Combining universal and odd RR axions for aligned natural inflation

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Abstract. We successfully embed the Kim-Nilles-Peloso (KNP) alignment mechanism for enhancing the axion decay constant in the context of large volume type IIB orientifolds. The flat direction is generated in the plane of $(C_0 - C_2)$ axions corresponding to the involutively even universal axion C_0 and odd axion C_2, respectively. The moduli stabilization with large volume scheme has been established as well.

Contents

1 Introduction and motivation

The recent BICEP2 results [1] have undoubtably shaken the status of inflationary model building in string cosmology. The discovery of primordial B-mode polarization of the cosmic microwave background has been recently claimed by the BICEP2 Collaboration. This claim can be understood as a signature of gravitational wave which is encoded in the so-called tensor-to-scalar ratio (r). The BICEP2 observations fix the inflationary scale by ensuring a large tensor-to-scalar ratio r as follows [1]

$$r = 0.20^{+0.07}_{-0.05} \ (68\% \,\mathrm{CL})\,,$$
$$H_{\mathrm{inf}} \simeq 1.2 \times 10^{14} \left(\frac{r}{0.16}\right)^{1/2} \mathrm{GeV}\,, \tag{1.1}$$

where H_{inf} denotes the Hubble parameter during the inflation. Subtracting the various dust models and re-deriving the r constraint still result in high significance of detection and one has $r = 0.16^{+0.06}_{-0.05}$. In order to reconcile the tension between BICEP2 [1] result and PLANCK [2], WMAP data [3], it demands the following windows for the other cosmological observables

$$\ln\left(10^{10} P_s\right) = 3.089^{+0.024}_{-0.027}\,, \quad n_s = 0.957 \pm 0.015\,, \quad \alpha_{n_s} = -0.022^{+0.020}_{-0.021}\,, \tag{1.2}$$

where P_s is the scalar power spectrum and α_{n_s} is the running of spectral index n_s. All these cosmological observables can be written out in terms of the inflationary potential and its various derivatives. Thus, with the available experimental data from various sources so far, the shape of a single field inflationary potential is significantly constrained. As a reverse computation, writing out various derivatives of inflationary potential in terms of the aforementioned cosmological observables, a generic single field inflationary potential can be locally reconstructed [4–6].

In order to realize the required large value of tensor-to-scalar ratio r, the inflaton field needs to travel over trans-Planckian distance according to the famous Lyth bound [7]. Further, it also suggests the inflationary process to be (a high scale process) near the scale of Grand Unified Theory (GUT). As a result, embedding inflationary models in a UV complete framework, such as string theory, is inevitable. The UV sensitivity in chaotic inflation class of models has been recently addressed in [8]. The BICEP2 claims, if proven, can also provide invaluable pieces of information in search of a consistent supersymmetry (SUSY) breaking scale [9, 10].

With large field excursions, the other relevant issues from higher order corrections should also be taken care of for the viability of the model [11–13]. If the BICEP2 result is confirmed, it would serve as a huge discriminator filtering out many among the plethora of inflationary models developed so far. However, it is interesting that the three classes of inflationary models, namely the chaotic-type [14–19], natural-type [20–23] as well as Assisted/N/M-flation type [24–31] are among the winners. In the context of models developed in a purely string framework prior to the BICEP2 results, the axion monodromy inflation [32, 33] was found to be much closer (but still insufficient) to fulfill the BICEP2 claims. There have been very vibrant and speedy progress on these lines of developing chaotic- or (multi)natural-type of inflationary models utilizing axion monodromy in a very short post-BICEP2 period so far [34–53].

Regarding the axionic inflation in Type IIB string framework, LARGE Volume Scenario (LVS) [54] provides a well-controlled moduli stabilization mechanism, which makes the lightest moduli a good candidate for being an inflaton. In the LVS mechanism, the exponentially large volume of the internal Calabi-Yau threefold $\mathcal{V}$ is favored as it also provides a control over the (un-)known α' [55] as well as string loop corrections g_s [56, 57]. In fact, it has been observed that the known/conjectured forms of these corrections, at the level of Kähler potential, appear with volume suppressed terms in the scalar potential [54, 56, 57], which makes large volume scenario more robust as well. Further due to the presence of (extended-)no-scale structure in the context of type IIB swiss-cheese compactification, various volume moduli directions orthogonal to the overall Calabi-Yau (CY) volume $\mathcal{V}$ remain flat. The breaking of flatness in the orthogonal direction via (non-)perturbative corrections leads to a flat enough inflationary potential; for example, see the models with inflaton being identified with divisor volume moduli [58–65].

In the context of LVS framework, embedding of axion monodromy type potential has been recently proposed in [35], in which the universal axion C_0 could drive the inflationary process. Based on certain assumptions on the background flux, the large volume expansion has been argued to be useful for trusting the effective field theory (EFT) description even in a non-perturbative regime where the string coupling g_s satisfies $1 < g_s < 10$ [35]. On the other hand, in the context of axionic inflationary models of natural-type inflation [20], a large decay constant has been proposed to be realized in a Kim-Nilles-Peloso (KNP)-type two-field potential [22]. The main idea is to align two sub-planckian decay constants such that with a certain rotation of field basis, one could create a hierarchy in the decay constant of the newly constructed axionic basis. The best advantage of this type of axionic inflation is that unlike N-flation [27, 28, 31] which requires a large number of $\left(\mathcal{O}\left(10^3 - 10^4\right)\right)$ axions assisting the inflationary process, this is a two-field model. However, the standard KNP-model with two fields usually requires large anomaly coefficients or equivalently large gauge groups of the non-perturbative effects to generate the potential, which is challenging to be embedded into a realistic particle physics or string model. On these lines, the standard KNP-model has been generalized to N-fields (with $N < 10$) [47, 66] to facilitate the axionic alignments (as well as to keep the number of axions less than those required in N-flation model).

Motivated by the KNP proposal for enhancing the decay constant to trans-planck scale, in this article, we propose a new class of inflationary potentials in the context of LVS framework. The inflationary direction lies in the plane of $(C_0 - C_2)$ axions, where C_0 corresponds to the involutively even universal axion while C_2 is involutively odd axion. If we restrict the orientifold to be divisor exchange or reflection, in order to support large volume scenarios with the orientifold odd axions, the underline Calabi-Yau threefold should

have $h^{1,1}(CY_3) \geq 3$ [67, 68]. Using two such involutively odd axions and magnetized non-perturbative effects, recently a KNP-type scenario has been proposed in [45]. Unlike this proposal, we utilize the universal axion C_0 along with a single odd axion C_2 to get the required alignment for the natural inflation. This engineering solves one of the major challenges of [35] by taking the framework within perturbative regime as large enough decay constant is realized within $g_s < 1$ in our model. Moreover, a combination of C_0 and C_2 axions provides a better decoupling in the kinetic sector unlike the case with two odd axions [45].

The article is organized as follows. In section 2 we provide a brief and relevant feature of type IIB orientifolds. Section 3 summarizes the original KNP formalism [22] for enhancing the decay constant in a two-field potential. In section 4, we provide a successful embedding of KNP-type potential in large volume scenarios with the inclusion of odd axion along with universal axion. Section 5 presents a detailed numerical analysis with a couple of benchmark models. Finally, in section 6 we provide a summary with possible open challenges.

2 Relevant ingredients of type IIB orientifolds

We consider type IIB superstring theory compactified on an orientifold of a Calabi-Yau threefold CY_3 with $O3/O7-$plane. The full orientifold action is $\mathcal{O} = (-)^{F_L}\Omega_p\sigma$, where the F_L is the spacetime fermion number in the left-moving sector, Ω_p denotes world-sheet parity while σ denotes a holomorphic and isometric involution. By performing the detailed dimensional reduction from ten to four dimensions [69], the low energy effective action at the second order in derivatives is given by a supergravity theory, whose dynamics is encoded in three building blocks; namely the Kähler potential K, the holomorphic superpotential W, and the holomorphic gauge kinetic functions. These building blocks can be generically written in terms of appropriate $\mathcal{N} = 1$ coordinates (S, G^a, T_α) defined as

$$S = i\,c_0 + e^{-\phi}\,, \qquad G^a = i\,c^a - S\,b^a\,,$$
$$T_\alpha = \frac{1}{2}\kappa_{\alpha\beta\gamma}\,t^\beta t^\gamma + i\left(\rho_\alpha - \frac{1}{2}\kappa_{\alpha ab}\,c^a b^b\right) - \frac{1}{4}\,e^{\phi}\,\kappa_{\alpha ab}\,\bar{G}^a(G + \bar{G})^b\,, \tag{2.1}$$

where t^α is the two-cycle volume while c_0, c^a and ρ_α correspond to RR axions C_0, C_2, and C_4, respectively. Further, $\kappa_{\alpha\beta\gamma}$ and $\kappa_{\alpha ab}$ are triple intersections numbers of the even/odd two cycle. Here, the indices α run in even (1,1)-cohomology of CY orientifold $\left(H^{1,1}_+(CY_3/\sigma)\right)$ while indices a are counted in odd (1,1)-cohomology $H^{1,1}_-(CY_3/\sigma)$.

The Kähler potential K. Generically, the Kähler potential is given as

$$K = -\ln\left(S + \bar{S}\right) - \ln\left(-i\int_X \Omega_3 \wedge \bar{\Omega}_3\right) - 2\ln\left(\mathcal{Y}\left(S, G^a, T_\alpha, \ldots\right)\right), \tag{2.2}$$

where $\mathcal{Y} = \frac{1}{6}\mathcal{K}_{ABC}\,t^A t^B t^C$ is the volume of the Calabi-Yau manifold expressed in terms of two-cycle volumes t^A. The dots in (2.2) denote the potential appearance of other moduli like D3/D7-brane fluctuations (and hence complex structure moduli which get coupled after including brane-fluctuations) or Wilson line moduli. Unfortunately, $\mathcal{Y}$ is only implicitly given in terms of the chiral superfields. It is in general non-trivial to invert the last relation in (2.1), and so it is not possible to write K in terms of T_α explicitly. Further, the most general Kähler potential can also depend on the derivatives of chiral superfield [11, 12]. However, we ignore such higher order corrections in the present analysis.

