

EXPERIMENTAL INVESTIGATIONS OF PHYSICAL PROCESSES FACILITATING THE CAPTURE OF ELECTRONS INJECTED INTO THE BETATRON *

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Summary

1. The maximum possible intensity obtained in betatrons is mainly determined by the magnitude of current limits, which can be kept on the stationary orbit by the quasi-elastic force of the magnetic field.
2. Most betatrons operate under conditions where the orbit current is close to its limit value. Hence a considerable increase in the intensity of betatron gamma radiation can be obtained only by increasing the current limit.
3. Particle capture in betatron acceleration is greatly influenced by the effect of collective particle interaction of electron beams with the local space-charge present in the chamber.

Introduction

The betatron, as a source of intense gamma radiation of high energy, has been widely used in physical researches and various fields of science and engineering. Nevertheless, there is still no complete theory of the betatron. This is particularly true as regards the theoretical problems of injection and the capture of particles in the stationary orbit, and the failure to solve these problems hampers betatron designing and construction. Due to the lack of possibility of immediately determining the most effective parameters and conditions for electron injection into the betatron, a good many accelerators of this kind are operating with lower efficiency than might otherwise be the case.

There are various hypotheses (D. Kerst, R. Wideröe et al.) concerning the mechanism of particle capture by betatron acceleration¹⁻⁴), but none of them completely agree with all the experimental data which, being also scrappy and often contradictory, give no reliable basis for formulating of the capture theory⁵⁻⁷).

To remedy this situation, the P.N. Lebedev Physics Institute in conjunction with the Institute for Scientific Research in Physics of the Moscow Lomonosov University, have conducted experimental investigations of the problems connected with the injection and capture of electrons in the betatron.

To widen experimental possibilities and to eliminate conclusions which might result from the particular properties of the accelerator, the investigations were conducted

on two different sets: a 30 Mev synchrotron and a 3 Mev betatron. Data obtained from the 250 Mev synchrotron of the USSR Academy of Sciences were also used⁸).

The extent of the work and the volume of experimental data make it impossible to give a full account in this paper which therefore includes only cardinal results obtained from the experimental data. A detailed report will be published in a series of articles elsewhere.

1. Two kinds of capture mechanism

The efficiency of electron capture in the stationary orbit is mainly determined by the decrease in radius of the instantaneous orbit of the electron per revolution ΔR .

The degree of efficiency depends substantially on the particle injection current in the betatron chamber i_{inj} .

This is expressed in the experiment by the variation of the function $I_{out} = f(i_{inj})$ when the current i_{inj} is changed within sufficiently wide limits, where I_{out} is the beam intensity of the accelerated particles. The shape of this curve is shown in fig. 1, which clearly indicates that the intensity increase is linear in the region of low injections current (section ab).

Owing to the fact that the shape of the I_{out} curve on section ab remains unchanged even when injection currents are very small ($i_{inj} \sim 0,01$ mA), it may be assumed that the capture of particles within this section is stimulated by the adiabatic contraction of instantaneous orbits and

* This paper was presented in title only.

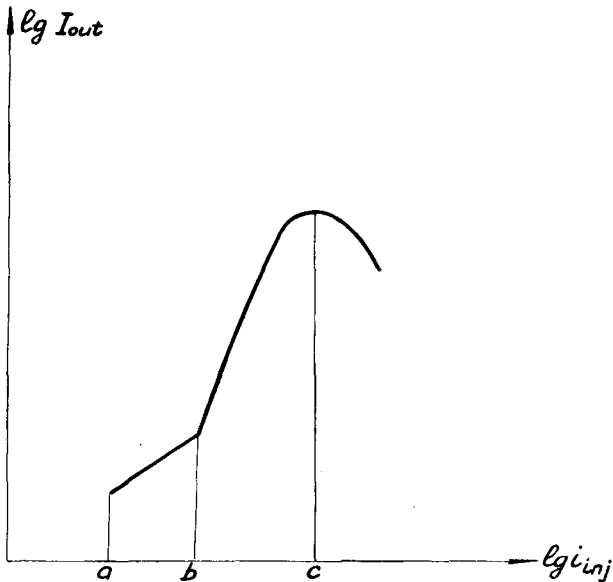


Fig. 1.

by the damping of free oscillations due to the increase in the magnetic field (adiabatic capture).

