

# IMPEDANCE MEASUREMENT FOR THE SPEAR3 STORAGE RING\*

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## Abstract

We studied the transverse impedance of the SPEAR3 storage ring with tune shift vs. beam intensity, head-tail instability and transverse mode coupling instability measurements. By taking measurements under different machine conditions, we probed the frequency dependence of the impedance, from which an impedance model was built. This model is consistent with instability measurements and previous bunch lengthening results.

## INTRODUCTION

Impedance of a storage ring is an important factor that determines the performance of the machine. For an operational machine, knowledge of the machine impedance helps one to understand the machine performance and provides input for upgrade considerations.

SPEAR3 is a third generation storage ring light source. Table 1 shows a list of selected parameters relevant to this study. Great care was given to controlling the machine impedance during the design and building phases. Consequently the ring is passively stable under normal operation conditions.

Table 1: Selected Parameters of SPEAR3

Parameters	Value	Unit
Energy	3	GeV
Circumference	234.1	m
Current	500	mA
Tune $\nu_{x,y}$	14.106, 6.177	
RF gap voltage	2.85	MV
Bunch length $\sigma_z$	6.3	mm
Phase slippage $\eta$	$1.62 \times 10^{-3}$	
Synchrotron tune $\nu_s$	0.0093	

During the 2014 and 2015 runs, we took various beam-based measurements in order to determine the transverse impedance of the machine. These include tune shifts vs. bunch current measurements, head-tail instability measurements and single bunch current threshold measurements. Machine conditions, such as momentum compaction factor, chromaticities, fill pattern, and RF gap voltages were changed as needed to probe the spectrum of the impedance. We tried to interpret the measured impedance with a model that includes resistive wall impedance and a broad-band resonator.

In the following we will show measurements and calculations and discuss the impedance model.

## TUNE SHIFT MEASUREMENT

### Vertical Plane

The betatron tune shift of a storage ring is related to the transverse impedance through [1, 2]

$$\frac{\Delta\nu_\beta}{\Delta I_b} = \frac{R}{4\sqrt{\pi}(E/e)\sigma_z} \sum_i \beta_i \text{Im}\{Z_{\perp,i}^{\text{eff}}\}, \quad (1)$$

where  $I_b$  is the bunch current,  $R$  the average ring radius,  $E/e$  beam energy in eV,  $\sigma_z$  the bunch length,  $\beta$  the beta function,  $Z_{\perp,i}^{\text{eff}}$  the effective impedance, and  $\text{Im}\{\cdot\}$  represents taking the imaginary part. The summation over  $i$  is for sections of the ring whose beta functions and impedance contributions may differ. The effective impedance is defined as

$$Z_{\perp}^{\text{eff}} = \frac{\sum_p Z_{\perp}(\omega_p) h(\omega_p - \omega_\xi)}{\sum_p h(\omega_p - \omega_\xi)}, \quad (2)$$

where  $\omega_p = (p + \nu_\beta)\omega_0$ ,  $\nu_\beta$  the betatron tune,  $\omega_0$  the angular revolution frequency,  $\omega_\xi = \xi\omega_0\nu_\beta/\eta$ ,  $\xi$  the chromaticity,  $\eta$  the phase slippage factor, and  $h(\omega)$  the spectral power density function for the  $m = 0$  azimuthal mode. For a Gaussian beam,  $h(\omega) = e^{-\omega^2\sigma_t^2}$ , with  $\sigma_t = \sigma_z/c$ ,  $c$  being the speed of light.

We measured tune shifts vs. bunch current for various machine conditions. Data for the vertical plane are shown in Figure 1. The vertical chromaticity was corrected to zero for the low emittance (LE) lattice data. One of the LE data points was measured with the RF gap voltage reduced to 1.40 MV in order to increase the bunch length. The LE data points at 2.85 MV were taken with 1, 6, 10, and 12 bunches filled, respectively. The low alpha data (LA4) were taken in a lattice for which the momentum compaction factor is reduced to  $2.73 \times 10^{-4}$ . The vertical chromaticity was  $\xi = 0.62$  for the low alpha lattice because we had exhausted our sextupole knobs for second order alpha control. The low alpha measurements were repeated on two shifts, each time with RF gap voltage settings of 3.2, 2.4, 2.1 and 1.4 MV, respectively. All data points were taken with the in-vacuum undulator BL12-2 gap open.