The superpotential W. The general schematic form of the superpotential W is given as

$$W = \int_X G_3 \wedge \Omega + \sum_D \mathcal{A}_D \left(z^{\tilde{a}}, G^a, \mathcal{F}_D, \ldots \right) e^{-a_D^\alpha T_\alpha}$$
$$= W_{\text{cs}} + W_{\text{np}}, \tag{2.3}$$

where the first term is the Gukov-Vafa-Witten (GVW) three-form flux induced tree-level superpotential [70] (see [71, 72] also for related work). The second term denotes a sum over non-perturbative corrections coming from the Euclidean $D3$-brane instantons or gaugino condensation on D7-branes [73]. Again the dots indicate a further dependence on e.g. D3/D7-brane fluctuations or Wilson line moduli. Further, the prefactor contains not only the one-loop Pfaffian for fluctuations around the instanton background but also contributions from so-called (gauge-)fluxed instantons [74, 75] and Euclidean $D1$-brane instantons [76]. The presence of gauge fluxes on the divisor contributing to the non-perturbative superpotential helps in alleviating [74] the chirality issue proposed in [77]. It also helps in stabilizing all the odd moduli, with or without the help of poly-instanton effects[1] [68]. Also, in principle one has to sum over all the possible instanton or gaugino condensation effects, and in the presence of extra magnetic fluxes turned-on on the relevant odd two-cycles sitting inside the relevant divisor, this issue becomes more delicate in terms of satisfying tadpole/anomaly cancellation conditions, etc [79, 80]; see also a related review in [81]. However in the present study, our approach would be rather phenomenological, and we consider the most suitable ansatz for the superpotential without getting into these technicalities.

We would consider the gauge flux effects on the orientifold invariant divisor D (having involutively odd two-cycles and) contributing to the non-perturbative superpotential. Usually, there are two kinds of non-perturbative corrections, one is induced through Euclidean D3-brane (E3-brane) instanton while the other one through gaugino condensation. For E3-brane instanton, we require the gauge flux turned-on on the brane to be $\mathcal{F}_E \in H^{1,1}_-(D_E)$ in order to ensure the instanton to be of $O(1)$-type. For gaugino condensation with a stack of $2N$ D7-branes, they should also be placed at orientifold invariant positions. If the D7-branes coincide with an O-plane, i.e. both N branes and their images are placed on top of an $O7^{-/+}$-plane, it provides $\mathrm{SO}(2N)/SP(2N)$ gauge group dynamics. If the $D7$-branes and their images wrap on the same internal geometry, it yields $SP(2N)/\mathrm{SO}(2N)$ gauge group. Turning on a gauge flux with $\mathcal{F}_D \in H^{1,1}_-(D)$, the fluxed brane can remain invariant under the orientifold projection, while turning on gauge flux $\mathcal{F}_D \in H^{1,1}_+(D)$ breaks the gauge symmetry to a unitary group [78, 82], then a D-term will be generated by the U(1) subgroup with Fayet-Iliopoulos terms. Since $O7^{-/+}$-plane carries $-8/+8$ times of the D7-brane charge, in the following, we always assume that we have $O7^-$-plane and turn-on only the odd gauge flux $\mathcal{F}_D \in H^{1,1}_-(D)$ on the branes. Also, for the time being, we concentrate on the F-term dynamics and just consider the suitable form of superpotential W with multiple gaugino condensation configuration. The D-brane tadpole cancellation as well as the zero-modes condition are assumed to be settled when addressed in concrete setup, and one can start with the following form of the (odd moduli G^a dependent) holomorphic prefactor $\mathcal{A}_D \left(z^{\tilde{a}}, G^a, \mathcal{F}_D, \ldots \right)$ of (2.3),

$$W_{\text{np}} = A \sum_{\mathcal{F}_D} e^{-a_D^\alpha T_\alpha} \exp\left[-a_D^\alpha h_1(\mathcal{F}_D) S - a_D^\alpha h_2(\mathcal{F}_D) G^a \right], \tag{2.4}$$

where $h_i(\mathcal{F}_D)$s are gauge flux dependent constants turned-on along the odd two-cycles of the divisor D supporting the non-perturbative superpotential contribution. This form of superpotential will be heavily utilized in the upcoming sections.

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[1]The proper zero mode structure of poly-instanton in type IIB orientifold has been clarified in [78].

The scalar potential V. From the Kähler potential and the superpotential one can compute the $\mathcal{N} = 1$ scalar potential

$$V = e^K \left(\sum_{I,J} K^{I\bar{J}} \mathcal{D}_I W \bar{\mathcal{D}}_{\bar{J}} \bar{W} - 3|W|^2 \right), \tag{2.5}$$

where the sum runs over all moduli. For studying the Kähler moduli dynamics, we will assume that the complex structure moduli and dilaton have already been stabilized supersymmetrically as $\mathcal{D}_{\text{c.s.}} W = 0$, $\mathcal{D}_S W = 0$. Further, on the lines of [35], we assume that with the freedom available through the landscape of background fluxes, one can still keep universal axion c_0 massless or 'nearly' massless. We will quantify what we mean by 'nearly' and elaborate on this point later while considering the explicit computations in sections 4 and 5.

3 Review of KNP-type natural inflation

Let us very briefly review the original KNP proposal for natural inflation [22]. We consider the following two-field inflationary potential

$$V(\phi_1, \phi_2) = \sum_{i=1}^{2} \Lambda_i \left(1 - \cos\left[\frac{\phi_1}{f_i} + \frac{\phi_2}{g_i} \right] \right), \tag{3.1}$$

where f_i and g_i's can be sub-Planckian decay constants as the most natural choice. The determinant of the Hessian of this potential is simplified to

$$\text{Det}\,(V_{ij}) = \frac{(f_2\, g_1 - f_1\, g_2)^2 \prod_{i=1}^{2} \Lambda_i \cos\left[\frac{\phi_1}{f_i} + \frac{\phi_2}{g_i} \right]}{f_1^2\, f_2^2\, g_1^2\, g_2^2}. \tag{3.2}$$

Thus, it will have a flat direction if the following condition holds

$$\frac{f_1}{f_2} = \frac{g_1}{g_2}. \tag{3.3}$$

Therefore, a small enough deviation from this condition can create a mass hierarchy between the two axions rotated in a new basis. As we will see explicitly in a moment, one can elegantly create a mass hierarchy and (with appropriate axionic rotation) an alignment leading to the enhancement of decay constant of the lighter combination also occurs. With the following rotation of axions

$$\psi_1 = \frac{g_1\, \phi_1 + f_1\, \phi_2}{\sqrt{f_1^2 + g_1^2}}, \qquad \psi_2 = \frac{f_1\, \phi_1 - g_1\, \phi_2}{\sqrt{f_1^2 + g_1^2}}, \tag{3.4}$$

we reformulate the expression eq. (3.1) as under

$$V(\psi_1, \psi_2) = \Lambda_1 \left(1 - \cos\left[\frac{\psi_1}{f_1'} \right] \right) + \Lambda_2 \left(1 - \cos\left[\frac{\psi_1}{f_2'} + \frac{\psi_2}{f_{\text{eff}}} \right] \right), \tag{3.5}$$

where f_1', f_2' and f_{eff} take the form as below

$$f_1' = \frac{f_1\, g_1}{\sqrt{f_1^2 + g_1^2}}, \quad f_2' = \frac{f_2\, g_2\, \sqrt{f_1^2 + g_1^2}}{f_1\, f_2 + g_1\, g_2}, \quad f_{\text{eff}} = \frac{f_2\, g_2\, \sqrt{f_1^2 + g_1^2}}{|f_1\, g_2 - g_1\, f_2|}. \tag{3.6}$$

Thus, if the deviation from the flatness condition eq. (3.3) is small enough, one can generate an 'effectively' large decay constant for ψ_2 combination. Further, together with eq. (3.3) and an appropriate hierarchy $\Lambda_2 \ll \Lambda_1$, one can make the field ψ_1 heavier than ψ_2 with the respective masses at the minimum given as

$$m_{\psi_1}^2 \simeq \Lambda_1 \left(\frac{1}{f_1^2} + \frac{1}{g_1^2} \right), \quad m_{\psi_2}^2 \simeq \frac{\Lambda_2 \left(f_2\, g_1 - f_1\, g_2 \right)^2}{g_2^2\, f_2^2\, \left(f_1^2 + g_1^2 \right)}\,. \tag{3.7}$$

Stabilizing ψ_1 at one of its minimum $\overline{\psi}_1 = 0$ would result in a single axion potential with large decay constant as below

$$V(\psi_2) = \Lambda_2 \left(1 - \cos\left[\frac{\psi_2}{f_{\text{eff}}} \right] \right)\,. \tag{3.8}$$

Now we turn to the embedding of KNP-type mechanism in large volume scenario. The main focus would be to utilize universal RR axion C_0 along with an involutively odd RR axion C_2.

4 Realizing natural inflation in large volume scenarios

Let us consider the following ansatz for the Kähler potential K motivated by the large volume scenarios. After introducing a single odd modulus G^1 via the appropriate choice of orientifold involution,[2] the Kähler potential becomes [68]

$$K \equiv K_{\text{cs}} - \ln(S + \overline{S}) - 2\ln \mathcal{Y} \tag{4.1}$$
$$= K_{\text{cs}} - \ln(S + \overline{S}) - 2\ln \left(\xi_B \Sigma_B^{3/2} - \xi_S \Sigma_S^{3/2} + \mathcal{C}_{\alpha'} \right),$$

where

$$\Sigma_\alpha = T_\alpha + \bar{T}_\alpha + \frac{\kappa_{\alpha 11}}{2\left(S + \bar{S} \right)} \left(G^1 + \bar{G}^1 \right) \left(G^1 + \bar{G}^1 \right) \quad \text{for} \quad \alpha \in \{B, S\}, \tag{4.2}$$

and $\mathcal{C}_{\alpha'} = -\frac{\chi(X)\xi(3)}{4(2\pi)^3\, g_s{}^{3/2}}$. This form of Kähler potential explicitly shows the shift symmetries in various RR axionic directions; namely the universal axion C_0, the involutively even axion C_4 and the involutively odd axion C_2. Although the presence of α'-corrections break the "no-scale structure", it still leaves the direction orthogonal to $\mathcal{V}$ (which is τ_s) to remain flat. This flatness and axionic shift symmetries are broken via the non-perturbative effects appearing in the following racetrack form of the superpotential which comes from eq. (2.4)

$$W = W_{\text{cs}} + A_0\, e^{-a_0 T_S} \tag{4.3}$$
$$+ A_s\, e^{-a_s \left(T_S + h_1(\mathcal{F})\, S + h_2(\mathcal{F})\, G^1 \right)} - B_s\, e^{-b_s \left(T_S + h_3(\mathcal{F})\, S + h_4(\mathcal{F})\, G^1 \right)},$$

where

$$W_{\text{cs}} = W_{\text{cs1}} + S\, W_{\text{cs2}}\,. \tag{4.4}$$

Such a form of superpotential eq. (4.3) could be thought of arising from different stacks of unfluxed and fluxed $D7$-branes wrapping the so-called small divisor in an orientifold invariant way as discussed before eq. (2.4). As a result, we can set the gaugino condensations effects with $a_0 = \frac{2\pi}{N_0}$, $a_s = \frac{2\pi}{N_1}$, $b_s = \frac{2\pi}{N_2}$, where N_0, N_1 and N_2 being the ranks of the corresponding gauge groups coming from different stacks of $D7$-branes. Further, W_{cs1}, W_{cs2}, A_0, A_s and

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[2]For constructing explicit examples of CY orientifold with $h_-^{11}(CY_3/\mathcal{O}) \neq 0$, see [67, 82].

