Universidad de Antioquia
Instituto de Física
Facultad de Ciencias Exactas y Naturales
Grupo de Fenomenología de Interacciones Fundamentales (GFIF)

Quantum Field Theory on Curved Spacetime and Black Holes

Quantum Effects on Spacetime and Black Holes

Undergraduate Thesis

JOSÉ LUIS BUILES CANCHALA

Advisor: Dr. Oscar Alberto Zapata N.

Medellín, 2025

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Abstract

This thesis presents a monograph based on some literature review about quantum field theory (QFT) in curved spacetime, with a special focus on the quantum effects that occurs on it and, mainly, in black holes. The relation between quantum mechanics and general relativity reveals insights into the behavior of quantum fields in non-flat spacetimes. The first section explores some aspects and principles of QFT, the significance of curved spacetime and general relativity foundations; looking for the way to combine both in search of some quantum effects in the universe and the implications for black hole physics.

The second section delves into specific quantum effects that arise in curved spacetime, which are the purpose of the work, including the Schwinger effect, the Unruh effect, the Hawking radiation, the Superradiance effect and the Memory Burden effect. Each phenomenon is examined physically and mathematically, emphasizing their theoretical principles and physical interpretations, thereby illustrating how quantum fluctuations can lead to observable consequences in gravitational contexts and particles interpretations, as the particle creation/annihilation in black hole horizons.

In this monograph, our aim is to understand theoretical predictions in astrophysics and cosmology, due to curved spacetime. This predictions underscore the necessity of integrating quantum field theory with general relativity to enhance our understanding of fundamental physics in extreme environments, such as those surrounding black holes. This work presents a base to exploring the implications of these quantum effects in cosmological scenarios and their relevance on future research and deepness into high-energy astrophysics and cosmology, as the observational data is growing up on this field.

Keywords

Quantum Field Theory, Curved Spacetime, Quantum Effects, Black Holes, Memory Burden Effect.

To my family, friends and professors.
Especially to Ana and Jonathan for all your support and friendship.

Contents

Chapter 1

Introduction

The present thesis presents a physical and theoretical framework, as a monograph, for understanding the relationship between quantum field theory (QFT) and general relativity, particularly focusing on the behavior of quantum fields in curved spacetime and the phenomena associated with black holes (BH). The quest to unify these two foundational pillars of modern physics remains one of the central challenges in theoretical physics, especially in extreme environments where both quantum effects and gravitational dynamics are dominant.

QFT, a cornerstone of modern physics, describes the behavior of particles and their interactions through the unification of quantum mechanics and special relativity. This theory has provided many results into particle interactions, electromagnetic fields and the fundamental forces shaping the universe [1]. However, its formulation in flat Minkowski flat spacetime inherently neglects gravitational effects. Yet, the universe we observe is inherently curved due to massive objects such as stars, galaxies, and, most notably, BH [2]. This raises a fundamental question: How does QFT adapt to curved spacetime, where gravitational effects cannot be ignored?

The relation between quantum mechanics and general relativity has several implications, extending beyond theoretical pursuits to experimental frontiers. By exploring QFT in curved spacetime, we aim to predict the mechanisms underlying these phenomena and their connections to some astrophysical objects, such as black holes. This synthesis of QFT and general relativity paves the way for advancements in astroparticles and quantum cosmology by describing the vacuum's ambiguity, the black hole's radiation and other quantum effects raised on curved spacetime.

The motivation on this work comes from an attempt to understand the big gravity sources behavior on quantum scales, the necessity for a coherent framework reconciling quantum mechanics with gravitational interactions becomes increasingly apparent. QFT in curved spacetime offers a robust theoretical tool to investigate phenomena occurring in strong gravitational fields, such as those near BH or during the early universe [3, 4]. Classical descriptions of spacetime and gravity, as provided by general relativity, are insufficient to explain the quantum nature of the universe, as they fail on small-scale, being not capable of making theoretical predictions.

These interactions give rise to phenomena such as vacuum polarization and the dynamic generation of particles, which redefine our understanding of the quantum vacuum [5, 6]. One of the most curious predictions of QFT in curved spacetime is the emergence of phenomena without counterparts in flat spacetime, such as particle creation in strong gravitational fields and quantum fluctuations that define the quantum vacuum [3]. Black holes serve as natural laboratories for these investigations; their extreme curvature and event horizons create environments where quantum effects, such as Hawking radiation, become theoretically significant [7].

The study of BH in this framework reveals some relations between the nature of spacetime, quantum mechanics and thermodynamics. Hawking radiation, for example, demonstrated that BH are not entirely "black" but emit thermal radiation due to quantum effects near the event horizon [6]. Other effects, such as the Schwinger effect, Unruh effect, Superradiance and Memory Burden effects, further illustrate the rich interplay between quantum fields and curved spacetime [8–10].

This thesis pursues two primary goals, these are the objectives for the project.

- To provide a brief theoretical framework for quantum field theory in curved spacetime, with an emphasis on its mathematical formulation and physical interpretation.
- To explore some quantum effects arising due to curved spacetime, particularly near black holes, through physical and mathematical analysis.

Structuring the thesis into two main sections to achieve these objectives:

Part I: Quantum Field Theory in Curved Spacetime

This section introduces the foundational concepts needed to reach QFT in curved spaces; as a brief introduction to QFT in Minkowski spacetime, the principles of general relativity and the relation between those. It includes the next topics:

- A brief theoretical development of QFT in Minkowski spacetime, by scalar field quantization, showing the basic tools to reach the canonical quantization [1].
- The mathematical formalism of curved spacetime, including metric tensors and covariant derivatives, based on general relativity theory [2, 11].
- The extension of QFT to curved geometries, emphasizing particle dynamics and the quantum vacuum [3, 4].

In addition, this section will explore how classical geometries influence quantum states, with particular emphasis on adiabatic vacua and the Bogoliubov transformations. These tools allow for the precise characterization of particle creation and annihilation processes in dynamic spacetimes [12], which leads to the quantum effects we want to explore.

Part II: Quantum Effects in Curved Spacetime and Black Holes

This section examines specific quantum phenomena arising from the interaction between quantum fields and curved spacetime, including:

- **Schwinger Effect:** Particle production in strong electric fields [8].
- **Unruh Effect:** Perception of thermal radiation by accelerating observers [10].
- **Hawking Effect:** Quantum radiation emitted by BH [7].

- **Superradiance Effect:** Amplification of waves interacting with rotating BH [13].
- **Memory Burden Effect:** Correction to Hawking radiation [9].

Each effect is analyzed by exploring its theoretical foundations. The mathematical frameworks employed in these analyses are grounded in some works of QFT, general relativity and the mix between both. Finally, it will given a shallow review on those quantum effects on Primordial Black Holes (PBH).

Understanding quantum effects in curved spacetime has profound implications for theoretical and experimental physics. As observational astrophysics, particularly gravitational-wave astronomy, advances, these insights are crucial for interpreting new data; this leaded to the theoretical predictions that allow to constrain some observational parameters. By exploring insights from literature [3, 4, 14] and recent research [9, 15–19], this thesis seeks to develop a useful summary for understanding this new physics.

Furthermore, the study of quantum effects near BH offers a potential window into resolving known paradoxes, such as the information loss problem, as we will see later. Investigating these issues advances our theoretical understanding and enhance our understanding of the fundamental interactions that govern the universe, particularly in the realm of BH.

Chapter 2

Quantum Field Theory and Curved Spacetime

This chapter delves into a brief framework of Quantum Field Theory and its generalization to curved spacetime, bridging the principles of quantum mechanics and general relativity. QFT, as the foundation for understanding particle interactions and fundamental forces, provides a synthesis of quantum mechanics and special relativity in flat Minkowski spacetime. However, the introduction of gravity into this framework necessitates a significant conceptual and mathematical extension to account for the curvature of spacetime induced by massive objects and energy distributions, as general relativity introduces, such as those near black hole horizons. The interaction between quantum fields and curved spacetime unveils phenomena absent in flat spacetime, including particle creation, vacuum fluctuations and horizon-related effects like Hawking radiation.

This chapter looks to provide a brief exploration of QFT, curved spacetime and the relation between both. It begins with an overview of QFT in flat spacetime, emphasizing the canonical quantization of the scalar field [14], which forms the cornerstone of quantum field formulations. Subsequently, the discussion transitions to curved spacetime [20], where the formalism of general relativity introduces the necessary tools for describing dynamic curved geometries [21]. The synthesis of these frameworks culminates in the study of QFT in curved spacetime [3], elucidating the modifications to quantum field dynamics and the emergence of novel quantum phenomena.

Through mathematical derivations and theoretical approaches, this chapter aims to establish a brief understanding of the concepts required for subsequent discussions on quantum effects on curved spacetime, as BH.

2.1 Quantum Field Theory

Quantum Field Theory is the framework that combines quantum mechanics with special relativity, providing a description of quantum fields [22]. In flat Minkowski spacetime, QFT is well understood, and in this section, we outline the basics of the canonical quantization procedure for the scalar field, which forms the foundation for later discussions on quantum fields in curved spacetime.

Initially, we begin with the simplest field, the real scalar field, denoted as $\phi(x)$. The field evolves according to the Klein-Gordon equation, which is derived from the Lagrangian density for the field. The Lagrangian density is given by [14]:

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu\phi\partial^\mu\phi - m^2\phi^2\right), \tag{2.1}$$

where $\phi(x)$ is the scalar field, a function of spacetime coordinates x^μ, m is the mass of the particle associated with the field, and ∂_μ is the partial derivative with respect to spacetime coordinates. The action S is defined [14] as the integral of the Lagrangian density over spacetime:

$$S = \int d^4x\, \mathcal{L}. \tag{2.2}$$

Using the principle of least action, the variation of the action with respect to the field $\phi(x)$ leads to the equation of motion for the field. Varying the action, we obtain:

$$\frac{\partial\mathcal{L}}{\partial\phi} - \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\right) = 0, \tag{2.3}$$

Substituting the Lagrangian density $\mathcal{L}$, we get the **Klein-Gordon equation**:

$$\partial_\mu\partial^\mu\phi + m^2\phi = 0; \tag{2.4}$$

This equation describes the evolution of a free scalar field in flat spacetime. It is the relativistic analog of the Schrödinger equation for a scalar particle.

In classical field theory, $\phi(x)$ is a classical field, but in quantum field theory, we promote $\phi(x)$ to a quantum operator [1]. The process of **canonical quantization** involves a number of steps which will be described below. Firstly, it is necessary to define the conjugate momentum $\pi(x)$ as the derivative of the Lagrangian with respect to the time derivative of the field $\phi(x)$:

$$\pi(x) = \frac{\partial\mathcal{L}}{\partial(\partial_0\phi)} = \dot{\phi}(x), \tag{2.5}$$

where $\partial_0\phi = \dot{\phi}$ is the time derivative of the field.

Secondly, the field $\phi(x)$ and its conjugate momentum $\pi(x)$ are promoted to quantum operators, and they must satisfy the following equal-time commutation relations [14]:

$$[\phi(\mathbf{x},t), \pi(\mathbf{y},t)] = i\hbar\delta^3(\mathbf{x}-\mathbf{y}), \tag{2.6}$$

$$[\phi(\mathbf{x},t), \phi(\mathbf{y},t)] = [\pi(\mathbf{x},t), \pi(\mathbf{y},t)] = 0. \tag{2.7}$$

These commutation relations encode the quantum nature of the field, similar to the position and momentum commutators in quantum mechanics. To solve the Klein-Gordon equation, we look for solutions in the form of plane waves. In flat spacetime, these solutions are written as [12]:

$$\phi(x) = \int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2\omega_\mathbf{k}}}\left(a_\mathbf{k}e^{-ik\cdot x} + a_\mathbf{k}^\dagger e^{ik\cdot x}\right), \tag{2.8}$$

where $k^\mu = (\omega_\mathbf{k}, \mathbf{k})$ is the four-momentum, with $\omega_\mathbf{k} = \sqrt{\mathbf{k}^2 + m^2}$; and $a_\mathbf{k}$ and $a_\mathbf{k}^\dagger$ are annihilation and creation operators, respectively.

This expansion represents the field as a sum over modes, where each mode is associated with a plane wave of momentum $\mathbf{k}$. The creation and annihilation operators satisfy the commutation relations [14]:

$$[a_{\mathbf{k}}, a_{\mathbf{p}}^\dagger] = (2\pi)^3 \delta^3(\mathbf{k} - \mathbf{p}), \tag{2.9}$$

$$[a_{\mathbf{k}}, a_{\mathbf{p}}] = [a_{\mathbf{k}}^\dagger, a_{\mathbf{p}}^\dagger] = 0. \tag{2.10}$$

These relations imply that $a_{\mathbf{k}}$ and $a_{\mathbf{k}}^\dagger$ are operators that annihilate and create quanta of the field with momentum $\mathbf{k}$, respectively.

The **vacuum state** $|0\rangle$ is defined as the state with no particles, or when energy is minimal, satisfying [14]:

$$a_{\mathbf{k}}|0\rangle = 0, \quad \text{for all}\, \mathbf{k}. \tag{2.11}$$

By applying creation operators to the vacuum state, we can build a Fock space of states with a definite number of particles. For example, a one-particle state with momentum $\mathbf{k}$ is given by [3]:

$$|\mathbf{k}\rangle = a_{\mathbf{k}}^\dagger |0\rangle. \tag{2.12}$$

More generally, a state with n particles can be constructed by applying n creation operators [3]:

$$|\mathbf{k}_1, \mathbf{k}_2, \ldots, \mathbf{k}_n\rangle = a_{\mathbf{k}_1}^\dagger a_{\mathbf{k}_2}^\dagger \ldots a_{\mathbf{k}_n}^\dagger |0\rangle. \tag{2.13}$$

This defines the structure of the Fock space, a many-particle Hilbert space where particles are excitations of the underlying field. Finally, the Hamiltonian and momentum operator of the field are derived from the components of the energy-momentum tensor $T_{\mu\nu}$. The Hamiltonian is given by [22]:

$$H = \int d^3x \left(\frac{1}{2}\pi^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}m^2\phi^2 \right); \tag{2.14}$$

Substituting the mode expansion of $\phi(x)$ and $\pi(x)$ into this expression, the Hamiltonian can be rewritten in terms of the creation and annihilation operators [22]:

$$H = \int \frac{d^3k}{(2\pi)^3}\, \omega_{\mathbf{k}} \left(a_{\mathbf{k}}^\dagger a_{\mathbf{k}} + \frac{1}{2} \right). \tag{2.15}$$

This shows that the energy of the field is the sum of the energies of individual quanta, with each quantum carrying energy $\omega_{\mathbf{k}}$. The term $\frac{1}{2}\omega_{\mathbf{k}}$ represents the **vacuum energy** [23].

In the context of this thesis, canonical quantization is the most practical tool for deriving explicit results in QFT on curved spacetime. As it was seen before, the scalar field quantization gives the needed process to quantize any field that has the same properties; we will show below, that in the extension of QFT to curved spacetime, the fields are, in general, scalar; therefore, this introduction to Klein-Gordon solution is enough for the subsequent understanding.

2.2 Curved Spacetime and Black Holes

The transition from flat spacetime to curved spacetime marks a generalization in theoretical physics, where the effects of gravity are incorporated into the geometric structure of spacetime. In general relativity, gravity is not a force but a manifestation of spacetime curvature caused by mass and energy [2]. This perspective revolutionized our understanding of gravitational interactions, replacing the Newtonian notion of gravitational forces with the Einsteinian framework of geodesic motion in curved spacetime [24].

Curved spacetime plays a critical role in describing extreme astrophysical phenomena such as black holes, neutron stars and other astrophysical objects. These environments exhibit gravitational fields strong enough to significantly distort the geometry of spacetime, demanding a treatment within the framework of general relativity.

This section explores the mathematical foundation of curved spacetime, beginning with the concept of the metric tensor and its role in describing spacetime geometry [21, 25]. Subsequent discussions introduce the geodesic equation, curvature tensors, and their implications for understanding black hole physics [26, 27].

2.2.1 Spacetime and the Metric Tensor

The foundation of general relativity is the concept of a four-dimensional spacetime, where the coordinates $x^\mu = (t, x, y, z)$ describe the temporal and spatial components of an event [24]. The structure of spacetime is encoded in the **metric tensor** $g_{\mu\nu}$, a symmetric second-rank tensor that defines the infinitesimal distance between two events in spacetime. The line element, which expresses the squared spacetime interval between nearby points, is given by:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu. \tag{2.16}$$

In flat spacetime, the metric tensor reduces to the Minkowski metric $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$, which corresponds to special relativity [27]. However, in the presence of gravitational fields, spacetime becomes curved, and the metric tensor $g_{\mu\nu}(x)$ varies with the spacetime coordinates x^μ.

The metric tensor determines the length of curves, the angles between vectors and the volume of regions in spacetime [24]. Some of its properties are:

- **Symmetry:** $g_{\mu\nu} = g_{\nu\mu}$, reflecting the fact that the spacetime interval does not depend on the order of the coordinates.
- **Inverse Metric:** The inverse metric $g^{\mu\nu}$ satisfies $g^{\mu\nu} g_{\nu\rho} = \delta^\mu_\rho$, where δ^μ_ρ is the Kronecker delta.
- **Determinant:** The determinant of the metric tensor $g = \det(g_{\mu\nu})$ defines the invariant volume element $\sqrt{-g} d^4x$.
- **Covariance:** The metric transforms covariantly under coordinate transformations, ensuring the invariance of physical laws.

In curved spacetime, the natural generalization of straight-line motion is given by geodesics [2]. A geodesic is the shortest path between two points in a curved manifold, and it describes the trajectory of a particle in the absence of any non-gravitational forces. The equation of motion for a particle moving along a geodesic is derived from the **geodesic equation** [27]:

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0, \tag{2.17}$$

Here, τ is the proper time along the particle's worldline, and $\Gamma^\mu_{\alpha\beta}$ are the **Christoffel symbols**, which represent the connection coefficients that define how vectors change as they are transported through spacetime [25]. The Christoffel symbols are computed from the metric tensor as follows:

$$\Gamma^\mu_{\alpha\beta} = \frac{1}{2}g^{\mu\nu}\left(\partial_\alpha g_{\nu\beta} + \partial_\beta g_{\nu\alpha} - \partial_\nu g_{\alpha\beta}\right). \tag{2.18}$$

These symbols are not tensors themselves but are indispensable in defining the covariant derivative, which generalizes partial derivatives in curved spacetime, and itself defines the curvature of the spacetime. The covariant derivative of a vector V^μ is [27].:

$$\nabla_\nu V^\mu = \partial_\nu V^\mu + \Gamma^\mu_{\nu\rho}V^\rho. \tag{2.19}$$

Defined using the Levi-Civita connection [21] which ensures metric compatibility:

$$\nabla_\lambda g_{\mu\nu} = 0. \tag{2.20}$$

This condition guarantees that lengths and angles are preserved under parallel transport, leading to the concept of conformal transformations. The Levi-Civita connection is torsion-free, satisfying $\Gamma^\rho_{\mu\nu} = \Gamma^\rho_{\nu\mu}$ [21]. Covariant derivatives ensure that geometric and physical quantities are properly defined in curved spacetime, preserving their form under coordinate transformations (covariance principle) [25].