With the transition into the region of large injection current (section bc), the dependence $I_{out} = f(i_{inj})$ seems to take the following form:

$$I_{out} = A i_{inj}^K \tag{1}$$

where $K \geq 2$. The particular shape of the curve on this section can only be due to the interaction of particles in the process of capture (collective capture).

The value of the injection current corresponding to the maximum yield of gamma radiation (point C) depends on the electron current limit on the orbit as restricted by the stabilising magnetic field. The magnitude of current limit can be approximately estimated on the basis of the well-known equation:

$$i_{lim} = \alpha \frac{\pi r^2 (1 - n) U v}{30cR^2} \tag{2}$$

where

- i_{lim} is the current limit (in ampères)
- r is the distance between the injector and the equilibrium orbit,
- U is the injection voltage,
- v is the injection electron velocity and
- R is the radius of the equilibrium orbit.

The correction factor (α) introduced in this equation takes account of the presence in the chamber of an electron "background" consisting of electrons not completing their

revolutions, as well as of electrons formed at the expense of the secondary and generally N-fold emission from the chamber walls.

The non-stationary electron current circulating in the chamber was measured directly by the induction method. The magnitude of this current at the moment of injection reached 0.1 amp. (The measurements were effected for $U = 15 + 20$ kV, $R = 20$ cm., $r = 2$ cm.)

This value conforms to the magnitude i_{lim} if $\alpha \sim 0.2 - 0.3$.

Additional experimental data also confirm the possibility of attaining the current limit in the accelerator chamber. It was ascertained that the position of the curve maximum $I_{out} = f(i_{inj})$ remained unchanged when various methods of increasing the efficiency of capture were used. The use in this connection of the well-known magnet contractor device (additional contraction of instantaneous orbits at the moment of injection) increased the intensity of the betatron radiation in the region of low and medium injection currents only without changing the position of the maximum (point C).

Similar results were obtained in various other experiments with artificially induced space charges into the chamber.

The mechanism of collective capture may cause an additional shift of the electron trajectory of the order of a few tenths of a millimeter or millimeters per revolution. Experiments performed with a movable radial screen showed that its insertion into the chamber to a depth of a few millimeters past the edge of the injector anode (3.5 mm., and under certain conditions up to 8 mm.) did not sub-

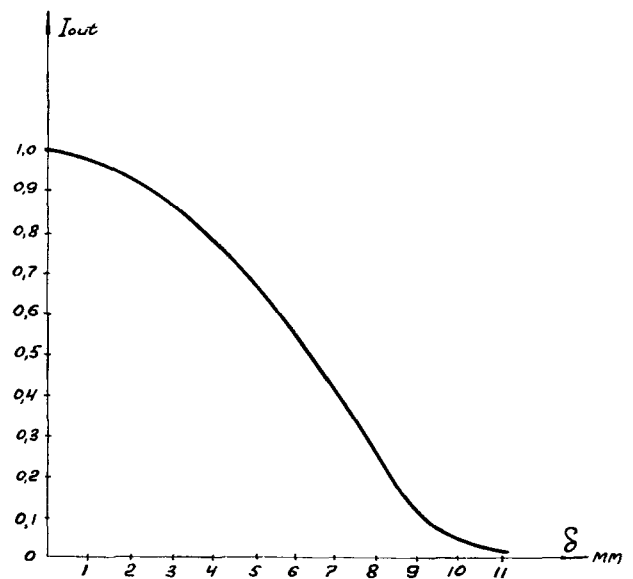


Fig. 2.

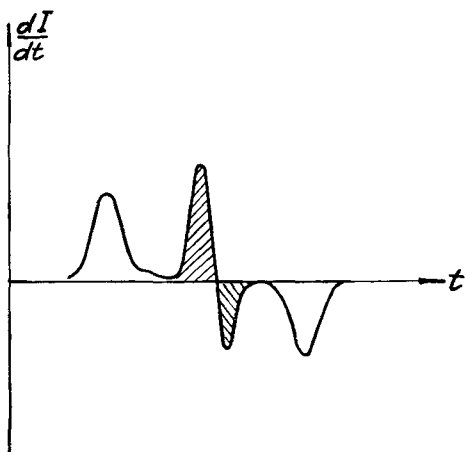


Fig. 3.

stantially change the intensity of the betatron yield. The general form of intensity depends on the position of a screen of this kind (δ mm.) (see fig. 2.).

