions to the scaling part of the Compton amplitude.
4. Typical hadronic corrections to the Compton amplitude that contribute at low t.
5. Correspondence between parton interchange and graphs duality diagrams. The circles represent Bethe-Salpeter wavefunctions (a) s-t contribution (b) u-t contribution.
6. Loop correction to the duality diagram for πp scattering corresponding to a hadron-reducible (st) interchange graph.
7. Sample duality diagrams for πp scattering generated by iteration of the interchange graphs.
8. The large transverse momentum reaction $A+B \rightarrow C+X$ in the parton interchange model. The basic large transverse momentum process is $q+H \rightarrow C+q'$, where the quark has a longitudinal momentum distribution given by $F_{2B}(x)/x$ and the virtual hadron H has a longitudinal momentum distribution $G_{H/B}(z)$. (See Eq. (12).)
9. Comparison of the parton interchange model (solid and dashed curves) with ISR data for $pp \rightarrow \pi^o X$ at 90^o (Ref. (13)). The quark distribution function

is taken as $f(x) \sim (1-x)^3/x$, and the momentum distribution of mesons in the proton is taken as $(1-z)^5/z$. These threshold behaviors are predicted by the theory. The meson form factor is assumed to be of the form $(1-t/m^2)^{-1}$ with $m^2 = .71\ \mathrm{GeV}^{-2}$.

10. Comparison of the parton interchange model predictions (dashed line) with the π^- production data points of Ref. (14). A factor of two difference in renormalization from the previous graph is assumed here. The lower energy points are not at $x_F = 0$ and equal amounts of (st) and (ut) contributions in the basic interchange process has been assumed. The solid line is the experimental fit of Ref. 14.