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B_s are generically complex structure moduli and background flux dependent quantities. For the time being, these are considered to be constants as in the standard moduli stabilization schemes. At the outset, let us clearly mention the following inherent assumptions before coming to the scalar potential computation

- in addition to background fluxes, there are gauge fluxes turned-on on the small divisor which induces axio-dilaton S and odd axion G^1 dependence on top of the non-perturbative effects. These are encoded in such gauge flux dependent quantities $h_i(\mathcal{F})$, $\forall i \in \{1, 2, 3, 4\}$. For the minimal setting $h_1(\mathcal{F})$ and $h_3(\mathcal{F})$ are quadratic in gauge flux while $h_2(\mathcal{F})$ and $h_4(\mathcal{F})$ are linear in gauge flux. As a result, we should keep $h_1 > h_2$ and $h_3 > h_4$.

- On the lines of [35], we assume that in the absence of non-perturbative corrections to the superpotential, the landscape of background fluxes can facilitate one with keeping the universal RR axion C_0 massless (or at least nearly massless) via creating a mass-hierarchy between dilaton mass and universal axion C_0. Although the univesal axion appears as a linear term in the superpotential, by tuning the background flux dependent parameters in the tree-level superpotential, the c_0 axion shift symmetry remain (nearly) unbroken via the quadratic term induced in the scalar potential. In order to restore large volume scenarios as well as a decoupled KNP-type inflationary potential of $c_0 - c^1$ axion, the coefficient w_2 in $W_{\rm cs} = w_1 + c_0\, w_2$ has to satisfy the following bound

$$|w_2|^2 \ll \mathcal{O}\left(\frac{e^{-a_s\, h_1/g_s}}{\mathcal{V}^3}\right) \sim \mathcal{O}\left(\frac{e^{-b_s\, h_3/g_s}}{\mathcal{V}^3}\right); \qquad w_1 \sim \mathcal{O}(1). \tag{4.5}$$

This is probably the strongest assumption in our model and should be examined to be realized in a concrete Calabi-Yau orientifold construction.

- Based on the aforementioned point, we assume the standard procedure to stabilize the complex structure moduli and dilation via the background flux superpotential. So we naively consider $W_{\rm cs} = w_1 + c_0\, w_2$ such that $w_2 \ll w_1$ and we will quantify how small w_2 should be to trustfully recover the large volume potential.

Utilizing these pieces of information, the F-term scalar potential can be computed from eq. (2.5) for the given ansatz of K and W, and various terms can be categorically collected as follows

$$V\left(\mathcal{V}, \tau_s; \rho_s, b^1, c^1, c_0\right) \simeq V_{\rm LVS}\left(\mathcal{V}, \tau_s; \rho_s, b^1\right) \tag{4.6}$$
$$+ V_{\rm rest}\left(\mathcal{V}, \tau_s\, \rho_s, b^1; c^1, c_0\right) ,$$

where $V_{\rm LVS}\left(\mathcal{V}, \tau_s; \rho_s, b^1\right)$ is the large volume potential contributing at the leading order $\mathcal{O}\left(\frac{1}{\mathcal{V}^3}\right)$. With stabilizing the axions at one of their minima $\rho_s = 0 = b^1$ [33, 45],[3] the stabilized values of volume moduli is obtained by the solutions of following coupled expressions

$$C_{\alpha'} \simeq \frac{32\sqrt{2}\, a_0\, \xi_S\, \overline{\tau}_s^{5/2}(a_0\overline{\tau}_s - 1)}{(1 - 4a_0\,\overline{\tau}_s)^2};$$
$$\overline{\mathcal{V}} \simeq -\frac{6\sqrt{2}\, \xi_S\, W_{\rm cs}\, \sqrt{\overline{\tau}_s}\,(a_0\,\overline{\tau}_s - 1)}{a_0\, A_0\,(4a_0\,\overline{\tau}_s - 1)}\, e^{a_0\,\overline{\tau}_s} . \tag{4.7}$$

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[3]This is also similar to the case of moduli stabilization via fluxed-instanton superpotential leading to the appearance of theta function in $W_{\rm np}$. In that case, there are several extrema in the axionic directions due to theta-function periodicities appearing in the potential [68], and $b^1 = 0$ is the simplest local minimum.

Let us mention an important point that in our approach of stepwise moduli stabilization, one has to maintain the hierarchy $|V_{\text{LVS}}| \gg |V_{\text{rest}}|$ throughout and so one has to be careful while samplings of model dependent parameters are made. As we will see later, relatively larger gauge groups N_1 and N_2 are needed for realizing large decay constant, and in order to stabilize the overall volume of the CY to order $\mathcal{O}(10^3)$, we need $N_0 < N_{1,2}$ and then to maintain the mass hierarchy between standard Kähler moduli and universal axion together with odd moduli, one has to appropriately choose the gauge flux parameters h_1 and h_3 large enough.

After stabilizing the heavier moduli and orientifold even axion C_4 along with odd axion B_2, the potential reduces to the form as below

$$
\begin{aligned}
V_{\text{rest}}(\mathcal{V}, \tau_s\, \rho_s, b^1; c^1, c_0) &\equiv V_{\text{rest}}\left(c^1, c_0\right) \\
&\simeq \Delta_0 + \Delta_1 \cos\left[a_s\, h_1\, c_0 + a_s\, h_2\, c^1\right] \\
&\quad + \Delta_2 \cos\left[b_s\, h_3\, c_0 + b_s\, h_4\, c^1\right] + \dots .
\end{aligned}
\tag{4.8}
$$

Thus, at the sub-leading order, the shift symmetry for the odd axion c^1 is broken. Here in the aforementioned simplification, the coefficients Δ_1 and Δ_2 are suppressed by factors $e^{-a_s\, h_1/g_s}$ and $e^{-b_s\, h_3/g_s}$ respectively as compared to $|V_{\text{LVS}}|$ while Δ_0 is the collection of all the terms independent of c^1 and c_0 axions. Given our assumption that the coefficient of quadratic potential for universal axion c_0 generated at tree level can be fairly negligible by utilizing the flux freedom, and so V_{rest} is effectively the leading contribution to break the c_0 flatness. Further, note that the model dependent parameters h_1 and h_3 depend on the gauge flux $\mathcal{F}$ supported on the divisor with odd two-cycles contributing to the non-perturbative superpotential. Further, the dots denote those terms which are doubly suppressed by flux dependent exponentials and hence are subleading for small string coupling regime. Now after using an appropriate uplifting mechanism, one can rearrange the terms to result in the desired KNP-type potential [22]

$$
V(\phi_1, \phi_2) \simeq \Lambda_1 \left(1 - \cos\left[\frac{n_1\, \phi_1}{f_1} + \frac{n_2\, \phi_2}{f_2}\right]\right) + \Lambda_2 \left(1 - \cos\left[\frac{m_1\, \phi_1}{f_1} + \frac{m_2\, \phi_2}{f_2}\right]\right),
\tag{4.9}
$$

where Λ_i's can be collected in terms of model dependent parameters given as below

$$
\begin{aligned}
\Lambda_1 &\simeq \frac{\sqrt{2}\, a_0\, a_s\, |A_0|\, |A_s|\, \overline{\tau}_s}{\xi_S\, \overline{\mathcal{V}}\, (a_0\, \overline{\tau}_s - 1)} \operatorname{Exp}\left[-a_0 \overline{\tau}_s - a_s \overline{\tau}_s - \frac{a_s\, h_1}{g_s}\right] \\
&\simeq \frac{12\, |W_{\text{cs}}|\, a_s\, |A_s|\, \overline{\tau}_s}{\overline{\mathcal{V}}^2\, \xi_S\, (4\, a_s\, \overline{\tau}_s - 1)}\, e^{-a_s \overline{\tau}_s - \frac{a_s\, h_1}{g_s}}, \\
\Lambda_2 &\simeq \frac{\sqrt{2}\, a_0\, b_s\, |A_0|\, |B_s|\, \overline{\tau}_s}{\xi_S\, \overline{\mathcal{V}}\, (a_0\, \overline{\tau}_s - 1)} \operatorname{Exp}\left[-a_0 \overline{\tau}_s - b_s \overline{\tau}_s - \frac{b_s\, h_3}{g_s}\right] \\
&\simeq \frac{12\, |W_{\text{cs}}|\, b_s\, |B_s|\, \overline{\tau}_s}{\overline{\mathcal{V}}^2\, \xi_S\, (4\, b_s\, \overline{\tau}_s - 1)}\, e^{-b_s \overline{\tau}_s - \frac{b_s\, h_3}{g_s}}.
\end{aligned}
\tag{4.10}
$$

Further, in expression eq. (4.9) of the potential, $n_i = a_s\, h_i$ and $m_i = b_s\, h_{i+2}$ for $i = 1, 2$. Subsequently, the canonically normalized fields ϕ_1 and ϕ_2 are defined as follows

$$
\phi_1 \equiv c_0\, f_1 \simeq c_0\, \frac{g_s}{\sqrt{2}}, \quad \phi_2 \equiv c^1\, f_2 \simeq c^1 \frac{\sqrt{-3\, g_s\, \kappa_{B11}}\, \xi_B^{1/3}}{\mathcal{V}^{1/3}}.
\tag{4.11}
$$

