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Astrophysical Tests of General Relativity

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Abstract—At the initial stage of its development, general relativity (GR) was verified and confirmed in a weak gravitational field limit. However, with the development of astronomical observation technologies, GR predictions in a strong gravitational field began to be discussed and confirmed, such as the profile of the X-ray iron $K\alpha$ line (in the case if the emission region is very close to the event horizon), the trajectories of stars near black holes and the shapes and sizes of shadows of supermassive black holes in M87* and Sgr A*. In 2005 it was predicted that a shadow formed near a supermassive black hole at the Galactic Center could be reconstructed from observations of ground based global VLBI system or ground—space interferometer acting in mm or sub-mm bands. In 2022 this prediction was confirmed since the Event Horizon Telescope (EHT) collaboration reported about a shadow reconstructions for Sgr A*. In 2019 the EHT collaboration presented the first image reconstruction around the shadow for the supermassive black hole in M87. In 2021 the EHT collaboration constrained parameters ("charges") of spherical symmetrical metrics of black holes from an allowed interval for shadow radius. In 2022 the EHT collaboration constrained charges of metrics for the supermassive black hole at the Galactic Center. Earlier, we obtained analytical expressions for the shadow radius as a function of charge (including a tidal one) in the case of Reissner–Nordström metric. Based on results of the shadow size evaluation for M87* done by the EHT collaboration we constrained a tidal charge. We discussed opportunities to use shadows to test alternative theories of gravity and alternative models for galactic centers.

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1. INTRODUCTION

It is well-known that general relativity (GR) was created in November 1915 as a result of joint discussions and correspondence between A. Einstein and D. Hilbert [1, 2]. Einstein used an heuristic approach while Hilbert derived corresponding equations using Lagrangian with natural properties (now this approach is widely used in theoretical physics to obtain Euler–Lagrange equations from given Lagrangians). Also in November 1915 Einstein calculated a perihelion shift for the Mercury orbit, that is, Einstein gave a solution to a problem that had not been solved since the middle of the XIX century, and in this work he also showed that the deflection of light in the vicinity of a gravitational body is twice the value of the angle of deflection of light calculated within the framework of the Newtonian theory of gravity. In 1919 Einstein's predictions on deflection of light were confirmed in observations of star positions near the solar disk during solar eclipse. After a meeting of Royal Society on November 6, 1919 where Sir Frank Dyson reported results of their observations, *The Times of London* for November 7, 1919 had a headline "Revolution in Science. Newtonian Ideas Overthrown" [3] (this Chandrasekhar's article contained also an interesting essay discussing a remarkable discovery confirming the GR predictions). Theory of gravitational lensing and its numerous applications are based on the fact that the light source deviates from a straight line in a gravitational field [4, 5] (in particular, microlensing made it possible to detect light exoplanets even in another galaxy, as shown in [6, 7]). Third GR test consisted in measurements of gravitational redshifts of spectral lines in gravitational field. In 1942 in his book on general relativity P. Bergmann noted that there are only three effects where it is necessary to take into account Einstein's theory of gravity. Due to the development of radar technology and related receiving devices, it has become possible to propose additional GR tests. In the 1960s Irwin Shapiro proposed the forth GR test, namely he suggested to consider a radar location of inner planets (Venus and Mercury) when Sun is located between Earth and one of these planets. Since a propagation of light in a gravitational field is different than a propagation in vacuum there are time delays which are approximately 10^{-4} s. These detected time delays were in a correspondence with GR estimates (now this gravitational delay is called the Shapiro time delay).