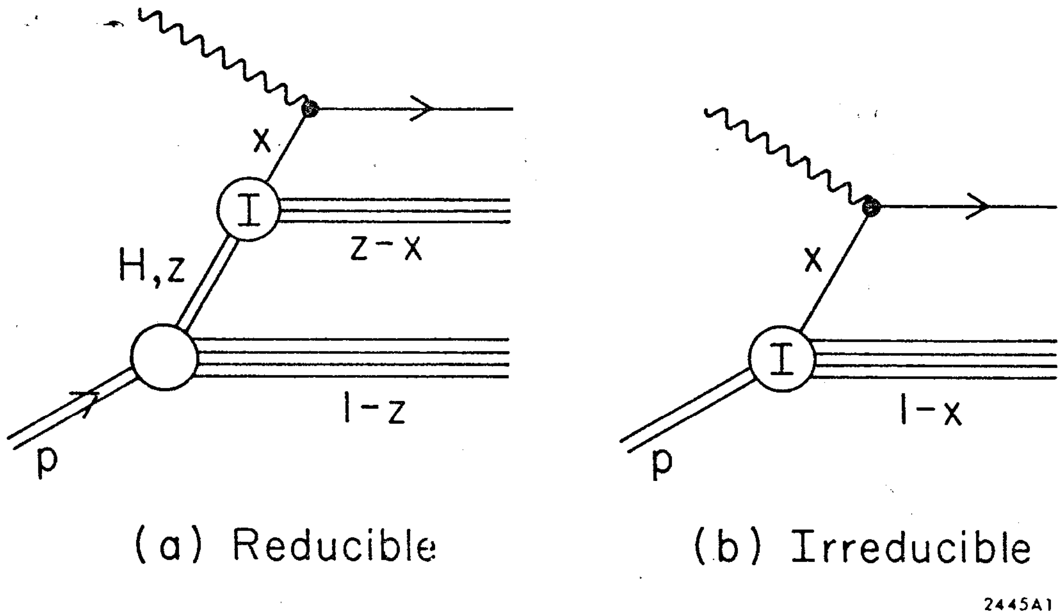

x
H,z
z−x
1−z
p
(a) Reducible
x
1−x
p
(b) Irreducible
2445A1

Fig. 1

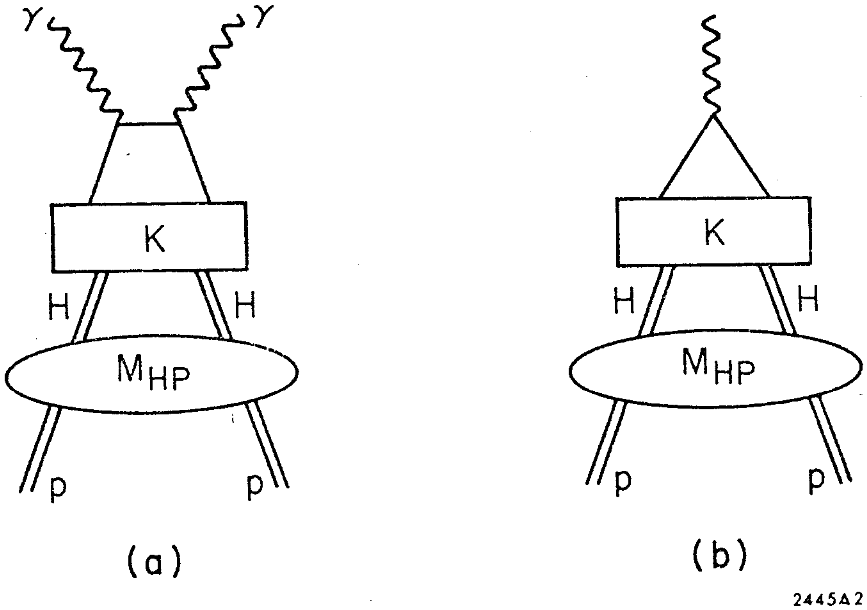
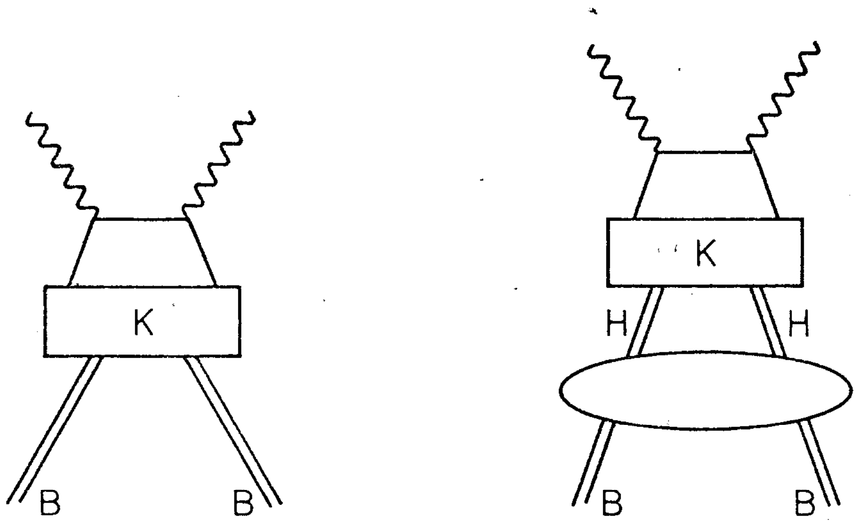