The geodesic equation ensures that particles in free-fall move along the natural curves dictated by spacetime geometry. This is a feature of general relativity, as gravity is understood not as a force, but as the effect of spacetime curvature on the motion of particles [27]. To describe how spacetime is curved, it is necessary to introduce some geometric objects that are derived from the metric tensor.

The first important quantity is the **Riemann curvature tensor** $R^\rho_{\mu\sigma\nu}$, which measures how much the spacetime deviates from being flat [26]. The Riemann tensor is defined in terms of the Christoffel symbols and their derivatives:

$$R^\rho_{\mu\sigma\nu} = \partial_\sigma\Gamma^\rho_{\mu\nu} - \partial_\nu\Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\sigma\lambda}\Gamma^\lambda_{\mu\nu} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma}. \tag{2.21}$$

The Riemann tensor encodes information about the tidal forces experienced by nearby particles moving through curved spacetime. Another way to show the curvature of the spacetime is to using a parameter Ω_0, which describes the curve of the manifold [28]. This is a cosmological point of view, and it could be neglected for the purpose of this work; however, looking for schematize the different types of curved spacetime, we present the Figure 2.1, showing the possibilities for the spacetime curvature.

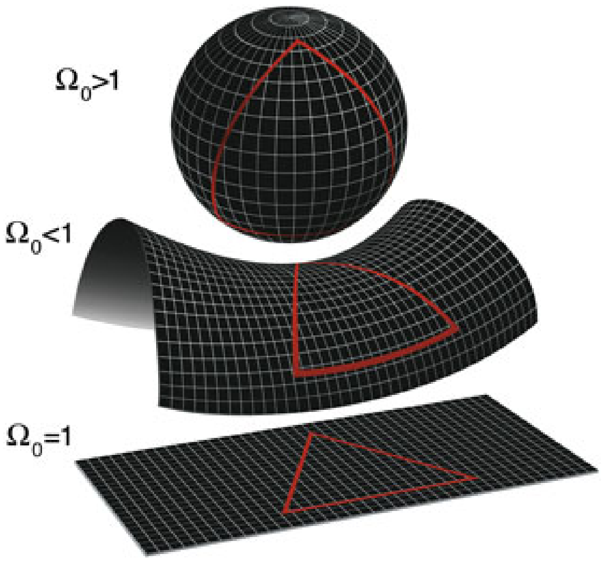

Figure 2.1: *Spacetime curvature possibilities. Flat, spherical or hyperbolic spacetime. Source: [29]*

Riemann tensor various contractions give rise to other important curvature tensors. The **Ricci tensor** $R_{\mu\nu}$ is obtained by contracting the Riemann tensor over its first and third indices [26]:

$$R_{\mu\nu} = R^{\rho}_{\mu\rho\nu}. \tag{2.22}$$

This tensor describes how volumes change as they are parallel transported around small closed loops in spacetime [21]. It plays a central role in Einstein's field equations, which relate the Ricci curvature to the distribution of matter and energy. Finally, the **Ricci scalar** R is obtained by contracting the Ricci tensor [26]:

$$R = g^{\mu\nu} R_{\mu\nu}. \tag{2.23}$$

This quantity provides a single scalar measure of the curvature of spacetime. It is often used to express the overall curvature of spacetime in a more concise form. The curvature of spacetime is intimately related to the distribution of matter and energy through **Einstein's field equations**, which serve as the cornerstone of general relativity. These equations are given by [26]:

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{2.24}$$

Where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$ is the **Einstein tensor**, which encapsulates the curvature of spacetime; $T_{\mu\nu}$ is the **stress-energy tensor**, which describes the energy density, momentum, and stresses of matter and radiation, it depends on the structure of the universe.

In this way, Einstein's field equations provide the link between spacetime curvature and the energy-momentum content of the universe. In the context of quantum field theory in curved spacetime, these equations form the backdrop against which quantum fields evolve, and the field we want to quantize corresponds to conformal transformations of Einstein's field.

2.2.2 Black Holes Physics

Black holes are among the most intriguing predictions of general relativity and play a central role in the study of both classical and quantum aspects of gravity [4]. BH are specific solutions to Einstein's field equations that describe regions of spacetime where the gravitational field is so strong that nothing, not even light, can escape; these are associated with regions where spacetime curvature becomes extreme due to the presence of compact mass concentrations.

As it was said before, BHs arise as solutions to Einstein's field equations by changing and imposing a unique metric, being the Schwarzschild and Kerr solutions two cases [20]. These solutions reveal unique geometric features such as singularities, horizons, and, in the case of Kerr black holes, rotational effects. The structure of these spacetimes has profound implications for our understanding of the causal and geometric properties of the universe.

There are some important solutions worth addressing, among them we have the Schwarzschild and Kerr metrics, which result in some BH solutions, as we will show below. It will be discussed the BH thermodynamics, as it brings Hawking ideas to relation this astrophysical objects with classical thermodynamics.

Schwarzschild Black Holes

The Schwarzschild BH is the simplest solution to Einstein's field equations, describing a static, spherically symmetric gravitational field generated by a non-rotating, uncharged mass [20]. The Schwarzschild metric, expressed in spherical coordinates (t, r, θ, ϕ), is given by [20]:

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right), \tag{2.25}$$

where M is the mass of the black hole, and G is the gravitational constant. This solution reveals geometric aspects of BHs like the **event horizon**; this is a boundary beyond which no information can escape to infinity [20]. For this metric, the event horizon is located at the Schwarzschild radius:

$$r_s = \frac{2GM}{c^2}, \tag{2.26}$$

where c is the speed of light. The presence of an event horizon fundamentally alters the causal structure of spacetime.

At the center of a BH lies a **singularity**, a point where the curvature of spacetime becomes infinite and the known laws of physics break down [30]. In the Schwarzschild solution, the singularity occurs at $r = 0$, where tidal forces become infinitely strong.

This solution also exhibits symmetries, including spherical symmetry and time-translation invariance. These symmetries simplify the analysis of particle motion and field propagation in the Schwarzschild spacetime. For massive particles, the effective potential V_{eff} determines the nature of particle orbits, including bound, unbound, and circular trajectories; it is given by [27]:

$$V_{\text{eff}}(r) = \left(1 - \frac{2GM}{r}\right)\left(1 + \frac{L^2}{r^2}\right), \tag{2.27}$$

where L is the angular momentum of the particle. This equation is important for calculating the kind of particles that BH should emit.

Kerr Black Holes

While the Schwarzschild solution describes non-rotating BH, more general BH solutions exist for rotating and charged objects. The most simple physically relevant is the Kerr BH, which describes a rotating and uncharged BH [2]. The Kerr metric is more complex than the Schwarzschild metric and introduces additional structure due to the BH's angular momentum J. In Boyer-Lindquist coordinates (t, r, θ, ϕ), the Kerr metric is [27]:

$$\begin{aligned} ds^2 = &-\left(1 - \frac{2GMr}{\rho^2}\right) dt^2 - \frac{4GMar\sin^2\theta}{\rho^2} dt\, d\phi + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 \\ &+ \left(r^2 + a^2 + \frac{2GMa^2 r\sin^2\theta}{\rho^2}\right)\sin^2\theta\, d\phi^2, \end{aligned} \tag{2.28}$$

where $\rho^2 = r^2 + a^2\cos^2\theta$, $\Delta = r^2 - 2GMr + a^2$ and $a = J/M$ is the specific angular momentum of the black hole, with J being its total angular momentum. The Kerr solution introduces several new properties compared to the Schwarzschild BH, like [30]:

- The Kerr black hole possesses two horizons, inner and outer, given by the roots of $\Delta = 0$ [27]:

  $$r_\pm = GM \pm \sqrt{G^2M^2 - a^2}, \tag{2.29}$$

  where r_+ corresponds to the event horizon, and r_- represents an instable horizon.

- Outside the event horizon lies the ergosphere [30], defined as the region where $g_{tt} > 0$. The ergosphere is bounded by the event horizon at r_+ and the static limit surface at:

  $$r_s = GM + \sqrt{G^2M^2 - a^2\cos^2\theta}. \tag{2.30}$$

  In this region no particle can remain stationary with respect to a distant observer.

The Kerr solution serves as a foundation for understanding astrophysical BHs, which are often rotating due to the angular momentum of their progenitor stars. The most general solution is the Kerr-Newman solution, incorporating the BH electrical charge; this will play a central role in Superradiance effect but the physics are analogous to Kerr BH, so it will not be showed.

Primordial Black Holes

Primordial black holes are theoretical BHs that could have formed in the early universe from the gravitational collapse of density perturbations [31]. Their formation does not rely on stellar processes but instead results from high-energy phenomena occurring shortly after the Big Bang [32]. The PBHs study offers insights into the physics of the early universe, particularly during inflation and the radiation-dominated epoch, and it's also a dark matter candidate [33].

PBHs form when local density perturbations exceed a critical threshold, δ_c, during the radiation-dominated era [31]. These perturbations collapse under their self-gravity, overcoming the pressure forces that otherwise maintain equilibrium in the hot, dense plasma.

Quantum fluctuations during inflation are a primary source of these density perturbations. Inflationary dynamics stretch these fluctuations to cosmological scales, seeding structures ranging from galaxies to PBHs [28]. The mass of a PBH is directly related to the mass within the cosmological horizon at the time of formation [32]:

$$M_{\mathrm{PBH}} \sim \gamma \frac{c^3}{G} t, \tag{2.31}$$

where t is the cosmic time of formation, and γ accounts for the efficiency of collapse. For PBHs formed at times ranging from 10^{-5} s to 10^{-3} s, the resulting masses span from $10^{-5} M_\odot$ to several solar masses [32].

Additionally, PBHs might act as seeds for supermassive black holes observed at high redshifts [31]. The rapid accretion of gas onto PBHs formed in the early universe provides a natural mechanism for explaining the existence of $10^9 M_\odot$ black holes at $z > 6$, a challenge for conventional formation scenarios relying solely on stellar-mass progenitors [32].

Detecting PBHs directly is challenging due to their diverse masses and weak interactions [31]. However, their gravitational and radiative effects provide several indirect observational pathways. Gamma-ray observations constrain the abundance of lighter PBHs through their contribution to the diffuse gamma-ray background [34]; the CMB places additional constraints, as accreting PBHs could alter the ionization history of the universe [35], leaving imprints detectable by missions like Planck experiment. Gravitational wave observatories such as LIGO, Virgo, and LISA offer another avenue for detection [36].

The potential evaporation of PBHs serves as a possible natural laboratory for studying QFT in curved spacetime, allowing us to predict some of the quantum effects we want to investigate in these astrophysical objects.

Black Hole Thermodynamics

BHs exhibit thermodynamic properties that link general relativity, quantum mechanics and statistical mechanics. These properties are governed by the four laws of BH thermodynamics [6], which are analogous to the classical laws of thermodynamics.

Therefore, it rises the Four Laws of BH Thermodynamics, which are listed below [3].

1. **Zeroth Law:** The surface gravity κ is constant across the event horizon of a BH in thermal equilibrium, analogous to the constancy of temperature in classical systems. Mathematically, this is expressed as [3]:

$$\kappa = \frac{1}{2} \left| g^{tt} \frac{\partial g_{tt}}{\partial r} \right|_{r=r_+}, \tag{2.32}$$

   where r_+ is the radius of the outer event horizon.

2. **First Law:** The variation in a BH's mass M relates to changes in its surface area A, angular momentum J, and electric charge Q [3]:

$$dM = \frac{\kappa}{8\pi} dA + \Omega dJ + \Phi dQ, \tag{2.33}$$

   Where $\Omega = \frac{a}{r_+^2 + a^2}$ is the angular velocity at the horizon (for Kerr BHs); Φ is the electrostatic potential (for charged BHs); and κ is the surface gravity, proportional to the temperature.

3. **Second Law:** The area A of a BH's event horizon never decreases in any classical process [3]:

$$\frac{dA}{dt} \geq 0, \tag{2.34}$$

Reflecting the irreversibility of BH dynamics. This law, often referred to as the *area theorem*, is analogous to the increase of entropy in classical thermodynamics.

4. **Third Law:** It is impossible to reduce a BH's surface gravity κ to zero in a finite number of physical processes, akin to the unattainability of absolute zero temperature.

Also, the entropy S is an important concept on this topic. For a BH, the entropy is proportional to the area A of its event horizon [3], revealing a deep connection between thermodynamics and spacetime geometry. The entropy S remains proportional to the area A of the event horizon, while the first law of thermodynamics incorporates angular momentum J [30]:

$$S = \frac{k_B A}{4\ell_p^2}; \quad A = 4\pi\left(r_+^2 + a^2\right). \tag{2.35}$$

The proportionality of entropy to horizon area indicates that BH entropy quantifies the number of microstates compatible with the macroscopic geometry [30].

Whilst the Schwarzschild BH is classically defined by its mass alone, then its thermodynamic properties emerge through quantum considerations. The entropy S of a Schwarzschild BH is proportional to the area A of its event horizon [30]:

$$S = \frac{k_B A}{4\ell_p^2} = \frac{k_B \pi r_s^2}{\ell_p^2}, \tag{2.36}$$

where ℓ_p is the Planck length. While, as a preview for the next section, the associated Hawking temperature T_H is given by [30]:

$$T_H = \frac{\hbar c^3}{8\pi G M k_B}. \tag{2.37}$$

The implications of BH thermodynamics arises on the unification of diverse areas of physics, as the entropy-area relationship and the connection between temperature and surface gravity suggest that spacetime itself has a microscopic structure, as statistical mechanics dictates. It helps to understand irreversibility, being that the second law of BH thermodynamics aligns with the arrow of time in the universe and classical thermodynamics. This section implies the Hawking Radiation [7], but this will be discuses forward.

2.3 Quantum Field Theory On Curved Spacetime

The extension of QFT from flat Minkowski spacetime to curved spacetime is a generalization that combines the principles of quantum mechanics with the geometrics of general relativity [4]. In curved spacetime, the fixed Minkowski metric $\eta_{\mu\nu}$ is replaced by a dynamic metric $g_{\mu\nu}(x)$ that evolves in response to matter and energy as described by Einstein's field equations. This transition requires adapting QFT to account for the more general diffeomorphism invariance inherent to general relativity [21], by making conformal transformations as it was discussed before.

The study of QFT in curved spacetime represents a significant historical milestone in theoretical physics, with its roots tracing back to the pioneering works of the 1960s and 1970s [4]. Seminal contributions by Leonard Parker, who demonstrated particle creation in expanding universes, and Stephen Hawking [7, 8], who revealed that BHs emit thermal radiation, laid the foundation for this field.

It also provides the study of BHs spacetimes; near the event horizons of BHs, the relation between quantum fields and the extreme curvature of spacetime gives rise to phenomena such as Hawking radiation [26]. These studies also illuminate the nature of singularities and the information paradox [37]. Another topic that appears to analyses are the cosmological horizons; in de Sitter-like spacetimes, associated with accelerated expansion, quantum effects near the cosmological horizon are crucial for understanding horizon entropy, temperature, and the thermodynamics of the universe as a whole [3].

A central challenge of QFT in curved spacetime is the absence of global symmetries [4], such as time translation invariance, which precludes the straightforward definition of concepts like energy and vacuum states. Unlike flat spacetime, where the vacuum is uniquely defined by the absence of particles, curved spacetime introduces vacuum ambiguity. The vacuum state becomes observer-dependent, leading to profound implications such as the prediction of particle creation in non-static spacetimes. For example, in an expanding universe, the rapid time-dependence of the metric can generate particles from the vacuum, different observers perceive distinct vacuum states, giving rise to thermal effects like the Unruh effect and Hawking radiation [7, 10].

The theoretical framework of QFT in curved spacetime builds upon the classical field theory of curved backgrounds and generalizes it to incorporate quantum aspects. This components includes, in brief [38], the metric tensor $g_{\mu\nu}$, covariant derivatives, and the spacetime volume element $\sqrt{-g}$, ensuring diffeomorphism invariance; the expression of the quantum fields in terms of mode functions that satisfy the curved spacetime wave equation, modes that form a complete orthonormal set with respect to the Klein-Gordon inner product [3].

In this section, we aim to review the theoretical point of view for QFT in curved spacetime. For this, we will take into account some of the results and discussions showed in previous sections, as the action formulation for a scalar field and the associated field equations, emphasizing the mathematical tools necessary to handle these equations in non-trivial geometries. We then explore the quantization of fields in curved backgrounds, with a focus on the implications of vacuum ambiguity and particle creation.

Mathematical Framework

The mathematical framework of quantum field theory in curved spacetime is built upon the principles of general covariance and diffeomorphism invariance [2, 21]. These principles makes general relativity compatible with QFT in dynamic, curved backgrounds.

The starting point for QFT in curved spacetime is the action for a scalar field $\phi(x)$, which generalizes the flat spacetime formulation to include the effects of spacetime curvature [22]. The action is given by:

$$S[\phi] = \int d^4x\sqrt{-g}\left(-\frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}m^2\phi^2\right), \tag{2.38}$$

where $g = \det(g_{\mu\nu})$ is the determinant of the metric tensor, ∇_μ is the covariant derivative associated with the metric $g_{\mu\nu}$, ensuring diffeomorphism invariance; and m is the mass of the scalar field.

The factor $\sqrt{-g}$ accounts for the proper volume element in curved spacetime, ensuring that the action is invariant under general coordinate transformations. This invariance, known as **diffeomorphism invariance**, is a key concept of general relativity and ensures that the laws of physics are independent of the choice of coordinates [25]. Mathematically, a diffeomorphism is a smooth, invertible transformation of the spacetime coordinates $x^\mu \to x'^\mu(x)$, it is also called conformal trasformation. Under such a transformation, the metric tensor transforms as:

$$g_{\mu\nu}(x) \to g'_{\mu\nu}(x') = \frac{\partial x^\rho}{\partial x'^\mu} \frac{\partial x^\sigma}{\partial x'^\nu} g_{\rho\sigma}(x). \tag{2.39}$$

For a scalar field $\phi(x)$, diffeomorphism invariance ensures that the value of the field remains unchanged under a coordinate transformation:

$$\phi'(x') = \phi(x). \tag{2.40}$$

The volume element of spacetime also transforms consistently:

$$\sqrt{-g}\, d^4x \to \sqrt{-g'}\, d^4x' = \sqrt{-g}\, d^4x, \tag{2.41}$$

Preserving the action:

$$S[\phi] = \int d^4x \sqrt{-g}\, \mathcal{L}[\phi, g_{\mu\nu}], \tag{2.42}$$

where $\mathcal{L}$ is the Lagrangian density of the scalar field. This invariance ensures that the equations of motion derived from the action, such as the Klein-Gordon equation, remain form-invariant under coordinate transformations.