2. RELATIVISTIC COSMOLOGY

In 1917 Einstein showed that GR could be applied to consider mathematical cosmological models and he introduced a static model. In 1930 A. Eddington showed that the Einstein's spherical world is unstable. In 1917 W. de Sitter proposed his cosmological model where there is no matter (and its density) but there is a positive Λ-term (this stage of the Universe evolution could be realized at the inflationary scenario). Realistic cosmological models were considered by Soviet mathematician A.A. Friedmann [8, 9] and these solutions are still the most important in modern physical cosmology. However, since Friedmann's models had the beginning and age of the Universe, Soviet philosophers treated these results as a purely mathematical approach which has no application for cosmology (in other words, for physical reality). In these circumstances G. Lemaitre's works started to play a dominant role in world cosmological studies. In particular G. Lemaitre derived a dependence of redshift on distance in [11][1] (now it is called the Hubble law which was obtained from observational data [12]). The English translation of the mentioned Lemaitre's paper published originally in French had no derivation of the Hubble law [13]. As it was found recently Lemaitre decided to omit this derivation of the Hubble, law in the English translation [14]. As a recognition of the Lemaitre contribution in the discovery of the Hubble law at the Thirtieth General Assembly of the International Astronomical Union in 2018 astronomers proposed to rename the Hubble law as the Hubble–Lemaitre law and this proposal was accepted that "from now on the expansion of the universe be referred to as the Hubble–Lemaitre law." In 1931 Lemaitre started to developed a hot Universe model which was named "Primeval Atom." In 1930s he published a book on the subject (the English edition was published in 1950 [15]) and several papers, in particular, in the framework of his approach he predicted an existence of background radiation with a temperature around a few kelvins [16]. Due to this reason many authors named G. Lemaitre as the Big Bang father. Lemaitre promoted his cosmological approach elsewhere, in particular, in 1933 he delivered his cosmological lecture at the Mount Wilson Observatory in a presence of A. Einstein. When journalists asked Einstein about his impression, Einstein replied "This is the most beautiful and satisfactory explanation of creation to which I have ever listened!"[2]. Journalist Duncan Aitkan published article "Lemaitre Follows Two Paths to Truth: The Famous Scientist, Who Is Also a Priest Tells Why He Finds No Conflict Between Science and Religion" in *New York Times* on February 19, 1933. This article was widely known in the world (including USSR). Since the Soviet Union had an official atheistic ideology, Lemaitre and Einstein's point of view that science and religion could develop without conflicting was unacceptable in the 1930s. Unlike Western cosmology, in which the universe originated and began to expand, Soviet cosmology claimed that both space and time are infinite. In this case, the Soviet Encyclopedia (Second Edition) in the article on Cosmology stated that Friedmann's solutions are too simplified models and can only be used for Metagalaxy, while the behavior of the Universe as a whole may be radically different from the predictions of Friedmann's models of the Universe. Practically, in Soviet Union it was a ban on dymanical cosmological models (including Friedmann's ones) in 1930–1960. In 1940 G. Gamow proposed his version of the hot Universe model where he estimated a helium nucleosynthesis in primordial Universe [17] and soon after that his students calculated Cosmic Microwave Background (CMB) radiation which should have now temperature around 5 K. In consequent calculations CMB temperature was slightly different but always it was a several kelvins. A presence of CMB temperature was considered as an important feature of the hot Universe model proposed by Gamow. For instance, as it is known that the CMB radiation (which is one of the signatures of hot Universe model developed by G. Gamow in 1940 and 1950) was discovered by T. Shmaonov at the Pulkovo Observatory several years before A. Penzias and R. Wilson (who were awarded the Nobel prize in 1978) [18]. For instance, there is a description of this Shmaonov's discovery in [19] and in many other books and articles. However, Shmaonov's achievements were not known for many years, since they did not have an appropriate cosmological interpretation. There is a popular opinion that no one in the Soviet Union knew about the Gamow model of the hot Universe and the predictions of this model. However, this interpretation is at least incomplete. As it was mentioned earlier, in the Soviet Union dynamic models of the Universe (including Gamow's models) were considered as inadequate descriptions of the Universe and their consideration was not welcomed by official ideology and philosophy. In addition, despite the fact that Gamow was one of the most famous Soviet theoretical physicists, he did not return from a business trip abroad without the permission of the authorities. Thus, the mention of Gamow's works could be interpreted as a support for his disloyal attitude towards the Soviet government. Therefore, even if some experts understood that Shmaonov's achievements had a cosmological interpretation, they preferred not to demonstrate their understanding in order to avoid the danger of being condemned for supporting the provisions of physical cosmology, which were criticized by Soviet philosophers. In 1963, apparently, Soviet authorities decided to reconsider the assessment of Friedmann's cosmological works and the

