

Breakdown of Global Fits to the Wilson Coefficients in Rare B Decays: A Left-Right Model Example *

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Abstract

In the Standard Model and many of its extensions, it is well known that all of the observables associated with the rare decays $b \rightarrow s\gamma$ and $b \rightarrow s\ell^+\ell^-$ can be expressed in terms of the three Wilson coefficients, $C_{7L,9L,10L}(\mu \sim m_b)$, together with several universal kinematic functions. In particular it has been shown that the numerical values of these coefficients can be uniquely extracted by a three parameter global fit to data obtainable at future B -factories given sufficient integrated luminosity. In this paper we examine if such global fits are also sensitive to new operators beyond those which correspond to the above coefficients, *i.e.*, whether is it possible that new operators can be of sufficient importance for the three parameter fit to fail and for this to be experimentally observable. Using the Left-Right Symmetric Model as an example of a scenario with an extended operator basis, we demonstrate via Monte Carlo techniques that such a possibility can indeed be realized. In some sense this potential failure of the global fit approach can actually be one of its greatest successes in identifying the existence of new physics.

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1 Introduction

Rare decays of heavy quarks which do not occur at the tree level in the Standard Model (SM) can provide a unique opportunity for new physics to reveal itself. When such decays occur through loops then the participating SM particles and those associated with the new interaction are placed on the same footing and may yield comparable contributions to the various decay amplitudes. In these cases it may be possible to isolate such additional contributions and learn something about the detailed nature of the new physics scenario.

Amongst the rare decays involving b quarks, two of the cleanest and most well understood inclusive processes are $b \rightarrow s\gamma$ and $b \rightarrow s\ell^+\ell^-$. At present, the branching fraction for the $b \rightarrow s\gamma$ mode has been measured by CLEO[1] to be $B(b \rightarrow s\gamma) = (2.32 \pm 0.57 \pm 0.35) \times 10^{-4}$ while a preliminary result from ALEPH[2] yields the value $(3.38 \pm 0.74 \pm 0.85) \times 10^{-4}$. On the other hand, there exist only upper bounds for the decay $b \rightarrow s\ell^+\ell^-$; the strongest constraint at present is the 90% CL limit $B(b \rightarrow s\ell^+\ell^-) < 4.2 \times 10^{-5}$ [3] from CLEO, which is obtained by combining both their di-electron and di-muon data samples. As we will see below, this is only a factor of ~ 6 above the expectations of the SM for this branching fraction so that we may expect this decay to be observed in the near future.

In the SM and in many of its extensions (including, *e.g.*, fourth generation models, models with an extra down-type quark, SUSY, extended Higgs sectors, Z' scenarios, theories with large anomalous gauge boson couplings, *etc.*) the phenomenology of both of these rare processes above are almost completely determined by the numerical values of the Wilson coefficients of only a small set of operators evaluated at the scale $\mu \sim m_b$. In our somewhat unconventional notation these are denoted as $C_{7L,9L,10L}(\mu)$. At the weak scale the operators corresponding to these coefficients arise in the SM from the usual γ and Z penguins as well as W box diagrams. It has been successfully argued in the literature[4]

that by combining observables associated with both the $b \rightarrow s\gamma$ and $b \rightarrow s\ell^+\ell^-$ processes, a model independent three dimensional global fit can be performed to numerically determine the values of these three Wilson coefficients. Indeed, given sufficient statistics at future B -factories this approach leads to only rather modest uncertainties in the fitted values of these coefficients allowing us to test the SM and look for new physics. We note that only the magnitude $|C_{7L}(\mu)|$ can be extracted from the $b \rightarrow s\gamma$ transition so that its sign would remain undetermined from this channel alone even if infinite precision were available. For the observables associated with the $b \rightarrow s\ell^+\ell^-$ decay all three of the coefficients contribute and therefore their relative signs as well as their magnitudes can be extracted from the data when combined with our knowledge of $B(b \rightarrow s\gamma)$.

In some ways the determination of the these three Wilson coefficients via a global fitting procedure in rare B decays is similar to the searches for new physics in precision electroweak measurements[5] through the use of the oblique parameters S, T, U [6]. As the reader may recall, in the SM (for a reference value of the top quark and Higgs boson masses) these parameters are all identically zero. For certain classes of new physics, such as a fourth generation of quarks and leptons, fits to precision data would then lead to some consistent set of non-zero values for these parameters with a good χ^2 . Of course, if these parameters are found to be non-zero and it is also found that different precision observables yielded statistically distinct S, T, U values then we would necessarily conclude that the new interactions are not describable by the oblique corrections alone. (As is well known, any class of new interactions that induce significant flavor dependent vertex corrections, such as a Z' , can not be portrayed solely in terms of S, T, U .) Such a situation would provide a unique window on the complex nature of the new physics scenario.