Physically, diffeomorphism invariance encapsulates the principle of general covariance, ensuring that the dynamics of any field depend only on the geometry of spacetime and not on the specific coordinates used [21]; the observer independence implies that physical phenomena, such as vacuum fluctuations and particle creation, are determined by the spacetime geometry rather than the coordinate system, this leads to observer-dependent effects [4].

There also exists gauge freedom, introducing a redundancy in the description of physical systems, analogous to gauge invariance in electromagnetism [22]. This requires careful treatment during quantization to define meaningful physical quantities.

The next step is find the equation of motion for the scalar field $\phi(x)$, which is derived by varying the action $S[\phi]$ with respect to $\phi(x)$. To describe a quantum scalar field $\phi(x)$ in curved spacetime, we must generalize the field equations to account for the non-trivial geometry. From the action [38]:

$$\delta S = \int d^4x \sqrt{-g} \left[-\nabla_\mu \left(\nabla^\mu \phi \right) + m^2 \phi \right] \delta\phi = 0. \tag{2.43}$$

Where, as proposed on previous sections, the covariant derivative ∇_μ generalizes the concept of partial derivatives to curved spacetime, incorporating the effects of spacetime curvature via the

Christoffel symbols $\Gamma^{\lambda}_{\mu\nu}$.

This action leads to the Klein-Gordon equation in curved spacetime [3], when the Euler-Lagrange equations are solved, equation that governs the dynamics of the scalar field, incorporating the effects of spacetime curvature.

$$\Box_g \phi - m^2 \phi = 0. \tag{2.44}$$

Where $\Box_g$ is the **D′Alembertian operator** in curved spacetime, which is defined as:

$$\Box_g \phi = \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} g^{\mu\nu} \partial_\nu \phi \right), \tag{2.45}$$

Where g is the determinant of the metric tensor, $g^{\mu\nu}$ is the inverse of the metric tensor, and ∂_μ is the partial derivative with respect to the spacetime coordinates x^μ.

Quantization

The D′Alembertian operator reduces to the ordinary wave operator $\Box = \eta^{\mu\nu}\partial_\mu\partial_\nu$ in flat (Minkowski) spacetime, when $g_{\mu\nu} \to \eta_{\mu\nu}$. The term $\sqrt{-g}$ accounts for the curved spacetime volume element, and the Klein-Gordon equation in this form governs the dynamics of the scalar field in a gravitationally curved background. It reduces to the standard flat-space Klein-Gordon equation when $g_{\mu\nu} \to \eta_{\mu\nu}$.

In flat spacetime, QFT relies on the symmetries of the Poincaré group, which provide a natural definition of energy, momentum and vacuum states [22]. In curved spacetime, the lack of global symmetries and a preferred reference frame poses challenges for quantization [4]. The key steps for quantization are:

- Decomposing the field $\phi(x)$ into mode functions $u_i(x)$ that satisfy the Klein-Gordon equation.
- Expressing the field as:
  $$\phi(x) = \sum_i \left(a_i u_i(x) + a_i^\dagger u_i^*(x) \right), \tag{2.46}$$
  where a_i and $a_i^\dagger$ are the annihilation and creation operators, respectively.
- Ensuring diffeomorphism invariance by constructing observables and quantities that are independent of coordinate choices; this depends on each type of spacetime, and the conformal mapping between the metrics is a mathematical procediment.

As shown before, the procedure for quantizing a field in curved spacetime follows the same basic steps as in flat spacetime (**Canonical Quantization**) but with key differences due to the absence of global symmetries such as time translation or Lorentz boosts [3]. In flat spacetime, one can expand a field $\phi(x)$ in terms of plane-wave modes with definite energy and momentum. However, in curved spacetime, such an expansion is not generally possible due to the lack of a global time-like symmetry that defines a preferred time direction [38]. Instead, we must work with a mode expansion in terms of local solutions to the field equation.

We express the quantum field $\phi(x)$ as a sum over mode functions $u_i(x)$ and their conjugates $u_i^*(x)$, which form a complete orthonormal set with respect to the Klein-Gordon inner product [3].

$$\phi(x) = \sum_i \left(a_i u_i(x) + a_i^\dagger u_i^*(x) \right); \tag{2.47}$$

Here, a_i and $a_i^\dagger$ are the annihilation and creation operators, satisfying the same canonical commutation relations as before [3]:

$$[a_k, a_l^\dagger] = \delta_{kl}, \quad [a_k, a_l] = [a_k^\dagger, a_l^\dagger] = 0. \tag{2.48}$$

This formalism allows us to build the Fock space in curved spacetime [4], but the key difference from flat spacetime arises in the concept of the vacuum state. This mode functions are chosen to satisfy the Klein-Gordon inner product, which is [3]:

$$(u_k, u_l) = -i \int_\Sigma d\Sigma^\mu \left(u_k \partial_\mu u_l^* - u_l^* \partial_\mu u_k \right), \tag{2.49}$$

Where Σ is a spacelike hypersurface. The choice of the mode functions $u_i(x)$ is crucial and depends on the background geometry. In general, different observers will select different sets of mode functions, leading to the observer dependence of particle content [3].

Bogoliubov Transformations and Particle Creation

As mentioned previously, one of the most striking features of QFT in curved spacetime is the phenomenon of particle creation in time-dependent geometries, leaded by the vacuum ambiguity. Unlike in flat spacetime, where the vacuum is uniquely defined and observer-independent, the lack of a global time symmetry in curved spacetime leads to the existence of multiple vacuums [3]. Then, the notion of a vacuum state becomes ambiguous due to the dependence of the field mode expansion on the spacetime geometry. Different observers may define different sets of mode functions, and thus different vacuum states.

In a time-dependent background, the mode functions that define the vacuum at one time may differ from those later, and as a result, an observer at a later time may detect particles in what was originally the vacuum state; this is the Unruh effect, which will be discussed later. Therefore, here is introduced the Bogoliubov transformations, as the way to describe the observer-dependent particle content of quantum fields.

Consider a quantum scalar field $\phi(x)$ in a curved spacetime that evolves between two asymptotically flat regions. Let the mode functions of the field at an initial time t_0 be denoted by $u_k^{(t_0)}(x)$, and at a later time t_1, by $u_k^{(t_1)}(x)$; corresponding to two different observers or two different vacuum. These mode functions form complete orthonormal sets, satisfying the Klein-Gordon equation (2.44). The relationship between the two sets of modes is expressed using Bogoliubov transformations [3]:

$$u_i^{(t_0)}(x) = \sum_j \left(\alpha_{ij} u_j^{(t_1)}(x) + \beta_{ij} u_j^{(t_1)*}(x) \right), \tag{2.50}$$

where α_{ij} and β_{ij} are the Bogoliubov coefficients. These coefficients encode how the modes $u_j^{(t_1)}$

and their complex conjugates contribute to the modes $u_i^{(t_0)}$ [3], encoding the mixing between positive and negative frequency modes [4].

The Bogoliubov coefficients satisfy the following normalization conditions, ensuring the preservation of the Klein-Gordon inner product [3]:

$$\sum_k \left(\alpha_{ik}\alpha^*_{jk} - \beta_{ik}\beta^*_{jk}\right) = \delta_{ij}, \tag{2.51}$$

$$\sum_k \left(\alpha_{ik}\beta_{jk} - \beta_{ik}\alpha_{jk}\right) = 0. \tag{2.52}$$

The number of particles created in mode j is proportional to $|\beta_{ij}|^2$, indicating that time-dependent gravitational fields can spontaneously generate particles from the vacuum [38]. The number of particles created in the v_k-vacuum, as observed in the u_k-vacuum, is given by:

$$N_j = \sum_i |\beta_{ij}|^2. \tag{2.53}$$

This transformation reflects the observer-dependence of the vacuum state in curved spacetime. Different observers, defined by their respective mode expansions, may disagree on the particle content of the same quantum field. Besides, near event horizons, the division of spacetime into causally disconnected regions leads to thermal radiation effects, such as Hawking radiation and the Unruh effect, which will be presented forward [7, 10].

Applications and Observations

QFT in curved spacetime is not merely a theoretical, it has implications for understanding the universe's most extreme environments and fundamental processes. Its applications span cosmology and BH physics.

Some observational advances are in observational astrophysics and cosmology, providing opportunities to test the predictions of QFT in curved spacetime, such as the detection of gravitational waves from BH mergers provides indirect evidence for horizon-related quantum phenomena, such as ringdown modes that may encode information about Hawking radiation [39–41]; and precision measurements of the CMB anisotropies confirm the quantum origin of primordial fluctuations during inflation, validating the ideas about the vacuum ambiguity [42–44].

Summary

This chapter has explored one of the most profound extensions of modern theoretical physics: the unification of QFT with the geometric principles of general relativity in curved spacetime. This generalization allows us to study quantum phenomena in the presence of strong gravitational fields, bridging two foundational pillars of physics.

A mathematical framework that describes quantum fields in curved spacetime was shown. Beginning with the action formulation for a scalar field, the role of the metric tensor, covariant derivatives and the spacetime volume element. These components ensure that the theory remains consistent under general coordinate transformations, embodying the principle of diffeomorphism invariance, or conformal mapping. The Klein-Gordon equation, generalized to curved backgrounds, became the foundation for understanding field dynamics under non-trivial geometric conditions.

The quantization of fields in curved spacetime introduced challenges do not present in flat spacetime, particularly the absence of global symmetries. This led to the concept of vacuum ambiguity, where the definition of the vacuum state depends on the observer's frame of reference. This ambiguity underpins much of the rich structure of QFT in curved spacetime, including phenomena such as particle creation. Bogoliubov transformations provided the mathematical machinery to relate the modes of quantum fields between different observers, illustrating how time-dependent spacetime geometries can generate particles.

The chapter also delved into the observer-dependent nature of quantum effects near horizons. In the vicinity of black hole event horizons or cosmological horizons, the perception of the vacuum and particle content changes depending on the observer's trajectory. This leads to phenomena such as the Unruh effect, where an accelerating observer detects a thermal bath of particles; and Hawking radiation, where black holes emit thermal radiation.

Chapter 3

Quantum Effects on Curved Spacetime and Black Holes

The interplay between quantum field theory (QFT) and general relativity has led to profound insights into the behavior of matter and energy in strong gravitational and electromagnetic fields. The curvature of spacetime, dictated by Einstein's equations, not only modifies the motion of particles and light but also alters the quantum vacuum itself. This chapter explores specific quantum phenomena that emerge in curved spacetime conditions, with a particular focus on the effects observed near black BH, where curvature reaches its most intense manifestations.

The origins of this field can be traced back to the mid-20th century, when the foundations of quantum field theory were established in flat spacetime, governed by the Poincaré group [22]. As the theoretical framework for general relativity matured, attention turned to understanding how quantum fields behave in curved spacetimes. Seminal works by Leonard Parker BH particle creation in expanding universes [5, 8] and Stephen Hawking on BH radiation [7] revealed that curved spacetime not only modifies the classical motion of particles but also creates entirely new quantum effects.

In this chapter, we investigate some of the quantum effects arising in curved spacetimes and their mathematical underpinnings. Among these phenomena, the Schwinger effect demonstrates how intense electromagnetic fields can destabilize the quantum vacuum, leading to the spontaneous creation of particle-antiparticle pairs [3, 8]. This process, initially formulated in flat spacetime by Julian Schwinger, becomes even more intricate in curved backgrounds due to the gravitational and electromagnetic fields.

Another critical phenomenon is the Unruh effect [10], where an observer undergoing constant acceleration in flat spacetime perceives a thermal bath of particles, in stark contrast to the inertial observer's perception of a vacuum. This effect shows the observer-dependence of the vacuum state in QFT and serves to understanding horizon-related phenomena.

Perhaps the most famous example of quantum effects in curved spacetime is Hawking radiation [3, 6], where BH are shown to emit thermal radiation due to quantum field interactions near the event horizon. This theoretical prediction challenged the classical view of BHs as perfectly absorbing objects and provided insights into the thermodynamic behavior of horizons, culminating in the formulation of BH thermodynamics.

The superradiance effect [45], another remarkable phenomenon, reveals how rotating BHs can amplify incoming waves, enabling energy extraction. This process, intimately tied to the structure of the Kerr metric, has implications for astrophysical jets and the dynamics of accretion disks around rotating BHs.

Finally, the chapter addresses the memory burden effect [9], a concept tied to the lasting imprints of gravitational waves on spacetime geometry. This phenomenon, linked to the non-linearities of Einstein's equations, offers new avenues for understanding radiation in BHs, due to the information paradox.

Physically, these effects are not limited to theoretical constructs but have observable consequences in high-energy astrophysics, cosmology, and gravitational wave astronomy; at least on predictions. The Schwinger effect provides insights into pair production near charged BHs [13], while Hawking radiation challenges the very foundations of quantum mechanics through the BH information paradox [16]. Superradiance and the memory burden effect have direct implications for the detection of gravitational waves and the dynamics of rotating compact objects as they could show evidences for BH- radiation [18, 45].

This chapter aims to provide a physical exploration of these phenomena, showing some of the mathematical principles and the physical interpretations. By delving into the interactions between quantum fields and the curvature of spacetime, we gain deeper understanding of these effects and the universe deepens. Finally, it will given a shallow review on those quantum effects applied to Primordial Black Holes.

3.1 Schwinger Effect

The Schwinger effect refers to the non-perturbative creation of particle-antiparticle pairs from the quantum vacuum under the influence of a strong electric field. Its origin came from quantum electrodynamics (QED) prediction and first derived by Julian Schwinger [8], this phenomenon is a prime example of how classical fields interact with quantum vacuum in ways that transcend perturbative approaches [46]. It occurs when the work done by the electric field over a Compton wavelength of a charged particle exceeds the particle's rest energy, leading to vacuum instability. In curved spacetimes, the Schwinger effect becomes more intricate, as gravitational fields contribute additional modifications to particle production [3].

The Schwinger effect is often derived using the formalism of the quantum effective action [47], which encodes vacuum fluctuations in the presence of a strong electric field. This action is obtained by applying quantum corrections to a classical action $S[\phi]$ for a field ϕ, for this we start by considering a linearly coupled field source $J(X)$, then we define a functional $Z[J]$ that depends of this source [22]:

$$Z[J] = e^{iW[J]} = \int \mathcal{D}\phi e^{iS[\phi]+i\int d^4xJ(x)\phi(x)}, \tag{3.1}$$

Where $\mathcal{D}\phi$ is the covariant derivative of the field. Then, we can write [22]:

$$\frac{\delta W}{\delta J(x)} = -i\frac{\delta \ln Z}{\delta J(x)} = \frac{\int \mathcal{D}\phi\phi(x)e^{iS[\phi]+i\int d^4xJ(x)\phi(x)}}{\delta J(x)}, \tag{3.2}$$

Which implies that [22]:

$$\frac{\delta W}{\delta J(x)} =< 0|\phi(x)|0 >\equiv \phi_c(x), \tag{3.3}$$

Where $\phi_c(x)$ is the expected value of the field, associated with the vacuum. Then, we can define the effective action, which is written as [22]:

$$\Gamma[\phi_c(x)] \equiv W[J] - \int d^4x J(x)\phi_c(x). \tag{3.4}$$

There we can see that $\Gamma[\phi_c(x)]$ is the Legendre transform of $W[J]$. If we assume the absence of the external source ($J(x) \to 0$), then $\phi_c(x)$ is constant [1], and the effective action turns into [22]:

$$\Gamma[\phi(x)] = -\int d^4x V_{eff}(\phi) = -VTV_{eff}(\phi), \tag{3.5}$$

Where VT is the volume of a finite spacetime box.

We will develop thorough a theoretical framework the Schwinger effect, deriving the pair production rate using the formalism of QFT and applying this to both flat and curved spacetime; also discussing how the Schwinger effect is modified in the presence of strong gravitational fields, such as near BHs, and how it relates to other quantum effects in curved spacetime.

First of all, we take a look at the vacuum instability in flat spacetime, let us review the derivation of the Schwinger effect [47]. The key idea is that a strong electric field can destabilize the quantum vacuum by providing enough energy for virtual particle-antiparticle pairs to materialize as real particles. This phenomenon can be understood semiclassically using a tunneling picture, where particles tunnel through the energy barrier separating the vacuum from the pair-production state [48].

We consider a uniform, constant electric field $\mathbf{E}$ in flat spacetime, aligned along the x-axis. The quantum field we focus on is a charged scalar field $\phi(x)$, with the Lagrangian density given by [47]:

$$\mathcal{L} = D_\mu\phi^* D^\mu\phi - m^2\phi^*\phi,$$

Where $D_\mu = \partial_\mu + ieA_\mu$ is the covariant derivative, e is the charge of the field, A_μ is the electromagnetic potential associated with the electric field $\mathbf{E}$, and m is the mass of the particles. In a gauge where the potential is $A_\mu = (-Et, 0, 0, 0)$, the equation of motion for the scalar field is [47]:

$$\left[(\partial_t + ieEt)^2 - \nabla^2 + m^2\right]\phi(x) = 0.$$

This is the Klein-Gordon equation for a charged particle in an external electric field. The physical interpretation of this equation is that the electric field imparts energy to the quantum vacuum, leading to the production of particle-antiparticle pairs [47]. To compute the pair-production rate, we follow [47] method which is based on the imaginary part of the effective action Γ_{eff}. The effective action In a flat spacetime and QED background, the effective action for a charged scalar field $\phi(x)$ in a constant electric field E is given by [49]:

$$\Gamma_{\text{eff}} = -i \ln\det\left(iD^\mu D_\mu - m^2\right), \tag{3.6}$$

where m is the particle mass. There, the imaginary part of the effective action provides a measure of the vacuum instability, signaling the production of real particle-antiparticle pairs [47]:

$$\mathrm{Im}\,\Gamma_{\text{eff}} = \frac{e^2E^2}{4\pi^3}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp\left(-\frac{n\pi m^2}{eE}\right), \tag{3.7}$$

The summation over n corresponds to the quantized energy levels contributing to the tunneling process. The exponential suppression factor $e^{(-\pi m^2/eE)}$ shows the non-perturbative nature of the Schwinger effect. For $eE \ll m^2$, the pair-production rate is negligible, but it becomes significant when $eE \sim m^2$, indicating that the field provides sufficient energy to overcome the particle rest mass. This expression shows that pair production is exponentially suppressed for weak electric fields but becomes significant when the electric field strength E approaches the critical value $E_{\text{crit}} = \frac{m^2}{e}$, where the energy supplied by the field over a Compton wavelength $\lambda_C = \frac{1}{m}$ is sufficient to produce real particles [47].