[1] On July 17, 2024 a scientific community will celebrate 130 years since Lemaitre's birthday and the Vatican Observatory announced the Lemaitre Conference—2024 on Black Holes, Gravitational Waves and Space-Time Singularities.

[2] https://inters.org/einstein-lemaitre.

Soviet Academy of Sciences held a session of the Department of Physical and Mathematical Sciences dedicated to the 75th anniversary of Friedmann's birth. In particular, P.L. Kapitsa said in his speech that "Friedmann's name has so far been undeserved oblivion. This is unfair and it needs to be fixed. We must perpetuate this name. After all, Friedmann is one of the pioneers of Soviet physics, a scientist who made a great contribution to domestic and world science." In July 1963, the journal "Soviet Physics—Uspekhi" published a special issue of the journal dedicated to the 75th anniversary of A.A. Friedmann, which contains articles by famous scientists such as P.Ya. Polubarinova-Kochina, V.A. Fock, Ya.B. Zeldovich, E.M. Lifshitz, and I.M. Khalatnikov, as well as Russian translations of Friedmann's articles on cosmology published in German in 1922 and 1924. Moreover, in his review Zeldovich started to quote Gamow's papers (earlier Soviet physicists did not mention Gamow's papers). Initially, Zeldovich criticized Gamow's papers on a hot Universe model while other Soviet researchers did not mention Gamow and his papers after 1932 when Gamow did not return to the Soviet Union from a foreign trip, but Zeldovich immediately recognized a hot Universe model as a correct cosmological approach after the CMB discovery by A. Penzias[3] and R. Wilson. Thus, it can be said that in 1963, the Soviet Union lifted the ban on discussing realistic cosmological models where it was considered the origin and evolution of the Universe. To mark the centenary of Friedmann's birth in 1988, the Soviet Academy of Sciences held a representative conference on gravity in Leningrad, which was attended by leading world experts in gravity and cosmology. In addition in 1988 a remarkable book describing a scientific Friedmann's biography was published [20], where, in particular, the authors wrote "He discards a centuries-old tradition that is notoriously, until all experience, it was considered the Universe eternal and forever motionless. He makes a real scientific revolution. How Copernicus forced Earth revolves around the Sun, so Friedmann forced the Universe to expand." At this time the Soviet Union stopped to be an atheistic state (in the USSR the millennium of the baptism of Russia was celebrated as a state holiday in 1988) and Soviet cosmology completely started to be a part of international science.

3. SHADOWS AS GR TEST

As it was noted in [21] opportunities to reconstruct shadows around supermassive black holes M87* and Sgr A* are based on three components, synchrotron emission which is a main source of electromagnetic radiation in many astronomical objects, VLBI technique which currently used not only in radio (but also in mm-band), GR effects considered in a strong gravitational field near black holes. Initially, the concept was introduced as a result of thought experiment by J.M. Bardeen [22]. He considered observational consequences for a distant equatorial observer in the case if there is a bright screen behind of a Kerr black hole. He assumed that light (photons) propagate freely along isotropic geodesics without an interaction with a matter around the black hole. Later, the theoretical concept has been transformed in GR test [23] due a great progress of observational facilities. Really, in [24] it was predicted that it would be possible to reconstruct a shadow around the black hole at the Galactic Center analyzing bright structures observed in mm and/or sub-mm bands with ground (or ground—space) VLBI interferometers. The spectral band was selected since in simulations done in [25] it was shown that a shadow could be reconstructed for mm wave band while in cm band could not be reconstructed due a Compton scatter of photons on electrons.

