A PROBABILITY MEASURE ON PARTON AND STRING STATES

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ABSTRACT: The states obtained in hard high energy processes can, in a complementary way, be described in terms of partonic QCD language or in terms of a stringlike confined force-field spanned by the partons. We define a surface characteristic of a confined force-field which breaks up into hadrons. The negative exponential of this surface multiplied by the appropriate phasespace defines a measure on the QCD-states which is everywhere finite and infrared stable. It reproduces the results of perturbative QCD in the kinematical regions where such results are relevant. It has simple mathematical properties, in particular it is factorizable and in that way generates a stochastic process.

The results I want to discuss in this talk are obtained in collaboration with B. Andersson and B. Söderberg[1].

Hard processes like e^+e^--annihilation or lepton-proton scattering are described as two-step processes

$$\left.\begin{matrix} e^+e^- \\ lp \end{matrix}\right\} \rightarrow \left.\begin{matrix} \text{parton state} \\ \text{quarks and gluons} \end{matrix}\right\} \rightarrow \text{hadron state}$$

The Lund hadronization model attempts to describe the second step, partons → hadrons, and for a complete description we also need the probability to obtain a definite parton state. This is given by perturbative QCD, which however is computationally very difficult. Interference effects are complicated and are often neglected in jet calculus calculations when one is satisfied by assuming the dominance of the poles in tree diagrams.

In this talk I want to propose a measure on a parton state for the relative probability to obtain different states. This measure has the following properties:

1. It coincides with perturbative QCD on the poles.
2. It gives a welldefined result in cases where many Feynman diagrams contribute and interfere, and is an infrared stable interpolation between the pole expressions.

The Lund hadronization model is based on the following assumptions[2]:

1. The colour field between a quark and an antiquark behaves like a vortex line in a superconductor or a relativistic string.
2. Gluons act as excitations on such a (colour triplet) string. Thus in the case $e^+e^- \rightarrow q\bar{q}g$ the string is stretched from the quark to the antiquark via the gluon, and for $\Upsilon \rightarrow ggg$ the string is a closed triangular loop.
3. The fragmentation of a linear stringlike colour field is described by the one-dimensional semiclassical model.

The fragmentation is illustrated in fig. 1. Quark-antiquark pairs are produced in the colour field and combine to the final state hadrons. In the recent version of the model[3] (called the symmetric Lund model) the probability P to obtain a certain state with n hadrons is given by

$$P \propto \int \prod_{i=1}^{n} [N d^2 p_i \delta(p_i^2 - m^2)] \delta(\Sigma p_i - P_{tot}) \exp(-bA) \qquad (1)$$

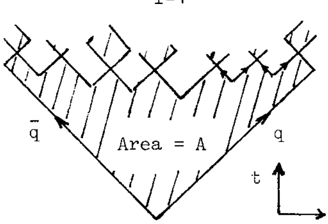

Fig. 1

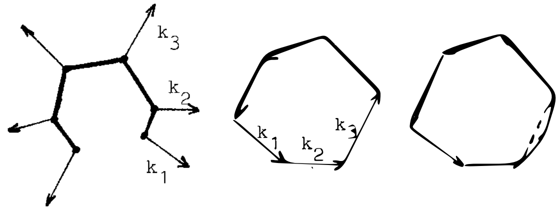

Fig. 2a 2b 2c

Here p_i are the hadron momenta and we assume here that there is only one type of hadron, which has mass m. The quantity A is the area indicated in fig. 1. Thus P contains a product of a phase space expression and an exponent which resembles a Wilson loop integral. The model contains two parameters, N and b. If N is large this favours the production of many hadrons and if b is large the hadrons are more strongly correlated in rapidity (a large b also favours fewer hadrons).

The particle distribution obtained from eq. (1) can also be generated as a jet cascade. It is called the symmetric Lund model because for a $q\bar{q}$ system it gives the same result if the cascade is generated from the quark end or from the antiquark end.