Classically, a pair of particles would not be able to overcome this potential barrier unless they had sufficient energy. However, in quantum mechanics, there is a finite probability for the particles to tunnel through the barrier, even if they do not have enough energy to classically overcome it [48]. The probability for this tunneling process can be calculated using the WKB (Wentzel–Kramers–Brillouin) approximation. The WKB approximation provides the tunneling probability [49]:

$$P \sim \exp\left(-2\int_0^{x_0}\sqrt{V_{\text{eff}}(x^0) - m^2}\,dx^0\right), \tag{3.8}$$

where x_0 represents the upper limit of the potential barrier.

In this case, due to the conditions, the pair-production rate per unit volume is more simple, derived again from the imaginary part of the effective action, is [48]:

$$\Gamma = \frac{e^2E^2}{4\pi^3}\exp\left(-\frac{\pi m^2}{eE}\right). \tag{3.9}$$

This rate matches the leading-order term of the exact result and captures the exponential suppression of the process for weak electric fields [48]; also it reveals the non-perturbative nature of the effect.

Thus, the Schwinger effect in flat spacetime provides a rigorous framework for understanding vacuum instability in the presence of strong fields. However, its generalization to curved spacetimes introduces additional factors, such as redshift effects and spacetime curvature, which modify the effective potential and tunneling dynamics.

3.1.1 Curved Spacetime Generalization

The extension of the Schwinger effect to curved spacetime introduces additional complexities due to the interplay between electromagnetic and gravitational fields. In this generalized framework, the creation of particle-antiparticle pairs is influenced by both the local curvature of spacetime and the redshift effects induced by gravity [50]. The Klein-Gordon equation, which governs the dynamics of a charged scalar field, must be adapted to account for the covariant structure of de Sitter spacetime.

In a curved spacetime background, the dynamics of a charged scalar field $\phi(x)$ interacting with a constant electromagnetic field in de Sitter space [2]:

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2, a(t) = e^{Ht} \tag{3.10}$$

where $a(t)$ is the scale factor and H is the Hubble's parameter. With a constant electric field which vector potential is $A(t) = -\frac{E}{H}\left(e^{Ht} - 1\right)dx$. Then the Hamiltonian is [50]:

$$H = \int dx \left[\frac{1}{a}|\pi|^2 + \frac{1}{a}|(\partial_x - iqA)\phi|^2 + m^2 a|\phi|^2\right]. \tag{3.11}$$

Where the conjugate momenta are $\pi = a\dot{\phi}^*$ and $\tau^* = a\dot{\phi}$. The Fourier modes are [50].

$$\phi(t,x) = \int \frac{dk}{2\pi}\phi_k e^{ikx}, \quad \phi^*(t,x) = \int \frac{dk}{2\pi}\phi_k^* e^{ikx} \tag{3.12}$$

Similarly for $\pi(t,x)$ and $\pi^*(t,x)$. By making the analysis with the Bogoliubov transformations, we obtain the Hamiltonian in terms of this Fourier modes [50]:

$$H(t) := \sum_k H_k(t) = \int \frac{dk}{2\pi}\left[\frac{1}{a}\pi_k^*\pi_k + a\omega_k^2\phi_k^*\phi_k\right]. \tag{3.13}$$

Where:

$$\omega_k^2 = \frac{1}{a^2}\left(k - \frac{qE}{H} + \frac{qE}{H}a\right)^2 + m^2, \tag{3.14}$$

Being visible that the scale factor induce changes on the pair-particle production. For this we have to make the process to find the Bogoliubov coefficients, but this is hard in general.

In the context of BH, the Schwinger effect is particularly relevant near the horizons of charged BH [13]; and it is closely related to Hawking radiation, as both effects involve the creation of particles from the quantum vacuum in the presence of a strong external field, although the Schwinger effect is driven by an electric field, while Hawking radiation is driven by spacetime curvature near the event horizon.

Exact solutions for the Schwinger effect in curved spacetime are rare and often require numerical methods [46–48], like solving the Klein-Gordon equation with curvature corrections using finite difference or spectral methods, where the created particles modify the spacetime geometry and electromagnetic field.

Now, despite the theoretical aspects of the Schwinger effect, direct observation remains challenging due to the immense field strengths required ($E_{\text{crit}} = m^2/e$) [3]. However, several potential avenues exist on astrophysical environments like charged BHs [51], where electromagnetic fields can approach or exceed E_{crit}, leading to observable high-energy emissions. It could be also seen on laboratory experiments using ultra-intense laser facilities, such as the Extreme Light Infrastructure (ELI) [52], aim to achieve field strengths close to E_{crit}, providing a potential means to observe pair production in controlled setting.

In summary, the Schwinger effect, though challenging to observe directly, provides a fascinating window into the non-perturbative dynamics of quantum fields. Its study not only deepens our understanding of QFT in curved spacetime but also bring insight into the theoretical predictions relation with experimental and astrophysical observations.

3.2 Unruh Effect

The Unruh effect is a phenomenon predicted in QFT demonstrating that an observer undergoing constant acceleration in flat spacetime perceives the vacuum as a thermal state [53]. First derived by William Unruh in 1976 [10], the effect has profound implications for understanding quantum phenomena in curved spacetime, where horizons introduce similar observer-dependent thermal effects, i.e. particle creation.

To develop the mathematical framework underlying the Unruh effect, is necessary to focus on the field quantization in both Minkowski and Rindler coordinates [3]. First, to derive the Unruh effect, we consider a quantum scalar field $\phi(x)$ in flat spacetime with the Minkowski metric, which is invariant under the symmetries of the Poincaré group [21]. A scalar field $\phi(x)$, representing a quantum excitation, satisfies the Klein-Gordon equation (2.44); the solutions to the Klein-Gordon equation are plane-wave modes:

$$u_{\mathbf{k}}(x) = \frac{1}{\sqrt{2\omega_{\mathbf{k}}}} e^{-i\omega_{\mathbf{k}} t + i\mathbf{k}\cdot\mathbf{x}}, \tag{3.15}$$

where $\mathbf{k} = (k_1, k_2, k_3)$ is the three-momentum and $\omega_{\mathbf{k}} = \sqrt{\mathbf{k}^2 + m^2}$ is the energy of the mode. The scalar field is quantized by promoting it to an operator $\hat{\phi}(x)$, applying the canonical quantization defined before. Then, the mode expansion:

$$\hat{\phi}(x) = \int d^3k \left(\hat{a}_{\mathbf{k}} u_{\mathbf{k}}(x) + \hat{a}^\dagger_{\mathbf{k}} u^*_{\mathbf{k}}(x) \right), \tag{3.16}$$

where $\hat{a}_{\mathbf{k}}$ and $\hat{a}^\dagger_{\mathbf{k}}$ are the annihilation and creation operators [3], respectively, and this operators obey the typical commutation relations for them.

The Minkowski vacuum state $|0\rangle_M$ is defined as the state annihilated by all $\hat{a}_{\mathbf{k}}$ [3] :

$$\hat{a}_{\mathbf{k}}|0\rangle_M = 0, \quad \forall \mathbf{k}. \tag{3.17}$$

This state represents the absence of particles as perceived by an inertial observer in Minkowski spacetime. The completeness and orthonormality of the mode functions are ensured by the Klein-Gordon inner product (2.47). The modes satisfy:

$$(u_{\mathbf{k}}, u_{\mathbf{k}'}) = \delta^3(\mathbf{k} - \mathbf{k}'), \quad (u^*_{\mathbf{k}}, u^*_{\mathbf{k}'}) = -\delta^3(\mathbf{k} - \mathbf{k}'), \quad (u_{\mathbf{k}}, u^*_{\mathbf{k}'}) = 0. \tag{3.18}$$

The quantized field $\hat{\phi}(x)$ represents a collection of harmonic oscillators, with each mode $\mathbf{k}$ corresponding to an independent degree of freedom [3]. The operator $\hat{a}^\dagger_{\mathbf{k}}$ creates a quantum of excitation (a particle) in the mode $\mathbf{k}$, while $\hat{a}_{\mathbf{k}}$ annihilates it. The Fock space is constructed by repeatedly applying the creation operators to the vacuum state [38], yielding multi-particle states:

$$|\mathbf{k}_1, \mathbf{k}_2, \ldots, \mathbf{k}_n\rangle = \hat{a}^\dagger_{\mathbf{k}_1} \hat{a}^\dagger_{\mathbf{k}_2} \cdots \hat{a}^\dagger_{\mathbf{k}_n} |0\rangle_M. \tag{3.19}$$

In Minkowski spacetime, the vacuum state $|0\rangle_M$ is uniquely defined by the symmetries of the spacetime, particularly time translation invariance [22]. This ensures that all inertial observers agree on the particle content of the field [54]. However, this property is lost in non-inertial frames or curved spacetimes, where the absence of global symmetries leads to observer-dependent vacuum states. This feature forms the basis of the Unruh effect and similar phenomena in curved spacetime.

Now, to describe the perspective of a constantly accelerating observer, it is suggested to adopt **Rindler coordinates** [3], which partition Minkowski spacetime into regions causally connected to the observer's trajectory. The relationship between Minkowski coordinates (t, z) and Rindler coordinates (η, ρ) is given by [2, 20, 27]:

$$t = \rho \sinh(a\eta) \quad \text{and} \quad z = \rho \cosh(a\eta), \tag{3.20}$$

where a is the constant proper acceleration of the observer, $\rho > 0$ is the proper distance from the Rindler horizon $(z = \pm t)$, η is the proper time of the accelerating observer. The Minkowski metric transforms into the following form in Rindler coordinates [20]:

$$ds^2 = -\rho^2 d\eta^2 + d\rho^2 + dx^2 + dy^2. \tag{3.21}$$

Here, $-\rho^2 d\eta^2$ indicates that ρ serves as the radial coordinate, and η is the temporal coordinate for the accelerated observer. The Rindler horizon, located at $\rho = 0$, acts as a boundary beyond which events are causally disconnected from the accelerated observer [55]. In Minkowski coordinates, this corresponds to the light-like surfaces $z = \pm t$, which the observer can never cross. The horizon introduces a causal asymmetry, analogous to the event horizon of a BH, and plays a critical role in the thermal perception of the vacuum by the accelerating observer [3].

The spacetime is divided into four distinct regions by the light-like surfaces $z = \pm t$ [55]:

- **Region I ($z > |t|$):** The right Rindler wedge, where the accelerated observer resides.
- **Region II ($z < -|t|$):** The left Rindler wedge, causally disconnected from Region I.
- **Regions III and IV:** The future and past wedges, outside the reach of the Rindler observer's trajectory.

Furthermore, the trajectory of the accelerating observer is given by [26]:

$$z^2 - t^2 = \rho^2 = \text{constant}. \tag{3.22}$$

The proper acceleration a is related to the radial coordinate ρ by [26]:

$$a = \frac{1}{\rho}. \tag{3.23}$$

Thus, observers closer to the Rindler horizon experience larger proper accelerations, while those further away have smaller accelerations. This dependence on ρ also affects the perceived temperature, as shown in the Unruh effect [53]:

$$T_U = \frac{\hbar}{2\pi k_B \rho}. \tag{3.24}$$

The Rindler horizon, acting as a causal barrier, is directly analogous to the event horizon of a BH. This analogy bridges the Unruh effect and Hawking radiation, as both involve thermal phenomena arising from horizon-related quantum effects. Furthermore the Doppler shift of modes near the

Rindler horizon resemble similar effects near BH horizons [53]; and the division of spacetime into causally disconnected regions reflects the observer-dependent nature of QFT.

3.2.1 Field Quantization in Rindler Coordinates

The quantization of a scalar field in Rindler coordinates follows a process analogous to Minkowski quantization but adapted to the non-inertial reference frame of an accelerated observer, it follows the canonical quantization by making a conformal mapping between both metrics. The Klein-Gordon equation in Rindler coordinates is [3]:

$$\Box_R \phi - m^2 \phi = 0, \tag{3.25}$$

where $\Box_R$ is the d'Alembertian operator in Rindler spacetime [3]:

$$\Box_R = \frac{1}{\rho^2}\frac{\partial^2}{\partial \eta^2} - \frac{1}{\rho}\frac{\partial}{\partial \rho} + \frac{\partial^2}{\partial \rho^2} + \nabla_\perp^2. \tag{3.26}$$

The solutions to the Klein-Gordon equation can be separated into modes using the ansatz [3]:

$$\phi(x) = \int d\omega \int d^2 k_\perp \left(b_{\omega,\mathbf{k}_\perp} v_{\omega,\mathbf{k}_\perp}(x) + b^\dagger_{\omega,\mathbf{k}_\perp} v^*_{\omega,\mathbf{k}_\perp}(x) \right), \tag{3.27}$$

where $v_{\omega,\mathbf{k}_\perp}(x)$ are the Rindler mode functions, given by:

$$v_{\omega,\mathbf{k}_\perp}(x) = \frac{1}{\sqrt{4\pi^3 \rho}} e^{i\mathbf{k}_\perp \cdot \mathbf{x}_\perp} e^{-i\omega\eta} K_{i\omega}\left(\sqrt{\mathbf{k}_\perp^2 + m^2}\rho \right). \tag{3.28}$$

Where $K_{i\omega}$ is the modified Bessel function of the second kind, ensuring regularity at the Rindler horizon ($\rho = 0$), $\mathbf{k}_\perp = (k_x, k_y)$ represents the transverse momentum [53]. The coefficients $b_{\omega,\mathbf{k}_\perp}$ and $b^\dagger_{\omega,\mathbf{k}_\perp}$ are the annihilation and creation operators associated with the Rindler modes. These operators obey the canonical commutation relations [38]:

$$[b_{\omega,\mathbf{k}_\perp}, b^\dagger_{\omega',\mathbf{k}'_\perp}] = \delta(\omega - \omega')\delta^2(\mathbf{k}_\perp - \mathbf{k}'_\perp), \quad [b_{\omega,\mathbf{k}_\perp}, b_{\omega',\mathbf{k}'_\perp}] = 0. \tag{3.29}$$

Defining the Rindler vacuum state $|0\rangle_R$ as [3]:

$$b_{\omega,\mathbf{k}_\perp}|0\rangle_R = 0, \quad \forall \omega, \mathbf{k}_\perp. \tag{3.30}$$

This vacuum reflects the absence of particles as perceived by an accelerating observer. Now, it's time to apply the Bogoliubov transformations, defined before; this formalism reveals that the Minkowski vacuum appears as a thermal state to an accelerating observer [3]. Thu, the Rindler modes are related to Minkowski modes via this transformations, as:

$$u_{\mathbf{k}}(x) = \int d\omega \left(\alpha_{\mathbf{k}\omega} v_{\omega,\mathbf{k}_\perp}(x) + \beta_{\mathbf{k}\omega} v^*_{\omega,\mathbf{k}_\perp}(x) \right), \tag{3.31}$$

These coefficients are determined by the overlap integrals using the Klein-Gordon inner product.

$$\alpha_{\mathbf{k}\omega} = (u_{\mathbf{k}}, v_{\omega,\mathbf{k}_\perp}), \quad \beta_{\mathbf{k}\omega} = -\left(u_{\mathbf{k}}, v^*_{\omega,\mathbf{k}_\perp}\right), \tag{3.32}$$

The mixing of positive and negative frequency components, quantified by $\beta_{\mathbf{k}\omega}$, leads to the perception of particles by a Rindler observer, even if the Minkowski observer perceives a vacuum [54].

3.2.2 Thermal Interpretation

The field quantization in Rindler coordinates remains an important phenomena, the thermal spectrum perceived by the accelerating observer. While an inertial observer in Minkowski spacetime perceives the vacuum state $|0\rangle_M$ as devoid of particles, an observer following a constant acceleration a detects a thermal distribution of particles. This phenomenon is encapsulated in the expectation value of the Rindler particle number operator [3]:

$$\langle 0|_M b^\dagger_{\omega,\mathbf{k}_\perp} b_{\omega,\mathbf{k}_\perp} |0\rangle_M = \frac{1}{e^{2\pi\omega/a} - 1}, \tag{3.33}$$

where ω is the mode frequency as measured by the Rindler observer. The result matches the Planck distribution for a thermal spectrum, by an analogy with statistical mechanics and its distribution functions, with the associated temperature:

$$T_U = \frac{\hbar a}{2\pi k_B c}. \tag{3.34}$$

This is known as the **Unruh temperature**, indicating that the accelerating observer experiences the Minkowski vacuum as a thermal bath of particles with a temperature proportional to their acceleration. The exponential factor in the denominator, $e^{2\pi\omega/a}$, arises from the Bogoliubov transformation, which mixes positive and negative frequency modes between the Minkowski and Rindler frames. The temperature T_U is proportional to the acceleration a, highlighting that stronger accelerations correspond to higher perceived temperatures. In the limit $a \to 0$, the Unruh temperature vanishes, recovering the inertial observer's perception of the vacuum.

The thermal spectrum is a direct consequence of the observer-dependent nature of the vacuum in QFT. For the Rindler observer, the Rindler horizon at $\rho = 0$ acts as a causal boundary, preventing information from beyond the horizon from influencing the observer's measurements [56]. This horizon introduces a division of modes, analogous to the division of modes near the event horizon of a BH. Thus, the thermal spectrum observed in the Unruh effect serves as a conceptual bridge to Hawking radiation [57]. In both cases, an observer perceives thermal radiation due to the presence of a horizon, the Rindler horizon in the Unruh effect and the event horizon in BH spacetimes.

Observing the Unruh effect directly is challenging due to the immense accelerations required to produce detectable Unruh temperatures [53]. For instance, an acceleration of $10^{24}\,\mathrm{m/s^2}$ corresponds to a temperature of approximately $473\,\mathrm{K}$ [57]. Although the Unruh temperature is minuscule for accelerations achievable in laboratory settings [56], the thermal spectrum provides a fundamental testable prediction of QFT in non-inertial frames.

3.3 Hawking Effect

The prediction of the Hawking effect in 1974 [7] marked an important moment in theoretical physics, bridging the domains of quantum mechanics, general relativity and thermodynamics.

Stephen Hawking demonstrated that BHs, as it was presented before, are not entirely "black" but emit thermal radiation due to quantum field effects near their event horizons [3]. This insight revolutionized our understanding of BH, challenging the classical notion of these objects as perfect absorbers.