Using the data in Figure 1 and Eq. (1) we can calculate the total effective impedance for the ring under the smooth optics approximation,

$$\text{Im}Z_{\perp}^{\text{eff}} = \frac{1}{R/\nu_y} \sum_i \beta_i \text{Im}\{Z_{\perp,i}^{\text{eff}}\}. \quad (3)$$

The total impedance includes the contribution of resistive wall (RW) impedance, which can be directly calculated, knowing the geometry and surface material of the vacuum chamber. For SPEAR3, the vacuum chamber height in the arcs and in straight sections without insertion devices (ID) is 34 mm. At the ID straight sections, the height ranges from

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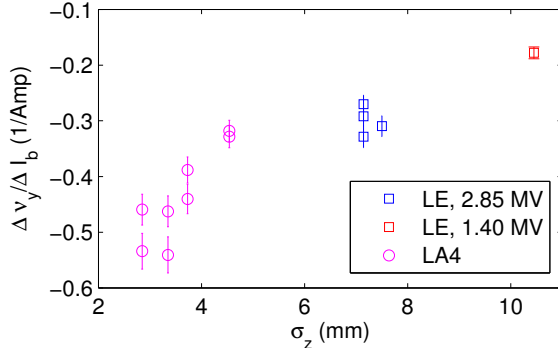


Figure 1: Vertical betatron tune vs. bunch current.

6 mm to 11 mm. The average geometric factor for resistive wall impedance is  $\langle \frac{\beta_y}{b^3} \rangle > \frac{\gamma_y}{R} = 8.06 \times 10^5 \text{ m}^{-3}$  with BL12-2 gap open (22 mm). The entire vacuum chamber is either copper or copper-plated. The RW impedance at the first revolution harmonic is  $Z_{\perp, \text{rw}}(\omega_0) = 1.16 \times (1 - i) \text{ M}\Omega/\text{m}$ . The RW contribution to the ring effective impedance is  $\text{Im}Z_{\perp, \text{rw}}^{\text{eff}} = -0.032 \text{ M}\Omega/\text{m}$  for the (LE, 2.85 MV) cases and  $-0.012 \text{ M}\Omega/\text{m}$  for the LA4 cases.

The differences between the measured total impedance and the RW impedance can be attributed to a broad-band resonator (BBR) impedance model which accounts for contributions of the discontinuities of the vacuum chamber, such as bellows, BPM buttons, stripline kickers, and tapers. The transverse impedance for a BBR model is

$$Z_{\perp}(\omega) = \frac{\omega_r}{\omega} \frac{R_{\perp}}{1 + iQ(\frac{\omega_r}{\omega} - \frac{\omega}{\omega_r})}, \quad (4)$$

where  $\omega_r$  is the resonant frequency,  $R_{\perp} = \frac{c}{\omega_r} R_1$ ,  $R_1$  the shunt impedance for the dipole mode, and  $Q$  the quality factor. The effective impedance for a Gaussian beam with the BBR model is calculated with

$$\frac{Z_{\perp, \text{BBR}}^{\text{eff}}}{R_{\perp}/Q} = \frac{\Delta}{\sqrt{\pi}} \int \frac{1}{x} \frac{e^{-(x-x_{\xi})^2 \Delta^2}}{1/Q + i(\frac{1}{x} - x)} dx, \quad (5)$$

where  $\Delta = \omega_r \sigma_t$  and  $x_{\xi} = \omega_{\xi}/\omega_r$ .

We use a BBR model with  $Q = 1$ . The shunt impedance can be determined from the data points taken with  $\xi = 0$  and taking advantage of the fact that the integral in Eq. (5) approaches  $-i$  when  $x_{\xi} = 0$  and  $\Delta > \sim 2$ . The latter condition is fulfilled for the resonant frequency required to explain the low alpha data points. Figure 2 shows the differences between the measured total vertical effective impedance and the calculated RW contribution. Using the data points from the low emittance lattice, it is found that  $R_{\perp} = 0.165 \pm 0.023 \text{ M}\Omega/\text{m}$ . Also plotted are calculated BBR effective impedances for a few possible resonant frequencies. The resonant frequency  $f_r = \omega_r/2\pi = 37 \text{ GHz}$  appears to fit the measurements well.

The longitudinal BBR impedance is approximately related to the transverse impedance with the relationship [2]

$$Z_{\perp, \text{BBR}}(\omega) = \frac{2c}{b^2} \frac{Z_{\parallel, \text{BBR}}(\omega)}{\omega}. \quad (6)$$

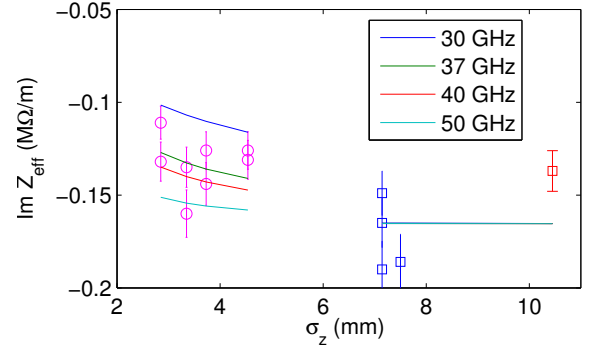


Figure 2: The vertical effective impedance attributed to the BBR model (i.e., the total minus the RW contribution) is compared to calculated values for various candidate resonant frequencies.

