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Tensor gauge boson dark matter extension of the electroweak sector

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Abstract The existence of dark matter is explained by a new, massive, neutral, non-symmetric, rank-2 tensor gauge boson ($Z_{\mu\nu}$-boson). The $Z_{\mu\nu}$-boson can be predicted by the tensor gauge boson extension of the Electro Weak (EW) theory, proposed by Savvidy (Phys Lett B 625:341, 2005). The non-symmetric rank-2 tensor $Z_{\mu\nu}$ can be decomposed into a symmetric ($Z_{(\mu\nu)}$) and anti-symmetric ($Z_{[\mu\nu]}$) part. Based on the non-Lagrangian formulation for the free sector of the R_2-theory proposed recently by Criado et al. (Phys Rev D 102:125031, arXiv:2010.02224, 2020), our massive anti-symmetric tensor field $Z_{[\mu\nu]}$ corresponds to the massive symmetric spinor field $Z_{\alpha\beta\gamma\delta}$ in the (2,0) irrep. For the massive $Z_{\alpha\beta\gamma\delta}$ with the Z_2-symmetric Higgs portal couplings to a Standard Model (SM) particle, we compute the self-annihilation cross-section of the $Z_{\alpha\beta\gamma\delta}$ dark matter and calculate its relic abundance. We also study the SM-SM particle scattering due to the exchange of the massive-$Z_{(\mu\nu)}$ symmetric field at a high energy scale. This proposition may have far reaching applications in astrophysics and cosmology.

1 Introduction

Dark matter was proposed in 1933 to explain why galaxies in some clusters move faster than their predicted speed if they contained only baryonic matter [1]. The nature of dark matter, however, is one of modern physics' persistent mysteries. Several candidate dark matter particles have been suggested, including Light Supersymmetric Particles [2–7], heavy fourth-generation neutrinos [8,9], Q-Balls [10,11], mirror particles [12–16], and axion particles – the latter introduced in an attempt to solve the Charge-Parity (CP) violation problem in particle physics [17,18]. Recently, the braneworld idea has been applied to furnish new solutions to old problems in particle physics and cosmology, including that of the nature of the dark matter [19–33]. Universal Extra Dimensions (UED) models allow all fields to propagate in the bulk [34,35]. These models provide a viable dark matter candidate, namely the Lightest Kaluza Klein particle (LKP) [36,37]. Gauge–Higgs unification models, based on grand unified gauge theories defined on six-dimensional space-time, have interesting properties: in these models, the extra-dimensional space has the topological structure of a two-sphere orbifold S^2/Z_2 [38–40]. Furthermore, (thin) braneworlds with conical singularities in six-dimensional Einstein-Gauss-Bonnet gravity with a bulk cosmological constant have been investigated [41]. For axially symmetric bulks, however, these models do not provide isotropic braneworld cosmological solutions [41]. Other stable or quasi-stable particles that could emerge in the string theory spectrum have also been suggested as dark matter candidates: modulinos [42], exotic gauge-charged matter [5], hidden-sector matter composites [2], hidden-sector gauge composites [43], and wrapped D-branes [44]. One or more of these (or other, not yet imagined) states could contribute to the cosmos' dark matter.

The author recently proposed that cold dark matter (in the form of heavy, neutral, non-regular leptons of an O-order mass (TeV)) can be produced from quarks and leptons through the process of Electric Charge Swap (ECS) symmetry [45–50]. Furthermore, the ECS symmetry could explain certain properties of lepton families within the framework of superstring theories [51–56].

Models with a vector dark matter, especially in the non-abelian case, are the least explored, despite the fact that the gauge principle can guide and constrain the possible theoretical constructions (for a discussion of non-abelian dark

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matter with non-renormalisable kinetic mixing terms or in Higgs portal scenarios, see [57–72]).

More recently, Masi [73] proposed a new criterion to extend the SM of particle physics from a straightforward algebraic conjecture: that the symmetries of physical microscopic forces originate from the automorphism groups of main Cayley–Dickson algebras, from complex numbers to octonions and sedenions. The exceptional symmetry group G(2) that could solve the dark matter problem can be identified from the automorphism of octonion (and sedenion) algebra.

Belyaev et al. [74,75] introduced the Fermion Portal Vector Dark Matter: a new class of renormalisable models, consisting of a dark $SU(2)_D$ (Dark-Isospin) gauge sector connected to the SM through a vector-like fermion mediator without the need for a Higgs portal. In these models, the dark matter candidate is a massive vector boson.

The Feynman rules for spin-2 fields were derived by Weinberg more than half a century ago in the R_2-representation [77]. No Lagrangian formulation for the free sector of this theory is known. Recently, Criado et al. [76] proposed a very useful reformulation of Weinberg's original idea based on the symmetric multispinor formulation: Criado's Effective Field Theory (EFT) allows for consistent computations of physical observables for general-spin dark matter particles, although it does not admit a Lagrangian description [76].

A satisfactory theory of higher-spin gauge fields was constructed by Savvidy [78–81]. Building on this theory, the present paper investigates the possibility that the occurrence of dark matter can be explained by a new, neutral, non-symmetric tensor gauge boson (the $Z_{\mu\nu}$-boson, of a mass of 2.85 TeV) that can be predicted by the tensor gauge boson extension of the Electro Weak (EW) [78–81]. Based on the non-Lagrangian formulation for the free sector of the R_2-theory proposed recently by Criado et al. [76], our massive anti-symmetric tensor field $Z_{[\mu\nu]}$ corresponds to the massive symmetric spinor field $Z_{\alpha\beta\gamma\delta}$ in the (2,0) irrep. For the massive $Z_{\alpha\beta\gamma\delta}$ with the Z_2-symmetric Higgs portal couplings to the SM particle, we compute the self-annihilation cross-section of the $Z_{\alpha\beta\gamma\delta}$-boson-dark matter, and calculate its relic abundance. We also study the SM-SM particles scattering due to the exchange of a massive symmetric tensor $Z_{\mu\nu}$ field at a high energy scale. The current proposition may have far reaching applications in astrophysics and cosmology.