It is then obvious that when sufficient statistics become available in the not too distant future for this type of analysis of rare B decays to be performed, there are only three

possible outcomes: (i) The numerical values for the coefficients are found to agree with the SM expectations with a good χ^2 . In this case the new physics is decoupled and either higher precision data or searches elsewhere are necessary to uncover it. (ii) A quality fit is obtained but the values one finds for the three Wilson coefficients are far from the SM expectations in χ^2 . This is the result usually discussed and anticipated in the literature[4] in the set of extended models listed above. (iii) As with the case of precision measurements, the last possibility is potentially the most interesting and the one we are interested in here: the value of χ^2 for the best three parameter fit is found to be very large and cannot be accounted for by an under estimation of systematic uncertainties. This represents in some sense a failure of the model independent approach in that it is clear that the true numerical values of the three Wilson coefficients are not being extracted from the data. However, another point of view is that this result is in fact this approach's greatest triumph since it is telling us that the new interactions necessarily involve an extension of the operator basis to include *new* operators beyond the usual set. This implies that the new physics scenario is richer than any of those in the list above.

Thus the question we wish to address here is whether new physics which does involve new operators in an extended basis can indeed manifest itself as a poor fit when we have the freedom to vary the 3 coefficients to obtain a good χ^2 . (Of course, $|C_{7L}(\mu)|$ cannot be freely varied by too large an amount due to the reasonable agreement between the present data and SM expectations for the $b \rightarrow s\gamma$ decay rate as discussed below.) The purpose of this paper is then to demonstrate this result by providing an existence proof that a new physics scenario of the desired type not only exists, in the form of the Left-Right Symmetric Model (LRM), but that it leads to poor values of χ^2 in the global fit when only the usual three operators are employed. The point we wish to stress here is not the particular physics of the LRM, or any other specific model, but that the existence of an extended operator

basis can indeed manifest itself in the poorness of the three parameter fit given reasonable integrated luminosities. We note, however, that without further analysis the failure of the fit itself will not yield information on which new operators would need to be introduced. We further note that the LRM is of course not the only new physics scenario with an extended operator basis[7].

The outline of this paper is as follows. In Section 2 we overview the status of the various pieces necessary for calculations of the $b \rightarrow s\gamma$ and $b \rightarrow s\ell^+\ell^-$ decay rates and distributions in the SM. In Section 3 we provide a background on the basics of the LRM and the parameters it contains which are relevant for the processes of interest here. We discuss and generate several possible forms of the right-handed CKM weak mixing matrix, V_R , following a systematic approach that maintains unitarity and wherein agreement with experimental constraints can be easily accommodated. Next we set up the LRM calculations for these decays and demonstrate that many LRM parameter space regions exist wherein the rate for $b \rightarrow s\gamma$ is essentially the same as in the SM. The general formulae for the $b \rightarrow s\ell^+\ell^-$ double differential distributions are discussed and it is shown that the LRM yields distinct results for observables associated with this decay even when the $b \rightarrow s\gamma$ rate duplicates the SM expectation. In Section 4 we discuss our Monte Carlo approach and generate a large number of data samples corresponding to each of the several models discussed in the previous section. We demonstrate that for high luminosities, corresponding to $5 \times 10^8 B\bar{B}$ pairs, typical of samples available at hadron colliders, the resulting fits to the conventional three Wilson coefficients lead to very large χ^2 values which clearly signal the failure of the fit. For lower luminosities, now corresponding to $5 \times 10^7 B\bar{B}$ pairs, typical of samples to be available at $\Upsilon(4S)$ machines, the χ^2 values are also found to be quite large in most, but not all, cases. A discussion of these results and our conclusions can be found in Section 5.

2 Rare Decays in the Standard Model

In order to be convinced that new physics has indeed been discovered it is necessary that the predictions for the rates and other observables associated with these decays in the SM be on firm ground. In particular, calculations of the decay rates and, in the case of $b \rightarrow s\ell^+\ell^-$, kinematic distributions have become increasingly sophisticated within the SM context. A “straightforward” next-to-leading order(NLO) calculation finds $B(b \rightarrow s\gamma) = (3.28 \pm 0.33) \times 10^{-4}$ [8]. However the inclusion of $1/m_b^2$ and $1/m_c^2$ corrections[9] increases this value by about 3%. A further enhancement of about 3% occurs when one systematically disregards any of the NNLO terms[10]. While closer to the central value of the preliminary ALEPH measurement these predictions are certainly consistent with the present CLEO data. For purposes of simplicity, in our numerical analysis below we make direct use the NLO result and ignore these additional small correction terms. This approximation will have no impact on our conclusions. One may anticipate that in the next few years this theoretical uncertainty may shrink to as low as 5% as the various input parameters are better determined. A comparable experimental determination of this branching fraction may also be possible at future B -factories since the measurements will be limited only by systematics. We will *assume* below that the SM value for this branching fraction is essentially realized by future experiments within the experimental and theoretical uncertainties.