Yet another important experimental confirmation of the high efficiency of the collective capture mechanism is obtained by direct measurements of the current circulating in the chamber at the moment of injection.

A device has been constructed whereby the e.m.f. induced by the circulation of the electron current in the chamber can be measured in a specially constructed measuring coil. The resolving time of the recording system allowed undistorted observance of the current change in the chamber during the period of a number of particle revolutions.

It became evident that the coefficient of transition of non-stationary into stationary current rapidly increases with the increase in magnitude of the non-stationary current circulating in the chamber, rising to 40-50% in the region adjacent to the limit value of this current.

A typical oscillogram of non-stationary current is shown in fig. 3.

It follows that in the region of large currents the injection efficiency is determined not only by the orbit contraction, which in this case is sufficiently large, but also by the perturbation influence of the space-charge on particle movements. It can be concluded on the ground of all the above considerations that the many attempts to increase the electron orbit contraction artificially at the moment of injection will not produce any noticeable increase of intensity as compared with the highest possible one obtained without using this auxiliary contraction. On the contrary, any method which increases the current limit determined by formula (2) will promote an increase in betatron efficiency at the moment of injection.

2. The collective capture hypothesis based on electromagnetic induction and electron interaction with the space-charge

As already mentioned, the main part in the betatron is played by collective capture. Special attention was therefore devoted in our work to the study of physical processes connected with this occurrence.

The majority of hypotheses concerning particle capture may be divided into two main groups: those connected with self-induction of the non-stationary current in the chamber, and those based on electron interaction with the Coulomb field of local space-charge.

Special devices were designed for the study of collective capture with artificially created conditions in which the decisive part is played by the one or other effect.

The effect of the contraction of the radius of the instantaneous electron orbit, due to non-stationary current self-induction in the chamber was studied with the aid of an artificial orbit contractor the loops of which were wound round the central core of the betatron. Measurements have shown that such a process promotes capture, although no quantitative agreement with experimental data has been found.

A considerable increase in injection efficiency was noted during the increase of current in the contractor loop at the rate of 5 amp/ μ sec.

Direct measurements of non-stationary circulating current at the orbit at the moment of injection during the maximum possible intensity showed a current value of 0.1 amp. with a rising time of 0.5 μ sec.

The following well-known equation may be used for obtaining a rough estimate of the instantaneous orbit contraction:

$$\delta R = -R \frac{eL \, dI/dt}{2(1-n)W} \quad (4)$$

where

R is the instantaneous orbit radius,
L is the beam induction (regarded as equal to the induction of a wire placed at the position of the equilibrium orbit),

dI/dt is the rate of change of the circulating current,

W is the injected energy, and

n is the magnetic field index.

Thus the magnitude of the radius shift per revolution for the measured non-stationary current on the orbit works out at hundredths of a millimeter (with $U = 15-20$ kV, $i_{inj} \sim 0,3$ amp.).

According to the capture hypothesis advanced, the effect of orbit contraction differs, therefore, only slightly in order of magnitude, from the radius decrease.

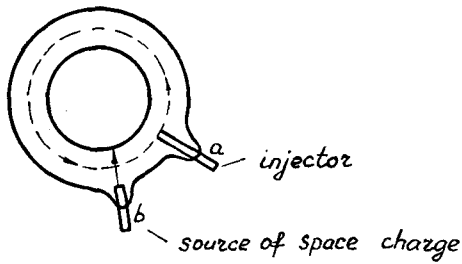


Fig. 4.

To study the influence of the Coulomb field on capture efficiency during the injection period, a space-charge was introduced into the chamber by a radial electron beam out of an additional injector (fig. 4).

The presence of a space-charge azimuthally localized increased the capture efficiency more than ten times in the region of low and medium injection currents (section a b in fig. 1). No noticeable influence on the intensity was detected with injection currents corresponding to current limits (point c).

