



Implications for vanishing complexity in dynamical spherically symmetric dissipative self-gravitating fluids

Robert S. Bogadi^{1,a}, Megandhren Govender^{1,b}, Sibusiso Moyo^{2,c}

¹ Department of Mathematics, Faculty of Applied Sciences, Durban University of Technology, Durban 4000, South Africa

² Institute of Systems Science, Durban University of Technology, Durban 4000, South Africa

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Abstract The complexity factor, originally based on a probabilistic description of a physical system, was re-defined by Herrera et al. for relativistic systems. This involves an assessment of the energy density inhomogeneity, anisotropic and shear stresses, and in the case of radiating collapse, the effects of heat flux. Already well integrated into the modelling of static configurations, the complexity factor is now being studied with respect to dynamical, self-gravitating systems. For static systems, the constraint of vanishing complexity is typically used however for the non-static case, the physical viability of the vanishing condition is less clear. To this end, we consider the ideal case of vanishing complexity in order to solve for the time-dependent gravitational potentials and generate models. We find that vanishing complexity constrains the metric to be of a form similar to that of Maiti's conformally flat metric.

1 Introduction

The process of gravitational collapse of massive objects is a challenging topic in relativistic astrophysics, having been pioneered by Oppenheimer and Snyder [1]. As the end state of the collapse process is approached, it is likely that the physical quantities calculated via modelling processes become inaccurate and less representative of the actual physics at work. Certainly, in the case where the remnant formed is a black hole, an accurate description of the final stage of collapse might be elusive. This is not due to shortcomings in the theory of general relativity, but rather a result of the modelling process which often involves setting up initial static configurations [2, 3] with associated boundary conditions and equations of state [4]. Initial conditions can also include an

inhomogeneous energy density [5] which is relevant to our study involving complexity. Constraints based on a quasi-static system in the distant past might lose relevance as the collapse process proceeds. The progression of the collapse is also assumed to proceed in a well-behaved manner, mainly governed by the heat flux boundary condition of Santos [6] in the case of non-adiabatic collapse. So far, this has been successful in generating models and may well be accurate for most of the collapse process. However, near the time of remnant formation where energy densities and pressures escalate to extreme measures, additional non-linear effects might arise, rendering the initial conditions and assumptions made in setting up the model of little relevance.

In an effort to generate models, other physical constructs such as gravitational decoupling [7] and complexity factor analysis are now being considered. A definition of complexity from the statistical point of view was developed by López-Ruiz et al. [8] and involves the interplay between entropy and information content. In the case of relativistic objects, it is more suitable to seek a measure of complexity in terms of the field equations of relativity. This was done by Herrera [9] for static systems and also for the dynamical case [10] which can be applied to the problem of gravitational collapse. With respect to relativistic objects, complexity was considered to assist in discerning equations of state of static compact objects [11]. The complexity factor focuses on the degree of energy inhomogeneity and can be coupled with pressure anisotropy, shear stress and in the case of radiating collapse, the heat flux. Much work has already been done in the case of static systems in which the constraint of vanishing complexity has been implemented. In the case of dynamical systems the situation is less clear and the complexity factor might likely have a specific, non-vanishing behaviour at certain stages during collapse. Initial investigations have already been conducted in this respect [12]. In this work, we assume the contrary that the dynamical complexity factor vanishes, in order to establish the restriction on the models obtainable

^a e-mail: bogadi.robert@gmail.com (corresponding author)

^b e-mail: megandhren@dur.ac.za

^c e-mail: moyos@dur.ac.za

under this regime. The framework has then been set up for future work whereby the complexity factor could be linked to a physically desirable non-vanishing function of spacetime. This could then lead to more suitable models in which improved stability and more realistic physical features are obtainable.

2 Dissipative collapse

For the modelling of a radiating star undergoing shear-free gravitational collapse, the interior spacetime is described by the spherically symmetric line element

$$ds^2 = -A^2(r, t)dt^2 + B^2(r, t) \times [dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)], \quad (1)$$

in which the metric functions A and B , describing the gravitational potentials, are yet to be determined. This shear-free metric was studied in detail by Bonnor et al. [13]. The energy momentum tensor for a fluid which dissipates heat to the exterior is given by

$$T_{ab} = (\rho + p)u_a u_b + p g_{ab} + q_a u_b + q_b u_a, \quad (2)$$

where ρ and p are the energy density and isotropic pressure of the fluid. The heat flow vector q^a is orthogonal to the velocity vector so that $q^a u_a = 0$. The Einstein field equations describing the interior of the stellar fluid are