This formula gives a longitudinal shunt impedance of  $Z_{\parallel, \text{BBR}} = 7.4 \text{ k}\Omega$ . By fitting measured bunch lengthening data for the LE lattice, Ref. [3] gave a longitudinal BBR model with the shunt impedance of  $10 \text{ k}\Omega$ ,  $Q = 0.7$  and  $f_r = 30 \text{ GHz}$ . A later study adjusted the shunt impedance to  $5.0 \text{ k}\Omega$  to better fit the LE data with the contribution of shielded coherent synchrotron radiation impedance included [4]. Considering  $R_{\perp}/Q$ , the longitudinal BBR model derived from bunch lengthening data is in excellent agreement with the recent transverse impedance measurement.

### Horizontal Plane

We also measured horizontal tune shifts vs. bunch current in the low emittance lattice. The slope showed a strong dependence on the fill pattern. With the RF gap voltage at 2.85 MV, horizontal chromaticity corrected to zero, and bunch length at 7.1 mm (w/ lengthening), the slope was  $\frac{\Delta v_x}{\Delta I_b} = -0.02 \pm 0.002$ ,  $0.071 \pm 0.004$ , and  $0.111 \pm 0.005 \text{ A}^{-1}$  for 1, 6, and 12 bunch uniform fills, respectively, corresponding to a horizontal effective impedance of  $0.031$ ,  $-0.109$ , and  $-0.171 \text{ M}\Omega/\text{m}$ . Note the sign for the single bunch case is opposite to the other two cases. The dependence on fill pattern indicates an effect of a long range wakefield. The magnitude of the horizontal effective impedance and the extent of dependence on fill pattern is inconsistent with a pure resistive wall impedance. We suspect a narrow-band resonant impedance. This topic needs further investigation.

## HEAD-TAIL INSTABILITY MEASUREMENT

The head-tail instability growth rate for the  $m = 0$  mode is given by (air bag model) [2]

$$\frac{1}{\tau} = \frac{1}{\pi} \frac{I_b c}{E e \eta} \xi \int_0^{\infty} dx \text{Re}\{Z_{\perp}(x/\sqrt{2}\sigma_t)\} J_0(x) J_0'(x), \quad (7)$$

where  $J_0(x)$  is the Bessel function, and  $\text{Re}\{\cdot\}$  indicates taking the real part. For our case with  $\eta > 0$ , the  $m = 0$

head-tail mode is stable with a positive chromaticity. With a negative chromaticity, it becomes unstable if the growth rate exceeds the damping rate.

In the experiments we filled the ring with a single bunch with a given current, then reduced the chromaticity until the beam became unstable as indicated by the growth of the tune signal on the spectrum analyzer. The products of the chromaticity and bunch current from the measurements are plotted against the bunch current in Figure 3 for both planes. For the horizontal plane,  $\xi I_b$  stays constant with a value of  $-6.1 \pm 0.3$  mA. For the vertical plane, the product increases nearly linearly with current. This occurs in part because the impedance integral in Eq. (7) decreases as the bunch lengthens. It may also indicate a damping mechanism becomes stronger with the increasing bunch current.

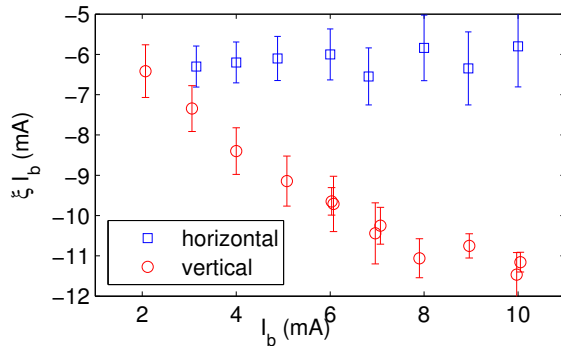


Figure 3: The products of chromaticity and bunch current at the head-tail instability threshold vs. bunch current.

