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PHASE STRUCTURE OF THE SO(3) GEORGI-GLASHOW MODEL

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1. In recent years the study of gauge-Higgs systems has drawn a great attention (see, e.g. refs. /1-14/). A special interest to theories including Higgs bosons is mainly due to the problem of the Higgs mechanism and of the continuum limit of such theories. Unlike pure gauge theories and the ones with fermions, in gauge-Higgs systems there is no asymptotic freedom. So, if in the case, say, of a pure gauge theory we know (or believe) that the continuum limit takes place when $g_0 \to 0$, where g_0 is a bare gauge coupling constant, in the case of a gauge-Higgs theory the problem concerning which set of bare parameters in the action permits one to achieve the continuum limit remains still open. We think that we may speak about the continuum limit of a lattice theory only in the vicinity of a critical point. Therefore, a first step towards undestanding the continuum limit is the study of phase structure of a lattice theory.

This note deals with studying phase diagrams of the SU(2) gauge-Higgs theory, Higgs bosons being in an adjoint representation (the Georgi-Glashow model). Qualitatively, this problem has been discussed in refs. /12-14/. The aim of this note is to establish an exact quantitative pattern and the nature of phase transitions observed in Monte-Carlo simulations. Here we do not consider the problem of classification of phases which requires an independent and more careful analysis. Our analysis is essentilly based on calculations by the Monte-Carlo method and with the use of an effective potential of the Coleman-Weinberg type.

2. The model under consideration couples an adjoint Higgs multiplet to a set of SU(2) non-abelian gauge fields. The Euclidean action is of the form

$$S = \beta \sum_{\square} S_{\square} + \sum_{L} S_{L} , \tag{2.1}$$

$$S_{\square} = 1 - 1/2 \, \mathrm{Tr}\, U_{\square} , \tag{2.2}$$

$$S_L = (1 + m^2/8)\, \mathrm{Tr}\, \Phi^2 + \lambda/4 \left(\mathrm{Tr}\, \Phi^2\right)^2 - \mathrm{Tr}\left(\Phi_i^{+} U_{i;\mu} \Phi_{i+\hat{\mu}} U^{+}_{i;\mu}\right) , \tag{2.3}$$

where $\beta = 4/g_0$, $U_\Box = U_{ij} U_{jk} U_{k\ell} U_{\ell i}$ and $U_{ij} \equiv U_L$ is a gauge field defined on the link $L \equiv (i,j) \equiv (i;\mu)$ which originates at the site i and ends at the site $j = i + \hat{\mu}$, $\mu = 1, \ldots 4$. The Higgs field Φ_i is defined at each site i , and we are using the 2x2 matrix representation: $\Phi_i = i \sum_{\alpha=1}^{3} \sigma_\alpha \Phi_i^\alpha$, where σ_α are Pauli matrices and Φ_i^α are real numbers. The average of any functional $\mathcal{O}\{\Phi; U\}$ is defined by

$$\langle \mathcal{O} \rangle \equiv Z^{-1} \int [dU][d^3\Phi]\, \mathcal{O} \exp\{-S\} . \qquad (2.4)$$

where $[d^3\Phi] = \prod_i d^3\Phi_i$; $[dU] = \prod_L dU_L$; dU_L

is the Haar measure and Z is a normalization factor. In our paper we calculated the averages of the following functionals:

$$1 - \Box \equiv 1 - \tfrac{1}{2} \mathrm{Tr}\, U_\Box \quad ; \quad R^2 \equiv \mathrm{Tr}(\Phi_i^+ \Phi_i) \quad ;$$

$$\mathcal{L} \equiv \mathrm{Tr}\, (\Phi_i^+ U_{i,\mu} \Phi_{i+\hat{\mu}} U_{i;\mu}^+) . \qquad (2.5)$$

Under gauge transformations the fields $U_{i;\mu}$ and Φ_i are transformed as follows

$$U_{i,\mu} \longrightarrow \omega_i U_{i;\mu} \omega_{i+\hat{\mu}}^+ ,$$

$$\Phi_i \longrightarrow \omega_i \Phi_i \omega_i^+ ,$$

where $\omega_i \in SU(2)$.