In the case of $b \rightarrow s\ell^+\ell^-$, a complete short-distance NLO calculation has been available for some time[11] and the $1/m_b^2$ correction terms are also known[12] but provide only very small modifications. One of the remaining difficulties is the inclusion of non-perturbative long distance pieces associated with the J/ψ and ψ' resonances and the corresponding $1/m_c^2$ corrections. Here some modeling uncertainties remain and the traditional approach has been to treat the resonance contributions phenomenologically, which is not without some

difficulties[13]. However, at least in the regions sufficiently below and above the resonances (*i.e.*, $s = q^2/m_b^2 \leq 0.3$ or ≥ 0.6 , where q^2 is the invariant mass of the lepton pair) Buchalla, Isidori and Rey[9] have shown that the heavy quark expansion in terms of $1/m_c^2$ leads to reliable predictions with only small corrections to the NLO results and that there are no difficulties associated with double counting. In the same kinematic regions the phenomenological resonance models give comparable numerical predictions. In many extensions to the SM, the $1/m_{c,b}^2$ and resonant contributions are either of the same form as in the SM or can be easily obtained via suitable modifications of the SM terms and we employ these results in our analysis below. We note that the SM predicts a branching fraction for $b \rightarrow s\mu^+\mu^-$ of $\simeq 6 \times 10^{-6}$, which is not too far from the present upper bound.

In order to obtain the complete parton level NLO predictions for these two processes in the SM (or in other models), several steps are necessary. First, the complete operator basis must be determined at the high (matching) scale, typically taken to be M_W . Secondly, the matching conditions for the coefficients of the operators at the high scale must be calculated at both the LO and NLO. Thirdly, the anomalous dimension matrices for the relevant operators at both LO and NLO are determined and the coefficients are evolved to the $\mu \sim m_b$ scale via the Renormalization Group Equations. Lastly, at the scale μ the matrix elements of the relevant operators need to be computed through NLO. For the SM all of these pieces are now essentially in place for both the $b \rightarrow s\gamma$ and $b \rightarrow s\ell^+\ell^-$ decays after an enormous amount of labor. Unfortunately, the corresponding results only partially exist for most of these pieces in almost all extensions to the SM[14].

3 The Left-Right Model

3.1 Model Background

In order to be self-contained we briefly review the relevant parts of the LRM needed for the discussion below; for details of the model the reader is referred to Ref. [15]. The LRM is based on the extended gauge group $SU(2)_L \times SU(2)_R \times U(1)$ and can lead to interesting new effects in the B system[16]. Due to the extended gauge structure there are both new neutral and charged gauge bosons, Z' and W_R^\pm , in addition to those present in the Standard Model. In this scenario the left-(right-)handed (LH, RH) fermions of the SM are assigned to doublets under the $SU(2)_{L(R)}$ group and a RH neutrino is introduced. The Higgs fields which can directly generate SM fermion masses are thus in bi-doublet representations, *i.e.*, they transform as doublets under both $SU(2)$ groups. The LRM is quite robust and possesses a large number of free parameters which play an interdependent role in the calculation of observables and in obtaining the existing constraints on the model resulting from various experiments.

As far as B physics and the subsequent discussion are concerned there are several parameters of direct interest, most of which result from the structure and spontaneous symmetry breaking of the extended gauge sector. The ratio of the $SU(2)_R$ and $SU(2)_L$ gauge couplings is bounded by $0.55 < \kappa = g_R/g_L \leq 2$, where the lower limit is a model constraint and the upper one is simply a naturalness assumption. Whereas g_L is directly related to e as usual through $\sin^2 \theta_W$, g_R is unconstrained except through the definition of electric charge and naturalness arguments; GUT embedding scenarios generally suggest that $\kappa \leq 1$. For simplicity we assume $\kappa = 1$ in most of our discussion below. The $SU(2)_L \times SU(2)_R \times U(1)$ extended symmetry is broken down to the SM via the action of Higgs fields that transform

either as doublets as discussed above, or also possibly as triplets under $SU(2)_R$. This choice of Higgs representation determines both the mass relationship between the Z' and W_R (analogous to the condition that $\rho = 1$ in the SM) as well as the nature of neutrino masses. In particular, the Higgs triplet choice allows for the implementation of the see-saw mechanism and yields a heavy RH neutrino. We assume triplet breaking below so that the Z' mass is completely specified by the W_R mass and the value of κ .

After complete symmetry breaking the W_L and W_R bosons mix; this mixing being described by two parameters, a real mixing angle ϕ and a phase ω . Note that it is usually $t_\phi = \tan \phi$ which appears in expressions directly related to observables. The additional phase, as always, can be a new source of CP violation. However, in discussing processes in which the RH neutrinos do not participate, as is the case in B decays, this angle can be thought of as an overall phase of the right-handed CKM matrix, V_R , and we will subsequently ignore it. The mixing between W_L and W_R results in the mass eigenstates $W_{1,2}$, with a ratio of masses $r = M_1^2/M_2^2$ (with $M_2 \simeq M_R$). In most models t_ϕ is then naturally of order a few times r , or less, in the large M_2 limit. Of course, W_1 is the state directly being produced at both the Tevatron and LEPII and is identical to the SM W in the limit $\phi \rightarrow 0$. We note that when ϕ is non-zero, W_1 no longer couples to a purely LH current. Of course if a heavy RH neutrino is indeed realized then the effective leptonic current coupling to W_1 remains purely LH as far as low energy experiments are concerned. As is well-known, one of the strongest classical constraints on this model arises from polarized μ decay[17], which is trivial to satisfy in this case.