With the following redefinitions of the two-fields (similar to the original KNP-formalism reviewed in the last section)

$$\psi_1 = \frac{n_1\,f_2\,\phi_1 + n_2\,f_1\,\phi_2}{\sqrt{n_1^2\,f_2^2 + n_2^2\,f_1^2}}\,, \quad \psi_2 = \frac{n_2\,f_1\,\phi_1 - n_1\,f_2\,\phi_2}{\sqrt{n_1^2\,f_2^2 + n_2^2\,f_1^2}}\,, \tag{4.12}$$

the expression of axionic potential eq. (4.9) can be adjusted into the form as below

$$V(\psi_1, \psi_2) = \Lambda_1 \left(1 - \cos\left[\frac{\psi_1}{f_1'}\right]\right) + \Lambda_2 \left(1 - \cos\left[\frac{\psi_1}{f_2'} + \frac{\psi_2}{f_{\mathrm{eff}}}\right]\right)\,, \tag{4.13}$$

where

$$f_1' = \frac{f_1\,f_2}{\sqrt{n_1^2\,f_2^2 + n_2^2\,f_1^2}}\,, \quad f_2' = \frac{f_1\,f_2\,\sqrt{n_1^2\,f_2^2 + n_2^2\,f_1^2}}{\left(n_1\,m_1\,f_2^2 + n_2\,m_2\,f_1^2\right)}\,,$$

and

$$f_{\mathrm{eff}} = \frac{\sqrt{n_1^2\,f_2^2 + n_2^2\,f_1^2}}{|n_1\,m_2 - n_2\,m_1|}\,. \tag{4.14}$$

Assuming a reasonable hierarchy $\Lambda_2 \ll \Lambda_1$, a justified diagonalization follows with a heavy (ψ_1) and a light (ψ_2) axionic combination. Further, stabilizing the heavier axion at its minimum $\overline{\psi}_1 = 0$ leads to a single-field natural inflation driven by a trans-Planckian axion as below

$$V(\psi_2) = \Lambda_2 \left(1 - \cos\left[\frac{\psi_2}{f_{\mathrm{eff}}}\right]\right)\,. \tag{4.15}$$

Before the detailed numerical analysis towards inflationary aspects, let us exemplify the moduli stabilization part by providing a benchmark sampling as below

$$W_{\mathrm{cs}} = -12,\quad N_0 = 15,\quad \xi_B = 1 = \xi_S,\quad C_{\alpha'} = 4.6,\quad A_0 = 0.1,\quad g_s = 0.35,\quad \kappa_{B11} = -1, \tag{4.16}$$

$$N_1 = 30,\quad N_2 = 32,\quad h_1 = 15,\quad h_2 = 1,\quad h_3 = 16,\quad h_4 = 1,\quad A_s = 10,\ B_s = 1.$$

Using these samplings in eq. (4.7), eq. (4.10), eq. (4.11) and eq. (4.14), one gets

$$\overline{\mathcal{V}} \simeq 925.7\,, \qquad \overline{\tau}_s \simeq 2.99\,, \qquad f_1 \simeq 0.248\,,\quad f_2 \simeq 0.105,$$

$$|V_{\mathrm{LVS}}| \simeq 5.0 \times 10^{-7}\,,\quad \Lambda_1 \simeq 1.78 \times 10^{-8}\,,\quad \Lambda_2 \simeq 1.73 \times 10^{-9}\,,$$

$$f_1' \simeq 0.078\,, \qquad f_2' \simeq 0.078\,, \qquad f_{\mathrm{eff}} \simeq 8.131. \tag{4.17}$$

Here, the parameters are chosen such that one gets

- the effective decay constant $f_{\mathrm{eff}} > 7$. It is the minimal values to fit the BICEP2 data. To match the PLANCK result, we can relax the constrain to $f_{\mathrm{eff}} > 4$ as we will systematically explore in the numerical section. For the sampling eq. (4.16) and eq. (4.17), the enhancement of decay constant can be seen from figure 1.

- $\Lambda_2 \simeq 10^{-9}$, which is needed to have a high inflationary Hubble scale $H_{\mathrm{inf}} \simeq 10^{14}\,\mathrm{GeV}$ as per the requirement of constraints given in eq. (1.1) or equivalently the magnitude of scalar power spectrum being $P_s \simeq 2.2 \times 10^{-9}$ given in eq. (1.2).

- Two hierarchies: $|V_{\mathrm{LVS}}| \gg V_{\mathrm{rest}}$ and $\Lambda_1 \gg \Lambda_2$. These are implicitly needed for reaching the single field inflationary potential via step-by-step route with hierarchial check-points.
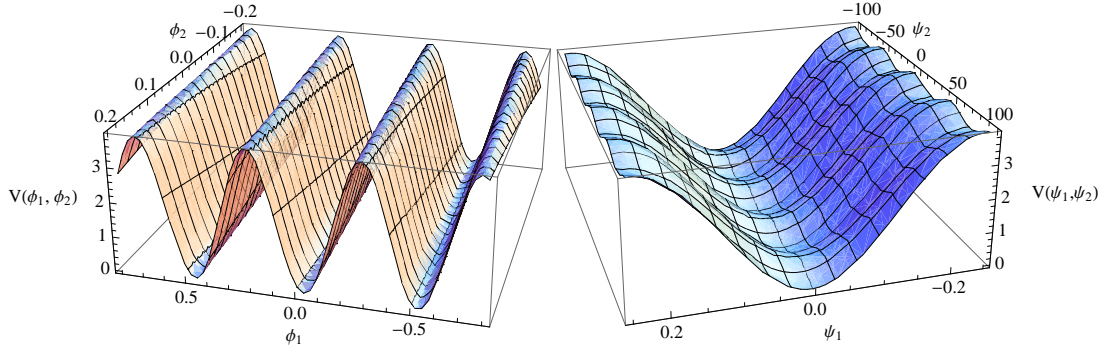
Figure 1. The two field potentials $V(\phi_1, \phi_2)$ and $V(\psi_1, \psi_2)$ (multiplied by 10^8) respectively given in eq. (4.9) and eq. (4.13) are plotted for the sampling eq. (4.16). The second figure shows the enhanced decay constant for ψ_2 direction as compared to the sub-Planckian ones shown in the first figure.

On validity of the single field inflationary approach. To justify that single field approach is a valid description, i.e. the heavier fields, set to their respective minima, do not significantly get shifted while the lightest axionic combination is in inflating phase, let us consider the dynamics of next-to-light field ψ_1 and perform the field evolution analysis of two-field inflationary potential (4.13). The dynamics is governed by the Einstein-Friedmann equations given as

$$\frac{d^2}{dN^2}\psi^a + \Gamma^a_{bc}\frac{d\psi^b}{dN}\frac{d\psi^c}{dN} + \left(3 + \frac{1}{H}\frac{dH}{dN}\right)\frac{d\psi^a}{dN} + \frac{\mathcal{G}^{ab}\partial_b V}{H^2} = 0, \tag{4.18a}$$

$$H^2 = \frac{1}{3}\left(V(\psi^a) + \frac{1}{2}H^2\,\mathcal{G}_{ab}\frac{d\psi^a}{dN}\frac{d\psi^b}{dN}\right), \tag{4.18b}$$

where we use the background N e-folding number as independent evolution coordinate with $dN = Hdt$. Here, let us recall that given the shift symmetry in C_0 and C_2 directions in the Kähler potential, for the canonically normalized fields ϕ^as (with $a = 1, 2$), one has the kinetic matrix $\mathcal{G}^{ab} = \delta^{ab}$. It holds in the new ψ^a basis as well.[4] So all Christophel connections vanish. In addition, using expressions (4.18a) and (4.18b), one can derive another useful expression for variation of Hubble rate in terms of e-folding,

$$\frac{1}{H}\frac{dH}{dN} = \frac{V}{H^2} - 3\,. \tag{4.19}$$

Now let us numerically solve these evolution equations for model dependent sampling given in (4.16)–(4.17), and see the dynamics of heavier (ψ_1) and lighter (ψ_2) axionic combinations during inflationary process. To get the full trajectories from a second order differential equations, we choose $\frac{d\psi^a}{dt}\frac{d\psi_a}{dt}|_{t=0} = 0$ along with following five initial conditions for $\psi^a(0)$, $\forall\, a \in \{1, 2\}$ as given in table 1. Utilizing the numerical solutions, various inflationary trajectories are plotted in figure 2. This figure also shows that depending on the initial condition, the field ψ_1 gets settled in its nearest minimum, for example trajectory with initial condition $\psi_1 = 0.1$ settles in the minimum at zero while the one with $\psi_1 = 0.3$ settles in a nearby minimum. Moreover, figure 2 also confirms that heavier axionic combination does not get involved

---

[4]A very quick check for kinetic sector metric to be diagonal with $\mathcal{G}_{ab} = \delta_{ab}$ in ψ^a basis is: $ds^2 = d\phi_1^2 + d\phi_2^2 = d\psi_1^2 + d\psi_2^2 = \delta_{ab}d\psi^a d\psi^b$ using relations in (4.12).

	I_1	I_2	I_3	I_4	I_5
$\psi_1(0)$	0.00	0.30	0.10	0.25	0.12
$\psi_2(0)$	14.0	15.0	16.0	16.8	17.5
N_F	57	69	79	89	98

Table 1. Five initial conditions studied for sampling given in (4.16)–(4.17).
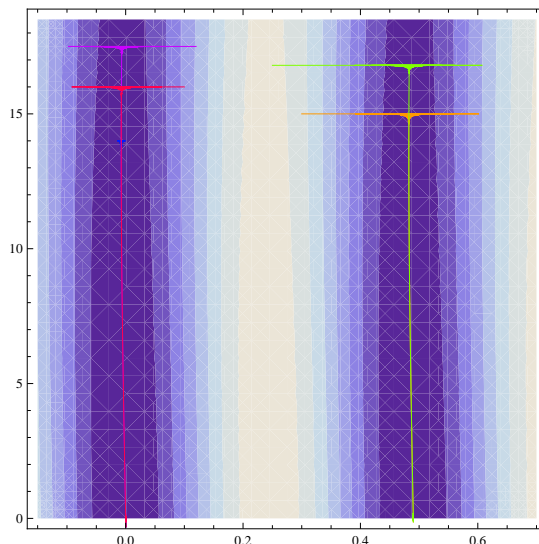

Figure 2. The various inflationary trajectories for five initial conditions. The two minima in dark blue are separated by maxima in light blue shade.

in the inflationary process, although it undergoes a negligible shift from its minimum. Except the first set of initial conditions I_1 which already corresponds to a single field inflation, for other trajectories $I_2 - I_5$ one observes some initial oscillations in ψ_1 direction which is just an artifact of initial conditions $\frac{d\psi^a}{dt}\frac{d\psi_a}{dt}|_{t=0} = 0$. These oscillations die off within a couple of e-foldings as seen in figures 3–4 and result in an effectively single field inflationary process.