Lattice field variables are connected with continuum ones in a standard manner

$$U_{i;\mu} \simeq \exp\{ig a A_\mu (a(i + \tfrac{1}{2}\hat{\mu}))\}$$

$$\Phi_i^\alpha = a \Phi^\alpha (ai) \qquad ; \qquad a \sim 0,$$

where a is a lattice step. Setting $m^2 = a^2 m_{cont}^2$ we obtain in the classical continuum limit $(a \to 0;\, g, \lambda, m_{cont}^2$ — fixed) from (2.1)-(2.3) the following

$$S_{cont} = \int d^4x \left\{ \tfrac{1}{4g^2} (\vec{F}_{\mu\nu}^{\,2}) + |D_\mu \vec{\Phi}|^2 + m_{cont}^2 \vec{\Phi}^2 + 4\lambda (\vec{\Phi}^2)^2 \right\} , \qquad (2.6)$$

where

$$D_\mu \Phi^\alpha = \partial_\mu \Phi^\alpha - g \varepsilon^{\alpha\beta\gamma} A_\mu^\beta \Phi^\gamma .$$

It may easily be verified that the quantity $S - \beta \sum_\Box 1$ is invariant under the transformations

$$\beta \longrightarrow -\beta$$

$$U_{n_1 n_2 n_3 n_4; 1} \longrightarrow U_{n_1 n_2 n_3 n_4; 1}$$

$$U_{n_1 \ldots n_4; 2} \longrightarrow (-1)^{n_1} U_{n_1 \ldots n_4; 2}$$

$$U_{n_1 \ldots n_4; 3} \longrightarrow (-1)^{n_1 + n_2} U_{n_1 \ldots n_4; 3}$$

$$U_{n_1 \ldots n_4; 4} \longrightarrow (-1)^{n_1 + n_2 + n_3} U_{n_1 \ldots n_4; 4} .$$

Hence it follows that at $\beta = 0$ the average $\langle \Box \rangle$ equals zero for all λ and m^2 and is an odd function of β , while the order parameters $\langle \Phi^+ \Phi \rangle$ and $\langle \mathcal{L} \rangle$ are even functions of β . When $m^2 \to \infty$ or $\lambda \to \infty$ the Higgs-field fluctuations die out, and we should come to a pure gauge theory. The limit of forzen radial mode is, obvilusly, established when $\lambda \to \infty$ and $m^2/4\lambda$ is fixed.

3. The numerical study of the model (2.1)-(2.3) was made by the Monte-Carlo method. All our numerical experiments have been performed on 4^4 and 6^4 lattices with periodic boundary conditions. According to our calculations the results on the 4^4 lattice do not in practice differ from those on the 6^4 lattice. In our calculations we used the Metropolis algorithm. The procedure of construction of the phase diagram is fully analogous to that described for instance in /4,10,11/.

Fig.1 presents our main result: the phase diagram of the model. Solid curves represent first-order phase transitions for different λ . These first-order phase-transitions are observed for all the three order parameters: $\langle 1 - \Box \rangle$, $\langle R^2 \rangle$ and $\langle \mathcal{L} \rangle$. In Figs. 2 a,b,c as an illustration, hysteresis loops are shown for

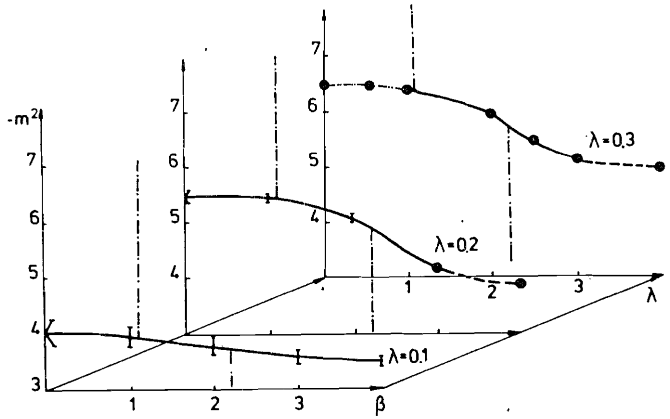
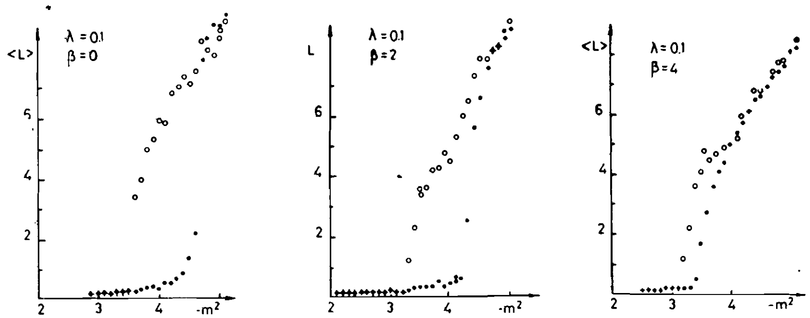