2 Non-Abelian tensor gauge bosons dark matter

Following Savvidy [78–81], we first consider a model whereby the $SU(2)_L$ group is extended to higher spins but the $U(1)_Y$ group is not extended. The $W^{\pm}$, Z gauge bosons receive their higher-spin descendance:

$$\left(W^{\pm}, Z\right)_{\mu}, \left(\tilde{W}^{\pm}, \tilde{Z}\right)_{\mu\lambda}, \ldots, \tag{1}$$

and the doublet of complex Higgs scalars appear together with their higher-spin partners:

$$\begin{pmatrix}\phi^{+}\\ \phi^{0}\end{pmatrix}, \begin{pmatrix}\phi^{+}\\ \phi^{0}\end{pmatrix}_{\lambda}, \begin{pmatrix}\phi^{+}\\ \phi^{0}\end{pmatrix}_{\lambda\rho}, \ldots . \quad Y = +1. \tag{2}$$

The Lagrangian that describes the interaction of the tensor gauge bosons with the scalar fields and tensor bosons is:

$$\begin{aligned}\Im = &-\frac{1}{4}G^{i}_{\mu\nu}G^{i}_{\mu\nu} - \frac{1}{4}G_{\mu\nu}G_{\mu\nu}\\ &-\left(\partial_\mu + \frac{ig'}{2}B_\mu + \frac{ig}{2}\tau^i A^i_\mu\right)\phi^\dagger\\ &\times\left(\partial_\mu - \frac{ig'}{2}B_\mu - \frac{ig}{2}\tau^i A^i_\mu\right)\phi\\ &+g_2\left\{-\frac{1}{4}G^{i}_{\mu\nu,\lambda}G^{i}_{\mu\nu,\lambda} - \frac{1}{4}G^{i}_{\mu\nu}G^{i}_{\mu\nu,\lambda\lambda}\right\}\\ &-b_2\left\{\frac{g^2}{4}\phi^\dagger\tau^i A^i_{\mu\lambda}\tau^j A^j_{\mu\lambda}\phi\right.\\ &+\nabla_\mu\phi^\dagger_\lambda\nabla_\mu\phi_\lambda + \frac{1}{2}\nabla_\mu\phi^\dagger_{\lambda\lambda}\nabla_\mu\phi + \frac{1}{2}\nabla_\mu\phi^\dagger\nabla_\mu\phi_{\lambda\lambda}\\ &-ig\nabla_\mu\phi^\dagger A_{\mu\lambda}\phi_\lambda + ig\phi^\dagger_\lambda A_{\mu\lambda}\nabla_\mu\phi\\ &-ig\nabla_\mu\phi^\dagger_\lambda A_{\mu\lambda}\phi + ig\phi^\dagger A_{\mu\lambda}\nabla_\mu\phi_\lambda\\ &\left.-\frac{1}{2}ig\nabla_\mu\phi^\dagger A_{\mu\lambda\lambda}\phi + \frac{1}{2}ig\phi^\dagger A_{\mu\lambda\lambda}\phi\nabla_\mu\phi\right\} - U(\phi),\end{aligned} \tag{3}$$

where

$$\nabla_\mu = \partial_\mu - \frac{ig'Y}{2}B_\mu - igT^i A^i_\mu. \tag{4}$$

In Eqs. (3) and (4), Y is the hypercharge, so the electric charge is $Q = T_3 + Y/2$, and, for isospinor fields, $T_i = \tau_i/2$. In the Lagrangian (3), g_2 is the tensor gauge boson coupling constant, and is a real positive parameter. The three terms in the first line of (3) represent the standard electroweak model, and the rest of the terms represent the higher-spin generalisation of this model. Therefore, all the parameters of the SM are incorporated in the tensor extension.

When the scalar fields acquire the vacuum expectation value η, then

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ \eta + \chi(x)\end{pmatrix}, \tag{5}$$

and

$$\tilde{Z}_{\mu\lambda} = A^3_{\mu\lambda}, \tag{6}$$

$$\tilde{W}^{\pm}_{\mu\lambda} = \frac{1}{\sqrt{2}}\left(A^1_{\mu\lambda} + iA^2_{\mu\lambda}\right), \tag{7}$$

the third term in the second line of Eq. (3) generates the masses of the tensor ($\tilde{W}^{\pm}$, $\tilde{Z}^0$) gauge bosons:

$$\frac{1}{8}\left(\frac{b_2}{g_2}\right) g^2\eta^2\left[\left(A^3_{\mu\lambda}\right)^2 + 2A^+_{\mu\lambda}A^-_{\mu\lambda}\right]. \tag{8}$$

Thus all the intermediate spin-2 bosons acquire the same mass:

$$M^2_{\tilde{W},\tilde{Z}} = \left(\frac{b_2}{g_2}\right) M^2_{W,Z} = g^2_{\tilde{W},\tilde{Z}} M^2_{W,Z}, \quad g^2_{\tilde{W},\tilde{Z}} \equiv \left(\frac{b_2}{g_2}\right). \tag{9}$$

The rest of the terms in Eq. (3) describe the interaction between old and new particles [78–81]. We note that the Lagrangian (3) is invariant under the simultaneous extended gauge transformations of the bosonic matter fields $\varphi_{\lambda 1}$ and tensor gauge fields $A_{\mu\lambda 1}$The transformation law of the bosonic matter fields $\varphi_{\lambda 1}$ is homogenous; however, that of the the tensor gauge fields $A_{\mu\lambda 1}$ is inhomogeneous. As shown from the higher-spin extension of the SM [78–81], the general formulation of the extended gauge transformation for the arbitrary tensor gauge fields $A_{\mu\lambda 1\ldots\lambda s}$ and bosonic matter fields $\varphi_{\lambda 1\ldots\lambda s}$ can correctly define the corresponding field strengths and the invariant Lagrangian.

The non-Abelian tensor gauge boson $Z_{\mu\nu}$, given by Eq. (7) is a real field. The tensor boson $Z_{\mu\nu}$, therefore, is its own antiparticle. For this reason, field $Z_{\mu\nu}$ has no electrical charge. In this article, we propose this neutral tensor gauge boson $Z_{\mu\nu}$ as a new dark matter candidate.