Hence the space-charge changing in time and azimuthally localized promoted the capture of electrons injected into the betatron.

The additional space-charge was produced over a short time interval (about 3 μ sec); moreover, the moment of its appearance in the chamber could be varied within wide limits.

It was found that the increase in capture efficiency occurs mainly with the decrease in the space-charge. It also appeared that the influence of this additional space-charge, with an increase in voltage on the main injector (leading edge of the pulse), is more marked than with an unchanged or decreasing injection voltage.

Similar experiments carried out with an ordinary magnet contractor have shown that its efficiency during capture on the leading edge of the injection is higher than for any other parts of the pulse (fig. 5).

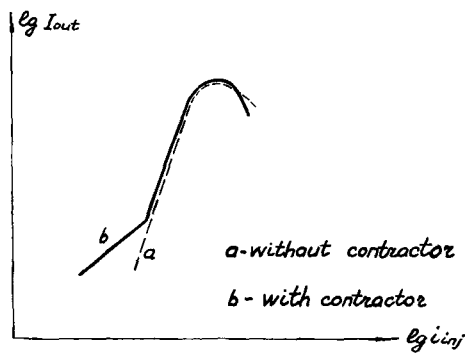


Fig. 5.

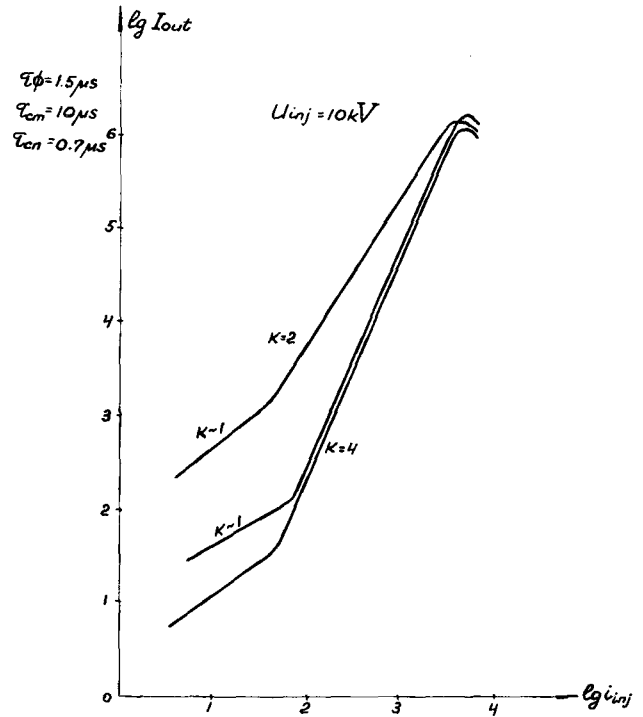


Fig. 6.

Moreover, investigations of the dependence $I_{out} = f(i_{inj})$ showed that on the leading edge of the injection pulse the increase of intensity with the increase of current proceeds considerably more slowly than on the tail of the pulse (fig. 6).

In studying the dependence $I_{out} = f(\omega_{mag})$ it was discovered that with the frequency alteration of the magnet feeding current ω_{mag} the intensity of betatron radiation I increases although the capture time decreases in proportion to the increase in frequency (fig. 7).

Finally, it was established that the decrease in the tail of the injection voltage pulse increases the intensity of the

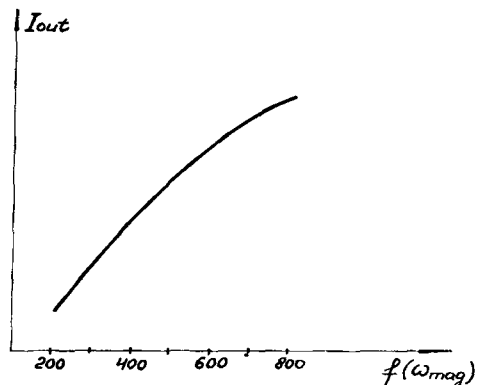


Fig. 7.

yield, in spite of the fact that the capture time $\Delta\tau$ is also decreasing. This phenomenon is especially marked when the injection current value approaches the limit.