Fig. 1. Phase diagram in three-parameter space, calculated by the Monte-Carlo method. The solid line **represents** the **phase** transitions of the first order; the dashed lines **denote** the region where the type of phase transition is not **clearly established;** the vertical dash-dotted lines represent the "crossovers"; the —·· — line marks region where the phase transition of the first order **disappears.**



a) b) c)

Fig. 2. Thermal cycles at λ =0.1 and various β.

the order parameter $\langle L\rangle$ at λ = 0 and three values of β : β = 0; 2; 4. Simulations from different types of initial configurations (starts) show that in the region of hysteresis a jump of the order parameter occurs.

It is to be noted that with growing β the hysteresis curve gets narrowing up to some large enough values of β where it vanishes at all (dashed lines in Fig.1). From the behaviour of the order parameter in this case we cannot yet conclude whether in that region the phase transition is of first or second order **or whether it** does exist at all. It is interesting that with increasing λ the picture of phase transitions abruptly changes: from the line of first-order phase transitions a segment "drops out" in the interval of β values between 0 and 1. (In Fig. 1 this is shown by a —·· — line for λ = 0.3). This occurs at $\lambda \simeq 0.22$, the value being in good agreement with that found by the effective potential method (see below). In Fig. 3 results are presented for the thermal cycles at λ = 0.3 and β = 0; 2; 4 illustrating this behaviour.

The vertical dash-dotted lines in Fig. 1 have been calculated with the use of the order parameter $\langle 1 - \square\rangle$ and they correspond, obviously, to the crossover.

4. The effective-potential method for investigating phase transitions turns out to be a useful supplement to the numerical Monte-Carlo method. At β =0 an explicit integration can be made over "angular" variables, and as a result, we obtain (like we have proceeded in ref. /4,10,11/) for the effective potential in the leading approximation the following expression:

$$V_{eff} = (8+m^2)\bar{R}^2 + 4\lambda\bar{R}^4 - 4\ln\left(\mathrm{sh}(2\bar{R}^2)/2\bar{R}^2\right) - \ln\bar{R}^2 . \tag{4.1}$$

The last term in (4.1) comes from the integration measure $d^3\phi$. In Fig. 4 a characteristic form of V_{eff} is shown for various values of the scalar-self-interaction constant λ at such $m_c^2(\lambda)$ at which values of the effective potential at minima coincide. At $\lambda < \lambda_c \simeq 0.22$ the effective potential has two local minima. At some value of $m^2 = m_c^2$ the V_{eff} values at these minima get equal, and consequently, a first-order transition occurs. With increasing λ the height of the barrier between minima lowers; at $\lambda = \lambda_c$ both the minima coincide, and the "well" bottom becomes flat; at $\lambda > \lambda_c$ the effective potential has a single minimum for all values of m^2 , and there is no first-order phase transition.

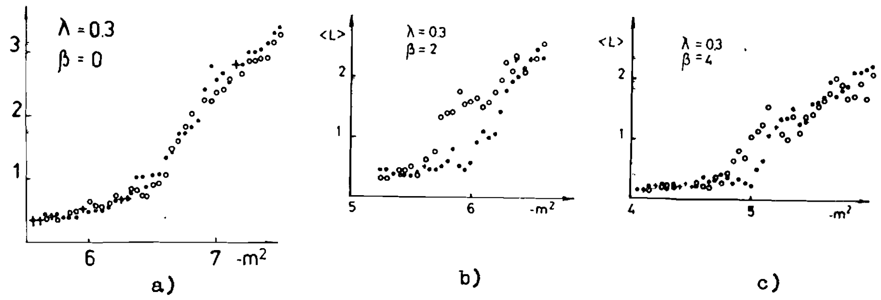


Fig. 3. Thermal cycles at λ =0.3 and varius β .

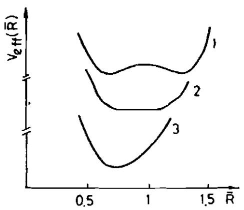


Fig. 4. Behaviour of the effective potential as a function of R at β =0. Curves $1 \div 3$ correspond to growing values of λ ; m^2 is equal to $m_c^2(\lambda)$.

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