$$8\pi\rho = \frac{3}{A^2} \frac{\dot{B}^2}{B^2} - \frac{1}{B^2} \left(\frac{2B''}{B} - \frac{B'^2}{B^2} + \frac{4}{r} \frac{B'}{B} \right), \quad (3)$$

$$8\pi p = \frac{1}{A^2} \left(-\frac{2\ddot{B}}{B} - \frac{\dot{B}^2}{B^2} + 2 \frac{\dot{A}}{A} \frac{\dot{B}}{B} \right) + \frac{1}{B^2} \left(\frac{B'^2}{B^2} + 2 \frac{A'}{A} \frac{B'}{B} + \frac{2}{r} \frac{A'}{A} + \frac{2}{r} \frac{B'}{B} \right), \quad (4)$$

$$8\pi p = \frac{1}{A^2} \left(-\frac{2\ddot{B}}{B} - \frac{\dot{B}^2}{B^2} + 2 \frac{\dot{A}}{A} \frac{\dot{B}}{B} \right) + \frac{1}{B^2} \left(-\frac{B'^2}{B^2} + \frac{1}{r} \frac{A'}{A} + \frac{1}{r} \frac{B'}{B} + \frac{A''}{A} + \frac{B''}{B} \right), \quad (5)$$

$$8\pi q = \frac{2}{B^2} \left(\frac{\dot{B}}{AB} \right)' \quad (6)$$

where the dot and prime are derivatives with respect to time and radial distance respectively. We obtain the condition of pressure isotropy by equating (4) and (5)

$$\left[\frac{A''}{A} + \frac{B''}{B} - \left(2 \frac{B'}{B} + \frac{1}{r} \right) \left(\frac{A'}{A} + \frac{B'}{B} \right) \right] = 0 \quad (7)$$

and we note that this is time-independent. Time dependence can then be introduced by promoting the constants of integration to functions of time. This was recognised by Ivanov [14].

Since the star is radiating energy, the exterior spacetime is described by the Vaidya metric [15]

$$ds^2 = - \left(1 - \frac{2m(v)}{r} \right) dv^2 - 2dvdr + r^2 [d\theta^2 + r^2 \sin^2\theta d\phi^2] \quad (8)$$

where v is the retarded time and m is the total mass inside the comoving surface Σ forming the boundary of the star. Matching of the interior line element (1) to the exterior spacetime (8) is achieved via the junction conditions, originally proposed by Santos [6]. These are

$$(rB)_\Sigma = r_\Sigma, \quad (9)$$

$$p_\Sigma = (qB)_\Sigma, \quad (10)$$

$$m_\Sigma = \left[\frac{r^3 B \dot{B}^2}{2A^2} - r^2 B' - \frac{r^3 B'^2}{2B} \right]_\Sigma, \quad (11)$$

where m_Σ is the total mass within a sphere of radius r_Σ and (10) represents the conservation of the momentum flux across the boundary Σ .

Lastly, we note that the shear-free approximation is prone to instability and in some respects, pressure anisotropy assists in overcoming this problem [16]. In our application to modelling, we evaluate the adiabatic index in order to highlight this aspect.

3 Complexity

The complexity factor for radiating, self-gravitating systems, as defined by Herrera [10] is given by

$$Y_{TF} = 8\pi\delta + \frac{4\pi}{(rB)^3} \int_0^r (rB)^3 \left(\rho' - 3qB \frac{\dot{B}}{A} \right) dr \quad (12)$$

which in terms of our shear-free metric and associated field equations (3)–(6) gives

$$Y_{TF} = 8\pi\delta - \frac{1}{(rB)^3} \int_0^r 2r \left[r \left(B'' + \frac{r}{2} B''' \right) - \frac{rB'}{B} (3B' + 2rB'') - B' + \frac{r^2(B')^3}{B^2} \right] dr. \quad (13)$$

The integration in (13) can be completed to yield

$$Y_{TF} = 8\pi\delta - \frac{1}{B^2} \left[\frac{B''}{B} - 2 \left(\frac{B'}{B} \right)^2 - \frac{1}{r} \frac{B'}{B} \right]. \quad (14)$$

3.1 Isotropic systems

In the case of isotropic systems ($\delta = 0$), we find that vanishing complexity is ensured if

$$B' = C(t)B^2r \quad (15)$$

where $C(t)$ is a promoted constant of integration. A general solution for B is then given by

$$B(r, t) = \frac{R(t)}{1 + k(t)r^2} \quad (16)$$

where $R(t)$ and $k(t)$ arise from integration and transformation.