The Hawking effect arises from the quantization of fields in the curved spacetime surrounding a BH. Near the event horizon, the strong curvature of spacetime creates a dynamic environment in which quantum fluctuations can generate particle-antiparticle pairs [4]. These fluctuations lead to the emission of particles that appear as thermal radiation to a distant observer, with a temperature proportional to the BH's surface gravity. This temperature, known as the Hawking temperature, established a direct link between the properties of BH and the laws of thermodynamics. This was showed in the previous chapter, but as one of the main results of quantum effects in curved spacetime, here we are going to show again some relevant results.

Historically, the idea of BH thermodynamics had been introduced earlier by Jacob Bekenstein [58], who proposed that BH possess an entropy proportional to the area of their event horizon. However, it was Hawking's derivation of BH radiation that provided the physical foundation for this idea [6], confirming that BH obey thermodynamic laws. This prediction also unveiled the tantalizing possibility of BH evaporation, as the radiation leads to a gradual loss of mass over time.

The Hawking effect arises from the quantization of a scalar field in the curved spacetime of a BH, leading to the emission of thermal radiation by the BH [38]. The quantization of a scalar field in Schwarzschild spacetime is the key to understanding the Hawking effect. It begins from the Schwarzschild metric, describing a spherically symmetric and static BH, is given by [26]:

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega^2, \tag{3.35}$$

where M is the BH mass, r is the radial coordinate, and $d\Omega^2$ is the line element on the unit sphere. The event horizon is located at $r_h = 2GM$. A scalar field $\phi(x)$ in this spacetime satisfies the Klein-Gordon equation (2.44). For the Schwarzschild metric, the equation explicitly becomes:

$$-\left(1 - \frac{2GM}{r}\right)^{-1} \frac{\partial^2 \phi}{\partial t^2} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \left(1 - \frac{2GM}{r}\right)\frac{\partial \phi}{\partial r}\right) + \frac{1}{r^2}\nabla^2_\Omega \phi - m^2\phi = 0, \tag{3.36}$$

Where ∇^2_Ω is the Laplacian on the two-sphere. To solve the Klein-Gordon equation, we assume a separable solution of the form:

$$\phi(t, r, \theta, \phi) = e^{-i\omega t} Y_{\ell m}(\theta, \phi) R_{\omega\ell}(r), \tag{3.37}$$

where $Y_{\ell m}(\theta, \phi)$ are the spherical harmonics, eigenfunctions of ∇^2_Ω, $R_{\omega\ell}(r)$ is the radial part of the solution, and ω is the frequency of the mode. Substituting this ansatz into the Klein-Gordon equation yields two independent equations:

$$\nabla^2_\Omega Y_{\ell m} = -\ell(\ell + 1) Y_{\ell m}, \tag{3.38}$$

$$\frac{d}{dr}\left(\Delta\frac{dR_{\omega\ell}}{dr}\right)+\left(\frac{\omega^2 r^4}{\Delta}-\ell(\ell+1)-m^2r^2\right)R_{\omega\ell}=0, \tag{3.39}$$

The first one is called the Angular equation, it allows to find the angular quantum number ℓ; whist second is the Radial equation, where $\Delta = r^2\left(1-\frac{2GM}{r}\right) = r^2 - 2GMr$ [59]. By solving this equations one can reach the expected result, but it is complicated in general. Then, there is a way to improve the solution.

Near the horizon, the Schwarzschild metric simplifies due to the vanishing of the lapse function $1 - 2GM/r$. To analyze this region, the radial coordinate r is transformed into the **tortoise coordinate** r_*, which accounts for the horizon's influence on wave propagation [3].

The tortoise coordinate provides a convenient framework for analyzing wave propagation in the Schwarzschild geometry. It regularizes the horizon behavior, maps the infinite redshift into a manageable coordinate divergence, and simplifies the connection between near-horizon dynamics and asymptotic observables [4]. The tortoise coordinate r_* is defined as [3]:

$$r_* = \int \frac{dr}{1-\frac{2GM}{r}}, \tag{3.40}$$

where the integrand diverges logarithmically as $r \to 2GM$. Explicitly:

$$r_* = r + 2GM\ln\left|\frac{r}{2GM}-1\right|, \tag{3.41}$$

Mapping the event horizon ($r = 2GM$) to $r_* \to -\infty$ and spatial infinity ($r \to \infty$) to $r_* \to \infty$. In terms of r_*, the Schwarzschild metric becomes [3]:

$$ds^2 = \left(1-\frac{2GM}{r}\right)\left(-dt^2 + dr_*^2\right), \tag{3.42}$$

Where the metric in the (t, r_*) plane approximates a conformally flat structure near the horizon. Rewriting the radial part of the Klein-Gordon equation in terms of r_* [3]:

$$\frac{d^2R_\omega}{dr_*^2} + \left[\omega^2 - V_{\text{eff}}(r)\right]R_\omega = 0, \tag{3.43}$$

where $V_{\text{eff}}(r)$ is the effective potential for the motion of a particle:

$$V_{\text{eff}}(r) = \left(1-\frac{2GM}{r}\right)\left[\frac{\ell(\ell+1)}{r^2}+\frac{2GM}{r^3}\right]. \tag{3.44}$$

Near the horizon ($r \to 2GM$), the effective potential V_{eff} vanishes, reducing the wave equation to:

$$\frac{d^2R_\omega}{dr_*^2} + \omega^2 R_\omega = 0. \tag{3.45}$$

Ergo, solutions are plane waves that represents ingoing ($-$) and outgoing ($+$) modes [4]:

$$R_\omega(r_*) \sim e^{\pm i\omega r_*}, \tag{3.46}$$

The vanishing of V_{eff} near the horizon reflects the absence of barriers for wave propagation, allowing ingoing modes to cross the horizon. However, outgoing modes experience a redshift as they propagate to infinity. This redshift significantly modifies the observed frequencies and contributes to the thermal nature of Hawking radiation [4]. The logarithmic divergence in r_* as $r \to 2GM$ introduces a branch cut in the complex plane, essential for deriving the thermal spectrum of Hawking radiation via Bogoliubov transformations [3]. The relationship between the ingoing and outgoing modes near the horizon captures the essence of particle creation in curved spacetime, which will be showed before.

The next step is to quantize this scalar field, to do this we follows the canonical quantization method. Then, expressing the scalar field $\phi(x)$ as an operator expanded in terms of mode functions $u_{\omega\ell m}(x)$ [3]:

$$\hat{\phi}(x) = \sum_{\ell,m} \int_0^\infty d\omega \left[\hat{a}_{\omega\ell m} u_{\omega\ell m}(x) + \hat{a}^\dagger_{\omega\ell m} u^*_{\omega\ell m}(x) \right], \tag{3.47}$$

where $\hat{a}_{\omega\ell m}$ and $\hat{a}^\dagger_{\omega\ell m}$ are the annihilation and creation operators for the mode (ω, ℓ, m); $u_{\omega\ell m}(x) = \frac{1}{\sqrt{4\pi\omega}} Y_{\ell m}(\theta,\phi) R_{\omega\ell}(r) e^{-i\omega t}$ are the mode functions; $Y_{\ell m}(\theta,\phi)$ are spherical harmonics; and $R_{\omega\ell}(r)$ are solutions to the radial wave equation. The operators satisfy the standard commutation relations:

$$\left[\hat{a}_{\omega\ell m}, \hat{a}^\dagger_{\omega'\ell' m'} \right] = \delta(\omega - \omega')\delta_{\ell\ell'}\delta_{mm'}, \tag{3.48}$$

With all other commutators vanishing. The vacuum state, known as the Boulware vacuum, is defined as [3]:

$$\hat{a}_{\omega\ell m}|0\rangle = 0, \quad \forall\, \omega, \ell, m. \tag{3.49}$$

Remembering the tortoise coordinates, the modes can be expressed as ingoing or outgoing depending on the position over the horizon. Thus, the scalar field can be written as [4]:

$$\phi(t,r,\theta,\phi) = \sum_{\ell,m} \int d\omega \, \left[\hat{a}^{\text{in}}_{\omega\ell m} u^{\text{in}}_{\omega\ell m}(x) + \hat{a}^{\text{out}}_{\omega\ell m} u^{\text{out}}_{\omega\ell m}(x) \right], \tag{3.50}$$

where $u^{\text{in}}_{\omega\ell m}(x)$ are ingoing modes regular at the horizon; $u^{\text{out}}_{\omega\ell m}(x)$ are outgoing modes observed at infinity; $\hat{a}^{\text{in}}_{\omega\ell m}$; and $\hat{a}^{\text{out}}_{\omega\ell m}$ are the corresponding annihilation operators. This ingoing and outgoing modes are related via a Bogoliubov transformation [60]:

$$u^{\text{out}}_{\omega\ell m} = \int d\omega' \left(\alpha_{\omega\omega'} u^{\text{in}}_{\omega'\ell m} + \beta_{\omega\omega'} u^{\text{in}*}_{\omega'\ell m} \right), \tag{3.51}$$

where $\alpha_{\omega\omega'}$ and $\beta_{\omega\omega'}$ are the Bogoliubov coefficients, encoding the mixing of positive and negative frequency components; and $\beta_{\omega\omega'} \neq 0$ indicates particle creation. The Bogoliubov coefficients satisfy the normalization condition ensuring the preservation of the Klein-Gordon inner product [3].

$$\int d\omega' \left(|\alpha_{\omega\omega'}|^2 - |\beta_{\omega\omega'}|^2 \right) = 1, \tag{3.52}$$

The Bogoliubov transformation formalism highlights the observer-dependent nature of the vacuum in curved spacetime. An asymptotic observer at infinity defines the vacuum state based on outgoing modes (u^{out}), perceiving a thermal spectrum. And an observer near the horizon perceives the vacuum as defined by ingoing modes (u^{in}), experiencing no particle creation [3]. The particle number detected at infinity is [3]:

$$\langle n_\omega \rangle = \sum_{\omega'} |\beta_{\omega\omega'}|^2. \tag{3.53}$$

Using the outgoing particle spectrum and the results for the Bogoliubov coefficients, it is possible to stablish the particle number relation [3]:

$$\langle 0|\hat{b}^\dagger_{\omega\ell m}\hat{b}_{\omega\ell m}|0\rangle = \langle n_\omega \rangle = \frac{1}{e^{\hbar\omega/k_B T_H} - 1}, \tag{3.54}$$

Where we can see that this distribution is identical to the Planck spectrum for blackbody radiation [61], confirming that the BH radiates thermally with temperature T_H, where T_H is the Hawking temperature [3]:

$$T_H = \frac{\hbar\kappa}{2\pi k_B} = \frac{\hbar c^3}{8\pi G M k_B}. \tag{3.55}$$

The thermal spectrum can be understood as the result of quantum fluctuations near the horizon. Implying virtual particle-antiparticle pairs are constantly produced due to vacuum fluctuations [3]. Also, near the horizon, the strong gravitational field separates the pairs; one particle escapes to infinity as Hawking radiation, while the other falls into the BH. This process causes the BH to lose mass, leading to gradual evaporation over time. The rate of mass loss is given by [3]:

$$\frac{dM}{dt} \propto -T_H \cdot A, \tag{3.56}$$

Where $A = 4\pi(2GM)^2$ is the surface area of the BH, and it coincides with the First Law of BH thermodynamics (for Schwarzschild BH).

3.3.1 Thermodynamic Implications

The theoretical foundation of Hawking radiation cemented the connection between BH mechanics and thermodynamics, transforming BH from purely gravitational objects into thermodynamic systems. Here we explore how Hawking radiation integrates BH physics into the broader framework of thermodynamics and quantum mechanics, elaborating on the connections previously introduced in Section 2.2.2.

Hawking's derivation of BH radiation revealed that the temperature of a BH is proportional to its surface gravity κ, as it showed before. For a Schwarzschild BH, $\kappa = \frac{1}{4GM}$, leading to:

$$T_H \propto \frac{1}{M}. \tag{3.57}$$

This inverse relationship implies that smaller BH are hotter, radiate more energetically, and evaporate faster than larger BH. Bekenstein's pioneering work suggested that BHs have an entropy proportional to the area of their event horizon [58]:

$$S_{\text{BH}} = \frac{k_B A}{4\ell_p^2}, \quad A = 4\pi(2GM)^2, \tag{3.58}$$

where $\ell_p = \sqrt{\hbar G/c^3}$ is the Planck length. The entropy is extraordinarily large compared to ordinary thermodynamic systems, due to BH mass, reflecting the immense information content that could be encoded in the event horizon. The Hawking temperature T_H and BH entropy S_{BH} together satisfy the first law of BH thermodynamics, previously shown:

$$dM = T_H dS_{\text{BH}}. \tag{3.59}$$

The inclusion of BH entropy in thermodynamics extends the second law to encompass both ordinary matter and the BH horizon. The generalized second law states [30]:

$$\Delta S_{\text{total}} = \Delta S_{\text{BH}} + \Delta S_{\text{outside}} \geq 0. \tag{3.60}$$

For astrophysical BH, the evaporation timescale greatly exceeds the age of the universe. However, for PBHs with masses below 10^{15} g, evaporation could be significant [15], being possible that several PBHs evaporated before our time, leading them to astrophysical objects which observations probes are hard to determine.

3.3.2 Observations and Perspectives

Detecting Hawking radiation directly remains an ambitious goal due to the minuscule temperatures associated with astrophysical BHs. For a solar-mass BH, the Hawking temperature is:

$$T_H \sim 10^{-7}\,\text{K}. \tag{3.61}$$

This is orders of magnitude below the CMB temperature, making direct observations exceedingly difficult [28]. However, potential avenues for indirect or analog observations includes the PBHs, which may have initial masses small enough to complete their evaporation within the current age of the universe [31].

The loss of mass due to radiation suggests that BH could completely evaporate over time. However, this poses a dilemma; if information about the matter that formed the BH is lost during evaporation, the principle of unitarity in quantum mechanics is violated [9]. Numerous resolutions to the information paradox have been proposed, each offering unique insights into the nature of quantum gravity as the holographic principle, topic that will be discussed on forward sections.

In summary, the field quantization in Schwarzschild spacetime reveals that the modes of the scalar field are influenced by the spacetime curvature. Near the horizon, there is a strong modification of the mode frequencies, leading to the emission of Hawking radiation. The particle creation process, as perceived by an observer at infinity, arises from the mismatch between the field modes near the horizon and those far from the BH.

3.4 Superradiance Effect

The superradiance effect represents a mechanism through which waves interacting with a rotating BH can extract its rotational energy [45] in form of radiation. The phenomenon is most famously associated with the Kerr BH, whose rotation allows for energy extraction under specific conditions. The effect is rooted in the interaction between waves and the ergoregion of a Kerr BH [13]. In this region, spacetime is dragged by the BH's angular momentum, leading to the possibility of amplifying incident waves under appropriate conditions. Specifically, for a wave with frequency ω and azimuthal quantum number m, energy extraction occurs when [62] tge superradiant condition is satisfied, given by the equation (3.62):

$$\omega < m\Omega_H, \tag{3.62}$$

where ω is the frequency of the wave, m is the azimuthal quantum number representing the wave's angular momentum about the axis of rotation, and $\Omega_H = \frac{a}{r_+^2+a^2}$ is the angular velocity of the BH's event horizon. This inequality signifies that the wave extracts rotational energy from the BH, growing in amplitude as it scatters [13].

Historically, the study of superradiance began with the work of Zel'dovich [63], who explored amplification mechanisms involving rotating cylinders and electromagnetic waves. These ideas were extended to BH spacetimes by Misner and later formalized in the context of Kerr BHs by Teukolsky and others [13]. Today, superradiance demonstrates how spacetime curvature and angular momentum influence wave dynamics.

The Kerr metric describes the geometry of spacetime around a rotating BH [26], having angular momentum as a fundamental parameter. Spinning BHs (Kerr BHs) possess a region outside the event horizon; due to the effect of inertial frame dragging the matter cannot remain in rest relative to an external observer; this is the **ergoregion** [64]. The superradiance effect occurs when waves interacting with a Kerr BH are amplified, extracting energy and angular momentum from the BH's rotation [13]. This amplification is rooted in the interaction of the wave with the ergoregion [65], where spacetime itself is dragged by the BH's angular momentum. The mathematical framework for superradiance begins with the Kerr metric and the wave equation governing perturbations in this geometry.

In Boyer-Lindquist coordinates (t, r, θ, ϕ), the Kerr metric is expressed as [30]:

$$ds^2 = -\left(1 - \frac{2GMr}{\rho^2}\right) dt^2 - \frac{4GMar\sin^2\theta}{\rho^2} dt\, d\phi + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 + \left(r^2 + a^2 + \frac{2GMa^2 r\sin^2\theta}{\rho^2}\right)\sin^2\theta\, d\phi^2, \tag{3.63}$$

Where $\rho^2 = r^2 + a^2\cos^2\theta$ and $\Delta = r^2 - 2GMr + a^2$. Here, M represents the mass of the BH, $a = J/M$ is the specific angular momentum, and J is the total angular momentum of the BH. The roots of $\Delta = 0$ define the horizons [30]:

$$r_\pm = GM \pm \sqrt{G^2M^2 - a^2}, \tag{3.64}$$

where r_+ corresponds to the event horizon and r_- to the Cauchy horizon [30]. The ergosphere is a region outside the event horizon where no static observer can remain stationary relative to a distant observer [66]. It is bounded by the event horizon (r_+) and the stationary limit surface, defined by the condition [13]:

$$g_{tt} = -\left(1 - \frac{2GMr}{\rho^2}\right) = 0. \tag{3.65}$$

The static limit horizon is given by the surface [66]:

$$r_{\text{erg}}(\theta) = GM + \sqrt{G^2M^2 - a^2\cos^2\theta}. \tag{3.66}$$

The ergosphere is oblate, touching the event horizon at the poles ($\theta = 0, \pi$) and extending outward at the equator ($\theta = \pi/2$) [67]. Within the ergosphere, the dragging of inertial frames by the BH's rotation forces all observers to co-rotate with the BH. This frame-dragging effect arises from the off-diagonal term $g_{t\phi}$, which couples the t and ϕ coordinates [13]:

$$g_{t\phi} = -\frac{2GMar\sin^2\theta}{\rho^2}. \tag{3.67}$$

The angular velocity of spacetime within the ergosphere is [13]:

$$\omega_{\text{drag}} = -\frac{g_{t\phi}}{g_{\phi\phi}} = \frac{2GMar}{r^2 + a^2 + \frac{2GMa^2r\sin^2\theta}{\rho^2}}. \tag{3.68}$$

This phenomenon becomes increasingly pronounced near the horizon. This energy extraction mechanism is closely related to the Penrose process [62], where particles entering the ergosphere can split, with one escaping to infinity with increased energy while the other falls into the BH with negative energy.

