

Emergency seismic sensing method of asteroid structure

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Abstract. The problem of the Earth protection against large celestial bodies, which have great kinetic energy and can pose a fatal threat to Earth civilization, is discussed. To select the method of threat elimination the information on the internal structure of a dangerous space object is necessary. The time deficiency complicates the problem. So, the method of emergency seismic sensing of the structure of a space body by a high-speed drummer is proposed.

1. Introduction

The problem of emergency choice of measures in order to protect the Earth from a large celestial body with high kinetic energy is under consideration for last time [1,2]. The asteroids of the Solar System are moving at a speeds of 10-72 km/s relative to the Earth [3]. The speed of asteroids and comets that enter the Solar System from other star systems and neighboring galaxies is determined by their origin and reaches 300 km/s. A direct hit of a body with size from kilometer across is fatal threat to life on Earth. Larger space objects can provoke disasters at a distance [3-6].

Information on the trajectory [7,8] and internal structure of a dangerous space object is necessary for choosing the means of preventing a catastrophe. It can be an ice or rock block, a composite of stones and ice, a metal asteroid, etc [5,6,9]. The problem is enhanced by the identification difficulty of such objects; especially when they approach the Earth from the side of the Sun. As a result, there is a severe time limit for studying the structure of the body and determining the associated danger, choosing the method for eliminating the threat and its implementation. Therefore, an urgent sounding of a celestial body is necessary.

The paper proposes a method of emergency seismic sensing of the structure of a space body by a high-speed drummer. The shell includes a drummer, a set of sensors and a seismic trace analyzer. A high-speed drummer excites seismic waves in a space body, recorded through a sensor system by a puller - an analyzer that processes information on a span. The structure of the investigated body is restored on the basis of the information obtained.

2. The method of emergency seismic sensing of high-speed space bodies

It is shown in [10] that the material particle should be considered as an element that accepts a wave packet of perturbations that changes the stress-strain state of a particle in dynamic non-stationary nonlinear problems for a stress-strain state. It is convenient to reduce the general system of equations to a system of wave equations, each of which describes a specific physical



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quantity, and together they form a description of the wave packet for a deformed particle. This wave packet determines the increment of parameters for a given particle (element), and the history of the passage of such packets completely determines the state of the physical continuum. It should be noted that the external impact on the physical body, as a rule, causes the passage of both elastic and plastic waves. Therefore, in order to set the problem in a wave form, it is necessary to have generalized equations of state so that they uniformly describe both elastic and plastic waves of a deformed physical continuum, and its boundaries, for this it is necessary to use the theory of distributions [11].

The wave equation for generalized mass velocity vector may be written in the following invariant form [10,12]:

$$\rho \frac{d^2 \vec{v}^*}{dt^2} = \left(k + \frac{1}{3} G \right) \nabla (\nabla \vec{v}^*) + G \Delta \vec{v}^* - 2G\Phi(\mathbf{A} \nabla) (\nabla \mathbf{A} \vec{v}) \quad (1)$$

where ρ is a density of medium, Φ is scalar function that takes into account the material state of medium deformed, $A_{ij} = s_{ij}^* - \rho_{ij}^*$, s_{ij}^* is generalized stress tensor, ρ_{ij}^* is generalized microstress tensor, G is metric tensor.

The resulting wave equation is not the only wave equation describing the dynamic behavior of the elements of engineering-physical systems and the physical continuum. To study the entire process, it is often convenient to have an equation for each of the parameters studied, so it is advisable to write wave equations in which the strain and force parameters are the main variables describing the process of deforming each individual element of the system. Equations for the stress and strain tensors may be written as [5,7]:

$$\frac{d^2 \sum^*}{dt^2} = \frac{G}{\rho} \left\{ \nabla (\nabla \sum^*) + (\nabla \sum^*) \nabla + \frac{2v}{1-2v} [\nabla (\nabla \sum^*)] \mathbf{G} - 2\Phi \mathbf{A} [(\mathbf{A} \nabla) \cdot (\nabla \sum^*)] \right\} \quad (2)$$

