CONSTRAINING THE DISTANCE TO INSPIRALING NS-NS WITH EINSTEIN TELESCOPE

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1 Introduction

Einstein Telescope (ET) is a planned third generation gravitational waves detector located in Europe[1]. Its design will be different from currently build interferometers: First, ET will be located underground in order to reduce the seismic noise. The arms length will be 10 km, and the configuration of arms will be different from all interferometers build so far i.e. there will be three tunnels in a triangular shape. ET will consist of three interferometers rotated by a 60deg with respect to each other in one plane. One of the biggest challenges for ET will be to determine sky position and distance to observed sources. If an object is observed in a few interferometers simultaneously one can estimate the position using traingulation from time delays[2], but so far there are no plans for a network of third generation detectors. Another possibility to deal with that problem is by using multimessenger approach, because redshift and sky position could be recovered from electromagnetic observations. However, in most cases of ET detection there will be only gravitational signal. In this paper we present a novel method of estimating distance and position in the sky of merging binaries. While our procedure is not as accurate as the multimessenger method, it can be applied to all observations, not just the ones with electromagnetic counterparts.

2 Distance estimation using one interferometer

For simplicity let us consider the case of observation of a double neutron star. In gravitational waves we will be observing directly two quantities: signal to noise ratio (ρ), which is a complicated function of the source properties, as well as the detector characterization, and redshifted chirp mass ($M_z = (1+z)M_{chirp}$, $M_{chirp} = (M_1 M_2)^{3/5}(M_1+M_2)^{-1/5}$). In this particular case we consider only binaries consisting of two neutron stars of equal masses $M_1 = M_2 = 1.4$ M$_\odot$, so $M_{chirp} = 1.2$ M$_\odot$. The signal to noise ratio in the quadruple approximation for merging double compact objects is well known[3]:

$$\rho \sim \frac{\Theta}{d_L(z)} (M_z)^{5/6} \sqrt{\xi(z)}, \quad (1)$$

where d_L is the luminosity distance, M_z is the redshifted chirp mass, z is the redshift, Θ is a function of sky position and orientation of the source, and ξ is the function that determines fraction of the sensitivity window filled by a signal (it depends on the chirp mass, and for NSNS binaries its value is close to unity). For a given binary that will be observed in the detector, we can measure ξ directly, by measuring the cutoff frequency when the inspiral ceases.

The function Θ depends on the sky position of the source $\Omega(\vartheta, \varphi)$ and on the orientation of the orbit with respect to the line of sight $\Omega_p(\Psi, i)$:

$$\begin{aligned}
\Theta &= 2\sqrt{(1+\cos^2 i)(F_+)^2 + 4\cos^2 i (F_x)^2}, \\
F_+ &= 0.5(1+\cos^2\vartheta)\cos 2\varphi \cos 2\Psi - \cos\vartheta \sin 2\varphi \sin 2\Psi, \\
F_x &= 0.5(1+\cos^2\vartheta)\cos 2\varphi \sin 2\Psi + \cos\vartheta \sin 2\varphi \cos 2\Psi.
\end{aligned} \tag{2}$$

The density of sources in a unit volume can be expressed by:

$$\frac{d^2 n}{dz d\Omega d\Omega_p} = \frac{n(z)}{1+z}\frac{dV}{dz}. \tag{3}$$

The comoving volume is $\frac{dV}{dz} = 4\pi \frac{c}{H_0}\frac{r^2(z)}{E(\Omega,z)}$, and $E(\Omega, z) = \sqrt{\Omega_\Lambda + \Omega_M(1+z)^3}$. Then we obtain for a single detector

$$\begin{aligned}
\frac{dn}{dz} &= \int d\Omega d\Omega_p \frac{n(z)}{1+z}\delta(\rho - \rho_m) \\
&= 4\pi \frac{n(z)}{1+z}\frac{c}{H_0}\frac{r^2(z)}{E(\Omega,z)}\frac{d_L}{8r_0}\left(\frac{1.2}{M_z}\right)^{5/6}\frac{1}{\sqrt{\xi}} \\
&\times P\left(\frac{\rho^m}{8r_0(\frac{M_z}{1.2})^{5/6}\sqrt{\xi}} d_L(z)\right),
\end{aligned} \tag{4}$$

where $n(z)$ is the merger rate, r_0 is the characteristic distance for a given detector (see Table 1 in paper by Taylor[4] for more details), ρ^m is the actual signal-to-noise ratio measured in the detector.

3 Distance estimation using three co-located interferometers

Design of the Einstein Telescope assumes three co-located interferometers lying in the same plane, so the methods for distance estimation based on triangulation will not be possible. However, a single source will be observed by each of the interferometer with a different orientation. There will be three different measurements of signal to noise ratio. That will provide additional information about the observed source and it allows to constrain the distributions obtained in previous section.