A synchrotron radiation (electromagnetic radiation of electrons moving in magnetic fields) was discovered by British theorist G.A. Schott more than 100 years ago. Later, the synchrotron radiation was re-discovered by I.Ya. Pomeranchuk and his co-authors in the 1940s [26–28]. The first detection of X-ray radiation from accelerated electrons in the General Electric 70 MeV synchrotron was reported in 1947 (so, it was a natural reason to call this kind of electromagnetic radiation as synchrotron one). In 1933 K. Jansky detected the first radio source (our Galactic Center). In 1940 astronomers detected radio emission from Sun and in 1946 I.S. Shklovsky and V.L. Ginzburg separately wrote papers where they claimed that this emission could be generated by electrons moving in solar magnetic fields or in other words, it is synchrotron radiation which was formed in solar corona. To check these conclusions and to observe displacements of background stars near Sun during the solar eclipse N.D. Papalexi proposed to detect solar radio emission during solar eclipse and he applied for permission from Soviet authorities to organize expedition in Brazil in May 1947 and Soviet Academy of Sciences supported this idea. However, at the stage of expedition preparation N.D. Papalexi died in February 1947 and S.E. Khaikin was chosen as a leader of this mission [29]. Famous theorists I.S. Shklovsky and V.L. Ginzburg participated in Soviet expedition to observe solar eclipse in Brazil where Khaikin and Chikhachev really confirmed that radio emission was generated in a solar corona (unfortunately optical observations were failed due to bad weather conditions [29]). In 1953 I.S. Shklovsky proposed a synchrotron radiation for electromagnetic emission of the Crab Nebula in a wide wave band. Later he reminded that the idea about a synchrotron radiation as a source of emission for many astronomical objects was the brightest insight in his entire scientific activity, however, V.L. Ginzburg had another opinion concerning Shklovsky's contribution in the development of a synchrotron radiation approach for astronomical applications.

[3] Arno Penzias passed away on January 22, 2024.

An early history of the VLBI (Very Long Based Interferometry) development was described in [30]. Really, ideas of VLBI observations were proposed by Soviet radio astronomer L.I. Matveenko in 1960. In fall 1962 L.I. Matveenko presented these ideas at Radioastronomical Observatory Seminar in Pushchino but astronomers met Matveenko's proposal without any enthusiasm and some of them said that this idea can not be realized. As a result the director of the Pushchino observatory V.V. Vitkevich did not recommend the corresponding paper for publication as it was needed at this time to submit a paper in journal. In summer 1963 Sir Bernard Lovell (Director of Jodrell Bank Radio Observatory) was an invited guest of Soviet Academy of Sciences and he visited several Soviet Institutes including Deep Space Network near Evpatoria. There I.S. Shklovsky invited Matveenko to present VLBI ideas for an audience where Lovell followed the Matveenko's talk. Lovell agreed the ideas but he noted that did not see astronomical problems where such a high resolution would be necessary [30]. After several years the corresponding paper was published [30] (as Matveenko reminded the idea about observations with a ground–space interferometer was proposed in the submitted version of the paper but this proposal was removed from the final version of the text under request of the editorial board of the journal). The results of the first joint Soviet–American VLBI observations were discussed in the popular Soviet journal "Science and Life" where Matveenko described nicely this VLBI as "a telescope with the Earth size" in 1973 (later many other authors used these words to characterize their global interferometric systems). As already noted, Matveenko (and his coauthors) also proposed to conduct ground–space based interferometric observations of astronomical objects already in their first paper [30] (but later this part of the text was removed from the article). However, later a project of ground–space interferometer has been proposed in the Soviet Union [30][4]. It was supposed that the interferometer will operate at $\lambda = 1.35$ cm, the apogee was planned to be 80000 km, while the perigee was 20000 km. The antenna diameter was planned to be 3.1 m (this size was constrained by launcher parameters). However, this project was practically stopped in "Perestroika" times and in 1990s [30]. In the beginning of 2000s the project was slowly re-started under a leadership of N.S. Kardashev as the Radioastron project (a budget of Russian science was very small at this period). An angular resolution of the ground–space interferometer at the shortest wave length (1.35 cm) was around 7 μas with Radioastron ground space interferometer (this small angle is comparable with an angular size of the Schwarzschild diameter for the black hole at the Galactic Center). Currently, the Astro Space Center of Lebedev Physics Institute develops the Millimetron (Spectr-M) project where it is planned to launch a cryogenic 10 meter telescope at Lagrange point L_2 which will be used as a component of ground–space interferometer at 0.5–10 mm wave band[5].