and

$$\frac{d^2 \mathbf{E}}{dt^2} = \frac{G}{\rho} \left\{ \nabla (\nabla \mathbf{E}) + (\nabla \mathbf{E}) \nabla + \frac{2\nu}{1-2\nu} \nabla [\nabla \mathbf{E} \cdot \mathbf{G}] - \Phi [(\mathbf{A} \nabla) \nabla + \nabla (\mathbf{A} \nabla)] (\mathbf{A} \cdot \mathbf{E}) \right\} \quad (3)$$

where $\delta(\vec{\xi}, t)$ is Dirac multidimensional function.

Imagine a multidimensional nonlinear nonstationary problem of bodies interaction in the form of one-dimensional wave problems set.

Vector equation (1) is most simply solved for the mass velocity vector, the fundamental solution of which is the matrix Φ defined by the equation:

$$\frac{d^2 \Phi}{dt^2} - \frac{1}{\rho} \left(k + \frac{1}{3} G \right) \nabla [\nabla \Phi] - \frac{G}{\rho} \Delta \Phi + \frac{2G\Phi}{\rho} (\mathbf{A} \nabla) [(\mathbf{A} \nabla) \Phi] = \mathbf{I} \delta(\vec{\xi}, t). \quad (4)$$

Substitution of the Fourier transform of the matrix Φ into equation (4) gives

$$\frac{d^2 \Phi}{dt^2} - \frac{1}{\rho} \left(k + \frac{1}{3} G \right) \vec{\eta} [\vec{\eta} \cdot \Phi] - \frac{G}{\rho} (\vec{\eta} \cdot \vec{\eta}) \Phi + \frac{2G\Phi}{\rho} (A \vec{\eta}) [(A \vec{\eta}) \Phi] = \mathbf{I} \delta(t). \quad (5)$$

The solution of the last equation can be represented as $\Phi(\vec{\eta}, t) = \Theta(t) \mathbf{P}(\vec{\eta}, t)$, where $\Theta(t)$ is the Heaviside function, and the matrix $\mathbf{P}(\vec{\eta}, t)$ is defined by the equation

$$\frac{d^2 \mathbf{P}}{dt^2} - \frac{1}{\rho} \left(k + \frac{1}{3} G \right) \vec{\eta} [\vec{\eta} \cdot \mathbf{P}] - \frac{G}{\rho} (\vec{\eta} \cdot \vec{\eta}) \mathbf{P} + \frac{2G\Phi}{\rho} (A \vec{\eta}) [(A \vec{\eta}) \mathbf{P}] = 0, \quad (6)$$

whose solution can be written as

$$\mathbf{P}(\vec{\eta}, t) = \mathbf{C}_1 \exp(i\sqrt{\mathbf{B}}t) + \mathbf{C}_2 \exp(-i\sqrt{\mathbf{B}}t), \quad (7)$$

Where \mathbf{B} is the symmetric coefficients' matrix, and $\mathbf{C}_1, \mathbf{C}_2$ are arbitrary matrix functions determined from the initial conditions. By setting the initial conditions in the form:

$$\mathbf{P}(\vec{\eta}, 0) = 0, \quad \left. \frac{d\mathbf{P}(\vec{\eta}, t)}{dt} \right|_{t=0} = \mathbf{I}.$$