The experimental results described here permit the conclusion that the efficiency of the collective mechanism of particle capture during the decrease in injection voltage is much higher than during its increase. It can therefore be concluded that the increase in efficiency of the maximum possible collective capture observed during the experiment is connected with the decrease of the space-charge. This conclusion may be supported also by the well-known fact that there is always a considerable non-uniformity in the betatron chamber in the azimuthal distribution of the space-charge. During the passage of charged particles through such a local space-charge in the decreasing period, their energy will decrease, and this promotes capture by the stationary orbit. There is also a possibility of a damping of the particle oscillation amplitude during its passage through the moving focus.

In that case, a redistribution of directions of oscillating particle pulses may occur.

The loss of particle energy during a single passage of a given local space-charge may be approximately conveyed by the formula :

$$(\Delta W) = 2 \frac{eQ}{v} K \ln \frac{R}{a} \quad (5)$$

where

- Q is the total charge,
 K is the rate of decrease in the charge density,
 v is the electron motion velocity on the orbit of radius R, and
 a is the dimension of the space-charge.

The numerical computations effected according to this formula for a case of azimuthal localization of the space-charge agrees with the actual artificial formation of space-charge by the additional electron source and shows a value of energy losses which conforms with an orbit contraction of ~ 0.3 mm. per revolution. (In these calculations, the current value of the auxiliary injector was taken at the rate ~ 0.1 amp., the injection amplitude pulse being 9 Kev.)

Space-charge diminution may take place not only during the reduction in voltage at injection but also with a constant and even increasing voltage.

In fact, at moments of time preceeding the formation in the chamber of a circulating current, the beam strikes the inner walls (leading edge of the injection pulse) or outer walls of the chamber (tail edge and plateau) and causes the creation of an intensive space-charge.

With the regeneration of non-stationary circular currents in the chamber, the space-charge rapidly decays. However, with the increase in injector voltage the effect of the space-charge is partly offset by the continuing charge increase of the electron cloud which, owing to the imperfection of beam focusing, is present in front of the injector.

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DISCUSSION

D. I. Blokhintsev: In the movement of an electron in an accelerator the quantum character of radiation should be considered. This radiation acts as a random force on the electron, and there is no doubt about the existence of such a force. However the quantum numbers are very large, and mistakes may easily be made when calculating the motion of an electron by the methods of quantum mechanics. It will be more correct to study this movement by the methods of classical mechanics, as Kolomenski has done. It therefore seems to me that Kolomenski's conclusions are more convincing.

M. Sands: I discussed with both Sokolov and Kolomenski and I would like to summarize the situation briefly. According to Sokolov and Kolomenski the oscillation amplitudes are proportional to:

$$\frac{1}{\sqrt{H}} e^{-\frac{1}{2} \frac{W}{E} \frac{(mc^2)^2}{E}} T$$

and

$$\frac{1}{\sqrt{H}} e^{-\frac{1}{2} \frac{n}{1-n} \frac{W}{E}} T \text{ respectively.}$$

If we confine ourselves to weak focusing machines, the essential difference is that Sokolov's exponent becomes very small for practical machines around 1 Bev. There is now some experimental evidence available from existing synchrotrons. For the Cal. Tech. 500 Mev synchrotron, that has $n = 0.6$ one finds experimentally $a^2 < 0.5 \text{ cm}^2$ for $T = 1/4 \text{ sec.}$, and $E = 500 \text{ Mev.}$

B. D. McDaniel: The Cornell A.G. electron synchrotron is operating around 900 Mev with an intensity of $Q = 10^{10}/\text{min.}$ At the top of the accelerating cycle, by looking at the radiation from the beam, one observes that the beam diameter is less than 2 mm. Pertinent data are: $n \approx 21$; $Q \approx 3.5$; $r \approx 150 \text{ inches}$; $W \approx 25 \text{ kV/turn}$ at 1 Bev, repetition frequency 30 c/sec.