As it was noted earlier an angular resolution of space-ground interferometer with the space Radioastron radio telescope was comparable with the event horizon diameter for the case of the supermassive black hole at the Galactic Center. It these circumstances it would be reasonable to think about opportunities to find GR signatures from observations of Sgr A*. Really, as it was noted in [24] a shadow at the Galactic Center could reconstructed from VLBI observations in mm or sub-mm bands (the shortest wave length of Radioastron was 1.3 cm and it was not suitable for shadow reconstruction since the Compton scatter could spoil a shadow image in cm band [25]). At the first glance, 50 years ago the Bardeen's idea looked rather artificial and a shadow can not be observed in the case of astrophysical black holes. First, actually there no a bright screen in realistic astronomical conditions but as it was discussed in [24] secondary images are located near shadow and analyzing bright structures around the shadow boundary it would be possible to reconstruct a shadow. Second, shadow sizes are too small to be detected since for black holes with stellar masses are around 10^{-10} *as* (a majority of black holes with such masses was found in binary star systems where black holes are companions of ordinary stars), while various estimates of the black hole mass in our Galactic Center were in very wide range, namely, in 1970s Soviet theorist L.M. Ozernoi and his group claimed that black hole at Sgr A* must be not very high $M_{\rm BH} < 3\times10^4 M_\odot$[6] (Ozernoi in particular, claimed that "Nevertheless, the Galactic Center appears to be the place where the existence of a massive black hole seems to be inconsistent with observational data coupled with theory, which together impose rather severe constraints on its mass") while British astronomers declared that the black hole mass should be around $M_{\rm BH} \approx 5\times10^6 M_\odot$ [33]. Consequent observations and their analysis showed that $M_{\rm BH} \approx 4\times10^6 M_\odot$ at Sgr A*. Later (in the 2000s)

[4] American astronomers also developed their own space orbiting VLBI (OVLBI) project in 1981 (initial ideas to launch a single satellite in a highly elliptical (300 km vs. 30000 km) orbit, carrying a 10-m dish, and operating at a wavelength near 1.3 cm were discussed in [31]. A test of ground–space interferometric observations was successfully done in 1986. Practically it was an experiment to probe operational facilities. The Tracking and Data Relay Satellite System (TDRSS) was used as the orbiting observatory. Everything was performed smoothly and this experiment showed the feasibility and potential of using a dedicated observatory in space (as it was planned in Japanese VSOP—HALCA and Soviet Radioastron projects in the mid of 1980s). The VSOP—HALCA was operated in 1997–2005, the Radioastron was in action in 2011–2019.

[5] https://millimetron.ru/en/.

[6] As it was claimed in [32] an existence of a black hole with a mass in the range 10^2–$10^3 M_\odot$ is the most probable and suitable approach for the Galactic Center.

astronomers started to discuss VLBI systems with angular resolution in several dozens of μas in mm band and in a few μas for Radioastron at the shortest wave length (1.35 cm), therefore, structures with these sizes may be observed with these facilities. Third, in observations it is very hard to distinguish darkness and faint objects but as since there is a gravitational focusing near shadows region near a shadow (but outside it) must be much brighter than background, therefore, there is an opportunity to reconstruct shadow from observations [24] (as we noted in [24] the predicted shadow size was $50\,\mu as$ it corresponded to its value obtained from observations of the Event Horizon Telescope Collaboration [34]). It means the prediction about an opportunity to reconstruct a shadow in Sgr A* was realized [34].