We get the solution for the matrix \mathbf{P} :

$$\mathbf{P}(\vec{\eta}, t) = \frac{\mathbf{B}(\vec{\eta})^{-\frac{1}{2}}}{2} \left(\exp(i\sqrt{\mathbf{B}}t) - \exp(-i\sqrt{\mathbf{B}}t) \right), \quad (8)$$

and the fundamental matrix of solutions of the wave equation $\Phi(\vec{\xi}, t)$ will look like the inverse Fourier transform:

$$\Phi(\vec{\xi}, t) = \frac{\Theta(t)}{(2\pi)^3} \int_{V_\eta} \frac{\sin \sqrt{\mathbf{B}(\vec{\eta})}t}{\sqrt{\mathbf{B}(\vec{\eta})}} \exp(i\vec{\xi}\vec{\eta}) dV_\eta, \quad (9)$$

or

$$\Phi(\vec{\xi}, t) = \frac{\Theta(t)}{(2\pi)^3} \int_{V_\eta} \mathbf{T}(\vec{\eta}) \begin{bmatrix} \Lambda(\lambda_1) & 0 & 0 \\ 0 & \Lambda(\lambda_2) & 0 \\ 0 & 0 & \Lambda(\lambda_3) \end{bmatrix} \mathbf{T}^T(\vec{\eta}) dV_\eta, \quad (10)$$

$$\Lambda(\lambda_i) = \frac{\exp(i\vec{\xi}\vec{\eta} + i\sqrt{\lambda_i}t) - \exp(i\vec{\xi}\vec{\eta} - i\sqrt{\lambda_i}t)}{2i\sqrt{\lambda_i(\vec{\eta})}}$$

where $\mathbf{T}(\vec{\eta})$ is orthogonal matrix whose columns are unit eigenvectors of the matrix \mathbf{B} , and λ_i are the corresponding eigenvalues.

Thus, the sought fundamental solution of the wave equation for mass velocity vector is represented as a superposition of six different solutions describing six types of waves and called modes, each mode has its own dispersion relation that determines the frequency of the i -th mode as a function of the wave vector $\vec{\eta}$:

$$\omega_{1,2} = \pm \sqrt{\lambda_1(\vec{\eta})}, \quad \omega_{3,4} = \pm \sqrt{\lambda_2(\vec{\eta})}, \quad \omega_{5,6} = \pm \sqrt{\lambda_3(\vec{\eta})}.$$

Representing the found fundamental solution of the equation for the mass velocity vector in the form of expansion in plane waves according to Radon, we obtain the solution of the Cauchy boundary value problem with the initial conditions:

$$\begin{aligned} \frac{d^2 \vec{v}}{dt^2} - \mathbf{B}(\nabla) \vec{v} &= f(\vec{\xi}, t), \\ \left. \frac{\partial \vec{v}(\vec{\xi}, t)}{\partial t} \right|_{t=0} &= \vec{v}_I(\vec{\xi}), \\ \left. \vec{v}(\vec{\xi}, t) \right|_{t=0} &= \vec{v}_{II}(\vec{\xi}) \end{aligned}$$

In the form

$$\vec{v}(\vec{\xi}, t) = \sum_{k=1}^3 \int_{\Omega} d\Omega \int_{-\infty}^{+\infty} d\tau \int_{V_p} \frac{\Theta(\tau)}{(2\pi)^3} \vartheta_k(\vec{p}, \vec{\alpha}, \tau) \times [\vec{R}_k(\vec{\alpha}) \vec{R}_k(\vec{\alpha})] \vec{f}(\vec{\xi} - \vec{p}, t - \tau) dV_p \quad (11)$$

where $\vec{R}_k(\vec{\alpha})$ are the eigenvectors of the matrix \mathbf{B} , $\vartheta_k(\vec{p}, \vec{\alpha}, \tau)$ are some functions.

Thus, the problem of the material element motion under the action of dynamic loads applied to each deformed particle is reduced to the problem of plane elastic-plastic waves propagation along a preloaded material of the deformed particle along a given direction of the wave vector, which is equivalent to the problem of the plane waves propagation over half space.

The inverse problem is solved by optimizing the system parameters.

3. Conclusions

It is proposed a method of emergency seismic sensing of dangerous space objects such as asteroids to protect the Earth. A sequence of protective measures has been developed. The problem of determining the structure and composition of asteroid materials has been posed and solved with aid of seismic express-analysis by the processing information satellite on a span.

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