The pressure isotropy condition, obtained via the Einstein field equations, gives

$$A'' - A' \left(2 \frac{B'}{B} + \frac{1}{r} \right) + A \left(\frac{B''}{B} - 2 \left(\frac{B'}{B} \right)^2 - \frac{B'}{rB} \right) = 0 \quad (17)$$

which may be integrated after substituting the form for $B(r, t)$ in (16). A solution is then given by

$$A(r, t) = \xi(t) + \frac{\zeta(t)}{1 + k(t)r^2} \quad (18)$$

where $\xi(t)$ and $\zeta(t)$ are further constants of integration which have been promoted to functions of time. These are to be set according to boundary conditions, equations of state, or additional physical constraints.

3.2 Anisotropic systems

For a more general, anisotropic system ($\delta \neq 0$), the complexity factor (14) is given by

$$Y_{TF} = \frac{1}{B^2} \left[\frac{1}{A} \left(A'' - \frac{A'}{r} \right) - 2 \frac{A'}{A} \frac{B'}{B} \right] \quad (19)$$

and for vanishing complexity, a first integration yields

$$A' = C(t)B^2r \quad (20)$$

which reduces the problem of finding solutions to a single-generating function.

4 Application to modelling

The application of the vanishing complexity condition has become an integral part of modelling relativistic objects within the static regime. For dynamical systems, the complexity factor as given according to the above definition is less likely to vanish especially in the case of gravitational collapse [12]. This is expected due to the causal nature of dynamical processes whereby the establishment of an equilibrium with respect to pressure anisotropy, energy density inhomogeneity and heat flux cannot occur instantaneously. Nevertheless, the vanishing complexity condition might still be valid for quasi-static systems or in our case, at early stages in the collapse process.

We see that for the isotropic case, vanishing complexity results in metrics which are similar to those of Maiti

and Bergmann [17–19]. On application to compact, radiating fluid spheres, the field equations give homogeneous energy density profiles as is the case with the simple Schwarzschild interior solution [13, 20].

Maiti's solution can represent a fluid with heat flux in a conformally flat spacetime and the general form is given by

$$A = 1 + \frac{M(t)}{1 + kr^2/4}, \quad B = \frac{R(t)}{1 + kr^2/4}.$$

Banerjee et al. [21] also considered this metric and explored its generality in terms of conformal flatness.

We make use of the results obtained from vanishing complexity for the isotropic case and set the gravitational potentials as follows,

$$A(r, t) = A_0(r)f(t) \quad (21)$$

$$B(r, t) = B_0(r)f(t) \quad (22)$$

where $f(t)$ is some function of time, and the static part is given by

$$A_0(r) = 1 + \frac{\zeta}{1 + r^2} \quad (23)$$

$$B_0(r) = \frac{2R}{1 + r^2}. \quad (24)$$

We thus chose ($k(t) \rightarrow 1$ and $R(t) \rightarrow 2Rf(t)$) in (16) and ($\xi(t) \rightarrow f(t)$ and $\zeta(t) \rightarrow \zeta f(t)$) in (18). This is almost that of the Maiti form, apart from the modified time-dependence on metric function A . Metric functions with the above separable form in radial distance and time have been utilised by Tewari [22] and is a more general form of that used by Bonnor et al. [13].

The associated field equations for the above system (21)–(22) are

$$\rho = \frac{\rho_s}{f^2} + \frac{3\dot{f}^2}{8\pi A_0^2 f^4} \quad (25)$$

$$p = \frac{p_s}{f^2} - \frac{1}{8\pi A_0^2} \left[2\frac{\ddot{f}}{f} - \frac{\dot{f}^2}{f^2} \right] \quad (26)$$

$$q = -\frac{A'_0 \dot{f}}{8\pi A_0^2 B_0^2 f^4} \quad (27)$$

where ρ_s and p_s denote the energy density and pressure respectively of the initial static configuration. These are given by

$$8\pi\rho_s = \frac{-1}{B_0^2} \left[\frac{2B''_0}{B_0} - \left(\frac{B'_0}{B_0} \right)^2 + \frac{4}{r} \frac{B'_0}{B_0} \right] \quad (28)$$

$$8\pi p_s = \frac{1}{B_0^2} \left[\left(\frac{B'_0}{B_0} \right)^2 + \frac{2}{r} \frac{B'_0}{B_0} + \frac{2A'_0 B'_0}{A_0 B_0} + \frac{2}{r} \frac{A'_0}{A_0} \right]. \quad (29)$$