It is important to recall that the extended Higgs sector associated with both the breaking of the full LRM gauge group down to $U(1)_{em}$ and with the complete generation of fermion masses may also play an important role in low energy physics through the existence of complex Yukawa and/or flavor-changing neutral current type couplings. However, this

sector of the LRM is highly model dependent and is of course quite sensitive to the detailed nature of the fermion mass generation problem. For purposes of brevity and simplicity these too will be ignored in the following discussion and we will focus solely on the effects associated with $W_{1,2}$ exchange.

Additional parameters arise in the quark sector. In principle the effective mass matrices for the SM fermions may be non-hermitian implying that the two matrices involved in the bi-unitary transformation needed to diagonalize them will be unrelated. This means that the elements of the mixing matrix V_R appearing in the RH charged current for quarks will be unrelated to the corresponding elements of $V_L = V_{CKM}$. V_R will then involve 3 new angles as well as 6 additional phases all of which are *a priori* unknown parameters. Needless to say the additional phases can be a further source of CP violation. The possibility that V_L and V_R may be unrelated is often overlooked when considering the potential impact of the LRM on low energy physics and there has been very little detailed exploration of this more general situation. Clearly as the elements of V_R are allowed to vary the impact of the extended gauge sector on B physics will be greatly affected. Other well-known constraints on the LRM, such as universality, the apparent observed unitarity of the CKM matrix, $B^0 - \bar{B}^0$ mixing, the $K_L - K_S$ mass difference[18], as well as Tevatron direct W' searches[19], are quite sensitive to variations in V_R [20], but W_2 masses as low as 450 – 500 GeV can be easily accommodated by the present data. To be safe and to keep future W_2 searches in mind, however, we will generally assume that $M_2 \geq 800$ GeV for any form of V_R , implying that the magnitude of t_ϕ is less than *a few* $\cdot 10^{-2}$. In our numerical study below we will assume various different forms for V_R ; in all cases we will assume for simplicity that the values of the elements of V_L as extracted by current experiment[21] are not much influenced by the existence of the new LRM interactions. An updated analysis on the possible general structure of V_R has yet to be performed.

3.2 Forms of V_R

In order to study the potential influence of the LRM on B physics various forms of V_R should be examined. In fact, the possibility that $V_L \neq V_R$ and that B decays are *purely* right-handed was entertained some time ago by Gronau and Wakaizumi[22]. Just how large the right-handed contribution to $b \rightarrow c$ transitions is allowed to be within the LRM context is not yet accurately known[23] but may be sizeable in magnitude with an unknown relative phase. The experimental bounds are as follows. L3 has compared[24] their measurements of both the lepton and missing energy spectra in semileptonic $b \rightarrow c$ decays with a number of different hypotheses, and have excluded both the $(V + A) \times (V - A)$ and $V \times (V - A)$ scenarios, clearly indicating that this coupling is dominantly left-handed. This qualitative result is confirmed by the sign of the Λ_b polarization observed by ALEPH[25] at the Z pole. The strongest constraint comes from CLEO[26], with measurements of both the leptonic forward-backward asymmetry as well as the D^* polarization in the decay $B \rightarrow D^* \ell \nu$. On the theoretical side, Voloshin[16] has recently considered how significant right-handed $b \rightarrow c$ currents, at the $\sim 15\%$ level, may assist in our understanding of the B semileptonic branching fraction. In a more general context, other forms of V_R have been discussed by Langacker and Sankar[27] upon which we generalize in the following analysis.

Since we are not concerned with CP violation in what follows, for numerical simplicity we will ignore the phases in this matrix in which case it can be completely described by three mixing angles. Even in this limiting scenario the set of possible forms for V_R is enormous; however, it is sufficient for our purposes here to simply examine some sample forms. We will assume that each row and column of V_R contains only one large element with a magnitude near unity as is the true for the conventional CKM matrix. In this case there are only six

matrices about which we can perturb; we write these symbolically as

$$\begin{aligned}
\mathcal{M}_A &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, & \mathcal{M}_D &= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \\
\mathcal{M}_B &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, & \mathcal{M}_E &= \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \\
\mathcal{M}_C &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, & \mathcal{M}_F &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}.
\end{aligned} \tag{1}$$

For example, matrix D corresponds, in the standard CKM parameterization[28], to the situation where s_{12}, s_{23} and c_{13} are all ~ 1 . Following Wolfenstein[29], this suggests taking $c_{12} \sim \lambda^n$, $s_{13} \sim A\lambda^m$, and $c_{23} \sim B\lambda^p$, where A, B are order unity, $n, m, p \geq 1$ and $\lambda \simeq 0.22$. Of course, all these parameters are not arbitrary since the experimental constraints from $K_L - K_S$ and $B - \bar{B}$ mixing must be satisfied. For $r \leq 10^{-2}$ and $|t_\phi| \leq a \text{ few} \times 10^{-2}$, these bounds are trivially fulfilled, without further fine tuning, if $p \geq 3$ and $n, m \geq 1$. For $p \geq 3$, B physics is not very sensitive to the value of m so we take for simplicity $p = 3$ and $m = 1$. This leaves us with a set of matrices we can label as $V_R = D(n)$, for $n=1,2,3,\dots$. Table 1 lists the complete set of parameterizations for all six types of V_R mixing matrices, which we arrive at by following a similar procedure, and the constrained range of the exponents we use in the analysis below. The values of these powers reflect simplicity as well as that required to satisfy the low energy experimental constraints.