3 The R_2-representation of the anti-symmetric second-rank tensor gauge field $\tilde{Z}_{[\mu\nu]}$

The second-rank tensor gauge field $\tilde{Z}^{\alpha}_{\mu\nu}$, which, according to the theory expounded in [78–81], is an arbitrary non-symmetric tensor$\tilde{Z}^{\alpha}_{\mu\nu} \neq \tilde{Z}^{\alpha}_{\nu\mu}$, does not coincide with the graviton. This is because has different gauge symmetries and interactions. Note that, for the spin-2 particles in the (1,1) representation of the Lorentz group L_0 (e.g., gravitons), the field is a symmetric rank-2 tensor [76]. The rank-2 tensor $\tilde{Z}_{\mu\nu}$ can be decomposed into a symmetric $\tilde{Z}_{(\mu\nu)})$ and an anti-symmetric ($\tilde{Z}_{[\mu\nu]}$) part:

$$\tilde{Z}_{\mu\nu} = \tilde{Z}_{(\mu\nu)} + \tilde{Z}_{[\mu\nu]}. \tag{10}$$

This decomposition is not generally true for tensors of rank 3 $\tilde{Z}_{\mu\nu\lambda}$ and above: tensors of these ranks have more complex symmetries. The symmetric and anti-symmetric parts have the same information of the arbitrary non-symmetric $\tilde{Z}_{\mu\nu}$ field. This means that $\tilde{Z}_{(\mu\nu)}$ and $\tilde{Z}_{[\mu\nu]}$ fields have the same spin (s), mass (m), coupling constant ($g_{\tilde{Z}}$), and all the other quantum numbers. Our anti-symmetric tensor field $\tilde{Z}^{\alpha}_{[\mu\nu]}$ will be in the R_2-representation. The symmetric multispinor formulation developed in Ref. [76] is based on the known two-component spinor formalism [82–84]. The proposed Effective Field Theory (EFT) allows for consistent computations of physical observables for general-spin dark matter particles, although it does not admit a Lagrangian description [76]. The representation R_2 to which spin-2 belongs is defined as the subspace of (2,0) ⊕ (0,2) for which $\tilde{Z}^{\dagger}_L = \tilde{Z}_R$. The irreps (2,0) and (0,2) are minimal in the sense that they contain exactly the necessary number of degrees of freedom [76]. In free R_2 theory, the massive spin-2 field in the (2,0) irrep is given by:

$$\tilde{Z}_{(\alpha)}(x) = \int \frac{d^3p}{(2\pi)^3(2E_p)} \sum_{\lambda}\left[c_{p\lambda}u_{(\alpha)}(p,\lambda)e^{ipx} + c^*_{p\lambda}\upsilon_{(\alpha)}(p,\lambda)e^{-ipx}\right], \tag{11}$$

where$E_p^2 = p^2 + m^2$, $p = \left(E_p, p\right)$, $(\alpha) \equiv a_1\ldots\alpha_{2s} = a_1\ldots\alpha_4$ is a symmetrized multi-index built from two-component spinor indices, and s is the particle spin. The indices (α) transform in the (1/2,0) irrep of the Lorentz group L_0, (for details of the symmetric multispinor formulation (see Appendix A, Refs. [76,84], and Refs. [82,83]).

Moreover, it is:

$$\sum_{\lambda} u_{(\alpha)}(p,\lambda)u^*_{(\dot{\alpha})}(p,\lambda) = \frac{p_{(\alpha)(\dot{\alpha})}}{m^4}, \tag{12}$$

$$\sum_{\lambda} \upsilon_{(\alpha)}(p,\lambda)\upsilon^*_{(\dot{\alpha})}(p,\lambda) = \frac{p_{(\alpha)(\dot{\alpha})}}{m^4}, \tag{13}$$

$$\sum_{\lambda} u_{(\alpha)}(p,\lambda)\upsilon^{(\beta)}(p,\lambda) = \delta^{(\beta)}_{(\alpha)}, \tag{14}$$

where$p_{(\alpha)(\dot{\alpha})} \equiv p_{\alpha_1\dot{\alpha}_1}\ldots p_{\alpha_4\dot{\alpha}_4}$, $\delta^{(\beta)}_{(\alpha)} \equiv \delta^{\beta_1}_{\alpha_1}\ldots\delta^{\beta_4}_{\alpha_4}$, with symmetrization over all indices of the same type at the same height being implied, and $\lambda = -2\ldots 2$ is the spin state. Undotted indices $(\alpha, \beta, \ldots)$ and dotted indices $\left(\dot{\alpha}, \dot{\beta}, \ldots\right)$ transform in the (1/2,0) and (0,1/2) irrep of the Lorentz group L_0, respectively. From Eq. (11), we compute the propagators:

$$\frac{ip_{(\alpha)(\dot{\alpha})}/m^4}{p^2-m^2}, \quad \frac{i\delta^{(\beta)}_{(\alpha)}}{p^2-m^2}, \quad \frac{ip^{(\alpha)(\dot{\alpha})}/m^4}{p^2-m^2}, \quad \frac{i\delta^{(\dot{\alpha})}_{(\dot{\beta})}}{p^2-m^2}[75]. \tag{15}$$

Furthermore, the spinor field $\tilde{Z}_{\alpha\beta\gamma\delta}$ in the (2,0) irrep can be written in terms of the vector-spinors $Y^{\mu}_{\alpha\beta}$, $Y^{\nu}_{\gamma\delta}$ and the anti-symmetric tensor gauge filed $\tilde{Z}_{[\mu\nu]}$, as follows:

$$\tilde{Z}_{\alpha\beta\gamma\delta} = Y^{\mu}_{\alpha\beta}Y^{\nu}_{\gamma\delta}\tilde{Z}_{[\mu\nu]}, \tag{16}$$

where

$$Y^{\mu}_{\alpha\beta} = \frac{1}{2}\left(\sigma^{\mu}_{\alpha\dot{\alpha}}\bar{\sigma}^{\nu\dot{\alpha}\gamma} + \sigma^{\nu}_{\alpha\dot{\alpha}}\bar{\sigma}^{\mu\dot{\alpha}\gamma}\right)Y_{\nu}\varepsilon_{\gamma\beta} = \eta^{\mu\nu}Y_{\nu}\delta^{\gamma}_{\alpha}\varepsilon_{\gamma\beta} = \delta^{\gamma}_{\alpha}\varepsilon_{\gamma\beta}Y^{\mu}, \tag{17}$$

and

$$\frac{1}{2}\left(\sigma^{\mu}_{\alpha\dot{\alpha}}\bar{\sigma}^{\nu\dot{\alpha}\gamma}+\sigma^{\nu}_{\alpha\dot{\alpha}}\bar{\sigma}^{\mu\dot{\alpha}\gamma}\right)\varepsilon_{\gamma\beta}=\eta^{\mu\nu}\delta^{\gamma}_{\alpha}\varepsilon_{\gamma\beta}, \tag{18}$$