This analysis has been done to justify a mass-hierarchy $m_{\psi_1} \gg m_{\psi_2}$ being maintained in such a way that the dynamics of two fields effectively lead to a single field inflationary process. The same has been done via designing a hierarchy $\Lambda_1 \gg \Lambda_2$ through appropriate sampling of model dependent parameters.

5 Detailed inflationary investigations

Revisiting the standard natural inflation. Let us recall the relevant features of standard natural inflation by checking the consistency requirements of cosmological observables from the PLANCK and BICEP2 data. Usually it is qualitatively mentioned that the decay constant for axion utilized in the natural inflation must be trans-Planckian, i.e., much larger than the reduced Planck scale $M_{\rm Pl}$. The realization of large decay constant in string models has always been a challenge, and in one way or the other, the choice of model dependent parameters are crucially affected (and in confrontation within) to accommodate the observables in best possible manner. Therefore, we revisit this aspect to quantify the decay constant window needed to fulfill the minimal experimental requirements. For a given single
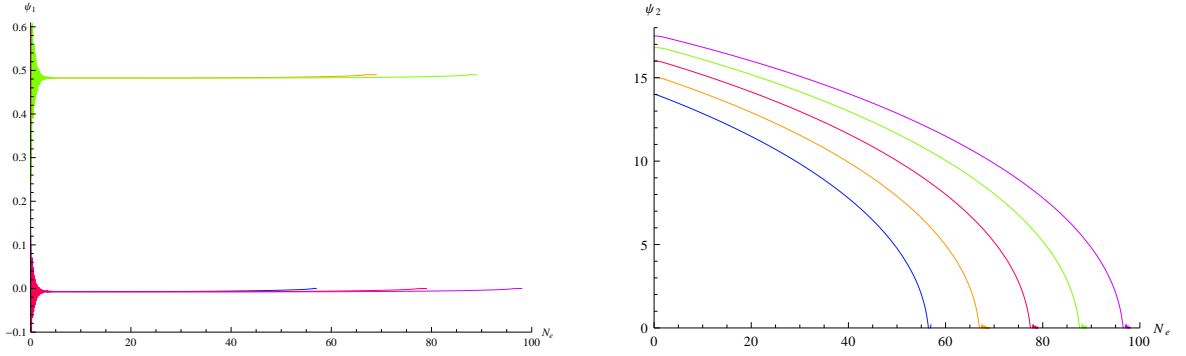
Figure 3. Evolution of heavier (ψ_1) and lighter (ψ_2) axionic combinations during inflationary process reflecting its single field nature.
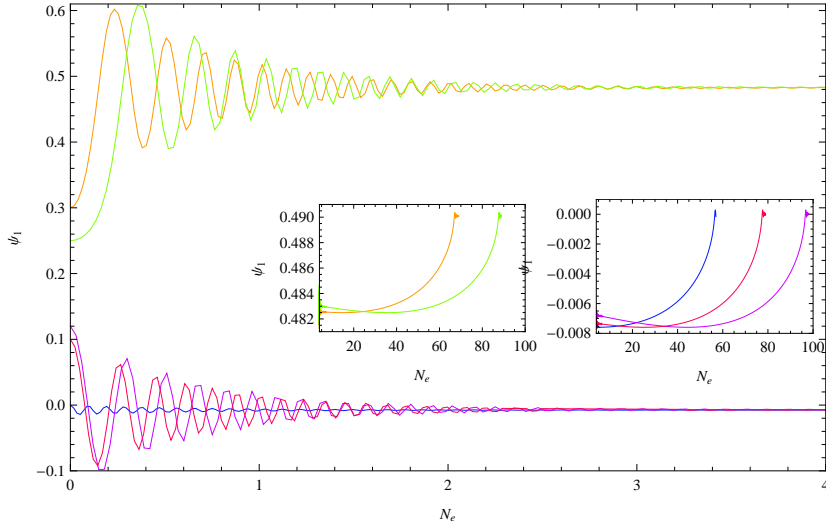
Figure 4. A closer look at the evolution of heavier field ψ_1 showing a negligible shift from their respective minima in each of the five inflationary trajectories. The shift in heavier field ψ_1 is less than 0.01 within the inflationary regime.

field potential $V(\phi)$, the sufficient condition for ensuring the slow-roll inflation is encoded in a set of so-called slow-roll parameters defined as below

$$\epsilon \equiv \frac{1}{2}\left(\frac{V'}{V}\right)^2 \ll 1\,, \quad \eta \equiv \frac{V''}{V} \ll 1\,, \quad \xi \equiv \frac{V'\,V'''}{V^2} \ll 1\,, \tag{5.1}$$

where ′ denotes the derivative of the potential w.r.t. the inflaton field ϕ. Also, the above expressions are defined in the units of reduced Planck mass $M_{\rm Pl}$ with $M_{\rm Pl} = 2.44 \times 10^{18}\,{\rm GeV}$.

The various cosmological observables such as the number of e-foldings N_e, scalar power spectrum P_s, tensorial power spectrum P_t, tensor-to-scalar ratio r, scalar spectral index n_s, and running of spectral index α_{n_s} can be written as the various derivatives of the inflationary

potential via introducing the aforementioned slow-roll parameters as follows

$$N_e \equiv \int_{\phi_{\text{end}}}^{\phi_*} \frac{1}{\sqrt{2\epsilon}}\, d\phi\,,$$

$$P_s \equiv \left[\frac{H^2}{4\,\pi^2\,(2\,\epsilon)} \left(1 - \left(2\,C_E - \frac{1}{6} \right) \epsilon + \left(C_E - \frac{1}{3} \right) \eta \right)^2 \right]\,,$$

$$r \simeq 16\,\epsilon \left[1 - \frac{4}{3}\,\epsilon + \frac{2}{3}\,\eta + 2\,C_E\,(2\,\epsilon - \eta) \right]\,,$$

$$n_s \equiv \frac{d\ln P_s}{d\ln k} \simeq 1 + 2\left[\eta - 3\,\epsilon - \left(\frac{5}{3} + 12\,C_E \right) \epsilon^2 + (8\,C_E - 1)\epsilon\,\eta \right.$$
$$\left. + \frac{1}{3}\,\eta^2 - \left(C_E - \frac{1}{3} \right) \xi \right]\,,$$

$$\alpha_{n_s} \equiv \frac{dn_s}{d\ln k} \simeq 16\,\epsilon\,\eta - 24\,\epsilon^2 - 2\,\xi\,,$$

where $C_E = -2 + 2\ln 2 + \gamma \simeq -0.73$, $\gamma = 0.57721$ being the Euler-Mascheroni constant.

For the standard singe field natural-inflation potential

$$V(\phi) = \Lambda_0 \left(1 - \cos\left[\frac{\phi}{f} \right] \right)\,, \tag{5.2}$$

the slow-roll parameters as well as the three main cosmological observables (n_s, r and α_{n_s}) to be constrained as per the relations in eq. (1.1)–(1.2) are simplified as below

$$\epsilon(\phi) = \frac{\cot\left[\frac{\phi}{2f} \right]^2}{2\,f^2}\,, \quad \eta(\phi) = \frac{\cos\left[\frac{\phi}{f} \right]\csc\left[\frac{\phi}{2f} \right]}{2\,f^2}\,, \quad \xi(\phi) = -\frac{\cot\left[\frac{\phi}{2f} \right]^2}{f^4}\,, \tag{5.3}$$

$$N_e(\phi) = -2\,f^2 \ln\left[\cos\left[\frac{\phi}{2\,f} \right] \right] - N_e^{\text{end}}\,,$$

$$r(\phi) = \frac{4\left(-2 + 6\,C_E + 3\,f^2 - 3\,f^2 \cos\left[\frac{\phi}{f} \right] \right)\cot\left[\frac{\phi}{2f} \right]^2 \csc\left[\frac{\phi}{2f} \right]^2}{3\,f^4}\,,$$

$$n_s(\phi) = -\left[\left\{ 17 + 60\,C_E + 30\,f^2 - 18\,f^4 + 8\left(4 + 6\,C_E - 3\,f^2 + 3\,f^4 \right)\cos\left[\frac{\phi}{f} \right] \right\} \right.$$
$$\left. - \left(-7 + 12C_E + 6f^2 + 6f^4 \right)\cos\left[\frac{2\,\phi}{f} \right] \right] \times \frac{\csc\left[\frac{\phi}{2f} \right]^4}{48\,f^4}\,,$$

$$\alpha_{n_s}(\phi) = -\frac{\csc\left[\frac{\phi}{2f} \right]^6 \sin\left[\frac{\phi}{f} \right]^2}{2\,f^4}\,,$$

where $N_e^{\text{end}} = f^2 \ln\left[1 - \frac{1}{2\,f^2} \right]$ is evaluated at $\epsilon = 1$ where inflation ends. Further, N_e^{end} values fall in the range $\{0.41, 0.50\}$ for decay constant lying inside $\{1, 16\}$.