Note that we have not employed any constraint on the allowed strength of the right-handed $b \rightarrow c$ coupling in our discussion. Since these bounds are relatively complicated their

Matrix	s_{12}	c_{12}	s_{13}	c_{13}	s_{23}	c_{23}	constraints
A	λ^n	1	$B\lambda^p$	1	$A\lambda^m$	1	$n = 2, p = 3, m = 1, 2, 3$
B	1	λ^n	$B\lambda^p$	1	$A\lambda^m$	1	$n = 2, m = 3, p = 1, 2, 3$
C	$A\lambda^m$	1	$B\lambda^p$	1	1	λ^n	$p = 1, m = 2, n = 1, 2, 3$
D	1	λ^n	$A\lambda^m$	1	1	$B\lambda^p$	$p = 3, m = 1, n = 1, 2, 3$
E	$A\lambda^m$	1	1	λ^n	1	$B\lambda^p$	$p, m \geq 2, n \geq 1$
F	$A\lambda^m$	1	1	λ^n	$B\lambda^p$	1	$p, m \geq 2, n \geq 4$

Table 1: Parameterizations of V_R in the absence of CP violation assuming $\lambda \sim 0.2$, A and B are of order unity, and $m, n, p \geq 1$.

detailed numerical impact will be discussed elsewhere[23], but will have the most impact in the cases of matrices C and D .

3.3 Rare Decay Formalism

The analysis of the decays $b \rightarrow s\gamma$ and $b \rightarrow s\ell^+\ell^-$ in the LRM begins with the following extended effective Hamiltonian,

$$\mathcal{H}_{eff} = \frac{4G_F}{\sqrt{2}} \sum_{i=1}^{12} C_{iL}(\mu) \mathcal{O}_{iL}(\mu) + L \rightarrow R, \quad (2)$$

The $\mathcal{O}_{iL,R}$ are the complete set of operators involving only the light fields which govern $b \rightarrow s$ transitions. The first thing to notice is that for convenience the conventional CKM factors in front have been absorbed into the values of the coefficients themselves. This is generally a useful approach to follow when multiple mixing angle structures appear simultaneously. The second point to note here is that whereas there are only 10 local operators describing $b \rightarrow s$ transitions in the SM, *i.e.*, $\mathcal{O}_{1L} - \mathcal{O}_{10L}$, here there are 24 operators. 10 of the additional operators correspond to the chiral partners, with $L \rightarrow R$, of those present in the SM. The

complete basis for each helicity structure then consists of the usual six 4-quark operators $\mathcal{O}_{1-6L,R}$, the penguin-induced electro- and chromo-magnetic operators respectively denoted as $\mathcal{O}_{7,8L,R}$, as well as $\mathcal{O}_{9L,R} \sim e\bar{s}_{L,R\alpha}\gamma_\mu b_{L,R\alpha}\bar{\ell}\gamma^\mu\ell$, and $\mathcal{O}_{10L,R} \sim e\bar{s}_{L,R\alpha}\gamma_\mu b_{L,R\alpha}\bar{\ell}\gamma^\mu\gamma_5\ell$ which arise from box diagrams and electroweak penguins. (Here the indices α label the color structure of the operators.) In the LRM we not only have the augmentation of the operator basis via the obvious doubling of $L \rightarrow R$, but two new additional 4-quark operators of each helicity structure are also present at the tree-level due to a possible mixing between the $W_{L,R}$ gauge bosons: $\mathcal{O}_{11L,R} \sim (\bar{s}_\alpha\gamma_\mu c_\beta)_{R,L}(\bar{c}_\beta\gamma^\mu b_\alpha)_{L,R}$ and $\mathcal{O}_{12L,R} \sim (\bar{s}_\alpha\gamma_\mu c_\alpha)_{R,L}(\bar{c}_\beta\gamma^\mu b_\beta)_{L,R}$. Operators $\mathcal{O}_{12L,R}$ occur at the tree level in a fashion analogous to operators $\mathcal{O}_{2L,R}$.

In evolving down from the weak scale to $\mu \sim m_b$ these operators mix under renormalization as usual. Fortunately the 24×24 anomalous dimension matrices split into two identical 12×12 chiral submatrices since the operators of each chirality do not mix under RGE evolution. The complete 12×12 anomalous dimension matrix at LO was first calculated by Cho and Misiak[30] and at NLO only the 10×10 submatrix corresponding to the SM operators is presently known.