The Lund model is infrared stable in the following sense. In case we have a state with many gluons we get a string with many corners as shown in fig. 2a. If we plot the momenta of all the partons after each other as in fig. 2b, we obtain a figure called the directrix, which fully determines the state of motion for the string (to be exact the directrix also contains the mirror image of fig. 2b). The directrix also describes the motion of the quark at the endpoint of the string, starting off with momentum k_1, provided the string does not break into pieces. We note that the directrix and the state of motion of the string is not changed much if one gluon with low mass is split into two as indicated in fig. 2c. We think that it should be possible to describe the physics in terms of asymptotic final states, i.e. hadronic states. Due to the finite hadron masses this implies a finite resolution power on the parton or string state. In this way we get a smooth transition between e.g. three- and two-jet events and between four- and three-jets.

In the fragmentation of a $q\bar{q}$ system as in fig. 1, the breaking points are located around a hyperbola. Thus the area is approximately given by

$$bA \sim \ln(s/m_o^2) \tag{2}$$

where the constant m_o is determined from the parameters N and b.

For a $q\bar{q}g$ system two straight string pieces are stretched. Thus for a hard gluon the corresponding area spanned in Minkowski space also contains two pieces A_1 and A_2 (cf fig. 3) which are given by

$$bA_1 \sim \ln(\tfrac{1}{2}s_{12}/m_o^2), \quad bA_2 \sim \ln(\tfrac{1}{2}s_{23}/m_o^2); \; s_{ij} \equiv (k_i+k_j)^2 \tag{3}$$

The factor $\frac{1}{2}$ in the argument of the logarithm comes because half of the gluon momentum, k_2, is used to stretch each of the two string pieces. If the gluon is soft, it cannot stretch out two string pieces as in fig. 3. Instead the retarded gluon has soon lost its energy and a straight string piece is stretched between the quark and the antiquark ends. Thus in the limit $k_2 \to 0$ the area approaches the expression in eq. (2). A smooth interpolation between the hard and soft gluon cases is given by

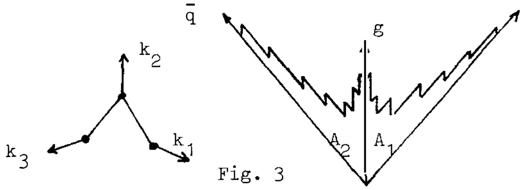

Fig. 3

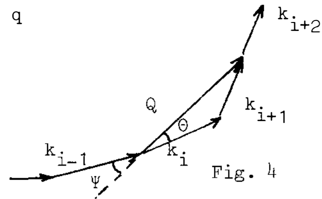

Fig. 4

$$bA \sim \ln\,[s/m_o^2 + s_{12}s_{23}/4m_o^4] \tag{4}$$

We now make the conjecture that the probability P to obtain a definite $q\bar{q}g$ parton state is given by the expression

$$P \sim \underbrace{\prod_{i=1}^{3} [dk_i\ \delta(k_i^2)]\ \delta(\Sigma k_i - P_{tot})}_{\sim\ dx_1 dx_3}\ \underbrace{\exp(-bA)}_{\sim\ \frac{1}{(1-x_1)(1-x_3)+4m_o^2/s}} \tag{5}$$

This expression has again the form of a phase space factor times an exponent of an area. This area can be interpreted as an effective action. It is given by the average size of a Wilson loop type area when the parton system fragments into hadrons. The quantities x_i are the usual scaled momenta, $x_i=2E_i/\sqrt{s}$, where E_i are the energies in the c.m. system.

When we compare eq. (5) with the result from perturbative QCD:

$$P \propto dx_1 dx_3\ \frac{x_1^2+x_3^2}{(1-x_1)(1-x_3)} \tag{6}$$

we note that the singularities at $x_1=1$ and $x_3=1$ are cut off by the term $4m_0^2/s$. We also see that the factor $x_1^2+x_3^2$ in the numerator is not reproduced. However, this factor follows because the $q\bar{q}g$ system originates from a virtual photon with spin one; such polarization properties are not included in eq. (5).