α, β, γ, δ are the two-spinor indices, Y_ν the real vector, and $\eta_{\mu\nu}$ the Lorentz metric. $\sigma^{\mu}_{\alpha\dot{\alpha}}$ are Pauli-spinor matrices [82,83], and $\varepsilon_{\beta\gamma}$ are the symmetric generalised symbols used to raise and lower the symmetric multispinor indices. Displaying the symmetric $\varepsilon_{\beta\gamma}$, as above is correct, as demonstrated in [76,84] on the basis of the symmetric multispinor formalism. In Eq. (17), the term (18) serves to convert the four-vector Y_ν into the vector-spinor $Y^{\mu}_{\alpha\beta}$. Therefore, the product $Y^{\mu}_{\alpha\beta}Y^{\nu}_{\gamma\delta}$ in Eq. (16) converts the anti-symmetric tensor gauge field $\tilde{Z}_{[\mu\nu]}$ into the spinor field $\tilde{Z}_{\alpha\beta\gamma\delta}$. Equation (16) establishes the correspondence between tensor gauge field $\tilde{Z}_{[\mu\nu]}$ and spinor field $\tilde{Z}_{\alpha\beta\gamma\delta}$ in the (2,0) irrep. The inverses $Y^{\alpha\beta}_{\mu}$ satisfy $Y^{\mu}_{\alpha\beta}Y^{\alpha\beta}_{\nu}=\delta^{\alpha}_{\alpha}\delta^{\beta}_{\beta}\delta^{\mu}_{\nu}, Y^{\mu}_{\alpha\beta}Y^{\gamma\delta}_{\mu}=\delta^{\gamma}_{\alpha}\delta^{\delta}_{\beta}\delta^{\mu}_{\mu}$. The spinor field $\tilde{Z}_{\alpha\beta\gamma\delta}$ in the (2,0) irrep equivalent of the anti-symmetric tensor gauge field $\tilde{Z}_{[\mu\nu]}$is given by $\tilde{Z}_{[\mu\nu]}=Y^{\mu\alpha\beta}_{\mu}Y^{\nu\gamma\delta}_{\nu}\tilde{Z}_{\alpha\beta\gamma\delta}$.

Following [78–81], to describe spin-2 quanta, we introduce the tensor gauge field $\tilde{Z}_{\mu\nu}$ together with the higher-tensor gauge field $\tilde{Z}_{\mu\nu\lambda}$. Furthermore, to describe the dynamics of the rank-3 gauge field $\tilde{Z}_{\mu\nu\lambda}$, we introduce the additional tensor gauge fields $\tilde{Z}_{\mu\nu\lambda\rho}$and $\tilde{Z}_{\mu\nu\lambda\rho\eta}$. Following [76], the massive spin-2 particle in (2,0)⊕(0,2) irrep is given by $\tilde{Z}_{\alpha\dot{\alpha}\beta\dot{\beta}\gamma\dot{\gamma}\delta\dot{\delta}}$. Therefore, our (2,0)⊕(0,2) irrep for massive spin-2 particle corresponds to a rank-4 tensor gauge filed $\tilde{Z}_{\mu\nu\lambda\rho}$:

$$\tilde{Z}_{\alpha\dot{\alpha}\beta\dot{\beta}\gamma\dot{\gamma}\delta\dot{\delta}}=\sigma^{\mu}_{\alpha\dot{\alpha}}\sigma^{\nu}_{\beta\dot{\beta}}\sigma^{\lambda}_{\gamma\dot{\gamma}}\sigma^{\rho}_{\delta\dot{\delta}}\tilde{Z}_{\mu\nu\lambda\rho}. \tag{19}$$

The rank-4 tensor $\tilde{Z}_{\mu\nu\lambda\rho}$ gauge field has the symmetries of the Weyl tensor [85], which transforms as (2,0)⊕(0,2). We now introduce the angular momentum and boost generators:

$$J^{i}=\frac{1}{2}\varepsilon^{ijk}M^{jk} \quad , \quad K^{i}=M^{0i}, \tag{20}$$

where M^{jk} are the infinitesimal generators of the Lorentz group L_0. The $\tilde{Z}_{\mu\nu\lambda\rho}$ tensor gauge field is given by:

$$\begin{aligned}\tilde{Z}_{\mu\nu\lambda\rho}&=4\left\{M^{\mu\nu},M^{\lambda\rho}\right\}+2\left\{M^{\mu\lambda},M^{\nu\rho}\right\}\\&\quad-2\left\{M^{\mu\rho},M^{\nu\lambda}\right\}-8\left(g_{\mu\lambda}g_{\nu\rho}-g_{\mu\rho}g_{\nu\lambda}\right),\end{aligned} \tag{21}$$

with the symmetry properties $Z_{\mu\nu\lambda\rho}=-Z_{\nu\mu\lambda\rho}=-Z_{\mu\nu\rho\lambda}; Z_{\mu\nu\lambda\rho}=Z_{\lambda\rho\mu\nu}$. It satisfies the Bianchi identity $Z_{\mu\lambda\rho\nu}+Z_{\mu\rho\nu\lambda}+Z_{\mu\nu\lambda\rho}=0$, and the contraction of any pair of indices vanishes ($\tilde{Z}^{\nu}_{\nu\lambda\rho}=0$) [85]. These constraints leave only 10 independent components. Now we have:

$$\tilde{Z}_{\alpha\beta\gamma\delta}=\sigma^{\mu\nu}_{\alpha\beta}\sigma^{\lambda\rho}_{\gamma\delta}\tilde{Z}_{\mu\nu\lambda\rho}. \tag{22}$$