The determination of the matching conditions for the 24 operators at the electroweak scale even at LO is already somewhat cumbersome since the LRM contains a large number of parameters and, in addition to new tree graphs, 116 one-loop graphs must also be calculated. (Additional diagrams due to possible physical Higgs exchange are not yet included and would introduce additional model dependence.) Some of these diagrams have already been calculated for the earlier analyses of the decay $b \rightarrow s\gamma$ in the LRM[30]. Clearly, NLO matching conditions do not yet exist for this model so we employ only the LO ones below. However we note that the numerical size of these NLO contributions in the SM case were found to be small. In addition one has to separately include the new LRM contributions to the semileptonic branching fraction, B_l , including both finite $m_c/m_b \simeq 0.29$ and LO

QCD corrections, since it is conventionally used to normalize both the $b \rightarrow s\gamma$ and $b \rightarrow s\ell^+\ell^-$ decay rates. (We assume that the relevant elements of V_L have the same numerical values in the LRM as in the SM in our numerical calculations; this need not be the case experimentally[23].) In the case of $b \rightarrow s\gamma$ the required NLO real and virtual corrections to the operator matrix elements for the LRM are almost completely obtainable from the SM results when augmented by the new terms in the LO anomalous dimension matrix and through the use of left \leftrightarrow right symmetry. Additional terms arising from operators $\mathcal{O}_{12L,R}$ have yet to be included. Clearly we cannot claim to be performing a complete NLO treatment of the LRM case until all of the missing pieces have been calculated. However, this complete calculation is not necessary to demonstrate our points but certainly our analysis should be repeated once all the NLO pieces are in place for the LRM. The advantage of the present approach, however, is that when we turn off the effects associated with the various LRM contributions to both $b \rightarrow s\gamma$ and $b \rightarrow s\ell^+\ell^-$ we recover the usual NLO SM results.

3.4 $b \rightarrow s\gamma$ in the LRM

Using the central values of the quantities $m_t(m_t) = 167$ GeV, $\mu = m_b$, $\alpha_s(M_Z) = 0.118$, $m_c/m_b = 0.29$, $\kappa = 1$ and $B_l = 0.1023$ we can calculate the rate for $b \rightarrow s\gamma$ in the LRM using the existing pieces of the NLO calculation that are presently available if we also provide sample values for the quantities M_{W_2} , t_ϕ and the relevant elements of the matrix V_R . Recall that we are looking here for particular non-decoupling regions in the LRM parameter space that essentially give the same result as the SM for the $b \rightarrow s\gamma$ branching fraction. (We are not interested in parameter space regions which differ only infinitesimally from the SM.) Let us first examine the simple scenario where $V_L = V_R$; this is the so-called manifest LRM. In this case, as discussed above, the $K_L - K_S$ mass difference and direct Tevatron collider searches require that W_2 be heavy; for purposes of demonstration we take $M_{W_2} = 1.6$ TeV

so that $t_\phi = \tan \phi$ is now the only free parameter as the W_2 contributions are now almost completely decoupled. Here we note that one of the interesting features uncovered by earlier analyses of $b \rightarrow s\gamma$ in the LRM[30] was that left-right mixing terms associated with $t_\phi \neq 0$ can be enhanced by a helicity flip factor of $\sim m_t/m_b$ and can lead to significantly different predictions than the SM even in this case where $V_L = V_R$ and W_2 is very heavy.

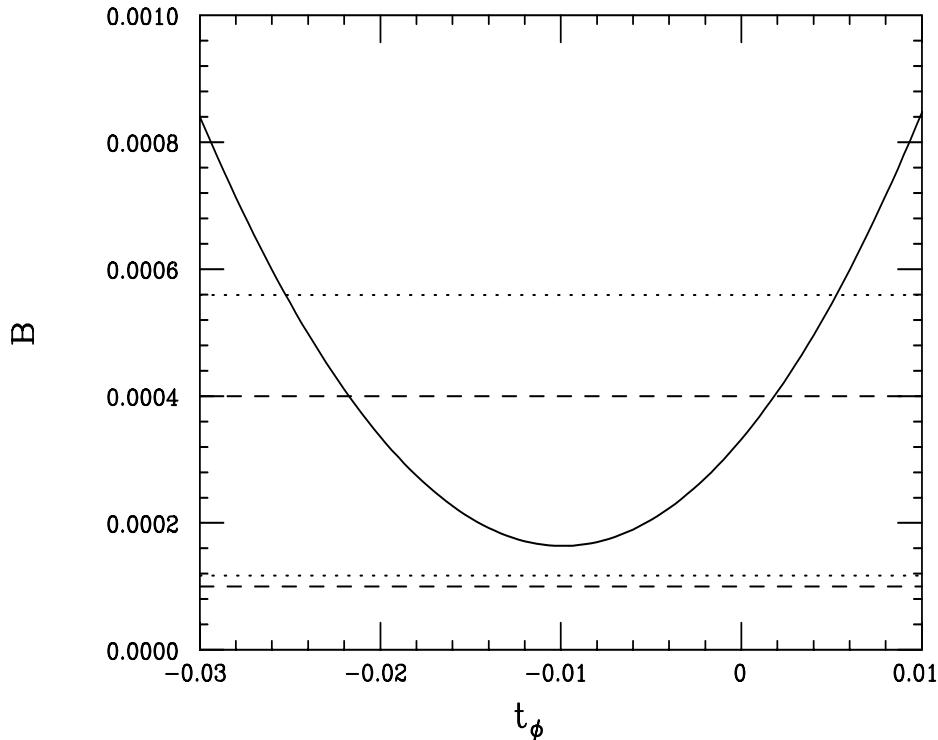


Figure 1: Prediction for the $b \rightarrow s\gamma$ branching fraction for the case $V_L = V_R$ and $M_{W_2}=1.6$ TeV as a function of t_ϕ in the LRM. The 95% CL CLEO(ALEPH) allowed range lies inside the dashed(dotted) lines.