The Kerr metric and ergosphere together define a spacetime that allows unique interactions between waves and the BH's rotation. Frame dragging ensures that energy and angular momentum transfer occurs between the BH and the field, making the ergosphere the engine of superradiance [65]. The ability to extract rotational energy from the BH challenges traditional notions of BHs as purely absorptive objects, showing that they can actively transfer energy to their surroundings under specific conditions [68].

The Kerr metric's structure introduces rotational and angular momentum effects, making the Klein-Gordon equation for a scalar field $\Phi(x)$ the foundation for understanding wave amplification. The scalar field $\Phi(x)$ satisfies the Klein-Gordon equation in the Kerr geometry [13]:

$$\Box_g\Phi = \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\Phi\right) = 0, \tag{3.69}$$

The explicit form of the Kerr metric, with its off-diagonal $g_{t\phi}$ term, introduces couplings between time and angular coordinates, which are essential for frame-dragging effects and superradiance [66]. The symmetry of the Kerr spacetime allows the separation of variables for the scalar field. Then [13]:

$$\Phi(t, r, \theta, \phi) = e^{-i\omega t}e^{im\phi}S_{lm}(\theta)R_{lm}(r), \tag{3.70}$$

where ω is the wave frequency, m is the azimuthal quantum number associated with rotation, $S_{lm}(\theta)$ are the spheroidal harmonics, $R_{lm}(r)$ is the radial wave function. Substituting this ansatz into the Klein-Gordon equation separates it into angular and radial components, as in the previous section [69]:

$$\text{Angular Equation:} \quad \frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{dS_{lm}}{d\theta}\right) + \left[\lambda_{lm} + a^2\omega^2\cos^2\theta - \frac{m^2}{\sin^2\theta}\right]S_{lm} = 0, \tag{3.71}$$

$$\text{Radial Equation:} \quad \Delta\frac{d}{dr}\left(\Delta\frac{dR_{lm}}{dr}\right) + \left[\frac{K^2}{\Delta} - \lambda_{lm} - a^2\omega^2 + 2ma\omega - m^2\right]R_{lm} = 0, \tag{3.72}$$

Where $K = (r^2 + a^2)\omega - ma$ and $\Delta = r^2 - 2GMr + a^2$. The separation constant λ_{lm} depends on the angular quantum numbers l and m, as well as the frequency ω. The angular equation governs the spheroidal harmonics $S_{lm}(\theta)$, which reduce to spherical harmonics in the Schwarzschild limit ($a = 0$) [69]. The radial equation governs the propagation of waves in the Kerr geometry and determines the conditions for superradiance.

3.4.1 Energy Flux, Amplification Factor and Superradiance Condition

The amplification occurs when specific conditions, derived from the interplay of the wave's properties and the BH's geometry, are met. As it was shown at the beginning, the central condition for superradiance is guven by equation (3.62) [62]. The superradiance condition can be understood as a consequence of the interaction between the wave's angular momentum and the BH's frame dragging.

To develop the derivation of the superradiance condition, we start by the flux of energy associated with a scalar field $\Phi(x)$, given by the stress-energy tensor [69]:

$$T^{\alpha}_{\beta} = -g^{\alpha\gamma}\partial_{(\beta}\Phi^*\partial_{\gamma)}\Phi - \delta^{\alpha}_{\beta}\mathcal{L}, \tag{3.73}$$

The energy flux across a hypersurface is proportional to the time component of the stress-energy tensor, being the energy flux of a wave interacting with a rotating Kerr BH. This flux determines whether energy is extracted from the BH's rotational energy or whether the wave is absorbed [13]. The flux through a hypersurface Σ with normal vector n^μ is given by [13]:

$$\delta\mathcal{E}_{\mathcal{H}} = \int_\Sigma T^t_\mu n^\mu d\Sigma, \tag{3.74}$$

where $d\Sigma$ is the volume element of the hypersurface. For the Kerr spacetime, the flux is evaluated at two key boundaries: near the horizon ($r = r_+$), where the wave is purely ingoing due to the event horizon acting as a one-way boundary; and at spatial infinity ($r \to \infty$), where the wave consists of both ingoing and outgoing components. The radial component of the flux is related to the energy-momentum components T^t_r, reflecting the energy flow through radial surfaces. Near the horizon, the radial part of the field satisfies the boundary condition [70]:

$$R_{lm}(r) \sim e^{-i\omega r_*}, \quad r_* \to -\infty, \tag{3.75}$$

where r_* is the tortoise coordinate mapping $r \to r_+$ to $r_* \to -\infty$. The energy flux at the horizon is proportional to [13]:

$$\delta\mathcal{E}_H \propto -\omega\left(\omega - m\Omega_H\right), \tag{3.76}$$

where Ω_H is the angular velocity of the BH's horizon, defined before. For $\omega < m\Omega_H$, the flux becomes negative, indicating that energy is extracted from the BH and transferred to the wave. This is the hallmark of superradiance. While at infinity ($r \to \infty$), the field consists of both ingoing and outgoing components [13]:

$$R_{lm}(r) \sim A_{\text{in}} e^{-i\omega r_*} + A_{\text{out}} e^{i\omega r_*}. \tag{3.77}$$

The energy flux at infinity is proportional to the difference in the amplitudes of the ingoing and outgoing waves [13]:

$$\delta\mathcal{E}_\infty \propto \omega \left(|A_{\text{out}}|^2 - |A_{\text{in}}|^2\right). \tag{3.78}$$

When the superradiance condition ($\omega < m\Omega_H$) is satisfied, $|A_{\text{out}}| > |A_{\text{in}}|$, leading to a net amplification of the wave [13]. The degree of amplification is characterized by the amplification factor $\mathcal{Z}$, defined as [13, 65]:

$$\mathcal{Z} = \frac{\mathcal{F}_\infty}{\mathcal{F}_H} - 1 = \frac{\text{Energy Outgoing at Infinity}}{\text{Energy Ingoing at Horizon}} - 1 = \frac{\omega^3}{(\ell+1)^2 4\pi}\sigma_{\ell m}. \tag{3.79}$$

Where the total absorption cross section superradiant modes are characterized by $\sigma_{\ell m}(\omega) = -\Gamma_{\ell m}(\omega)/\mathcal{F}_{\text{in}} < 0$. For superradiance, $\mathcal{Z} > 0$, confirming that the outgoing wave has greater energy than the ingoing wave. This factor measures the relative increase in the wave's energy due to its interaction with the BH. For superradiant scattering, $\mathcal{Z} > 0$, confirming that the outgoing wave carries more energy than the ingoing wave [13].

For scalar waves, $\mathcal{Z}$ is typically small but becomes significant for finely tuned frequencies near the superradiant threshold. For electromagnetic and gravitational waves, the amplification can be more pronounced due to their higher spin [68].

3.4.2 Extensions to Massive Fields

While the superradiance condition is often discussed in the context of scalar fields, it extends to other types of fields, such as electromagnetic and gravitational waves [71]. For electromagnetic fields, the azimuthal quantum number m corresponds to the helicity of the wave, and for gravitational waves, it relates to the spin-weighted spherical harmonics [68]. The general condition remains on the standard superradiance condition showed in equation (3.62).

But the amplification factor $\mathcal{Z}$ depends on the field's spin and its interaction with the BH's geometry. Superradiance is also relevant for massive fields. For a massive scalar field with mass μ_S, the condition is modified to include a term that accounts for the field's rest energy [13]:

$$\mu_S < \sqrt{2} m\Omega_H. \tag{3.80}$$

This modification leads to new phenomena, such as the instability of Kerr BHs surrounded by massive fields, forming what are known as “BH bombs" [13].

3.4.3 Numerical Studies and Resonances

The superradiance effect, while theoretically well-established, often requires numerical analysis to determine the precise amplification factor $\mathcal{Z}$ and its dependence on the wave frequency, BH spin and field type. These studies reveal intricate behaviors, including resonances that significantly enhance the amplification for certain parameter regimes [70].

Numerical computations show that $\mathcal{Z}$ depends sensitively on the wave frequency ω, the azimuthal quantum number m, and the BH spin parameter a/M [45]. Near the superradiant threshold $\omega \approx m\Omega_H$, $\mathcal{Z}$ reaches its maximum value. For highly spinning BHs ($a \to M$), the amplification can exceed 10% for scalar waves with finely tuned frequencies [13].

A typical result for the amplification factor is [45]:

$$\mathcal{Z} \propto \left(1 - \frac{\omega}{m\Omega_H}\right)^2, \tag{3.81}$$

Illustrating the sharp peak near the threshold. The numerical studies also reveal that higher m values lead to stronger amplification, as the interaction with the BH's angular momentum becomes more pronounced [45].

For massive scalar fields, the presence of a mass term μ_S modifies the dynamics, introducing quasi-bound states [69]. The radial wave equation for massive fields can exhibit trapping behavior, leading to resonances where the field's amplitude grows exponentially. Numerical results for quasi-bound states show that the growth rate depends on both the field mass and the BH spin [68].

3.4.4 Quantum Field Perspective

The superradiance effect extends beyond classical wave mechanics to the realm of QFT in curved spacetime, which is our interest for the present work. In this quantum framework, superradiance is interpreted as the stimulated emission of particles due to the interaction of quantum fields with the ergoregion of a rotating black BH [13]. This perspective connects superradiance to fundamental processes such as Hawking radiation and spontaneous emission.

The quantization of a scalar field $\Phi(x)$ in Kerr spacetime follows the same principles as in flat spacetime but accounts for the spacetime's rotational and curved geometry. Using the Kerr metric, developed previously, we construct the wave functions. Then there are a lot of steps than can be neglected, and start with the mode functions. The mode functions $u_{\omega\ell m}(x)$ are solutions to the Klein-Gordon equation and take the form:

$$u_{\omega\ell m}(x) = e^{-i\omega t} e^{im\phi} S_{lm}(\theta) R_{lm}(r), \tag{3.82}$$

The mode functions $u_{\omega\ell m}(x)$ are orthonormal with respect to the usual Klein-Gordon inner product (2.47). The inner product is conserved due to the divergence-free nature of the Klein-Gordon current [22]:

$$j^\mu = -i\left(u_1 \partial^\mu u_2^* - u_2^* \partial^\mu u_1\right), \quad \nabla_\mu j^\mu = 0. \tag{3.83}$$

This conservation ensures that the orthogonality of modes persists across spacetime. Explicitly, the orthonormality condition is the same as showed before:

$$(u_{\omega\ell m}, u_{\omega'\ell'm'}) = \delta(\omega-\omega')\delta_{\ell\ell'}\delta_{mm'}. \tag{3.84}$$

The mode functions also exhibit distinct behavior in two asymptotic regions, as was showed before. The modes defined near the horizon (ingoing basis) and at infinity (outgoing basis) are related through the typical Bogoliubov transformations and its relation between the Bogoliubov coefficients [3]:

$$\int d\omega' \left(|\alpha_{\omega\omega'}|^2 - |\beta_{\omega\omega'}|^2\right) = 1, \tag{3.85}$$

Ensuring unitarity in the transformation between modes. The number of particles observed in a given mode is determined by the Bogoliubov coefficient $\beta_{\omega\omega'}$, which quantifies the mixing of positive and negative frequency components [3], analogous to the other cases:

$$\langle n_\omega \rangle = \sum_{\omega'} |\beta_{\omega\omega'}|^2. \tag{3.86}$$

For superradiant modes, where $\omega < m\Omega_H$, the negative energy contribution from the ergoregion amplifies the outgoing wave, resulting in $|\beta_{\omega\omega'}|^2 > 0$. This leads to a net particle creation, interpreted as the quantum analog of wave amplification in the classical superradiance effect [13]. The observer-dependence of the mode functions should show the ambiguity in defining the vacuum state. For an observer at infinity, the vacuum is defined with respect to the outgoing modes:

$$a^{\text{out}}_{\omega\ell m}|0_{\text{out}}\rangle = 0, \quad \forall\, \omega, \ell, m. \tag{3.87}$$

Near the horizon, the vacuum is defined with respect to ingoing modes:

$$a^{\text{in}}_{\omega\ell m}|0_{\text{in}}\rangle = 0, \quad \forall\, \omega, \ell, m. \tag{3.88}$$

The Bogoliubov transformation between these vacuum leads to the thermal spectrum of particles detected at infinity [13]:

$$\langle n_\omega \rangle = \frac{1}{e^{(\omega - m\Omega_H)/T_H} - 1}, \tag{3.89}$$

where $T_H = \frac{\hbar\kappa}{2\pi k_B}$ is the Hawking temperature and κ is the surface gravity of the BH. The quantum analog of superradiance is the stimulated emission of particles from the BH. This process is analogous to the interaction of light with an excited atom, where incoming photons stimulate the emission of additional photons [71]. In the case of Kerr BH, the rotational energy acts as the reservoir that fuels this amplification.

The relationship between the two is particularly evident in the context of the black BH's entropy and thermodynamics. Superradiance contributes to BH's angular momentum loss, while Hawking radiation reduces its mass, leading to the gradual evolution of the BH's state. From a thermodynamic viewpoint, superradiance and Hawking radiation are complementary. Superradiance reduces the BH's spin, while Hawking radiation decreases its mass. Together, they enforce the laws of BH thermodynamics, particularly the generalized second law [30]:

$$\Delta S_{\text{total}} = \Delta S_{\text{BH}} + \Delta S_{\text{radiation}} \geq 0, \tag{3.90}$$

3.4.5 Observational Perspectives

One of the most profound applications of superradiance is its role in the evolution of rotating BH. The extraction of rotational energy through superradiant scattering affects the spin distribution of BH over time. Observational data from X-ray binaries and gravitational wave detectors, such as LIGO and Virgo, provide evidence for spin limits in astrophysical BH that align with predictions from superradiance [13, 64, 72]. The process is particularly significant in the presence of massive fields, where it can lead to instabilities that amplify the energy extraction.

The superradiance effect encapsulates the richness of BH physics, blending classical wave dynamics with QFT in curved spacetime. Its role in extracting rotational energy from BH gave the dynamic nature of horizons and ergoregions, changing traditional notions of BH as purely absorptive objects.

3.5 Memory Burden Effect

The memory burden effect represents an interesting concept in modern theoretical physics. It emerges as a consequence of spacetime memory effects, in which a system with high capacity of information storage, as a BH, is stabilized by the load of information it carries [73]. This effect rise as a correction to Hawking radiation, looking to solve the information paradox on BH.

Historically, memory effects have been studied through the Bondi-Metzner-Sachs symmetry group and its implications for asymptotically flat spacetimes [74]. Classical memory effects, such as the displacement memory, reveal how waves leave permanent displacements in test masses after they pass. The Memory Burden Effect generalizes this concept, and acquires particular significance in the study of BH, where it intersects with the BH information paradox [73]. The classical understanding of BH, governed by general relativity, suggests that all information falling into a BH is irretrievably lost beyond the event horizon [26]. However, QFT in curved spacetime predicts subtle imprints of the information in the emitted Hawking radiation or Superradiance. The Memory Burden Effect bridges this divide, providing a mechanism through which information about infalling matter and radiation is preserved in the form of memory modes [**?**].

This effect is also deeply connected to the holographic principle [16], which posits that all the information contained within a region of spacetime can be encoded on its boundary. Memory modes represent one way in which this encoding manifests, particularly in burdened spacetimes where the dynamics of horizons and the persistence of memory play critical roles.

This section explore some theoretical foundations and implications of the Memory Burden Effect. Starting with classical memory effects, we extend our discussion to quantum corrections and the role of quantum field theory in describing these phenomena. We then examine the implications for BH information, entropy bounds and holography principle.

3.5.1 Theoretical Foundations

The Memory Burden Effect rests on the intersection of general relativity, QFT and holography, offering a picture to explore the retention of information in spacetime and its implications for BH physics.

Classical memory effects are fundamental for our understanding of spacetime dynamics in general relativity. These effects arise in asymptotically flat spacetimes, where the passage of gravitational waves leaves a permanent change in the relative positions of test masses. This phenomenon, known as the gravitational wave memory effect [75], is deeply connected to the underlying symmetries of spacetime.

The displacement memory effect manifests as a permanent relative displacement between freely falling test particles after the passage of a gravitational wave [76]. This effect is mathematically described by solving Einstein's field equations in the linearized regime, where the metric perturbation $h_{\mu\nu}$ encodes the gravitational wave's effects. The asymptotic displacement is given by [77]:

$$\Delta h_{ij} = \int_{-\infty}^{\infty} dt\, \mathcal{F}_{ij}(t), \tag{3.91}$$

where $\mathcal{F}_{ij}(t)$ represents the gravitational wave's energy flux. This memory effect originates from the linear memory, associated with the flux of energy and momentum carried by gravitational waves themselves; and the non-linear memory, rising from the stress-energy tensor of matter and radiation on spacetime [78]. This term reflects the non-linear nature of Einstein's equations and adds a cumulative effect to the memory [76].

The extension of classical memory effects to the quantum regime introduces new dynamics. In QFT, memory effects manifest as lasting changes in the quantum states of fields due to their interaction with spacetime geometry [79]. Near BH, these effects become particularly pronounced, as quantum fluctuations are influenced by the intense curvature and the presence of an event horizon. Quantum memory arises when quantum states evolve in response to gravitational and quantum interactions, leaving behind detectable changes in observables.

Mathematically, quantum memory is encoded in the quantum states of the field, $|\psi\rangle$; the resulting change in the quantum state, $|\psi\rangle \to |\psi'\rangle$, encodes the memory of the interaction, with the persistence of this change reflecting the quantum memory effect [18].

In the context of BH evaporation, quantum memory modes ensure that the emitted Hawking radiation retains correlations with the infalling matter [18]. This retention of information addresses the BH information paradox, suggesting that memory modes serve as a mechanism for preserving unitarity.

The quantum memory effect is intimately connected to the entropy and information content of BH. Memory modes contribute to the BH entropy by encoding information about infalling matter and radiation.

Black Hole Information Paradox

The BH information paradox emerges as a conflict between the predictions of quantum mechanics and the classical description of BHs in general relativity. It stems from the apparent incompatibility of unitarity the preservation of information in quantum evolution, with the thermal nature of Hawking radiation emitted by BHs [37].

In general relativity, BHs are described by the no-hair theorem [26], which asserts that the spacetime outside a BH is fully characterized by only three macroscopic parameters: mass (M), charge (Q), and angular momentum (J). Mathematically, this property of BHs is reflected in the uniqueness of solutions to Einstein's field equations, where the final state of a collapsing star, for instance, is completely independent of the internal structure or composition of the progenitor object [37]. For an external observer, all details about the infalling matter are encoded solely in the BH's M, Q, and J, with no mechanism for recovering the original information.