Number of e-foldings N_e. Because the natural inflation potential has a maximum at $\phi = \pi f$, depending on the decay constant, there is an upper limit on N_e which can be realized for a given f. It can be shown that even $f = 1$ can generate around 20 e-foldings as shown in figure 5, while $f = 2$ can result in a maximal value of N_e around 80. However, in order to have $|n_s - 1| < 0.05$ one needs larger decay constant.
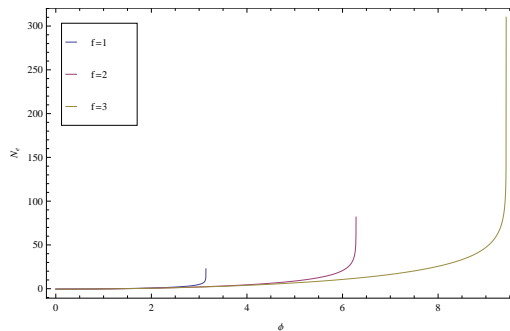
Figure 5. Number of e-foldings N_e versus inflaton field. Here, the largest possible values of N_e are shown and the fast enhancement at the end is due to the field values approaching towards the maxima of the potential ($\phi_{\max} = \pi f$). Here f varies from 1 to 3 in the upward direction.
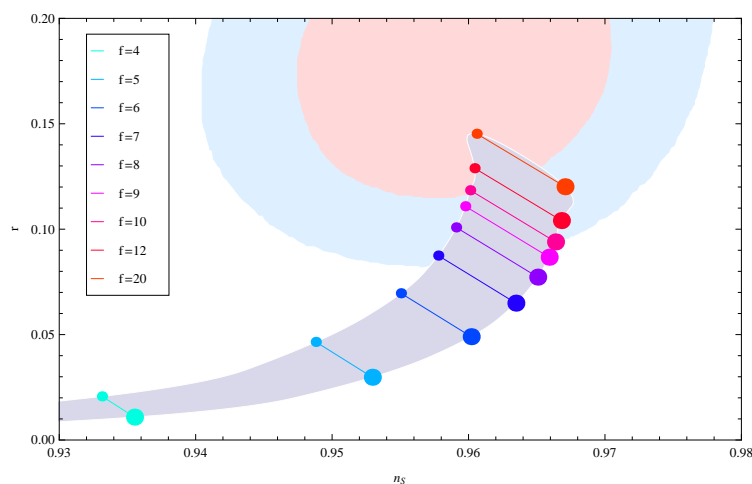
Figure 6. n_s versus r for various effective decay constants from $f = 4$ to $f = 20$. The blue and red region are respectively the 2σ and the 1σ regions of r and n_s for BICEP2. The number of e-folding is from 50 (small circle) to 60 (big circle).

The spectral index n_s and tensor-to-scalar ratio r. Although more than sixty number of e-foldings can be generated even with the decay constant in the range $1 < f < 2$, the fitting of the spectral index n_s and tensor-to-scalar ratio r pushes the f window towards $f > 4$. To be more precise, one finds that for $1 < f < 4$, the spectral index lies in the range $0.1 < n_s < 0.9$ while increasing the decay constant values enhances the spectral index. As can be seen from the figure 6, for e-foldings $50 < N_e < 60$, the decay constant f should be in the range of $4 < f < 12$ in order to be consistent with PLANCK result, and larger than $f > 7$ in order to fall in the 2σ regions of r and n_s for the BICEP2 data.

The running of spectral index α_{n_s} is small. It needs to be of the same order (10^{-2}) to reconcile the PLANCK and BICEP2 data eq. (1.2). This confrontation has been investigated recently in [83] as can be also seen from figure 7. Again, it shows that in order to be consistent with both the PLANCK and BICEP2 data, one needs larger decay constant. However, getting larger decay constant always results in a larger rank of gauge group for gaugino condensation. Of course, f should not be too large. In our case, we constrain the decay constant f to be less than 20 for a natural choice of parameters.
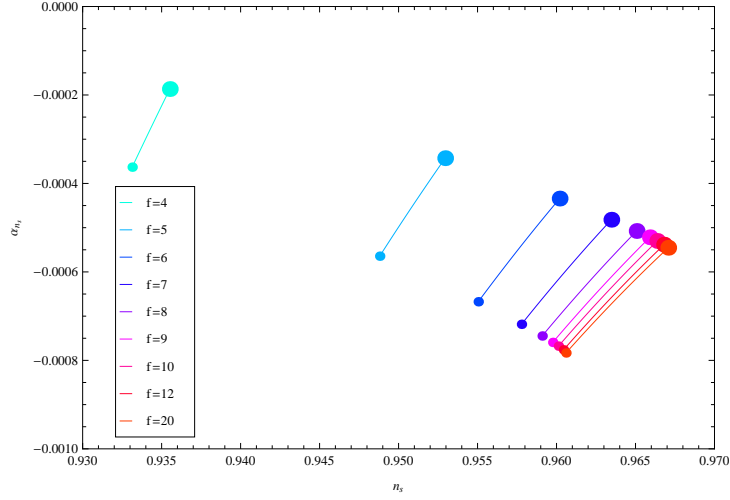
Figure 7. α_{n_s} versus r for decay constants varying from 4 to 20. The number of e-folding is from 50 (small circle) to 60 (big circle).

Benchmark points in our aligned natural inflation models. As shown in figure 6 and the analysis done in the previous subsection, the best fit requirement for n_s and r values (from the PLANCK and BICEP2 observations) demands the decay constant to be within $4 < f_{\rm eff} < 20$. Further, as we have already matched our aligned natural inflationary potential (4.15) with the standard form given in (5.2), and having all the cosmological observables related analysis been revisited already, now all we need to do is to realize a large decay constant. For our samplings we would focus in the range $7 < f_{\rm eff} < 12$.

Before the explicit numerical analysis and sampling of model dependent parameters, let us make the following important points

- although the form of scalar potential suggests that the two fields ϕ_1 and ϕ_2 in eq. (4.9) or ψ_1 and ψ_2 in eq. (4.13) should be arbitrarily interchangable, the canonical normalizations fix the choice for a given sampling. This argument is in the sense that the decay constants of the two axions are different

$$f_1 \equiv f_{c_0} \simeq \frac{g_s}{\sqrt{2}}\,, \qquad f_2 \equiv f_{c^1} \simeq \frac{\sqrt{-3\,g_s\,\kappa_{B11}}\,\xi_B^{1/3}}{\mathcal{V}^{1/3}}\,.$$

  In the large volume limit, one naturally expects $f_1 > f_2$, and this hierarchy restricts the interchangeability of the two fields in eq. (4.9). The reason for considering this constraint $f_1 > f_2$ is to put a lower bound on the volume of the CY such that $\mathcal{V} > g_s^{-3/2}$ for natural orientifold constructions. The positive definiteness of kinetic sector demands $\kappa_{B11} < 0$, and some explicit examples of swiss-cheese Calabi-Yau orientifolds with such intersections can be found in [67].

- As discussed below eq. (4.8), if we want to neglected the subleading corrections which are doubly suppressed in $e^{-a_s\,h_1/g_s}$ or $e^{-b_s\,h_3/g_s}$ (or a product of the two factors) with an inherent assumption that $e^{-a_s\,h_1/g_s} \simeq e^{-b_s\,h_3/g_s}$, we have to choose $a_s\,h_1 \simeq b_s\,h_3$ for consistancy, or equivalently

$$\frac{h_1}{N_1} \simeq \frac{h_3}{N_2}\,,$$

  where N_1 and N_2 are the ranks of the gauge groups corresponding to the gaugino condensations.

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	W_{cs}	N_0	A_0	$C_{\alpha'}$	g_s	$\overline{\mathcal{V}}$	$\overline{\tau}_s$	$|V_{\mathrm{LVS}}|$	f_1	f_2
$S1$	-12	15	0.1	4.6	0.35	925.7	2.99	5.0×10^{-7}	0.248	0.105
$S2$	-10	3	0.4	5.1	0.35	849.6	1.68	1.8×10^{-7}	0.248	0.108
$S3$	-14	6	0.8	7.5	0.28	421.3	2.35	6.0×10^{-6}	0.198	0.122
$S4$	-20	14	0.1	2.8	0.40	909.5	2.61	9.5×10^{-7}	0.283	0.113
$S5$	-18	16	0.1	3.5	0.30	1024.7	2.99	6.7×10^{-7}	0.212	0.094
$S6$	-11	8	0.2	5.8	0.29	688.6	2.28	9.0×10^{-7}	0.205	0.105

Table 2. The six benchmark samplings for model dependent parameters to stabilize the moduli at large volume minima. Here, $\xi_B = 1 = \xi_S$ and $\kappa_{B11} = -1$ have been used.

	A_s	B_s	h_1	h_3	Λ_1	Λ_2	f'_1	f'_2	f_{eff}
$S1$	10	1	15	16	1.8×10^{-8}	1.7×10^{-9}	0.078	0.078	8.131
$S2$	14	3	17	18	1.7×10^{-9}	3.7×10^{-10}	0.069	0.069	9.452
$S3$	12	4	15	16	5.2×10^{-9}	1.7×10^{-9}	0.063	0.063	9.394
$S4$	8	1	19	20	9.5×10^{-9}	1.3×10^{-9}	0.071	0.072	11.035
$S5$	25	5	18	19	1.6×10^{-9}	3.6×10^{-10}	0.056	0.057	8.694
$S6$	10	2	13	14	1.1×10^{-8}	2.0×10^{-9}	0.075	0.074	7.071

Table 3. The manifestation of effective large decay constant and the hierarchial scales Λ_i's for the six benchmark samplings presented in table 2. Here, the ranks of gauge groups are chosen to be $N_1 = 30$ and $N_2 = 32$ while the additional flux parameters are set to be $h_2 = 1 = h_4$.

- Further, while choosing the flux parameters, one has to take care of the requirement of significant suppressions from factors $e^{-a_s h_1/g_s}$ as well as $e^{-b_s h_3/g_s}$ to trust the hierarchy of masses used for reaching the single field potential. This requirement usually results in a larger value of h_1 and h_3. Also, h_1 and h_3 should be larger than h_2 and h_4 from the different flux dependence on S and G^1.

Several benchmark points in table 2 and 3 have been presented for various model dependent parameters to realize consistent r and n_s values as shown in figure 6.

6 Open challenges and conclusion

In this paper, we have successfully embedded the idea of KNP [22] for the enhancement of axion decay constant relevant for realizing the Natural inflation. The inflaton is identified with a linear combination of the universal axion c_0 and an involutively odd axion c^1. The expressions of decay constants for these two axions enjoy appearance of string coupling g_s and the Calabi-Yau volume $\mathcal{V}$ with a less suppressed factor as compared to the C_4 axions. Moreover, their decouplings in the kinetic sector via the Kähler potential are more natural in large volume limit as compared to the case of considering two C_2 axionic setup as then, one has to diagonalize the intersection matrix κ_{Sab} along the odd directions a and b. Despite of the several nice features of our model, there are certain assumptions to be consistently realized in concrete setups, especially on the technical grounds. On these lines, let us recall that the original universal axion monodromy inflation [35] has two delicate issues as below.

- The decay constant for universal axion c_0 is given as $f_{c_0} = \frac{g_s}{\sqrt{2}}$, and so natural inflation embedding demands string coupling to be in the window $1 < g_s < 10$ and thus pushing the whole description into the non-perturbative regime.

  Our approach of realizing the KNP-type inflation with inflaton being a combination of the universal axion c_0 and the odd axion c^1 provides a natural way of enhancing the decay constant in the regime where the perturbative description remains trustfully valid along with the support of large volume scenarios.