Figure 1 shows the prediction for the $b \rightarrow s\gamma$ branching fraction in this case and we see that the SM result is reproduced when $t_\phi = 0$, as expected, apart from a very small correction of order $M_{W_1}^2/M_{W_2}^2$. However we also see that a non-decoupling conspiratorial solution occurs when $t_\phi \simeq -0.02$, which yields a result which is exactly the same as the SM.

From using $b \rightarrow s\gamma$ alone, the SM and this LRM case are indistinguishable, independent of what further improvements can be made in the branching fraction determination. This means that additional measurements would be necessary to separate these two cases. Of course, given present experimental data both the ALEPH and CLEO results allow for a rather broad range of t_ϕ .

We can now ask if corresponding conspiratorial regions exist for any of the more general forms of V_R considered above. For example, if we calculate the $b \rightarrow s\gamma$ rate with the $A(m)$ and $D(n)$ matrices we obtain the results in Fig. 2. For the $D(n)$ case, we see that only $n = 2$ provides such a region. For the other matrices we find conspiratorial regions when $m = 1, 2, 3$ for matrix A and $p = 2, 3$ for matrix B; no such regions are found for matrices C, E and F. Note that $m = 2$ in case A essentially corresponds to $V_L = V_R$. We denote these six conspiratorial cases as $V_L = V_R$, $A(1, 3)$, $B(2, 3)$ and $D(2)$ for later purposes. It is interesting to note that all of these cases lead to modest increases[31] in $B(b \rightarrow sg)$ by as much as 75% above the SM prediction at LO. Of course, in addition to these conspiratorial solutions, we note from Fig. 2 that a rather wide range of t_ϕ is again allowed by current data from CLEO and ALEPH.

3.5 $b \rightarrow s\ell^+\ell^-$ in the LRM

For $b \rightarrow s\ell^+\ell^-$, the effective Hamiltonian leads to the matrix element (neglecting the strange quark mass but keeping the mass of the leptons)

$$\begin{aligned} \mathcal{M} = & \frac{\sqrt{2}G_F\alpha}{\pi} \left[C_{9L}^{eff} \bar{s}_L \gamma_\mu b_L \bar{\ell} \gamma^\mu \ell + C_{10L} \bar{s}_L \gamma_\mu b_L \bar{\ell} \gamma^\mu \gamma_5 \ell \right. \\ & \left. - 2C_{7L}^{eff} m_b \bar{s}_L i\sigma_{\mu\nu} \frac{q^\nu}{q^2} b_R \bar{\ell} \gamma^\mu \ell + L \rightarrow R \right], \end{aligned} \quad (3)$$

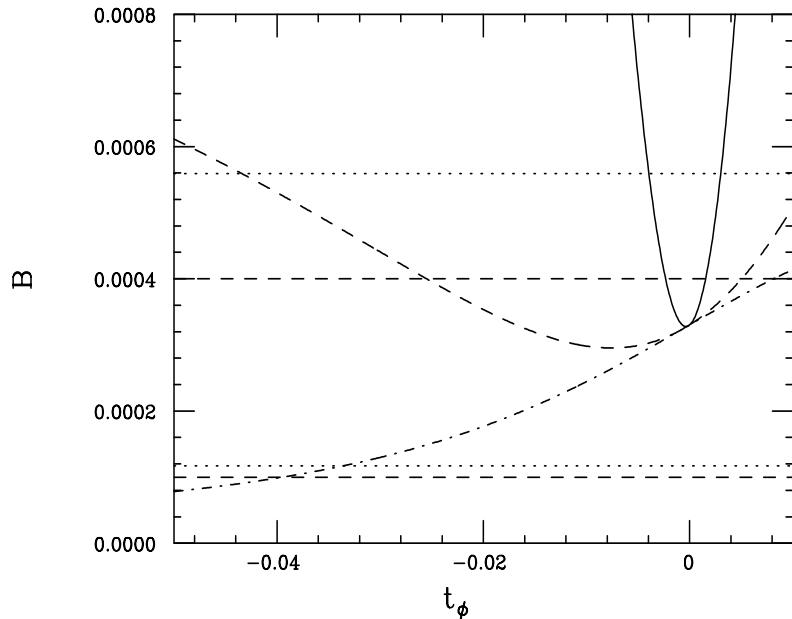
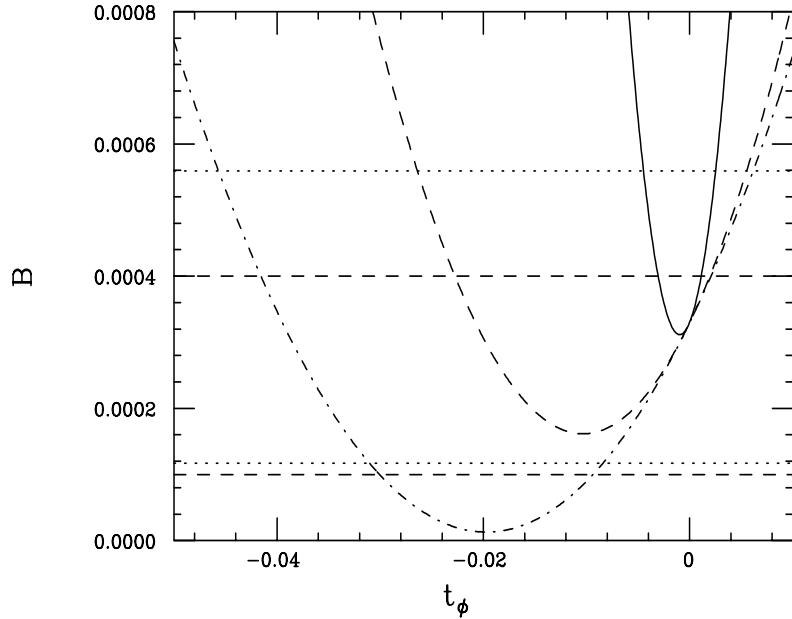