The object:

$$\sigma^{\mu\nu}_{\alpha\beta}=\frac{i}{4}\left(\sigma^{\mu}_{\alpha\dot{\alpha}}\bar{\sigma}^{\nu\dot{\alpha}\gamma}-\sigma^{\nu}_{\alpha\dot{\alpha}}\bar{\sigma}^{\mu\dot{\alpha}\gamma}\right)\varepsilon_{\beta\gamma}, \tag{23}$$

is symmetric in the two-spinor indices (αβ) and antisymmetric in the Lorentz indices [μν]. The product $\sigma^{\mu\nu}_{\alpha\beta}\sigma^{\lambda\rho}_{\gamma\delta}$ projects the rank-4 tensor gauge filed $\tilde{Z}_{\mu\nu\lambda\rho}$ into the (2,0) subspace. From Eqs. (22) and (16), we derive the following relation:

$$\tilde{Z}_{\alpha\beta\gamma\delta}=\sigma^{\mu\nu}_{\alpha\beta}\sigma^{\lambda\rho}_{\gamma\delta}\tilde{Z}_{\mu\nu\lambda\rho}=Y^{\mu}_{\alpha\beta}Y^{\nu}_{\gamma\delta}\tilde{Z}_{[\mu\nu]}. \tag{24}$$

4 The interactions of the $\tilde{Z}$-dark matter field

Konitopoulos and Savvidy [86] analyzed the interaction between two tensor currents caused by the exchange of these tensor gauge bosons. They found that all the negative-norm states are excluded from the spectrum of the second-rank massless non-symmetric tensor gauge field $A_{\mu\nu}$, due to the gauge invariance of the theory. They thus came to the conclusion that the theory does indeed respect unitarily at the free level. In our description, hovered, perturbative unitarity is unavoidably broken at some high energy scale Λ, above the $Z_{\mu\nu}$ particle mass. Following [76], below the energy scale Λ – much higher than the particle mass, m, and the electroweak energy scale, Λ_{EW} – the only degrees of freedom present are those of the SM and the spin-2 dark matter particles. The interactions between the spin-2 dark matter particles and the SM particles can be described by an EFT that incorporates the effects of the new physics at Λ through Lorentz-invariant local operators whose effects are suppressed by inverse powers of Λ. We define the effective cutoff scale Λ as follows:

$$\left(\frac{M_{\tilde{Z}}}{\Lambda}\right)^{2s}=\left(\frac{M_{\tilde{Z}}}{\Lambda}\right)^{4}=\frac{g_{\tilde{Z}}}{4\pi}, \qquad (\text{for spin } s=2), \tag{25}$$

where $g_{\tilde{Z}}$ is the dark matter coupling, and the mass $M_{\tilde{Z}}$ is derived from Eq. (9). We expect that the perturbative unitarity is broken in processes of energy $E\approx\Lambda$. From Eq. (25), we obtain the upper limit for the validity of the theory. Furthermore, the perturbative unitarity condition $g_{\tilde{Z}}<4\pi$is given by Eq. (25) for $M_{\tilde{Z}}<\Lambda$. The $Z_{\alpha\beta\gamma\delta}$-dark matter particle must be stable. This can be achieved by imposing the $\mathbb{Z}_2$-symmetry. The $Z_{\alpha\beta\gamma\delta}$ natural dark matter candidate is $\mathbb{Z}_2$-odd, while all the SM particles are $\mathbb{Z}_2$-even. Now, the effective interacting Hamiltonian density is given by:

$$H_{\text{Int}}=H_{Linear}+H_{Portal}. \tag{26}$$

Based on [76], since a spin-s field carries an effective dimension of $\Delta=s+1$, the lowest dimension of the Lorentz-invariant local operators linear in the spinor $\tilde{Z}$ field is $N=1+3s$. Hence the $\mathbb{Z}_2$-violating interaction H_{Linear} in Eq. (26)

is:

$$H_{Linear} = \frac{\tilde{Z}^{\alpha\beta\gamma\delta}}{\Lambda_{Linear}^{N-4}}\Big[c_B\sigma_{\alpha\beta}^{\mu\nu}\sigma_{\gamma\delta}^{\lambda\rho}B_{\mu\nu}B_{\lambda\rho} + c_W\sigma_{\alpha\beta}^{\mu\nu}\sigma_{\gamma\delta}^{\lambda\rho}W_{i\mu\nu}W_{\lambda\rho}^{i}\Big] = \frac{\tilde{Z}^{\alpha\beta\gamma\delta}}{\Lambda_{Linear}^{3}}\Big[c_B\sigma_{\alpha\beta}^{\mu\nu}\sigma_{\gamma\delta}^{\lambda\rho}B_{\mu\nu}B_{\lambda\rho} + c_W\sigma_{\alpha\beta}^{\mu\nu}\sigma_{\gamma\delta}^{\lambda\rho}W_{i\mu\nu}W_{\lambda\rho}^{i}\Big] \quad (27)$$

where

$$\mathrm{N} = 1 + 3\mathrm{s} = 7, \qquad (\text{for spin s} = 2). \quad (28)$$

$\tilde{Z}^{\alpha\beta\gamma\delta}$ is the spin-2 dark matter field in the (2,0) irrep; Λ_{Linear} is an energy scale of the $\mathbb{Z}_2$-violating linear interactions of the $\tilde{Z}$ dark matter field with the SM particles; $B_{\mu\nu}$ and $W^i_{\mu\nu}$ are the $U(1)_Y$ and $SU(2)_L$ field strengths, respectively. The coefficients c_B and c_W are arbitrary in principle. The object $\sigma_{\alpha\beta}^{\mu\nu}$ projects rank-2 tensors $B_{\mu\nu}$ and $W^i_{\mu\nu}$ into their (1,0) subspace. Note that, unlike the case for spin-2 particles in the (1,1) representation (e.g., gravitons), in Eq. (27), linear interactions with fermions and scalars are absent at the leading order due to Lorentz symmetry [84]. Interaction (27) yields to the decay of the $Z_{\alpha\beta\gamma\delta}$ field to EW vector gauge bosons VV. The total decay width of the massive spin-2 particle to EW vector gauge bosons VV is given by:

$$\Gamma(\tilde{Z} \to VV) \approx M_{\tilde{Z}}^{7}/\Lambda_{Linear}^{6} \quad [84]. \quad (29)$$

For $\mathbb{Z}_2$-violating interactions at a large energy scale Λ_{Linear}, the spin-2 particle becomes metastable. The term H_{Portal} in Eq. (26) is the $\mathbb{Z}_2$-symmetric Higgs portal:

$$H_{Portal} = g_{\tilde{Z}}^{2}\tilde{Z}^{\alpha\beta\gamma\delta}\tilde{Z}_{\alpha\beta\gamma\delta}|\phi|^2, \quad (30)$$

where $\tilde{Z}^{\alpha\beta\gamma\delta}$is the spin-2 dark matter field in the (2,0) irrep, ϕ is the Higgs doublet, and $g_{\tilde{Z}}$ is the dark matter coupling. The interaction is mediated by a quartic coupling between two Higgs and two spin-2 dark matter particles [76].