Table 1 Characteristics of shear-free, radiating fluid collapse following Maiti-type metric. ($f_{Lmax} = 0.0004113$; $f_{BH} = 0.00004696$)

f	t (s)	M/M_\odot	ρ (MeV/fm 3)	p_Σ (MeV/fm 3)	L_∞ ($\times 10^{54}$ erg/s)	z
1	$-\infty$	5.01	1.33E-4	0	0	0.00344
0.7	-7.60	3.51	2.71E-4	1.22E-7	1.66	0.00480
0.4	-4.20	2.01	8.31E-4	1.11E-6	4.91	0.00748
0.1	-1.59	0.517	1.37E-2	6.59E-5	17.9	0.0186
f_{Lmax}	-85.9E-3	0.0352	1.34E+4	87.1	183	0.505
f_{BH}	-28.8E-3	0.0343	8.77E+6	20.0E+3	0	9E+15

Using the boundary condition ($p_s|_\Sigma = 0$) for the initial static configuration, we obtain

$$\zeta = \frac{1 + r_\Sigma^2}{-2 + r_\Sigma^2}. \quad (30)$$

Considering the time-dependent, collapsing phase, the appropriate boundary condition is ($p_\Sigma = (qB)_\Sigma$) which provides an equation for the temporal dependence,

$$2f\ddot{f} - \dot{f}^2 - 2\alpha f\dot{f} = 0 \quad (31)$$

where

$$\alpha = \left(\frac{A'_0}{B_0} \right)_\Sigma. \quad (32)$$

A first integration of (31) yields

$$\dot{f} = -2\alpha\sqrt{f} \left(1 - \sqrt{f} \right) \quad (33)$$

and further integration,

$$t = \frac{1}{\alpha} \ln(1 - \sqrt{f}). \quad (34)$$

We now consider the collapse of a $5M_\odot$ shear-free fluid of initial radius $r_\Sigma B_0(r_\Sigma) = 2.159 \times 10^3$ km and initial density $\rho_0 = 2.363 \times 10^8$ g/cm 3 . The initial density then determines the value of R to be $R = 2.608 \times 10^4$ km. These parameters may be compared with data from the extensive and in-depth study done on Supernova 1987A [23]. The associated model parameters and calculated quantities are given in Table 1.

5 Discussion

As noted in application to modelling, imposing a vanishing complexity constraint results in metric line elements of the Maiti and Bergmann form.

The Maiti form was selected with the gravitational potentials set in variable separable form, providing a static configuration consistent with the Schwarzschild interior solution. This method has been used by Pinheiro and Chan [24] with metric functions chosen such that only the spatial part incorporated time dependence. We followed the approach of Tewari and

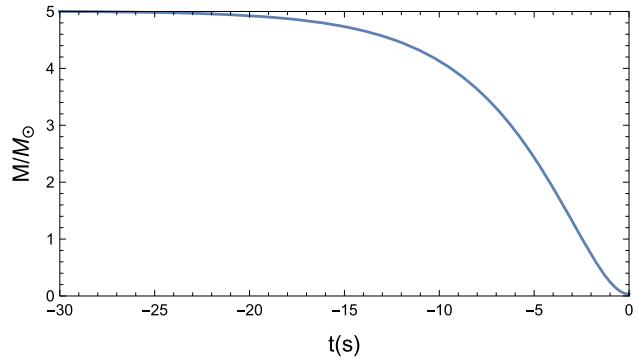
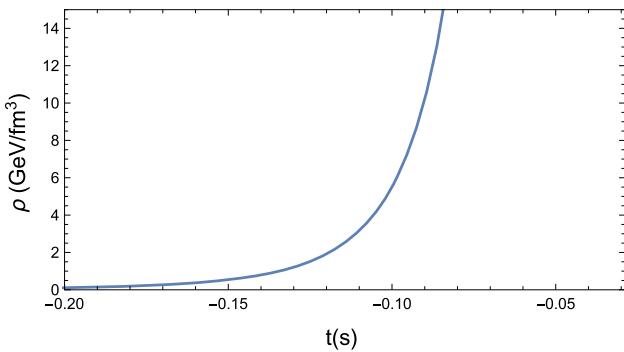
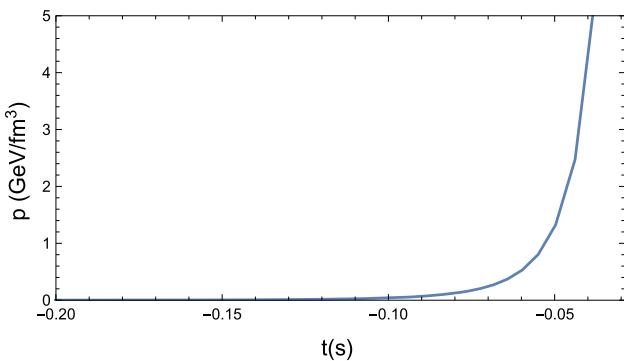
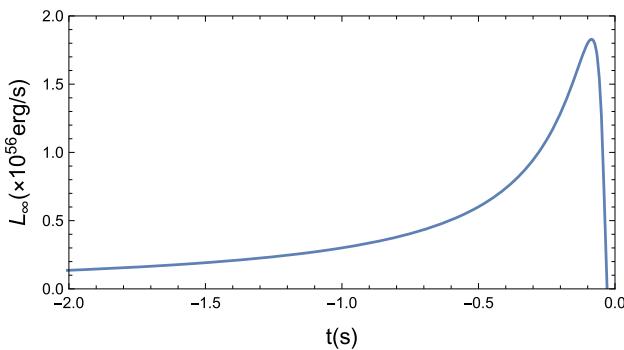