When we plug in the calculated RW impedance and the measured BBR impedance from the last section into Eq. (7) for the vertical plane, the contributions to the impedance integral are found to be  $-16.5$  k $\Omega$ /m for the BBR and  $-9.0$  k $\Omega$ /m for the RW, respectively. Therefore, the growth time for the case with  $I_b = 1$  mA and  $\xi = -1$  is  $\tau_y = 2.0$  ms. If we extrapolate the vertical data in Figure 3 to  $I_b = 1$  mA, the product  $\xi I_b$  would be  $-5.6$  mA. Thus the vertical growth time is 5.6 times smaller, at 0.36 ms. This is 15 times faster than the radiation damping for the vertical plane. This result may need a numerical correction as the calculation is based on the air bag model, which tends to over estimate the results [2]. The rest of the discrepancy may be explained by Landau damping, although further study is required.

## TRANSVERSE MODE COUPLING INSTABILITY MEASUREMENT

The transverse mode coupling instability (TMCI) threshold is given by [1],

$$I_{th} = \frac{8\nu_s f_0 E/e}{<\beta> k_{\perp}(\sigma_t)}, \quad (8)$$

where  $\nu_s$  is the synchrotron tune,  $<\beta> = R/\nu_{\beta}$  the average beta function, and the kick factor is defined as

$$k_{\perp}(\sigma_t) = \frac{1}{\pi} \int_0^{\infty} d\omega \text{Im}\{Z_{\perp}(\omega)\} h(\omega, \sigma_t). \quad (9)$$

The kick factors for the BBR, RW, and total impedances for SPEAR3 are plotted in Figure 4 against the bunch length.

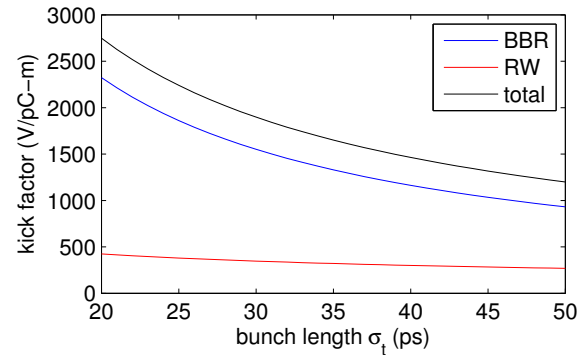


Figure 4: The vertical kick factor vs. bunch length.

In the experiment we measured the single bunch current threshold for SPEAR3 by injecting into one bunch until accumulation stopped, at which point there was typically a step loss of a few milliamps. The bunch current threshold for a few different RF gap voltage levels is plotted in Figure 5. The instability that limits the bunch current is consistent with TMCI. For example, the coherent betatron tune shift at  $I_b = 27$  mA for the case with an RF voltage of 2.85 MV would be 0.008, close to the synchrotron tune of 0.0093. We calculated the TMCI threshold with Eq. (8) and the vertical impedance model and found good agreement. The result is also plotted in Figure 5. The calculation includes the bunch lengthening effect using measurements reported in Ref. [3]. As a rough approximation, the bunch length at the threshold is considered 50% longer than the zero-current limit.

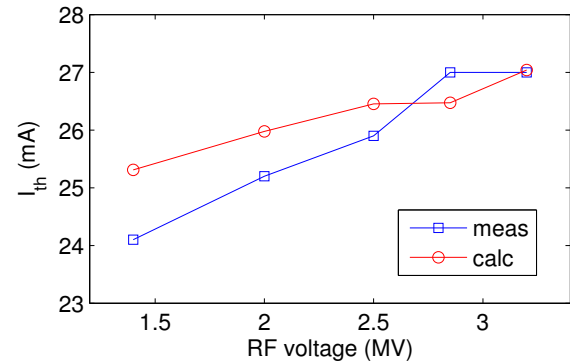


Figure 5: The measured single bunch current threshold for SPEAR3 at various RF gap voltage.

## CONCLUSION

A broad-band resonator impedance model of  $R_{\perp} = 0.165$  M $\Omega$ /m,  $Q = 1$ , and  $f_r = 37$  GHz for the vertical plane is derived from tune shift with beam intensity measurements for SPEAR3. This model agrees with head-tail and TMCI instability threshold measurements. It is also consistent with a previous study of the SPEAR3 longitudinal impedance. Further study is needed to understand the horizontal impedance.

## REFERENCES

- [1] A. Chao, M. Tigner, Handbook of Accelerator Physics and Engineering, World Scientific (1999).
- [2] A. Chao, Physics of Collective Beam Instabilities in High Energy Accelerators, John Wiley & Sons, Inc (1993).
- [3] J. Corbett, W. Cheng, A. Fisher, X. Huang, EPAC08, Genoa, Italy (2008)
- [4] X. Huang, SSRL-AP-Note 47, (April 2013)