The paradox takes on a quantum dimension with Hawking's theoretical work in 1974 [7]. The Hawking radiation arises from the quantum fluctuations of the vacuum, with particle-antiparticle pairs being created near the horizon. One of these particles escapes to infinity as radiation, while the other falls into the BH, reducing its mass. The emitted radiation follows a nearly thermal spectrum (as it was showed in previous equations), with no apparent dependence on the detailed structure of the matter that formed the BH, depending only on the BH area. This thermality implies a loss of correlations between the emitted quanta and the initial state of the system [80].

In quantum mechanics, the evolution of a closed system is governed by unitary operators, preserving the purity of the system's state. A pure initial state $|\psi\rangle$ remains pure under unitary evolution [61]:

$$|\psi(t)\rangle = U(t)|\psi(0)\rangle, \quad \text{with} \quad U^\dagger U = I. \tag{3.92}$$

Hawking's result, however, suggests that the final state of the BH after complete evaporation is a thermal mixed state, described by the density matrix [80]:

$$\rho_{\text{final}} = \sum_i p_i |\phi_i\rangle\langle\phi_i|, \tag{3.93}$$

where $\{|\phi_i\rangle\}$ are orthonormal basis states, and p_i are thermal probabilities. Such a transition from a pure state to a mixed state violates unitarity, as the entropy of the final state [58]:

$$S_{\text{final}} = S_{\text{BH}} + S_{\text{IP}}(\rho_{\text{final}}). \tag{3.94}$$

Where $S_{\text{IP}}(\rho_{\text{final}}) \propto < Tr(\rho \log \rho) >$ [37]. It is non-zero, unlike the zero entropy of the pure initial state. The Bekenstein-Hawking entropy is proportional to the horizon area A, $S_{\text{BH}} = \frac{k_B A}{4\ell_p^2}$, with $\ell_p = \sqrt{\hbar G/c^3}$ the Planck length. As the BH evaporates due to Hawking radiation, its area decreases, reducing the associated entropy. However, the thermal nature of the radiation implies that the information about the initial state of the BH is not encoded in the radiation, leading to an apparent loss of information as the BH evaporates completely [37].

This creates a tension between the generalized second law of thermodynamics, which suggests that the total entropy of the system (BH + radiation) must increase or remain constant [58]; and the unitarity of quantum mechanics, which requires that the evolution of the system preserves the information content and entropy of the initial state.

The paradox is further complicated by the role of vacuum states in QFT in curved spacetime. Near the event horizon, the vacuum state for an inertial observer differs from that for a stationary observer at infinity, leading to the apparent creation of thermal radiation [3]. This observer-dependence contributes to the perceived loss of information in Hawking radiation.

The Memory Burden Effect reframes the paradox by introducing quantum memory modes that encode information about the infalling matter and radiation. These modes persist at the horizon and interact with the outgoing radiation, ensuring that the emitted spectrum is not perfectly thermal. By incorporating these memory effects, the evolution of the BH system can remain unitary, preserving the information content [18]. The role of memory modes and quantum corrections extends beyond modifying the radiation spectrum. They provide a concrete mechanism for information recovery, suggesting that the information about the initial state of the BH is encoded in the radiation and the horizon memory.

Asymptotic Symmetries and Soft Theorems

Asymptotic symmetries and soft theorems are fundamental for understanding the memory burden effect in BH spacetimes [81]. These concepts reveal how information about physical processes, encoded in low-energy modes and spacetime symmetries, persists and contributes to the structure of the emitted radiation and the dynamics of the BH.

In asymptotically flat spacetimes, the symmetries at null infinity extend beyond the familiar Poincaré group [82]. The Bondi-Metzner-Sachs group governs the asymptotic structure of spacetime and includes supertranslations [83], being an infinite-dimensional generalization of translations, associated with shifts in the retarded time coordinate u at null infinity; and superrotations, as extensions of Lorentz transformations.

The presence of these symmetries modifies the behavior of fields and metrics at null infinity [16]. The metric perturbation near null infinity in Bondi gauge is expressed as [37], coinciding with Minkowski spacetime (asymptotically flat):

$$ds^2 \underset{r\to\infty}{\longrightarrow} -du^2 - 2dudr + r^2\gamma_{AB}d\Omega^A d\Omega^B + h_{\mu\nu}, \tag{3.95}$$

Supertranslations induce shifts in the Bondi mass aspect [37]:

$$m_B(u,\Omega) = \frac{r}{2}C_{AB} = \frac{1}{2}\lim_{r\to\infty} r h_{uu}(r,u,\Omega) \tag{3.96}$$

Where m_B is the Bondi mass aspect, and C_{AB} is the shear tensor, encoding gravitational radiation.. These shifts create memory effects by leaving permanent imprints on the spacetime structure, even after the gravitational wave has passed.

The connection between asymptotic symmetries and soft theorems directly links to memory effects in spacetime. In the context of BHs, asymptotic symmetries and soft theorems explain how information about the infalling matter and radiation is preserved [37]. Supertranslations at the horizon encode changes in the Bondi mass and angular momentum aspects, creating memory modes that persist throughout the BH's evolution [84].

Holographic Principle

The holographic principle provides a path for understanding how information is retained in BH spacetimes. This principle posits that all information within a volume of spacetime can be encoded on its boundary [85]. For BHs, the event horizon serves as this boundary, acting as a repository for information about the system's internal states [80]. The Memory Burden Effect finds a natural home in the holographic framework, where memory modes represent physical carriers of information that ensure unitarity and address the BH information paradox [73].

The holographic principle originated from the study of BH entropy, where the Bekenstein-Hawking formula, showed before, for entropy demonstrated that the information content of a BH scales with its horizon area, not its volume; this area law suggests that the degrees of freedom of the BH are encoded on its two-dimensional boundary, supporting the notion that spacetime itself may have a holographic nature [9]. In the AdS/CFT correspondence, a concrete realization of the holographic principle, the dynamics of bulk spacetime (AdS space) are encoded in a CFT on its boundary [16].

In this correspondence, the entropy of the BH in the bulk is identified with the entropy of the thermal state in the CFT [86]:

$$S_{\text{bulk}} = S_{\text{boundary}} = K \ln d + 1. \tag{3.97}$$

Where d is maximal occupation of the memory modes. This duality ensures that information is preserved in the boundary description, resolving the information paradox within the AdS/CFT framework.

Memory modes naturally align with the holographic principle, acting as degrees of freedom that encode information about spacetime events and interactions [87]. The event horizon of a BH acts as a dynamic boundary, encoding information about the interior states through its degrees of freedom. Memory modes at the horizon, influenced by soft theorems and asymptotic symmetries, record the interaction history of infalling matter and radiation [88]. These modes ensure that the outgoing Hawking radiation retains correlations with the BH's past, addressing the information paradox.

Holography also provides insights into the dynamic nature of the event horizon. Quantum fluctuations at the horizon, driven by interactions with infalling and outgoing fields, manifest as boundary dynamics in the dual theory. These fluctuations influence the scattering amplitudes of soft modes and contribute to the memory burden effect by modifying the horizon's structure [18].

3.5.2 Physical-Mathematical Aspects

Now we present classical memory effects from Einstein's field equations and explore their connection to spacetime symmetries. Next, we extend these ideas to quantum fields in curved spacetimes, introducing quantum memory modes description.

Classical Memory Effects

Classical memory effects in general relativity describe the lasting imprints left on spacetime by transient gravitational phenomena. These effects are directly connected to the symmetries of spacetime at null infinity, particularly the Bondi-Metzner-Sachs group, and manifest as permanent displacements, rotations, or shifts in the relative positions or momenta of test particles [89].

Gravitational wave memory is the most well-studied classical memory effect, arising from the passage of gravitational waves through spacetime. To derive this effect, consider the metric perturbation $h_{\mu\nu}$ in an asymptotically flat spacetime, where the background metric is flat Minkowski [90]:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1. \tag{3.98}$$

The perturbed Einstein field equations in the transverse-traceless gauge are [44]:

$$\Box h_{\mu\nu} = -16\pi G T_{\mu\nu}, \tag{3.99}$$

where $T_{\mu\nu}$ is the stress-energy tensor of the source, and $\Box$ is the d'Alembert operator in flat spacetime. At null infinity, the metric perturbation is dominated by the Bondi shear tensor C_{AB}, which encodes the gravitational radiation [91]:

$$h_{AB} = \frac{C_{AB}}{r}, \tag{3.100}$$

where r is the radial coordinate, and A, B index the transverse angular directions. The shear evolves according to the Bondi news tensor N_{AB}, defined as [91]:

$$N_{AB} = \partial_u C_{AB}, \tag{3.101}$$

where u is the retarded time. Integrating over u, the displacement memory effect is given by [91]:

$$\Delta C_{AB} = \int_{-\infty}^{\infty} N_{AB}\, du. \tag{3.102}$$

This result indicates a permanent change in the spacetime geometry caused by the gravitational wave, observable as a displacement in the relative positions of test particles.

QFT and the Memory Burden Effect

The Memory Burden Effect encapsulates the persistence of quantum memory modes in BH spacetimes, emphasizing their role in encoding and preserving information about the system's history. The core mechanism for quantum memory in curved spacetime is the mixing of modes at the BH horizon, described by Bogoliubov transformations [3]. Consider a quantum scalar field $\phi(x)$ in a curved spacetime background. The field is decomposed into a basis of mode functions $\{u_{\omega\ell m}(x)\}$, analogous to the other cases, associated with different regions of spacetime:

$$\phi(x) = \sum_i \left(a_i u_{\omega\ell m}(x) + a_i^\dagger u^*_{\omega\ell m}(x) \right), \tag{3.103}$$

with all stuff it implies, as we saw on previous sections. In the presence of a BH, the mode functions can be classified into those defined near the horizon ($u^{\text{hor}}_{\omega\ell m}$) and those observed at infinity ($u^{\infty}_{\omega\ell m}$).

The relation between the horizon modes and the asymptotic modes is expressed through Bogoliubov transformations:

$$u^{\infty}_{\omega\ell m} = \int d\omega' \left(\alpha_{\omega\omega'} u^{\text{hor}}_{\omega'\ell m} + \beta_{\omega\omega'} u^{\text{hor}*}_{\omega'\ell m} \right), \tag{3.104}$$

where $\alpha_{\omega\omega'}$ and $\beta_{\omega\omega'}$ are the Bogoliubov coefficients, encoding the mixing of positive- and negative-frequency modes. These coefficients are determined by solving the Klein-Gordon equation in the curved spacetime background and matching boundary conditions at the horizon and infinity.

The coefficients $\beta_{\omega\omega'}$ quantify the creation of particles in the asymptotic region:

$$\langle n_\omega \rangle = \int |\beta_{\omega\omega'}|^2 \, d\omega', \tag{3.105}$$

where $\langle N_\omega \rangle$ is the number of particles observed in mode ω. The non-zero value of $|\beta_{\omega\omega'}|^2$ reflects the spontaneous creation of particle-antiparticle pairs, a hallmark of curved spacetime dynamics. The memory burden effect is derived as a proposal from this number of particles, and it is defined as [18]:

$$\mu = -\sum_{k=1}^{K} \epsilon_k \frac{n_k}{N_c} \tag{3.106}$$

Where N refers to the occupation modes of the system, n the particle number and ϵ the energy gap between the possible microstates. A detailed derivation of this can be founded on [16, 18].

The Bogoliubov coefficients encode information about the spacetime's history, providing a mechanism for memory effects. The coefficients $\beta_{\omega\omega'}$ reflect the interaction of quantum fields with the spacetime curvature and the dynamics of the horizon [16]. For instance, changes in the BH's mass, spin, or charge influence the mode mixing and are imprinted in the coefficients. In general this way to obtain the particle number is difficult, due to it is preferred to take Memory Burden as a correction for mass loss.

3.6 Hawking, Superradiance, and Memory Burden Effect in Primordial Black Holes

PBHs offers a possible testing ground for quantum gravitational phenomena. Their formation mechanisms and small masses make PBHs particularly sensitive to quantum effects such as Hawking radiation, superradiance, and the Memory Burden Effect. The purpose of use PBHs instead classical BHs is to connect these quantum effects with cosmological observables and dark matter candidates. While there are the same effects in both astrophysical objects, the reason to choice PBHs are to explore another kind of applications.

Hawking radiation, extensively discussed in earlier sections, describes the thermal emission of particles from the event horizon of a BH due to quantum field fluctuations in curved spacetime. This radiation leads to gradual BH evaporation, with the mass-loss rate scaling inversely with the square of the BH mass. For PBHs, the small initial masses significantly amplify this process, making Hawking radiation a dominant factor in their evolution [32].

Superradiance, another fundamental quantum phenomenon, occurs in rotating BHs, where certain modes of incident waves are amplified by extracting angular momentum and energy from the BH. While previously explored in the context of astrophysical BHs, superradiance is particularly relevant for PBHs with non-negligible spin, as it can significantly influence their mass-loss dynamics and observational signatures [13].

The Memory Burden Effect, introduced in prior section, modifies these phenomena by incorporating persistent quantum corrections from memory modes. These corrections influence the emitted radiation spectrum, entropy evolution, and gravitational wave production, particularly during the late stages of PBH evaporation [86].

Here we focus on the connection between these three effects in PBHs. By combining the frameworks of these quantum effects, we aim to construct a unified shallow description of PBH evolution and explore the observational consequences of these intertwined processes. For detailed discussions of individual effects, we refer to earlier sections and concentrate here on their interactions and cumulative implications. Also it is discussed some phenomena that appear when the quantum effects are taking into account on BHs and PBHs, as the greybody factors, used to calculate and understand the quantum effects.

Greybody Factors

Greybody factors quantify the deviation of BH radiation from a perfect blackbody spectrum [3]. These arise from the scattering of emitted particles by the curved spacetime surrounding the BH, which modifies the transmission probabilities of radiation to infinity [92]. Understanding greybody factors is critical for accurately modeling the radiation emitted by BHs.

For a scalar field ϕ emitted from a BH, the transmission probability is encapsulated by the greybody factor $\Gamma_{\ell m\omega}$, where ℓ, m, and ω denote the angular momentum quantum numbers and frequency of the mode. The absorption cross-section is [93]:

$$\sigma(\omega) = \Gamma(\omega)|\Psi(\omega)|^2, \tag{3.107}$$

where $\Psi(\omega)$ is the flux near the horizon. The greybody factor $\Gamma(\omega)$ represent the effective area through which radiation interacts with the BH's geometry. The greybody factor is computed by solving the wave equation [93]:

$$\frac{d^2R}{dr_*^2} + \left[\omega^2 - V_\ell(r)\right] R = 0, \tag{3.108}$$

where $R(r_*)$ is the radial wave function, r_* is the tortoise coordinate, and $V_\ell(r)$ is the effective potential, which depends on the BH parameters and mode quantum numbers. For Schwarzschild BHs, the potential is [94]:

$$V_\ell(r) = \left(1 - \frac{2GM}{r}\right) \left[\frac{\ell(\ell+1)}{r^2} + \frac{2GM}{r^3}\right]. \tag{3.109}$$

In the high-energy (geometrical optics) limit, the greybody factor simplifies, allowing analytical approximations. For a Schwarzschild BH, the absorption cross-section for high-frequency modes becomes [93]:

$$\sigma_{\text{geo}} = 27\pi(GM)^2, \tag{3.110}$$

which corresponds to the effective area of the photon sphere. This approximation provides valuable insights into the high-energy behavior of the emitted spectrum, though it fails to capture low-energy effects dominated by the potential barrier [95].

The calculation of greybody factors in general settings, particularly for rotating BHs, is notoriously difficult [93]. The effective potential $V_\ell(r)$ in the Kerr metric is complex and depends non-trivially on the spin parameter $a_\star$, leading to coupled differential equations that require numerical solutions [95].

The need for numerical precision is especially important when incorporating greybody factors into Hawking radiation spectra, as they directly influence the emitted particle flux, as we will saw below [15].

3.6.1 Connection of Hawking Radiation and Superradiance

The evolution of PBHs, and even any BH, is governed by competing quantum phenomena. These effects act in opposing directions: Hawking radiation drives mass loss due to thermal emission, while superradiance extracts angular momentum and energy through wave amplification. The Memory Burden Effect further modifies these processes, introducing quantum corrections that influence the radiation spectrum, energy extraction, and entropy dynamics [87].

The mass-loss rate of a PBH, incorporating both Hawking radiation and superradiance, can be expressed as:

$$\frac{dM}{dt} = -(\Gamma_{\text{Hawking}} + \Gamma_{\text{superradiance}}), \tag{3.111}$$

where Γ_{Hawking} is the mass-loss rate due to Hawking radiation, and $\Gamma_{\text{superradiance}}$ is the energy extraction rate via superradiance. This is a general vision of the situation, but in this case we take one specific way to develop the BH evolution.

When we combine both effects, it have to consider the spin loss for a Kerr PBH, the equations that governs that kind of situation are showed below. Then, the evolution of PBHS under the influence of Hawking radiation and superradiance effects is governed by a coupled system of differential equations for mass and spin loss. These equations account for the quantum mechanical processes of energy and angular momentum extraction, as well as persistent quantum memory effects that influence the overall dynamics. The mass loss due to Hawking radiation is given by [15]:

$$\frac{dM_{\text{PBH}}}{dt} = -\frac{64\pi^2 \epsilon(M_{\text{PBH}}, a_\star)}{M_{\text{PBH}}^2 M_p^4}, \tag{3.112}$$

where $\epsilon(M_{\text{PBH}}, a_\star)$ is the emission efficiency dependent on the PBH mass M_{PBH} and the dimensionless spin parameter $a_\star$, M_p is the Planck mass. The parameter $\epsilon(M_{\text{PBH}}, a_\star)$ encapsulates contributions from all emitted particle species, with higher-spin particles being preferentially emitted by rotating PBHs. The emission efficiency is integrated over the spectrum of emitted particles, accounting for greybody factors [15]:

$$\epsilon(M_{\text{PBH}}, a_\star) = \sum_j \frac{g_j}{2\pi^2} \int_0^\infty dE\, E \sum_{l=m} \frac{\sigma_{l,m}(M_{\text{PBH}}, E, a_\star)}{e^{\chi_j} - (-1)^{2s_j}}, \tag{3.113}$$

where g_j is the spin-weighted degrees of freedom for particle species j, $\sigma_{l,m}(M_{\text{PBH}}, E, a_\star)$ is the greybody factor for each mode, $\chi_j = (E - m\Omega_H)/T_H$, Ω_H is the horizon angular velocity.