γ
γ
K
H
H
M_{HP}
p
p
(a)
K
H
H
M_{HP}
p
p
(b)
2445A2

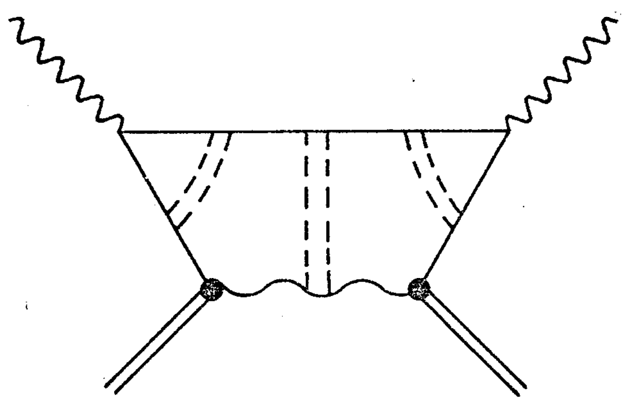
Fig. 2



(a) Irreducible (b) Reducible

2445A3

Fig. 3



2445A4

Fig. 4

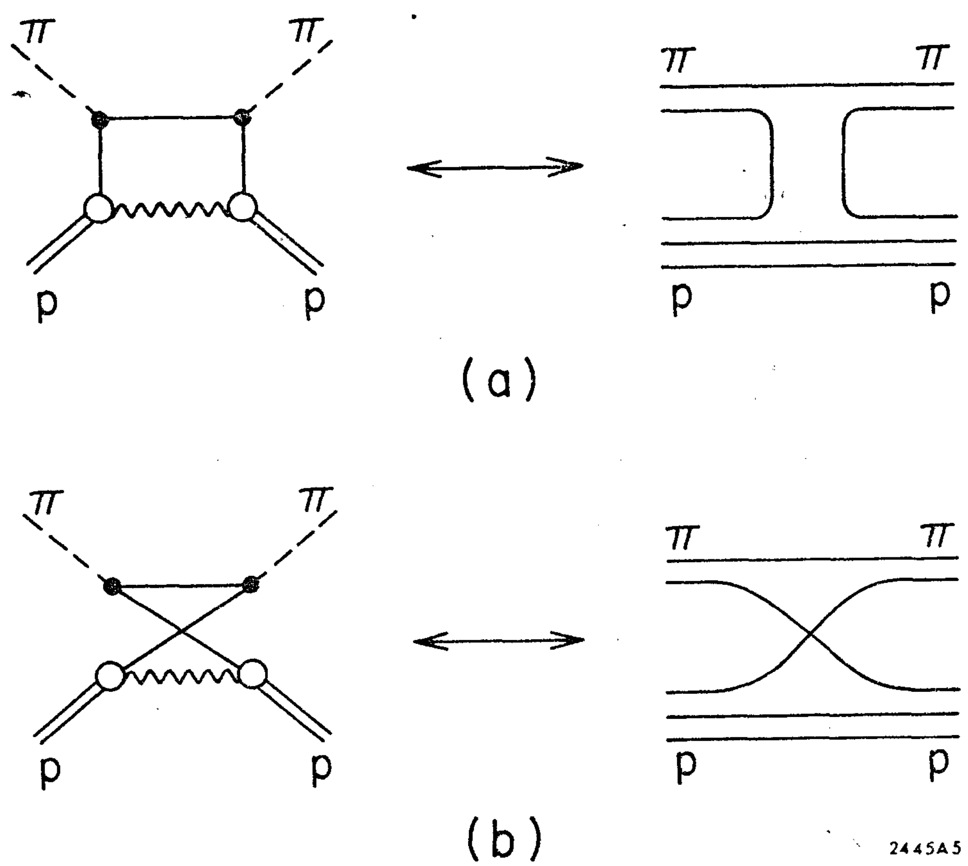


Fig. 5

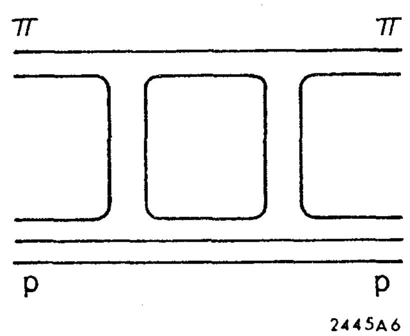


Fig. 6

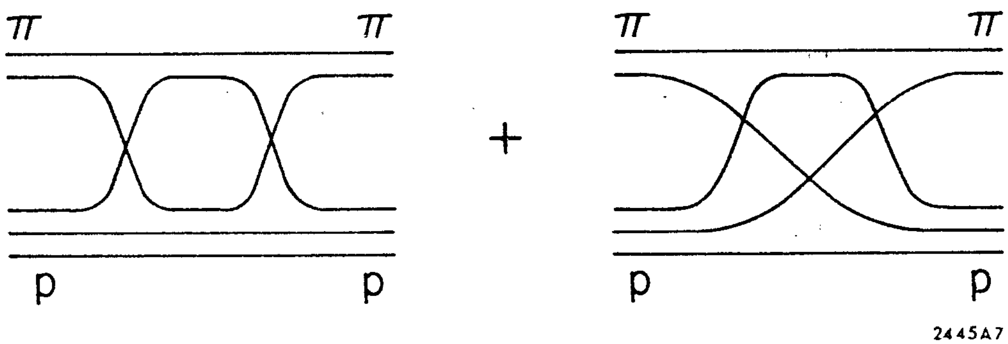