V. I. Veksler: Do you use D.C. biasing?

B. D. McDaniel: Yes, so the accelerating time is about 1/60 sec.

L. S. Osborne: There are points of agreement between the various theories. I made calculations together with D. M. Ritson on the quantum fluctuations in connection with the design of the M.I.T. synchrotron. The results agree with both the results from Schwinger's and Sokolov's theories. The elegance of the method used by Sokolov should be emphasized, especially if one wants to worry about the fluctuations due to the magnetic moment of the electron. The agreement means that the electron behaves like a point charge and that its Dirac

nature plays no part in this problem. I should like some comment from Sokolov on the fact, that the betatron oscillations cannot be considered as arising from an adiabatic process, since the changes in the path of the electron due to photon radiation occur within one betatron wavelength.

A. A. Sokolov: I agree with Osborne's remarks about the agreement of classical and quantum effect, because its magnitude is proportional to \hbar . In this method the influence of the quantum theory comes out in the relation $a^2 H = \hbar S$. S increases by quantum transitions and so does a^2 . The quantum effect on the intensity of the radiation is felt only above 1000 Bev, but the quantum effect on the radial oscillations sets in above 1 Bev. This is the new point of our work.

D. W. Kerst: B. D. McDaniel's experiment gives a relaxation time of $\sim 10^{-2} \text{ sec.}$, which is too large, so that it does not help on this point.

T. A. Welton: Although the radiation may become quite hard, it is still soft in the rest system of the electron so that its wavelength is always much larger than the Compton wavelength of the electron. In the classical calculation the average value of a field coordinate, or of a particle coordinate or velocity is given correctly, but the average of the square of those quantities is not, due to the fluctuations arising from quantum theory. The classical calculation of Čerenkov radiation by N.H. Frank and E. I. Tamm gives an identical result as a careful quantum mechanical treatment, although some broadening of trajectory with respect to the classical theory occurs.

E. G. Komar (to M. S. Livingston): What are the effects of the remanent field and how is the heat taken away from the resonators?

M. S. Livingston: The remanent field, measured on models is about 18 gauss. It has the same radial distribution as the powered field. The one turn alignment procedure should help in correcting the azimuthal variation. Each magnet will have its own bias winding for correcting. The copper R.F. resonators will be water cooled and thermostatically controlled. The phase of the R.F. voltage with respect to a master phase will be measured by a sensitive phase sensing device and will be controlled by a servosystem.

V. Migulin (to M. S. Livingston): In your report phase shifts up to 20° due to quantum effects were mentioned. What is the stable phase at the end of the cycle? What is the voltage, on the inflector, and what is its time dependence and stability?

M. S. Livingston: The inflector is electrostatic and gives a deflection of $3-5^\circ$. Combined with a previous magnetic deflection gives an opposite deflection, the net deflection is zero. So one obtains accurate inflection in

spite of energy spread of the injected electrons. The voltage in the inflector is 70 kV. The accuracy of this voltage is much better than the energy spread of the injected particles. Corrections of the radial betatron amplitude can be made in two ways. Damping magnets decrease the amplitude of the radial oscillations, but increase that of the synchrotron oscillations. Therefore one must increase the R.F. voltage and decrease the stable phase angle. Normally one has $\varphi_0 \approx 45^\circ$ and at the end of the cycle $\varphi_0 \approx 30^\circ$. A coupling of the radial to the vertical oscillations gives no increase in amplitude of the synchrotron oscillations. We are not yet sure which solution we shall adopt.

B. D. McDaniel: How is the ratio DC/AC current controlled?

M. S. Livingston: We have not yet decided how to do this, but we hope to lean upon experience obtained on the Cornell synchrotron.

E. G. Komar: How is the beam extracted?

M. S. Livingston: Only X-ray beams from an internal target are foreseen at the start.

W. Paul: The remanent field in models of the Bonn 500 Mev synchrotron was 18 gauss, with an azimuthal variation of less than 1 gauss, due to mixing of the steel plates. The ratio DC/AC current is kept constant by use of rectifiers.

G. Salvini: Our magnet is operated on 20 cycles per second with d.c. bias. On a full scale model with a bias of - 700 gauss and full excitation (10,000 gauss) the remanent field was practically reduced to zero (less than 0.1 gauss). It will probably be possible to go down to a biasing of only -250 gauss.

D. W. Kerst: It would be very interesting, if one could make two measurements of the beam diameter of the Cornell synchrotron at maximum energy. The diameter should double according to Kolomenski's formula, but not according to Sokolov's formula.