The density of sources per unit volume given by Eq. 3 has to be integrated taking into account that we have three conditions to satisfy.We assume that each signal to noise ratio is measured with perfect accuracy:

$$\begin{aligned}
\frac{dn}{dz} &= \int d\Omega d\Omega_p \frac{n(z)}{1+z}\delta(\rho_1 - \rho_1^m)\delta(\rho_2 - \rho_2^m)\delta(\rho_3 - \rho_3^m) \\
&= \int \frac{d^2 n}{dz d\Omega d\Omega_p} 4\pi \frac{n(z)}{1+z}\frac{c}{H_0}\frac{r^2(z)}{E(\Omega,z)}\frac{d_L}{8r_0}\left(\frac{1.2}{M_z}\right)^{5/6}\frac{1}{\sqrt{\xi}(\rho_1^m)^2} \\
&\times \delta\left(\Theta_1 - \frac{\rho_1^m}{8r_0(\frac{M_z}{1.2})^{5/6}\sqrt{\xi}} d_L(z)\right)\delta\left(\frac{\Theta_2}{\Theta 1} - \frac{\rho_2^m}{\rho_1^m}\right)\delta\left(\frac{\Theta_3}{\Theta 1} - \frac{\rho_3^m}{\rho_1^m}\right),
\end{aligned} \tag{5}$$

For illustration we present four cases of binary neutron stars simulated ET observations. The physical parameters of those sources, as well as the observed quantities are shown in Table 1.

Table 1: Physical parameters and observed quantities of four sources. For first three of them (A, B, C) sky position and orientation were chosen randomly from uniform distributions, while in the last case (D) they where choosen to maximize the obtained signal to noise ratio (the binary is optimally oriented).

	Physical parameters						Observed quantities			
	$M_1 = M_2$ [$M_\odot$]	z	ϑ [rad]	ϕ [rad]	Ψ [rad]	i [rad]	M_z [$M_\odot$]	ρ_1^m	ρ_2^m	ρ_3^m
A		0.1	0.53 π	0.82 π	1.30 π	0.70 π	1.34	44.04	94.42	55.95
B	1.4	0.5	0.71 π	0.18 π	1.38 π	0.08 π	1.83	41.46	42.42	45.35
C		1.0	0.34 π	1.61 π	0.64 π	0.66 π	2.44	9.99	10.57	12.69
D		1.0	0	π	π	0	2.44	36.90	36.90	36.90

The results are shown in Fig. 1. It can be clearly seen that our method can constrain distances to with the accuracy of about 20%.

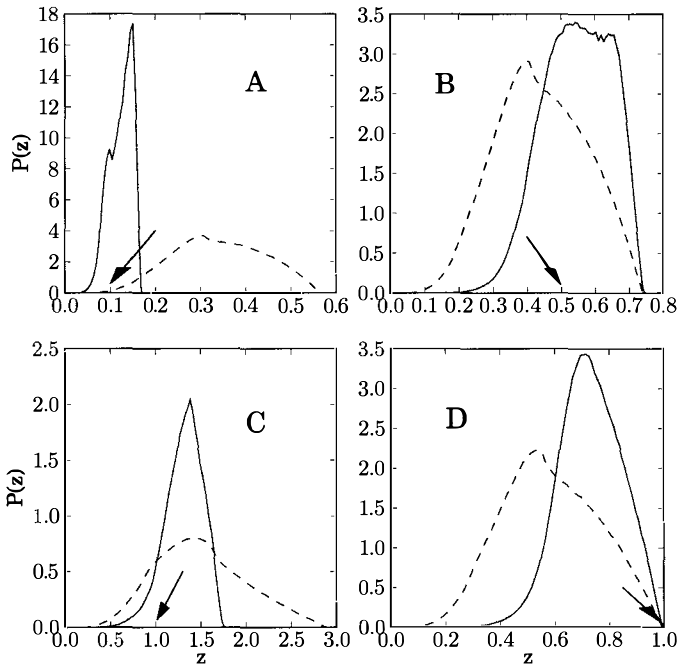

Figure 1 – Normalized redshift distributions for four NSNS system listed in Table 1. Dashed line represent distribution obtained using only one interferometer, while solid line represent distribution taking into account information from all three interferometers. The arrows indicate the position of the source.

4 Summary

Distance measurements to merging binaries will be very challenging in the third generation detectors era. So far, there are no plans for any other detector than Einstein Telescope. In this paper, we presented a method that can be used to constrain distance distribution for a given double neutron star observation. We have shown that it is possible to significantly improve distance estimates using the measurements of the signal to noise ratio from all three interferometers .

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