Fig. 7

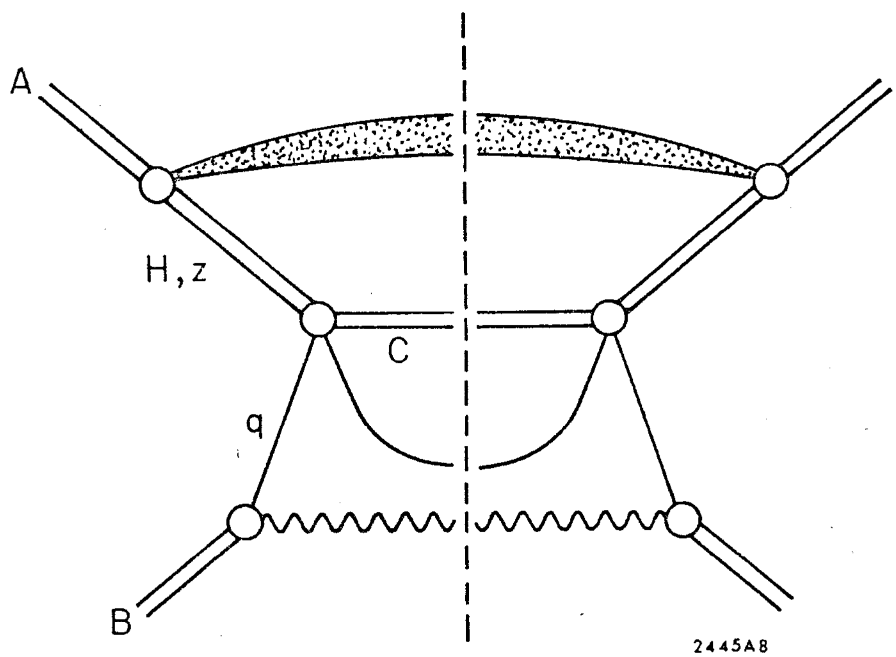
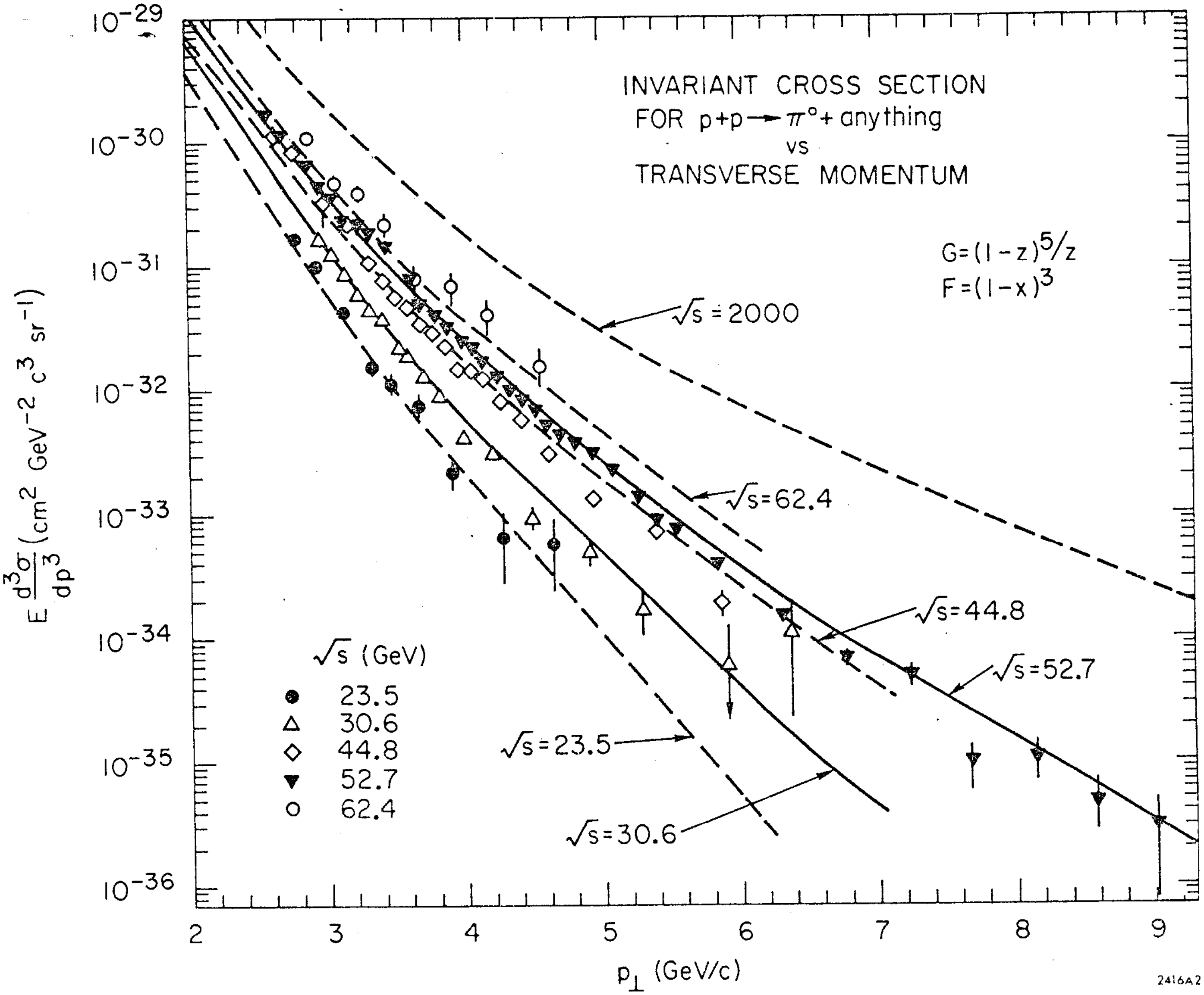
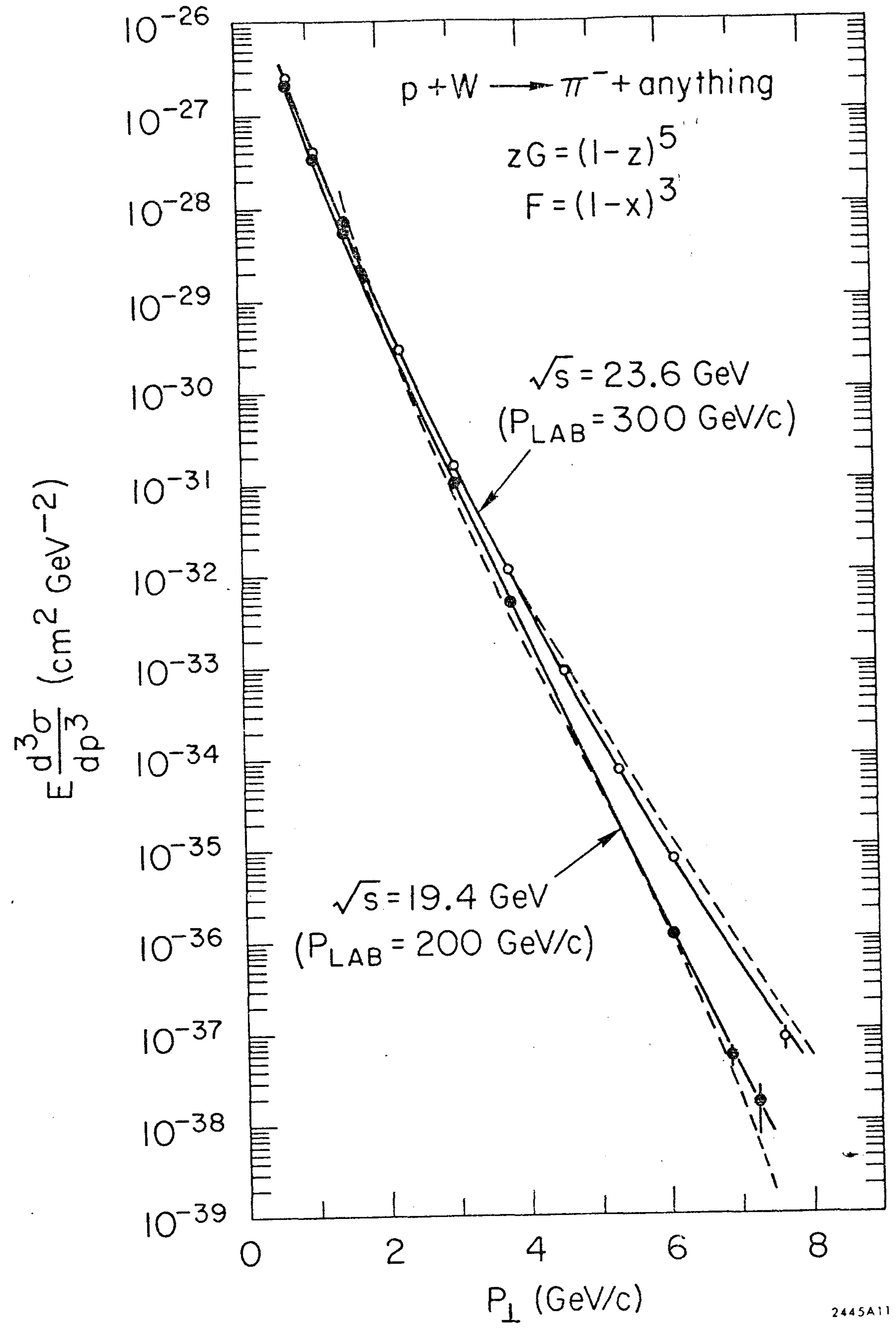