5 The annihilation cross-section of the $\tilde{Z}$-dark matter field

Based on [76], the total annihilation cross-section times the relative velocity of $\tilde{Z}$-dark matter to SM particles is given by:

$$(\sigma u_{rel})_{total} = \sigma u_{rel}\left(\tilde{Z}\tilde{Z} \to \bar{f}f\right) + \sigma u_{rel}\left(\tilde{Z}\tilde{Z} \to VV\right) + \sigma u_{rel}\left(\tilde{Z}\tilde{Z} \to hh\right), \quad (31)$$

where

$$\sigma u_{rel}\left(\tilde{Z}\tilde{Z} \to \bar{f}f\right) \approx \frac{u_{rel}^2}{5}\frac{g_{\tilde{Z}}^4 m_f^2}{8\pi M_{\tilde{Z}}^4}, \quad (32)$$

$$\sigma u_{rel}\left(\tilde{Z}\tilde{Z} \to VV\right) \approx \frac{g_v g_{\tilde{Z}}^4}{10\pi M_{\tilde{Z}}^2}, \quad (33)$$

$$\sigma u_{rel}\left(\tilde{Z}\tilde{Z} \to hh\right) \approx \frac{g_{\tilde{Z}}^4}{20\pi M_{\tilde{Z}}^2}. \quad (34)$$

$M_{\tilde{Z}}$ is derived from Eq. (9); $g_{\tilde{Z}}$ is the coupling constant, and m_f, and g_v are the SM-fermion mass and SM-vector gauge boson coupling, respectively. The annihilation cross-sections (32), (33), and (34) follow from the $\mathbb{Z}_2$-symmetric Higgs portal given by Eq. (30), and calculated from Reference [76] for the case of real coupling constant $g_{\tilde{Z}}$ and spin-2 dark matter particles. Equations (33) and (34) correspond to the s-wave ($\sim u_{rel}^0$) dark matter annihilation for bosons; Eq. (32) corresponds to the p-wave annihilation ($\sim u_{rel}^2$) for fermions. The annihilation of $\tilde{Z}$-dark matter particle to SM-fermions is thus velocity-suppressed ($u_{rel}^2 < 1$). Therefore, the total annihilation cross section (31) becomes:

$$(\sigma u_{rel})_{total} \approx (g_v + 1/2)\,\frac{g_{\tilde{Z}}^4}{10\pi M_{\tilde{Z}}^2}. \quad (35)$$

Following [87], for the proposed $\tilde{Z}$-dark matter, the relic density should be:

$$\Omega_{\tilde{Z}}h^2 = \frac{0.1\,pb}{\langle\sigma u_{rel}\rangle} \approx 0.12, \quad (36)$$

where$\langle\sigma u_{rel}\rangle \approx 0.83\,pb$. Analysis of the three-year Wilkinson Microwave Anisotropy Probe (WMAP) data suggests that the density of dark matter is $\Omega_{DM}\,h^2 = 0.102 \pm 0.009$ (where $\Omega_{DM} = \rho_{DM}/\rho_{crit}$, with ρ_{crit} being the density corresponding to a flat universe [88], and h being the Hubble constant, in units of 100 km $s^{-1} \cdot Mpc^{-1}$) [89].

More recently, Aghanim et al. (Planck Collaboration, 2020) [87] suggested that the density of dark matter is about $\Omega_{DM}h^2 \sim 0.12$. A cold dark matter candidate produced at the LHC should, therefore, have this annihilation cross section. This quantity leads us to the second method of measuring the coupling of dark matter from SM particles: through the search for the products of dark matter annihilation or decay originating from high-density regions of the Universe, such as the center of galaxies [90]. Since the WMAP results provide good information about $\langle\sigma u_{rel}\rangle$, the uncertainties in this approach stem from our sketchy knowledge of the exact density of dark matter in the center of galaxies, and from the difficulty of separating the dark matter annihilation signal from possible background signals. From the total annihilation cross section Eq. (35), the mass of the $\tilde{Z}$-dark matter is given by:

$$M_{\tilde{Z}} \approx g_{\tilde{Z}}^2\sqrt{\frac{(g_v + 1/2)}{10\pi(\sigma u_{rel})_{total}}}. \quad (37)$$

Assuming $(\sigma u_{rel})_{total} \approx 0.83 pb = 2,1315977 \times 10^{-9}$ GeV^{-2}, from the requirement of relic abundance and $g_{\tilde{Z}}^2 = 0.4$, $g_V = 0.6$, we obtain the following value for the mass of $\tilde{Z}$-dark matter:

$$M_{\tilde{Z}} \approx 2.85\,\mathrm{TeV}. \tag{38}$$

6 SM-SM particle scattering due to the exchange of a massive, symmetric second-rank tensor gauge field $\tilde{Z}_{(\mu\nu)}$ at a high energy scale

The symmetric part of Eq. (10) is the symmetric gauge field $\tilde{Z}_{\mu\nu} = \tilde{Z}_{\nu\mu}$. The spin (s), mass (m), coupling constant (g_2), and all the other quantum numbers of this field are the same as those of the anti-symmetric tensor field $\tilde{Z}_{[\mu\nu]}$. The symmetric gauge field $\tilde{Z}_{(\mu\nu)}$is in the (1,1) representation of the Lorentz group L_0. The $\tilde{Z}_{(\mu\nu)}$field does not coincide with the graviton because its gauge symmetries and interactions differ from those of the graviton. Based on Savvidy [78–81,86], the Lagrangian for the symmetric spin-2 particle of mass $M_{\tilde{Z}_{Sym}} \approx 2.85 TeV$ has the Fierz–Pauli form:

$$\Im = \frac{1}{2}\Big(\partial_\lambda \tilde{Z}^{\mu\nu}\partial^\lambda \tilde{Z}_{\mu\nu} - \partial_\lambda \tilde{Z}^\mu_\mu \partial^\lambda \tilde{Z}^\nu_\nu - 2\partial_\lambda \tilde{Z}^{\lambda\nu}\partial^\mu \tilde{Z}_{\mu\nu} + 2\partial^\nu \tilde{Z}^\lambda_\lambda \partial^\mu \tilde{Z}_{\mu\nu} + M^2_{\tilde{Z}}(\tilde{Z}^\mu_\mu \tilde{Z}^\nu_\nu - \tilde{Z}^{\mu\nu}\tilde{Z}_{\mu\nu})\Big), \tag{39}$$

where $\tilde{Z}_{(\mu\nu)}$ is the linearized field. The higher-order terms describe the self-interactions of the $\tilde{Z}_{(\mu\nu)}$ field. Furthermore, for the symmetric $\tilde{Z}_{(\mu\nu)}$ field, we impose the traceless condition $\tilde{Z}^\mu_\mu = 0$. The interaction term with the SM fields is:

$$\Im_{eff} = \sum_{s=spin} G_{\tilde{Z}_{Sym}} Z^{\mu\nu} \times \left(2\frac{\delta \Im^{(s)}}{\delta \hat{g}^{\mu\nu}} - \eta_{\mu\nu}\Im^{(s)}\right)|_{\hat{g}} = \eta \quad [91], \tag{40}$$

where,

$$G_{\tilde{Z}_{Sym}} = \frac{g_{\tilde{Z}}}{\Lambda_{Sym}}, \tag{41}$$

$\eta_{\mu\nu}$ is the Minkowski metric, and the factors within the parentheses are the stress-energy tensors $T^{(s)}_{\mu\nu}$ for the SM particles of different spins($s = 0, 1/2, 1$) [91]. As such, $\Im^{(s)}$ is the SM Lagrangian for the particles of spin-s. Since the spin sum of the $\tilde{Z}_{Sym}$ polarisation:

$$P_{\mu\nu,\lambda\rho} = \frac{1}{2}\left(P_{\mu\lambda}P_{\nu\rho} + P_{\nu\lambda}P_{\mu\rho} - \frac{2}{3}P_{\mu\nu}P_{\lambda\rho}\right), \tag{42}$$

with

$$P_{\mu\nu} = \eta_{\mu\nu} - \left(\frac{k_\mu k_\nu}{M^2_{\tilde{Z}}}\right), \tag{43}$$

is traceless, the terms proportional to $\Im^{(s)}$ in Eq. (40) do not contribute [91] (Explicit expressions for the various spins can be found in Ref. [92]). Equation (41) is the effective coupling constant, and Λ_{Sym} is the cut-off energy scale of the EFT symmetric tensor generalisations of the EW theory. We note that Λ_{Sym} does not need to be the same with the cut-off energy scale Λ of the EFT symmetric spinor version of the non-symmetric tensor generalisations of the EW theory (see above). Based on Haiying Cai et al. [91], for a generic scattering process $B_1 + B_2 \to B_3\tilde{Z}_{Sym}$ where $B_{1,2,3}$ are the SM particles, the amplitude squared after the solid angle integration of the process is given by:

$$A_{B_1B_2\to B_3\tilde{Z}_{Sym}} = \int d\Omega |M|^2_{B_1B_2\to B_3\tilde{Z}_{Sym}}. \tag{44}$$

At high energies, above the EW scale Λ_{EW}, we find that all amplitudes squared scale as follows:

$$A_{B_1B_2\to B_3\tilde{Z}_{Sym}} \sim \frac{g^2_{\tilde{Z}} g_s s}{\Lambda^2_{Sym}}, \tag{45}$$

where g_s is an appropriate SM coupling, and $s = (p_{B_1} + p_{B_2})^2$. The only exception is the process $hh \to h\tilde{Z}_{Sym}$, for which the amplitude is a constant:

$$A_{hh\to h\tilde{Z}_{Sym}} \sim \frac{g^2_{\tilde{Z}} m_h^4}{\Lambda^2_{Sym}\upsilon^2}, \tag{46}$$

where the Higgs mass $m_h = 125$ GeV, and the EW vacuum expectation value $\upsilon \approx 246$ GeV. For the SM fermion–fermion process $\bar{f}_1 f_2 \to V\tilde{Z}_{Sym}$ with the SM vector gauge bosons V, the total amplitude squared is calculated using the Feynman Rules [93,94]:

$$A_{\bar{f}f\to V\tilde{Z}_{Sym}} \sim \frac{g^2_{\tilde{Z}} g^2_V s}{\Lambda^2_{Sym}}, \tag{47}$$

where g_v is the weak coupling. The decay width of the symmetric spin-2 particle Z_{Sym} of a mass of 2.85 TeV is governed by the Lagrangian in Eq. (40):

$$\Gamma\left(\tilde{Z}_{Sym} \to hh\right) \approx \frac{g^2_{\tilde{Z}} M^3_{\tilde{Z}}}{960\pi\Lambda^2_{Sym}}, \tag{48}$$

$$\Gamma\left(\tilde{Z}_{Sym} \to \bar{f}f\right) \approx \frac{3g^2_{\tilde{Z}} M^3_{\tilde{Z}}}{320\pi\Lambda^2_{Sym}}, \tag{49}$$

$$\Gamma\left(\tilde{Z}_{Sym} \to VV\right) \approx \frac{4g^2_{\tilde{Z}} M^3_{\tilde{Z}}}{320\pi\Lambda^2_{Sym}}. \tag{50}$$

The three partial decay widths describe decay to scalar h, fermion f and vector particles V respectively.

7 Discussion

The second-rank tensor gauge field $Z_{\mu\nu}$, which in the Lagrangian theory [78–81] described in Sect. 2 is an arbitrary non-symmetric tensor $Z_{\mu\nu} \neq Z_{\nu\mu}$, does not coincide with the graviton, because it has different gauge symmetries and interactions. Note that, for the spin-2 particles in the (1,1) representation of the Lorentz group L_0 (e.g., gravitons), the field is a symmetric rank-2 tensor [76]. The rank-2 tensor $Z_{\mu\nu}$ can be decomposed into a symmetric, $Z_{(\mu\nu)}$ and an anti-symmetric, $Z_{[\mu\nu]}$ part.