Fig. 1 Mass

Charan [25] in which both metric functions A and B are spatially and temporally separable. In particular, the time dependence was not removed from gravitational potential $A(r, t) = A_0(r)f(t)$ as it is common to set $A(r, t) \rightarrow A_0(r)$ [13, 24]. By including time dependence, we incorporate horizon formation and comparison may be made with systems which follow the perturbative approach [26]. Gravitational collapse models with heat flow and without horizon formation have also been studied [27].

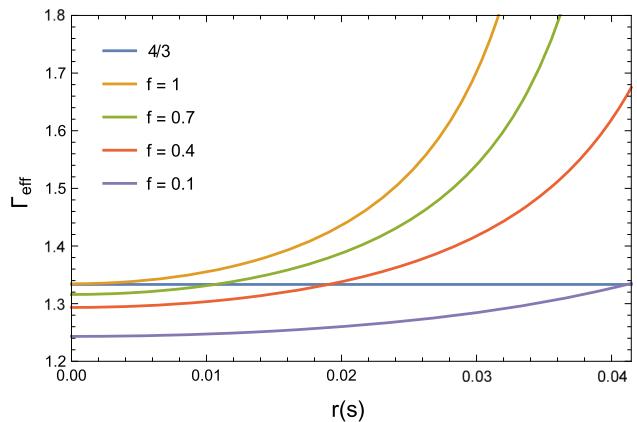
From Table 1, we see that horizon formation occurs at $f \approx 0$ when most of the mass (99.3%) has been radiated as seen in Fig. 1. In Figs. 2 and 3 we see that the energy density and pressure become exceedingly large within the last 100 ms. The luminosity peaks at about 57 ms before horizon formation as seen in Fig. 4, which is about two orders of magnitude slower than that predicted for the gravitational collapse of unstable neutron stars [4]. It could be that a non-vanishing complexity might provide for additional processes or pathways which are conducive to more rapid gravitational collapse. The inclusion of a string field has already shown that inhomogeneity promotes earlier horizon formation [26]. In addition, non-vanishing complexity might assist with stability. In Fig. 5, it is evident that our radiating model suffers from instability towards the centre and instability appears to grow as the collapse proceeds. This is in contrast to the adiabatic case in which isotropic pressure and homogeneous energy density lead to the stability of the vanishing complexity condition

**Fig. 2** Density**Fig. 3** Isotropic pressure at the co-moving boundary**Fig. 4** Luminosity at infinity

[28]. It has been noted that the condition of zero complexity is affected by heat dissipation [29] and initial investigations into non-vanishing complexity for self-gravitating systems have recently been made [12].

6 Conclusion

We have generated a model for the collapse of a self-gravitating, shear-free and isotropic fluid with heat flux, based only on the constraint of vanishing complexity. No initial static configuration was used apart from the specification of the initial mass and radius. Vanishing complexity restricts

**Fig. 5** Effective adiabatic index

the gravitational potentials to that of a Maiti type metric. The potentials were assumed to be separable in radial and time coordinates and a temporal behaviour similar to that of Tewari was followed. In other studies, temporal dependence is only included on the spatial part of the metric which can lead to models without horizon formation. In our model, temporal dependence on both metric functions ensured the formation of a horizon, albeit at a point where most of the mass has been radiated.

We also concluded a relationship between the metric functions for the anisotropic case with vanishing complexity. Specification of one of the metric functions is then required, allowing for the inclusion of an initial state configuration. The modelling process then depends on the successful determination of the other metric function. It is expected that anisotropy would assist in improved stability.

Lastly, future work on the complexity factor might result in non-vanishing trends in the factor during certain stages of the collapse process. The resulting differential equation in the metric functions would no doubt be much more complex and likely require numerical techniques.

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