- The second delicate assumption of inflationary model in [35] is related to facilitate a hierarchy in the dilaton and universal axion at the tree-level superpotential. This flux superpotential depends on the landscape of background fluxes and it would be interesting to construct the explicit models in which this requirement could be satisfied.

In addition to the second point, it would be interesting to address more technical issues like the tadpole/anomaly cancellations in concrete Calabi-Yau orientifold examples with all the suitable gauge fluxes arranged through the incorporation of relevant involutively odd two-cycles to contribute the non-perturbative effects. Further, the trans-Planckian nature of the inflaton opens up some more challenges and hence there are some cautionary concerns on the lines of [11–13, 84–86]. One of such concerns could be the inflaton coupling to the gauge degrees of freedom living on the two stacks of $D7$-brane wrappings with magnetic-fluxes turned-on, and those could be of the following kind

$$\mathcal{L} \supset \frac{\phi_1}{f_1} \left[\frac{a_s\, h_1}{32\,\pi^2}\, F_{\mu\nu 1}\, F^{\mu\nu 1} + \frac{b_s\, h_3}{32\,\pi^2}\, F_{\mu\nu 2}\, F^{\mu\nu 2} \right] \tag{6.1}$$
$$+ \frac{\phi_2}{f_2} \left[\frac{a_s\, h_2}{32\,\pi^2}\, F_{\mu\nu 1}\, F^{\mu\nu 1} + \frac{b_s\, h_4}{32\,\pi^2}\, F_{\mu\nu 2}\, F^{\mu\nu 2} \right],$$

where $a_s = \frac{2\pi}{N_1}$ and $b_s = \frac{2\pi}{N_2}$ with N_i's being the ranks of gauge groups. In order to avoid the gauge degrees of freedom being supermassive by acquiring the trans-Planckian masses out of the axion vacuum expectation values, one has to ensure that the overall coupling still remains under control. For that one has to have large rank of gauge groups which comes out to be an unnatural requirement beyond a certain value. Also in our setup, we have required some of gauge flux parameters (h_1 and h_3) to be relatively large (of order 10) to sustain the mass-hierarchy, and this has to be done in a consistent manner by not letting these fluxes become very large. Although in our samplings we have successfully realized large effective decay constant with the rank of the gauge groups being not too large ($N_1 = 30$ and $N_2 = 32$), it would be interesting to increase the number of the odd axions and take the requirement for the rank of gauge group to be below ten on the lines of [47]. It suggests an exponential enhancement of the decay constant with increasing the number of axions in the KNP formalism.

Acknowledgments

References

[1] BICEP2 collaboration, P.A.R. Ade et al., *Detection of B-Mode Polarization at Degree Angular Scales by BICEP2*, *Phys. Rev. Lett.* **112** (2014) 241101 [`arXiv:1403.3985`] [INSPIRE].

[2] PLANCK collaboration, P.A.R. Ade et al., *Planck 2013 results. XVI. Cosmological parameters*, *Astron. Astrophys.* (2014) [`arXiv:1303.5076`] [INSPIRE].

[3] WMAP collaboration, G. Hinshaw et al., *Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Parameter Results*, *Astrophys. J. Suppl.* **208** (2013) 19 [`arXiv:1212.5226`] [INSPIRE].

[4] S. Choudhury and A. Mazumdar, *Reconstructing inflationary potential from BICEP2 and running of tensor modes*, `arXiv:1403.5549` [INSPIRE].

[5] Y.-Z. Ma and Y. Wang, *Reconstructing the Local Potential of Inflation with BICEP2 data*, *JCAP* **09** (2014) 041 [`arXiv:1403.4585`] [INSPIRE].

[6] Q. Gao, Y. Gong and T. Li, *The Modified Lyth Bound and Implications of BICEP2 Results*, `arXiv:1405.6451` [INSPIRE].

[7] D.H. Lyth, *What would we learn by detecting a gravitational wave signal in the cosmic microwave background anisotropy?*, *Phys. Rev. Lett.* **78** (1997) 1861 [`hep-ph/9606387`] [INSPIRE].

[8] N. Kaloper and A. Lawrence, *Natural Chaotic Inflation and UV Sensitivity*, *Phys. Rev.* **D 90** (2014) 023506 [`arXiv:1404.2912`] [INSPIRE].

[9] L.E. Ibáñez and I. Valenzuela, *BICEP2, the Higgs Mass and the SUSY-breaking Scale*, `arXiv:1403.6081` [INSPIRE].

[10] K. Harigaya, M. Ibe, K. Ichikawa, K. Kaneta and S. Matsumoto, *High-Scale SUSY Breaking Models in light of the BICEP2 Result*, *JHEP* **07** (2014) 093 [`arXiv:1403.5880`] [INSPIRE].

[11] D. Chialva and A. Mazumdar, *Super-Planckian excursions of the inflaton and quantum corrections*, `arXiv:1405.0513` [INSPIRE].

[12] D. Baumann and D. Green, *Supergravity for Effective Theories*, *JHEP* **03** (2012) 001 [`arXiv:1109.0293`] [INSPIRE].

[13] C.P. Burgess, M. Cicoli, F. Quevedo and M. Williams, *Inflating with Large Effective Fields*, `arXiv:1404.6236` [INSPIRE].

[14] A.D. Linde, *Chaotic Inflation*, *Phys. Lett.* **B 129** (1983) 177 [INSPIRE].

[15] M. Kawasaki, M. Yamaguchi and T. Yanagida, *Natural chaotic inflation in supergravity*, *Phys. Rev. Lett.* **85** (2000) 3572 [`hep-ph/0004243`] [INSPIRE].

[16] R. Kallosh and A. Linde, *New models of chaotic inflation in supergravity*, *JCAP* **11** (2010) 011 [`arXiv:1008.3375`] [INSPIRE].

[17] K. Nakayama, F. Takahashi and T.T. Yanagida, *Polynomial Chaotic Inflation in Supergravity*, *JCAP* **08** (2013) 038 [`arXiv:1305.5099`] [INSPIRE].

[18] T. Kobayashi and O. Seto, *Polynomial inflation models after BICEP2*, *Phys. Rev.* **D 89** (2014) 103524 [`arXiv:1403.5055`] [INSPIRE].

[19] X. Gao, T. Li and P. Shukla, *Fractional chaotic inflation in the lights of PLANCK and BICEP2*, `arXiv:1404.5230` [INSPIRE].

[20] K. Freese, J.A. Frieman and A.V. Olinto, *Natural inflation with pseudo Nambu-Goldstone bosons*, *Phys. Rev. Lett.* **65** (1990) 3233 [INSPIRE].

[21] N. Arkani-Hamed, H.-C. Cheng, P. Creminelli and L. Randall, *Extra natural inflation*, *Phys. Rev. Lett.* **90** (2003) 221302 [`hep-th/0301218`] [INSPIRE].

[22] J.E. Kim, H.P. Nilles and M. Peloso, *Completing natural inflation*, *JCAP* **01** (2005) 005 [hep-ph/0409138] [INSPIRE].

[23] M. Czerny and F. Takahashi, *Multi-Natural Inflation*, *Phys. Lett.* **B 733** (2014) 241 [arXiv:1401.5212] [INSPIRE].

[24] A.R. Liddle, A. Mazumdar and F.E. Schunck, *Assisted inflation*, *Phys. Rev.* **D 58** (1998) 061301 [astro-ph/9804177] [INSPIRE].

[25] E.J. Copeland, A. Mazumdar and N.J. Nunes, *Generalized assisted inflation*, *Phys. Rev.* **D 60** (1999) 083506 [astro-ph/9904309] [INSPIRE].

[26] A. Jokinen and A. Mazumdar, *Inflation in large-N limit of supersymmetric gauge theories*, *Phys. Lett.* **B 597** (2004) 222 [hep-th/0406074] [INSPIRE].

[27] S. Dimopoulos, S. Kachru, J. McGreevy and J.G. Wacker, *N-flation*, *JCAP* **08** (2008) 003 [hep-th/0507205] [INSPIRE].

[28] T.W. Grimm, *Axion inflation in type-II string theory*, *Phys. Rev.* **D 77** (2008) 126007 [arXiv:0710.3883] [INSPIRE].

[29] A. Ashoorioon, H. Firouzjahi and M.M. Sheikh-Jabbari, *M-flation: Inflation From Matrix Valued Scalar Fields*, *JCAP* **06** (2009) 018 [arXiv:0903.1481] [INSPIRE].

[30] A. Ashoorioon and M.M. Sheikh-Jabbari, *Gauged M-flation, its UV sensitivity and Spectator Species*, *JCAP* **06** (2011) 014 [arXiv:1101.0048] [INSPIRE].

[31] M. Cicoli, K. Dutta and A. Maharana, *N-flation with Hierarchically Light Axions in String Compactifications*, *JCAP* **08** (2014) 012 [arXiv:1401.2579] [INSPIRE].

[32] L. McAllister, E. Silverstein and A. Westphal, *Gravity Waves and Linear Inflation from Axion Monodromy*, *Phys. Rev.* **D 82** (2010) 046003 [arXiv:0808.0706] [INSPIRE].

[33] R. Flauger, L. McAllister, E. Pajer, A. Westphal and G. Xu, *Oscillations in the CMB from Axion Monodromy Inflation*, *JCAP* **06** (2010) 009 [arXiv:0907.2916] [INSPIRE].

[34] E. Palti and T. Weigand, *Towards large r from $[p, q]$-inflation*, *JHEP* **04** (2014) 155 [arXiv:1403.7507] [INSPIRE].

[35] R. Blumenhagen and E. Plauschinn, *Towards Universal Axion Inflation and Reheating in String Theory*, *Phys. Lett.* **B 736** (2014) 482 [arXiv:1404.3542] [INSPIRE].

[36] T.W. Grimm, *Axion Inflation in F-theory*, arXiv:1404.4268 [INSPIRE].

[37] M. Arends et al., *D7-Brane Moduli Space in Axion Monodromy and Fluxbrane Inflation*, *Fortsch. Phys.* **62** (2014) 647 [arXiv:1405.0283] [INSPIRE].

[38] F. Hassler, D. Lüst and S. Massai, *On Inflation and de Sitter in Non-Geometric String Backgrounds*, arXiv:1405.2325 [INSPIRE].

[39] M. Czerny, T. Higaki and F. Takahashi, *Multi-Natural Inflation in Supergravity and BICEP2*, arXiv:1403.5883 [INSPIRE].