The proposed EFT allows for consistent computations of physical observables for general-spin dark matter particles, although it does not admit a Lagrangian description [76]. Our heavy anti-symmetric tensor field $Z_{[\mu\nu]}$ corresponds to the massive symmetric spinor field $Z_{\alpha\beta\gamma\delta}$ in the (2,0) irrep by Eq. (16). For this reason, for the symmetric spinor field $Z_{\alpha\beta\gamma\delta}$, we follow a non-Lagrangian formulation of the EFT in Sects. 3–5.

On the other hand, the symmetric gauge field $Z_{(\mu\nu)}$ is in the (1,1) representation of the Lorentz group L_0. Therefore, in Sect. 6, we follow a Lagrangian formulation of the EFT.

Based on [76], the collider constraints of our non-Lagrangian effective framework for the symmetric spinor field $\tilde{Z}$ are quite similar to the usual Higgs-portal DM models [95–98]. In these models, the only way to produce DM in colliders is by first producing Higgs bosons, either on-shell or off-shell, which subsequently decay into DM: $pp \to hB \to \tilde{Z}\tilde{Z}B$ (where B represents visible SM states). The prospect of a DM signal then crucially depends on the mass of the DM. If the DM mass is $M_{DM} \leq m_h/2$, the Higgs boson can decay to DM on-shell. This is an invisible decay. The SM Higgs boson decays predominantly to visible channels. The only invisible decay channel of the Higgs boson is to neutrinos: $BR_{SM}(h \to inv) = BR_{SM}(h \to 4\nu) \approx 10^{-3}$. This decay can be neglected. BSM contributions can significantly alter the invisible decay rate of the Higgs boson. Here, the symmetric spinor field $\tilde{Z}$ is much heavier than $M_{DM} > m_h/2$, and the Higgs boson in $pp \to hB \to \tilde{Z}\tilde{Z}B$ has to be virtual. In this case, DM production is suppressed by $g^2_{\tilde{Z}}$.

For the symmetric spin-2 particle Z_{Symm}, the Lagrangian formulations allow. The collider constraints are quite similar to those of the massive Kaluza–Klein (KK) graviton models [25,30,31,99]. The heavy symmetric spin-2 particle Z_{Symm} can be produced from quark/anti-quark scattering at the LHC, and then decay into the SM particles (as we showed in Sect. 6). Based on Kang and Lee [99], we may also constrain non-universal lepton and photon couplings by the photon energy distribution from the process $e^+e^- \to \gamma Z_{Symm}$. Following [99], the squared amplitude for $e^+e^- \to \gamma Z_{Symm}$, is as follows:

$$|M|^2 = \frac{e^2c_e^2}{4\Lambda^2_{Symm}\left(s+t-M^2_{\tilde{Z}}\right)} \times \left(s^2+2t(s+t)-2M^2_{\tilde{Z}}t+M^4_{\tilde{Z}}\right) \times \left(4t(s+t)-M^2_{\tilde{Z}}(s+4t)\right)+\frac{e^2c_e^2}{\Lambda^2_{Symm}s} \times \left(\frac{c_\gamma}{c_e}-1\right)\left((s+2t)^2-M^2_{\tilde{Z}}(s+4t)+2M^4_{\tilde{Z}}\right) + \frac{e^2c_e^2}{6\Lambda^2_{Symm}M^4_{\tilde{Z}}s}\left(\frac{c_\gamma}{c_e}-1\right)^2\left\{s^2\left(s^2+2st+t^2\right) -2M^2_{\tilde{Z}}s(s+t)(s+6t)+M^4_{\tilde{Z}}(7s^2+24st +12t^2)-12M^6_{\tilde{Z}}(s+t)+6M^8_{\tilde{Z}}\right\}, \tag{51}$$

where,

$$t = -\frac{1}{2}\left(s-M^2_{\tilde{Z}}\right)(1-\cos\theta). \tag{52}$$

For $c_\gamma = c_e$, the squared amplitude behaves like s/Λ^2_{Symm} for $s \gg M^2_{\tilde{Z}}$ [65–69]. For $c_\gamma \neq c_e$, the squared amplitude becomes $s^3/M^4_{\tilde{Z}}\Lambda^2_{Symm}$. This shows that unitarity is violated at a lower energy. A similar phenomenon was observed in the Quantum Chromodynamics (QCD) process, $q\bar{q} \to gZ_{Symm}$ [100–104]: in the latter, $c_g \neq c_q$ would give rise to a similar dependence of the corresponding squared amplitude on the centre of mass energy [99].

The Dijet and dilepton searches at the LHC can constrain relatively heavy spin-2 resonances [105,106]. Although not sensitive enough, the Initial State Radiation (ISR) photon or jet + heavy Dijet resonances may have the potential to constrain non-universal quark and gluon couplings by the jet-transverse momentum p_T distribution from the $q\bar{q} \to g\, Z_{Symm}$ process at the LHC and future hadron colliders [105].

Direct detection bounds from XENON1T [107,108], LUX [109], PandaX [110], etc., are most stringent for weak-scale or heavier dark matter. As a general feature, the necessary values of coupling $g_{\tilde{Z}}$ for the correct DM abundance to be generated through freeze-out are excluded by the bounds on invisible decays of the Higgs boson for $M_{DM} < mh/2$, and by direct detection experiments for 6 GeV $< M_{DM} <$ 1 TeV.

8 Conclusions

We suggest that a new, neutral, non-symmetric tensor gauge boson ($Z_{\mu\nu}$-boson) can explain the existence of dark matter in our Universe. The non-symmetric $Z_{\mu\nu}$-boson can be predicted by the tensor gauge boson extension of the Electro Weak (EW) theory proposed by Savvidy. Based on the non-Lagrangian formulation for the free sector of the R_2-

theory proposed recently by Criado et al. our massive antisymmetric tensor field $Z_{[\mu\nu]}$ corresponds to the massive symmetric spinor field $Z_{\alpha\beta\gamma\delta}$ in the (2,0) irrep. For the massive $Z_{\alpha\beta\gamma\delta}$ with the Z_2-symmetric Higgs portal couplings to the SM particle, we compute the self-annihilation cross-section of the $Z_{\alpha\beta\gamma\delta}$-boson-dark matter, and calculate its relic abundance. We also study the SM-SM particle scattering due to the exchange of a massive, $Z_{\mu\nu}$-symmetric boson at a high energy scale. This proposition may have far-reaching applications in astrophysics and cosmology.

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