Fig. 8



INVARIANT CROSS SECTION
FOR p+p → π°+ anything
vs
TRANSVERSE MOMENTUM
G=(1-z)^5/z
F=(1-x)^3
√s = 2000
√s=62.4
√s=44.8
√s=52.7
√s=23.5
√s=30.6
√s (GeV)
23.5
30.6
44.8
52.7
62.4
E d^3σ/dp^3 (cm^2 GeV^-2 c^3 sr^-1)
10^-29
10^-30
10^-31
10^-32
10^-33
10^-34
10^-35
10^-36
2
3
4
5
6
7
8
9
p⊥ (GeV/c)
2416A2

Fig. 9



$p + W \longrightarrow \pi^- +$ anything
$zG = (1-z)^5$
$F = (1-x)^3$
$\sqrt{s} = 23.6$ GeV
($P_{LAB} = 300$ GeV/c)
$\sqrt{s} = 19.4$ GeV
($P_{LAB} = 200$ GeV/c)
$E\frac{d^3\sigma}{dp^3}$ ($cm^2\ GeV^{-2}$)
10^{-26}
10^{-27}
10^{-28}
10^{-29}
10^{-30}
10^{-31}
10^{-32}
10^{-33}
10^{-34}
10^{-35}
10^{-36}
10^{-37}
10^{-38}
10^{-39}
0
2
4
6
8
$P_\perp$ (GeV/c)
2445A11

Fig. 10