[40] J. Ellis, M.A.G. García, D.V. Nanopoulos and K.A. Olive, *Resurrecting Quadratic Inflation in No-Scale Supergravity in Light of BICEP2*, *JCAP* **05** (2014) 037 [arXiv:1403.7518] [INSPIRE].

[41] A. Hebecker, S.C. Kraus and L.T. Witkowski, *D7-Brane Chaotic Inflation*, *Phys. Lett.* **B 737** (2014) 16 [arXiv:1404.3711] [INSPIRE].

[42] F. Marchesano, G. Shiu and A.M. Uranga, *F-term Axion Monodromy Inflation*, arXiv:1404.3040 [INSPIRE].

[43] S.H.H. Tye and S.S.C. Wong, *Helical Inflation and Cosmic Strings*, arXiv:1404.6988 [INSPIRE].

[44] I. Ben-Dayan, F.G. Pedro and A. Westphal, *Hierarchical Axion Inflation*, `arXiv:1404.7773` [InSPIRE].

[45] C. Long, L. McAllister and P. McGuirk, *Aligned Natural Inflation in String Theory*, *Phys. Rev.* **D 90** (2014) 023501 [`arXiv:1404.7852`] [InSPIRE].

[46] R. Kappl, S. Krippendorf and H.P. Nilles, *Aligned Natural Inflation: Monodromies of two Axions*, *Phys. Lett.* **B 737** (2014) 124 [`arXiv:1404.7127`] [InSPIRE].

[47] K. Choi, H. Kim and S. Yun, *Natural inflation with multiple sub-Planckian axions*, *Phys. Rev.* **D 90** (2014) 023545 [`arXiv:1404.6209`] [InSPIRE].

[48] T. Li, Z. Li and D.V. Nanopoulos, *Chaotic Inflation in No-Scale Supergravity with String Inspired Moduli Stabilization*, `arXiv:1405.0197` [InSPIRE].

[49] T. Li, Z. Li and D.V. Nanopoulos, *Natural Inflation with Natural Trans-Planckian Axion Decay Constant from Anomalous* U(1)$_X$, *JHEP* **07** (2014) 052 [`arXiv:1405.1804`] [InSPIRE].

[50] L. McAllister, E. Silverstein, A. Westphal and T. Wrase, *The Powers of Monodromy*, *JHEP* **09** (2014) 123 [`arXiv:1405.3652`] [InSPIRE].

[51] A. Ashoorioon and M.M. Sheikh-Jabbari, *Gauged M-flation After BICEP2*, `arXiv:1405.1685` [InSPIRE].

[52] T. Kobayashi, O. Seto and Y. Yamaguchi, *Axion monodromy inflation with sinusoidal corrections*, `arXiv:1404.5518` [InSPIRE].

[53] T. Higaki, T. Kobayashi, O. Seto and Y. Yamaguchi, *Axion monodromy inflation with multi-natural modulations*, `arXiv:1405.0775` [InSPIRE].

[54] V. Balasubramanian, P. Berglund, J.P. Conlon and F. Quevedo, *Systematics of moduli stabilisation in Calabi-Yau flux compactifications*, *JHEP* **03** (2005) 007 [`hep-th/0502058`] [InSPIRE].

[55] K. Becker, M. Becker, M. Haack and J. Louis, *Supersymmetry breaking and alpha-prime corrections to flux induced potentials*, *JHEP* **06** (2002) 060 [`hep-th/0204254`] [InSPIRE].

[56] M. Berg, M. Haack and E. Pajer, *Jumping Through Loops: On Soft Terms from Large Volume Compactifications*, *JHEP* **09** (2007) 031 [`arXiv:0704.0737`] [InSPIRE].

[57] M. Cicoli, J.P. Conlon and F. Quevedo, *Systematics of String Loop Corrections in Type IIB Calabi-Yau Flux Compactifications*, *JHEP* **01** (2008) 052 [`arXiv:0708.1873`] [InSPIRE].

[58] J.P. Conlon and F. Quevedo, *Kähler moduli inflation*, *JHEP* **01** (2006) 146 [`hep-th/0509012`] [InSPIRE].

[59] J.R. Bond, L. Kofman, S. Prokushkin and P.M. Vaudrevange, *Roulette inflation with Kähler moduli and their axions*, *Phys. Rev.* **D 75** (2007) 123511 [`hep-th/0612197`] [InSPIRE].

[60] J.J. Blanco-Pillado, D. Buck, E.J. Copeland, M. Gomez-Reino and N.J. Nunes, *Kähler Moduli Inflation Revisited*, *JHEP* **01** (2010) 081 [`arXiv:0906.3711`] [InSPIRE].

[61] M. Cicoli, C.P. Burgess and F. Quevedo, *Fibre Inflation: Observable Gravity Waves from IIB String Compactifications*, *JCAP* **03** (2009) 013 [`arXiv:0808.0691`] [InSPIRE].

[62] M. Cicoli, F.G. Pedro and G. Tasinato, *Poly-instanton Inflation*, *JCAP* **12** (2011) 022 [`arXiv:1110.6182`] [InSPIRE].

[63] R. Blumenhagen, X. Gao, T. Rahn and P. Shukla, *Moduli Stabilization and Inflationary Cosmology with Poly-Instantons in Type IIB Orientifolds*, *JHEP* **11** (2012) 101 [`arXiv:1208.1160`] [InSPIRE].

[64] D. Lüst and X. Zhang, *Four Kähler Moduli Stabilisation in type IIB Orientifolds with K3-fibred Calabi-Yau threefold compactification*, *JHEP* **05** (2013) 051 [`arXiv:1301.7280`] [InSPIRE].

[65] X. Gao, T. Li and P. Shukla, *Cosmological observables in multi-field inflation with a non-flat field space*, *JCAP* **10** (2014) 008 [arXiv:1403.0654] [inSPIRE].

[66] T. Higaki and F. Takahashi, *Natural and Multi-Natural Inflation in Axion Landscape*, *JHEP* **07** (2014) 074 [arXiv:1404.6923] [inSPIRE].

[67] X. Gao and P. Shukla, *On Classifying the Divisor Involutions in Calabi-Yau Threefolds*, *JHEP* **11** (2013) 170 [arXiv:1307.1139] [inSPIRE].

[68] X. Gao and P. Shukla, *F-term Stabilization of Odd Axions in LARGE Volume Scenario*, *Nucl. Phys.* **B 878** (2014) 269 [arXiv:1307.1141] [inSPIRE].

[69] D. Lüst, S. Reffert, E. Scheidegger, W. Schulgin and S. Stieberger, *Moduli Stabilization in Type IIB Orientifolds (II)*, *Nucl. Phys.* **B 766** (2007) 178 [hep-th/0609013] [inSPIRE].

[70] S. Gukov, C. Vafa and E. Witten, *CFT's from Calabi-Yau four folds*, *Nucl. Phys.* **B 584** (2000) 69 [*Erratum ibid.* **B 608** (2001) 477-478] [hep-th/9906070] [inSPIRE].

[71] K. Dasgupta, G. Rajesh and S. Sethi, *M theory, orientifolds and G-flux*, *JHEP* **08** (1999) 023 [hep-th/9908088] [inSPIRE].

[72] T.R. Taylor and C. Vafa, *RR flux on Calabi-Yau and partial supersymmetry breaking*, *Phys. Lett.* **B 474** (2000) 130 [hep-th/9912152] [inSPIRE].

[73] E. Witten, *Nonperturbative superpotentials in string theory*, *Nucl. Phys.* **B 474** (1996) 343 [hep-th/9604030] [inSPIRE].

[74] T.W. Grimm, M. Kerstan, E. Palti and T. Weigand, *On Fluxed Instantons and Moduli Stabilisation in IIB Orientifolds and F-theory*, *Phys. Rev.* **D 84** (2011) 066001 [arXiv:1105.3193] [inSPIRE].

[75] M. Kerstan and T. Weigand, *Fluxed M5-instantons in F-theory*, *Nucl. Phys.* **B 864** (2012) 597 [arXiv:1205.4720] [inSPIRE].

[76] T.W. Grimm, *Non-Perturbative Corrections and Modularity in $N = 1$ Type IIB Compactifications*, *JHEP* **10** (2007) 004 [arXiv:0705.3253] [inSPIRE].

[77] R. Blumenhagen, S. Moster and E. Plauschinn, *Moduli Stabilisation versus Chirality for MSSM like Type IIB Orientifolds*, *JHEP* **01** (2008) 058 [arXiv:0711.3389] [inSPIRE].

[78] R. Blumenhagen, X. Gao, T. Rahn and P. Shukla, *A Note on Poly-Instanton Effects in Type IIB Orientifolds on Calabi-Yau Threefolds*, *JHEP* **06** (2012) 162 [arXiv:1205.2485] [inSPIRE].

[79] M. Bianchi, A. Collinucci and L. Martucci, *Magnetized E3-brane instantons in F-theory*, *JHEP* **12** (2011) 045 [arXiv:1107.3732] [inSPIRE].

[80] P. Berglund and I. Garcia-Etxebarria, *D-brane instantons on non-Spin cycles*, *JHEP* **01** (2013) 056 [arXiv:1210.1221] [inSPIRE].

[81] R. Blumenhagen, M. Cvetič, S. Kachru and T. Weigand, *D-Brane Instantons in Type II Orientifolds*, *Ann. Rev. Nucl. Part. Sci.* **59** (2009) 269 [arXiv:0902.3251] [inSPIRE].

[82] R. Blumenhagen, V. Braun, T.W. Grimm and T. Weigand, *GUTs in Type IIB Orientifold Compactifications*, *Nucl. Phys.* **B 815** (2009) 1 [arXiv:0811.2936] [inSPIRE].

[83] Y. Gong and Y. Gong, *The challenge for single field inflation with BICEP2 result*, *Phys. Lett.* **B 734** (2014) 41 [arXiv:1403.5716] [inSPIRE].

[84] J. Martin and R.H. Brandenberger, *The TransPlanckian problem of inflationary cosmology*, *Phys. Rev.* **D 63** (2001) 123501 [hep-th/0005209] [inSPIRE].

[85] R.H. Brandenberger and J. Martin, *Back-reaction and the trans-Planckian problem of inflation revisited*, *Phys. Rev.* **D 71** (2005) 023504 [hep-th/0410223] [inSPIRE].

[86] R.H. Brandenberger and J. Martin, *Trans-Planckian Issues for Inflationary Cosmology*, *Class. Quant. Grav.* **30** (2013) 113001 [arXiv:1211.6753] [inSPIRE].