For Kerr PBHs, the spin parameter evolves due to the extraction of angular momentum via superradiance. The spin-loss rate is expressed as [15]:

$$\frac{da_\star}{dt} = -\frac{64\pi^2 a_\star[\gamma(M_{\rm PBH}, a_\star) - 2\epsilon(M_{\rm PBH}, a_\star)]}{M_{\rm PBH}^3 M_p^4}, \tag{3.114}$$

Where $\gamma(M_{\rm PBH}, a_\star)$ quantifies the angular momentum flux due to superradiance. Superradiance occurs when the superradiance condition is satisfied. The next figures, Figure 3.1 and 3.2, shows the solution for the BH mass evolution in terms of time, represented with the scale factor a by replacing it on , and the spin loss; both for Schwarzschild ($a_* \to 0$) and Kerr ($a_* \to 1$) BH, taking $M_{in} = 10^4\ g$. The first one match with the analogous presented Barman's, et. al. paper [15]; but the another one diverges from their results, this is due to numerical simulations and may be caused by a mistake.

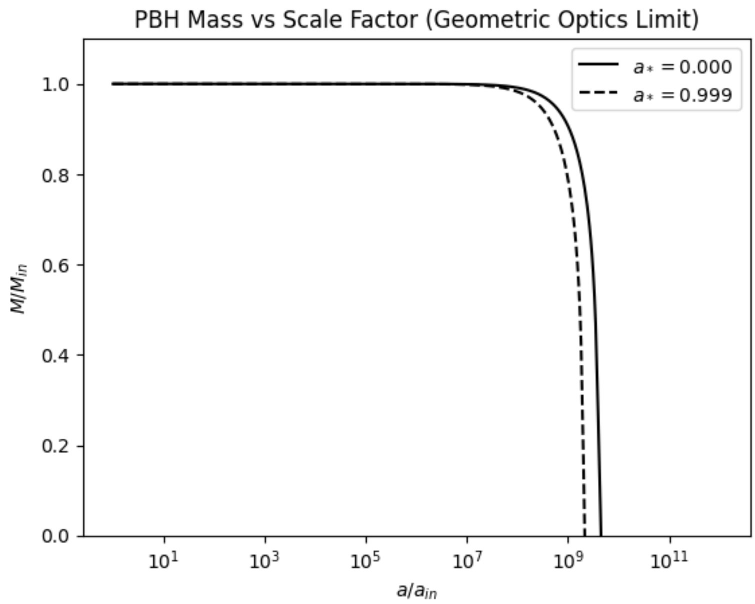

Figure 3.1: *M_{BH}/M_{in} as a function of a/a_{in} for Schwarzschild (solid line) and nearly extremal Kerr (dashed line) PBH with $M_{in} = 10^4\ g$.*

Memory Burden Corrections

The Memory Burden Effect introduces quantum corrections to the mass and spin evolution of black holes, influencing their radiation spectra, entropy dynamics, and lifetimes. These corrections arise from the interaction of quantum fields with spacetime geometry, imprinting persistent memory modes in the radiation and horizon dynamics [60]. In the context of PBHs, memory burden corrections play a pivotal role during the late stages of evaporation, significantly extending their lifetimes [31].

As the PBHs evolves, quantum corrections from memory modes alter both the mass and spin loss rates. There exists many kind of proposal to solve this topic, being a starring feature on the current literature. One of this proposal is made by Barman et al. [15], their mass-loss rate of a PBH is primarily driven by Hawking radiation, is corrected to account for the entropy contribution from

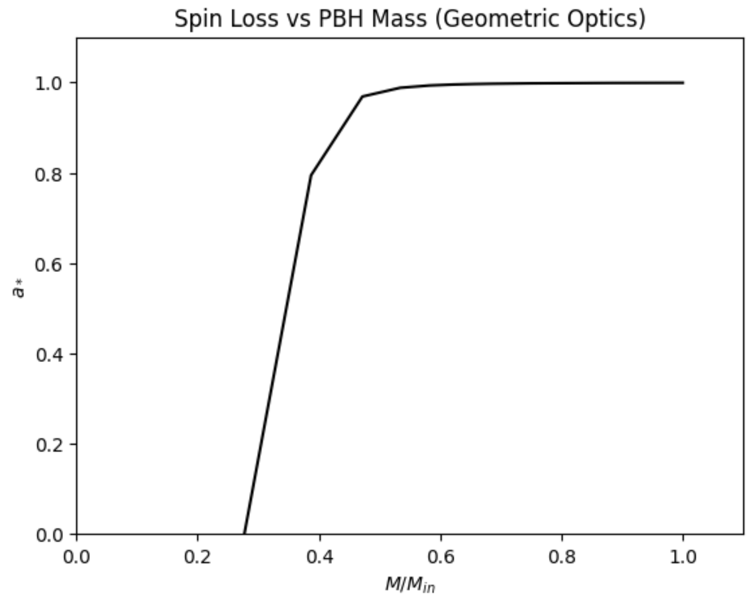

Figure 3.2: *Mass and spin loss rate for a Kerr PBH with $M_{in} = 10^4$ g.*

memory modes. Once the mass of the PBH reaches $M_{PBH} = qM_{\rm in}$, with q a parameter between 0 to 1, memory corrections modify the equation. The corrected mass-loss equation is [15],:

$$\frac{dM_{\rm PBH}}{dt} = -\frac{64\pi^2\epsilon(M_{\rm PBH}, a_\star)}{M_{\rm PBH}^2 M_p^4}\left[S_{\rm BH}\right]^{-k}, \tag{3.115}$$

where $S_{\rm BH} = \frac{k_B A}{4\ell_p^2} = \frac{\pi M_{\rm PBH}^2}{M_p^2}$ is the Bekenstein-Hawking entropy, and k is a parameter quantifying the efficiency of backreaction.

The memory contribution arises from persistent quantum correlations between the horizon and emitted radiation. These correlations reduce the effective emission rate, slowing the mass loss and extending the PBH's lifetime. For Kerr PBHs, memory effects similarly modify the spin evolution, which is influenced by superradiance and Hawking radiation. However, the effect that we take into account does not change the spin loss, as memory burden effect just affects the loss mass [86]. So the equation doesn't change. These is due to the role of memory modes in preserving angular momentum, potentially delaying the onset of superradiance instabilities and extending the PBH's rotational lifetime.

Solving the differential equation system provided by equations (3.115) and (3.114), presents the mass and spin evolution of PBHs, including the effects of Hawking radiation, superradiance, and memory burden corrections. Implications of the memory burden corrections include the extension of BH lifetimes, as memory corrections suppress the Hawking radiation flux, significantly extending the PBH's lifetime, particularly for small masses nearing the Planck scale.

3.6.2 Observational Prospects

The connection between Hawking radiation, Superradiance, and the Memory Burden effect in PBHs opens up intriguing avenues for observational and experimental investigation. One of the most promising observational avenues for probing PBHs and their associated quantum effects lies in the detection of gravitational waves [96]. PBHs, particularly those with significant angular momentum (Kerr PBHs), are expected to generate distinctive gravitational wave signals due to their interaction with surrounding matter and fields.

The energy density evolution of PBHs and radiation, as influenced by memory burden effects, directly impacts the thermal history of the universe. Observational imprints of these effects may be found in the CMB, as the delayed evaporation of PBHs alters the spectrum of Hawking radiation and the resulting contribution to it. Subtle distortions in the CMB, such as spectral features or anisotropies, could provide evidence of memory burden effects [33, 97]. These distortions would be particularly prominent in scenarios where PBHs dominated the energy density for extended periods. In neutrino observations, as PBHs emitting neutrinos over extended timescales, could contribute to the high-energy neutrino background. Experiments such as IceCube or KM3NeT may be sensitive to such contributions, providing an indirect probe of memory burden effects [98].

The memory burden effect implies a residual imprint of the PBH's initial state, contributing to the so-called "quantum hair" hypothesis [86]. Detecting evidence of such quantum imprints would have profound implications for the BH information paradox and holographic principles. Memory burden corrections introduce profound modifications to the dynamics of Hawking radiation and superradiance in PBHs. By incorporating quantum memory effects into the mass, spin, and entropy evolution, these corrections provide a way for understanding the relation of quantum and classical processes in BH physics.

Numerical simulations play a crucial role in bridging theoretical predictions and observational data. Simulating the coupled differential equations governing PBH mass, spin, and energy density evolution, particularly with memory burden corrections, allows for more precise predictions of observational signatures and to constrain several properties of BH [15, 99].

Summary

This chapter explored the relation between quantum fields and curved spacetime, focusing on the quantum effects that arise in the presence of strong gravitational fields, particularly around BHs. The analysis provided a path for understanding the fundamental phenomena that emerge from the interaction of quantum mechanics and general relativity.

The Schwinger effect illustrated the non-perturbative production of particle-antiparticle pairs in the presence of strong electric fields, demonstrating the ability of external fields to destabilize the quantum vacuum. This effect, extended to curved spacetime, revealed how electromagnetic and gravitational fields jointly influence quantum processes.

The Unruh effect highlighted the observer-dependent nature of the quantum vacuum, showing that an accelerating observer perceives a thermal bath of particles even when an inertial observer detects no particles. This phenomenon provided an essential conceptual bridge to Hawking radiation, emphasizing the relativistic redefinition of particle content in QFT and the intimate connection between acceleration, temperature and spacetime curvature.

Hawking radiation emerged as a key effect of QFT in curved spacetime, demonstrating that BHs emit thermal radiation due to quantum processes near their event horizons. This theoretical prediction relation quantum mechanics, general relativity and statistical mechanics, fundamentally altering our understanding of BHs as dynamic and evolving objects. The mathematical derivation of Hawking radiation, supported by Bogoliubov transformations and field quantization in Schwarzschild spacetime, reveals this exciting result, in which our understanding of BHs physics change due to their mass loss.

The superradiance effect, a unique feature of rotating BHs, revealed how energy and angular momentum could be extracted from Kerr BHs under specific conditions. By amplifying incident waves that satisfy the superradiance condition, this phenomenon demonstrated the dynamic coupling between BH spin, spacetime curvature and quantum fields.

Finally, the Memory Burden effect introduced a novel quantum correction to BH evolution, emphasizing the role of quantum memory modes in modifying Hawking radiation and extending BH lifetimes. This effect, analyzed in the context of both Schwarzschild and Kerr spacetimes, provided new insights into BH entropy, the information paradox, and the persistence of quantum information. Together, the quantum effects analyzed in this chapter illuminate the fundamental nature of spacetime, quantum fields and BH dynamics.

Conclusions

This thesis has explored the relation between quantum field theory and curved spacetime, delving into quantum effects near black holes and their implications for fundamental physics. Developing aa brief foundation for Hawking radiation, the Schwinger effect, the Unruh effect, superradiance, and the Memory Burden effect.

The extension of QFT to curved spacetime represents a profound generalization of our understanding of quantum fields in the presence of strong gravitational fields. By moving beyond the assumptions of flat Minkowski spacetime, this formalism enables the study of phenomena such as particle creation in expanding universes, the thermal properties of BHs, and the emergence of observer-dependent vacuum. These effects illustrates the connections between quantum mechanics, general relativity and thermodynamics. Through a brief physical-mathematical analysis, this monograph presents how the absence of global symmetries (like time and Lorentz boosts), the presence of event horizons (depending on each solution), and the ambiguity of vacuum states play a central role in shaping the dynamics of quantum fields in these settings.

The Schwinger and Unruh effects exemplify the ways in which QFT transcends its origins in flat spacetime. The Schwinger effect demonstrates how strong electric fields can destabilize the quantum vacuum, leading to the creation of particle-antiparticle pairs through a non-perturbative tunneling process. The Unruh effect, by contrast, shows the observer-dependent nature of reality in QFT, where an accelerating observer perceives a thermal bath of particles in what an inertial observer would consider a vacuum. Together, these effects gives some light on the fundamental nature of quantum vacuum, the role of observer dependence and the emergence of thermal phenomena.

The study of BH physics has reaffirmed the significance of thermodynamic principles in describing the behavior of these enigmatic objects. Hawking radiation, as derived, has demonstrated that BHs are not completely isolated but gradually lose mass through quantum effects. This evaporation process raises profound questions about the ultimate fate of BHs and the fate of the information they contain. Superradiance reveals that rotating (Kerr) BHs can lose both mass and angular momentum through interactions with external fields, making their spin-down and mass loss more significant than in Schwarzschild BHs.

A key contribution of this thesis is the study and shallow review of the Memory Burden effect, by examining how BH spacetimes encode quantum information in memory modes and the significant implications of the BH information paradox. The integration of quantum corrections, holographic principles, and memory effects suggests that BHs may retain or encode information in subtle ways, challenging traditional notions of irreversibility, information loss and BH evaporation.

The thesis has also highlighted the role of primordial black holes (PBHs) as natural laboratories for testing these quantum effects ideas. As relics of the early universe, PBHs offer a unique setting to study the combined effects of Hawking radiation, superradiance and Memory Burden effects.

In conclusion, this monograph presents a brief preamble to understand the quantum behavior of curved spacetime and BHs, thanks to the application of QFT to curved spacetime. By showing the mathematical formalism, theoretical insights, and observational perspectives of some quantum effects that we predict will occur in some astrophysical objects, this work contributes to the ongoing effort to understand current physical advances and to the path toward learning and developing new physics. Future research based on these ideas will constitute a path to progress in early physics career.

Appendices

Appendix A

Python Code - Chapter 3 Figures

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import solve_ivp
from scipy.interpolate import interp1d

# Constants
M_in = 1e4  # PBH initial mass.
q = 0.5  # Memory burden mass threshold.
k = 0.2  # Memory burden parameter.
epsilon_geom_optics = 27 * np.pi / 480  # Geometric optics limit for epsilon.
a_star_kerr = 0.999  # Spin Kerr BH.
t_span = (1.0, 1e12)  # Time span.
t_eval = np.logspace(0, 12, 500)  # Logarithmic evaluation points.

# Emission efficiency.
def emission_efficiency_geometric(M, a_star):
    return epsilon_geom_optics * (1 + a_star**2)

# Mass-spin differential equations system.
def hawking_mass_spin_evolution(t, y, q, k, memory_burden=False):
    M, a_star = y
    epsilon = emission_efficiency_geometric(M, a_star)

    if memory_burden and M <= q * M_in: # Mass evolution
        dM_dt = -epsilon / M**2 * (M / (q * M_in))**(-k)
    else:
        dM_dt = -epsilon / M**2

    if M > 0: # Spin evolution
        da_star_dt = -epsilon * a_star / M**3 * (1 - a_star**2) * (M / M_in)**(-2
                                                        .0)
    else:
        da_star_dt = 0
    return [dM_dt, da_star_dt]

# Solving mass evolution equations dynamically both regimes.
def solve_mass_dynamic(a_span, M0, q, k, memory_burden):
    sol = solve_ivp(hawking_mass_spin_evolution, a_span, [M0, 0.0], args=(q, k,
                                                memory_burden), method="RK45",
```

```
                                                      t_eval=t_eval, rtol=1e-7)
    return sol.t, sol.y[0]

# Resampling.
def resample_solution(solution, t_eval):
    M_interp = interp1d(solution.t, solution.y[0], bounds_error=False, fill_value
                                                      ="extrapolate")
    a_star_interp = interp1d(solution.t, solution.y[1], bounds_error=False,
                                                      fill_value="extrapolate")
    return M_interp(t_eval), a_star_interp(t_eval)

# Solving Kerr BH.
sol_kerr_spin_no_mb = solve_ivp(
    hawking_mass_spin_evolution, t_span, [M_in, a_star_kerr], t_eval=t_eval,
    args=(1, 0, False), method="RK45", rtol=1e-7)

sol_kerr_spin_mb = solve_ivp(
    hawking_mass_spin_evolution, t_span, [M_in, a_star_kerr], t_eval=t_eval,
    args=(q, k, True), method="RK45", rtol=1e-7)

# Normalize solution.
M_kerr_spin_no_mb, a_star_kerr_spin_no_mb = resample_solution(sol_kerr_spin_no_mb
                                                  , t_eval)
M_kerr_spin_mb, a_star_kerr_spin_mb = resample_solution(sol_kerr_spin_mb, t_eval)

# Normalize mass.
M_kerr_spin_no_mb /= M_in
M_kerr_spin_mb /= M_in

# Solving Schwarzschild BH with memory burden.
t_dynamic, M_memory_burden = solve_mass_dynamic(t_span, M_in, q, k, True)
M_memory_burden /= M_in

# Solving Schwarzschild BH standard.
t_no_mb, M_no_memory_burden = solve_mass_dynamic(t_span, M_in, q, k, False)
M_no_memory_burden /= M_in

# Plotting.
plt.figure(1)

# Top Left: Mass evolution for Schwarzschild and Kerr without memory burden
plt.plot(t_eval, M_no_memory_burden, label="$a_* = 0.000$", color="black")
plt.plot(t_eval, M_kerr_spin_no_mb, label="$a_* = 0.999$", linestyle="dashed",
                                                  color="black")
plt.xscale("log")
plt.ylim(0, 1.1)
plt.ylabel("$M / M_{in}$")
plt.xlabel("$a / a_{in}$")
plt.legend()
plt.title("PBH Mass vs Scale Factor (Geometric Optics Limit)")

plt.figure(2)
```

```
# Top Right: Mass evolution with memory burden
plt.plot(t_eval, M_no_memory_burden, label="$a_* = 0.000$", color="black")
plt.plot(t_dynamic, M_memory_burden, label="$a_* = 0.999$", linestyle="dashed",
                                          color="black")
plt.xscale("log")
plt.ylim(0, 1.1)
plt.ylabel("$M / M_{in}$")
plt.xlabel("$a / a_{in}$")
plt.legend()
plt.title("PBH Mass vs Scale Factor (Memory Burden, Geometric Optics Limit)")

plt.figure(3)

# Bottom Left: Spin evolution without memory burden
plt.plot(M_kerr_spin_no_mb, a_star_kerr_spin_no_mb, color="black")
plt.ylim(0, 1.1)
plt.xlim(0, 1.1)
plt.ylabel("$a_*$")
plt.xlabel("$M / M_{in}$")
plt.title("Spin Loss vs PBH Mass (Geometric Optics)")

plt.figure(4)

# Bottom Right: Spin evolution with memory burden
plt.plot(M_kerr_spin_mb, a_star_kerr_spin_mb, color="black")
plt.ylim(0, 1.1)
plt.xlim(0, 1.1)
plt.ylabel("$a_*$")
plt.xlabel("$M / M_{in}$")
plt.title("Spin Loss vs PBH Mass (Memory Burden, Geometric Optics)")

plt.tight_layout()
plt.show()
```

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José Luis Builes Canchala

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Abbreviations

QFT	Quantum Field Theory
BH	Black Holes
FLRW	Friedmann-Lemaître-Robertson-Walker
IEEE	Institute of Electrical and Electronics Engineers
PhD	Philosophiae Doctor
UdeA	Universidad de Antioquia
CFT	Conformal Field Theory
Ads	Anti-de Sitter
CMB	Cosmic Microwave Background
QED	Quantum Electrodynamics
PBH	Primordial Black Hole