For astrophysical black holes electric charges should be very small in black hole mass units, however, shadows for Reissner–Nordström black holes were considered and it was found that there is an analytical expression for a shadow radius as a function of charge [35] (we used an analysis of cross-sections for neutral slow moving particles and photons in Reissner–Nordström done in [36]). L. Randall and R. Sundrum consider a theory with additional non-compact dimension, soon after that Dadhich et al. found a solution which looks like a Reissner–Nordström metric where parameter q may be negative. The authors proposed to call this solution a Reissner–Nordström metric with a tidal charge. Later, people started to use these solutions for astronomical objects, in particular, it was proposed to apply this solution for the black hole at the Galactic Center to test observational signatures but it was noted that in the case of a significant negative tidal charge q the shadow size is so large that it is not consistent with existed observational constraints on the shadow size at Sgr A* [37]. A derivation of a shadow size as a function of a tidal charge was given in [38].

4. CONCLUSIONS

The Event Horizon Telescope Collaboration observed M87* and Sgr A* in April 2017. Results of shadow reconstruction for M87* were presented in [39] after three years of data analysis and later observational constraints on "charges" from the shadow size evaluations in M87* were obtained. Based on these estimates of shadow size for M87* it was constrained a tidal charge, namely $q \in [-1.22, 0.814]$ at 68% C.L. in [40]. The Event Horizon Telescope Collaboration reconstructed a shadow at the Sgr A* (after 5 yr of analysis of observational data) and evaluated a shadow size in [34] and if similarly to this paper we adopted a shadow diameter $\theta_{\mathrm{sh\ SgrA^*}} \approx (51.8 \pm 2.3)\mu as$ at 68% confidence level at the Galactic Center then as it was done in [21] constraints on a tidal charge were obtained $q \in [-0.27, 0.25]$. Therefore, for M87* and Sgr A* constraints on a tidal charge (and/or a parameter of scalar-tensor gravity of Horndeski type theory) were found. So, bounds for the Randall–Sundrum theory with extra dimension for M87* and Sgr A* were obtained. Thus, as it was predicted in 2005 the shadow reconstructions around supermassive black holes provide a new test of GR and some alternative theories of gravity (see also recent related papers on the subject [23, 41–43]).

Since the 1980 and the 1990s massive gravity theories were actively developed by A.A. Logunov and his co-authors [44, 45]. At this period studies of these alternative theories of gravity were not active, however, in the last years this class of gravity theories started to be very popular basically due to researches by C. de Rham and her co-authors [46]. For instance, in the first LIGO publication [47] where the authors reported on the discoveries of gravitational waves and binary black holes, they also constrained a graviton mass (it means that these authors considered massive gravity theories as reasonable alternatives for GR). Analyzing the S2 star trajectory near the Galactic Center in [48] it was constrained a graviton mass at a level which is comparable with the LIGO graviton mass bound. Recently based on new observational data for the S2 star orbit constraints of Yukawa gravity parameters [49] and on graviton mass [50] were improved. However, we have to note that the graviton mass constraint found using relativistic theory of gravity in [45] is still the best among other upper graviton mass estimates done in PDG.

In a recent paper [51] the authors constrained a graviton mass based on C. Will approximation of gravitational potential while in [52,53] new tests of GR were considered. A remarkable review on astrophysical black holes was published recently in [54].

In conclusion of the article, I would like to honor the bright memory of two great scientists, Abram Isaakovich Alikhanov (the founder of the Institute of Theoretical and Experimental Physics, Moscow) and George Gamow (the founder of physical cosmology), who were born on the same day (March 4, 1904, or now it is 120 years since their birth) and they were both outstanding representatives of the Leningrad school of physics.

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CONFLICT OF INTEREST

The author of this work declares that he has no conflicts of interest.

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