For a generalization of the result in eq. (5) to the general case with many gluons we demand that the result shall be infrared stable. Thus when the invariant mass of two (consecutive) gluons goes to zero, then the measure for n partons should go over into the measure for n-1 partons. This is directly achieved if it is expressed in terms of the string state.

If we add to the directrix curve in fig. 2b a time component we obtain a four-vector function $P_\mu(t)$ which describes the state of the string and also the motion of the quark at the endpoint of the string. In case of a three-jet system the expression in eq. (4) can be written in the following form

$$T \equiv 1 + \tfrac{1}{2}\exp(bA) = 1 + \frac{1}{2m_o^2}\int_o^E -\frac{ds(E,t)}{dt}\,dt + \frac{1}{4m_o^2}\int_o^E -\frac{ds(E,t_1)}{dt_1}\,dt_1 \int_o^{t_1} -\frac{ds(t_1,t_2)}{dt_2}\,dt_2; \quad s(t_1,t_2) \equiv (P(t_1)-P(t_2))^2 \tag{7}$$

A generalization to many gluons is obtained by the expression

$$T = 1 + \sum_{j=1}^{\infty} \int_0^E - \frac{ds(E,t_1)}{2m_o^2} \int_0^{t_1} - \frac{ds(t_1,t_2)}{2m_0^2} \int \ldots \int_0^{t_{j-1}} - \frac{ds(t_{j-1},t_j)}{2m_o^2} \tag{8}$$

The function T can be interpreted as the exponent of an area in Minkowski space, which in turn can be interpreted as an effective action. As for the three-jet events above we make the conjecture that the probability P_n to obtain a certain state with n partons is given by a phase space factor (where ρ is a constant) times T^{-1}:

$$P_n \propto \prod_{j=1}^{n} [\rho dk_j \delta(k_j^2)] \delta(\Sigma k_j - P_{tot}) \; T^{-1}(k_1,\ldots,k_n) \tag{10}$$

For the simple example when all the invariant masses $k_i k_{i+1}$ are large compared to the parameter m_o^2 it is easy to show that if $k_i + k_{i+1} = Q$ as in fig. 4, then

$$P_n \approx P_{n-1} \cdot 2\pi\rho m_o^2 \frac{dQ^2 \cdot dz}{Q^2 \cdot z(1-z) + m_o^2} \tag{11}$$

Here P_{n-1} is the probability to obtain a state with n-1 partons having momenta $k_1, \ldots k_{i-1}, Q, k_{i+2}, \ldots k_n$. The parameter z is as usual k_i's lightcone fraction of Q. This result obviously very much resembles the corresponding result in perturbative QCD. However, for z very close to 0 or 1 the approximate expression in eq. (11) needs corrections. Small values of z correspond to large values of Θ and the corrections effectively cut off the pole at $z \approx 0$ when the angle Θ in fig. 4 is larger than ψ. This corresponds exactly to the angular cut off due to soft gluon interference obtained in perturbative QCD and discussed by Mueller, Marchesini, Webber and others[4].

A similar result is obtained also when the two massless gluons with momenta k_i and k_{i+1} are replaced by subsystems of gluons, so that a large subsystem is divided into two smaller subsystems.

An important property of the measure is that events distributed according to eq (10) can be generated as in iterative stochastic processes. A Monte Carlo simulation program is being developed by M. Bengtsson and M. v Zijl. There seems to be a characteristic "curvature". Smoothly bent directrices are favoured; strong bends give small values for T^{-1} whereas for long straight sections the phase space is not fully utilized. In e.g. an high $E_\perp$ event at the collider this might correspond to coherent initial and final state bremsstrahlung.

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