Figure 2: Prediction for the $b \rightarrow s\gamma$ branching fraction for the cases $V_R = A(m)$ (top) and $V_R = D(n)$ (bottom) with $\kappa = 1$ and $M_{W_2} = 0.8$ TeV as a functions of t_ϕ in the LRM. The solid, dashed, dash-dotted curves correspond to $m, n=1, 2$ and 3 , respectively. As before, the 95% CL CLEO(ALEPH) allowed range lies inside horizontal the dashed(dotted) lines.

where q^2 is the momentum transferred to the lepton pair. Note that $C_{9L,R}^{eff}$ contains the usual phenomenological long distance terms and that all the CKM elements are now contained in the coefficients themselves. From here we can directly obtain the expression for the double differential decay distribution

$$\begin{aligned}
\frac{dB}{dzds} = & B_l K \frac{3\alpha^2}{16\pi^2} \beta (1-s)^2 \left\{ \left[(a_-^2 + a_+^2) + (b_-^2 + b_+^2) \right] \frac{1}{2} \left[(1+s) - (1-s)\beta^2 z^2 \right] \right. \\
& \left[(a_-^2 - a_+^2) - (b_-^2 - b_+^2) \right] \beta z s + 4x(a_- a_+ + b_- b_+) \\
& + \frac{4}{s^2} (C_{7L}^2 + C_{7R}^2) (1-s)^2 (1 - \beta^2 z^2) \\
& \left. - \frac{2}{s} \text{Re} [C_{7L}(a_- + a_+) + C_{7R}(b_- + b_+)] (1-s)(1 - \beta^2 z^2) \right\}, \tag{4}
\end{aligned}$$

where $z = \cos \theta_{\ell^+\ell^-}$, $s = q^2/m_b^2$, $x = m_\ell^2/m_b^2$, $\beta = \sqrt{1 - 4x/s}$, $a_\pm = C_{9L}^{eff} \pm C_{10L} + 2C_{7L}/s$ and $b_\pm = a_\pm$ with the replacement $L \rightarrow R$. As usual $\theta_{\ell^+\ell^-}$ is the angle between the lepton direction and that of the original b quark in the lepton pair center of mass frame. Multiplication of the above expression by the total number of produced B mesons gives the double differential event distribution. We normalize this rate to the usual semileptonic branching fraction ($B_l = 0.1023$), including finite $m_c/m_b = 0.29$ and QCD corrections with $\alpha_s(M_Z) = 0.118$, which are fully included in the overall normalization parameter K (see Ali *et al.*, in Ref. [4]). LRM corrections to the semileptonic rate are, of course, also included; here the assumption that the mass of the right-handed neutrino $m_N > m_b$ becomes relevant. For numerical purposes we take $m_N = 250$ GeV in the calculation of the right-handed box diagram for $b \rightarrow s\ell^+\ell^-$, but the precise value of m_N is not important for our purposes of demonstration.

From this double differential distribution we can compute both the lepton pair invariant mass distribution by integration over z as well as the leptonic forward-backward

asymmetry. The asymmetry is given by the expression

$$A(s) = \frac{\int_0^1 \frac{dB}{dsdz} dz - \int_{-1}^0 \frac{dB}{dsdz} dz}{\int_{-1}^1 \frac{dB}{dsdz} dz}. \quad (5)$$

These two observables are shown in Fig. 3 for the SM as well as for four of the LRM conspiratorial cases discussed above, assuming massless leptons in the final state. As can be easily seen here the predictions of the four LRM cases are quite distinct from those of the SM even though they all yield the same prediction for $B(b \rightarrow s\gamma)$.

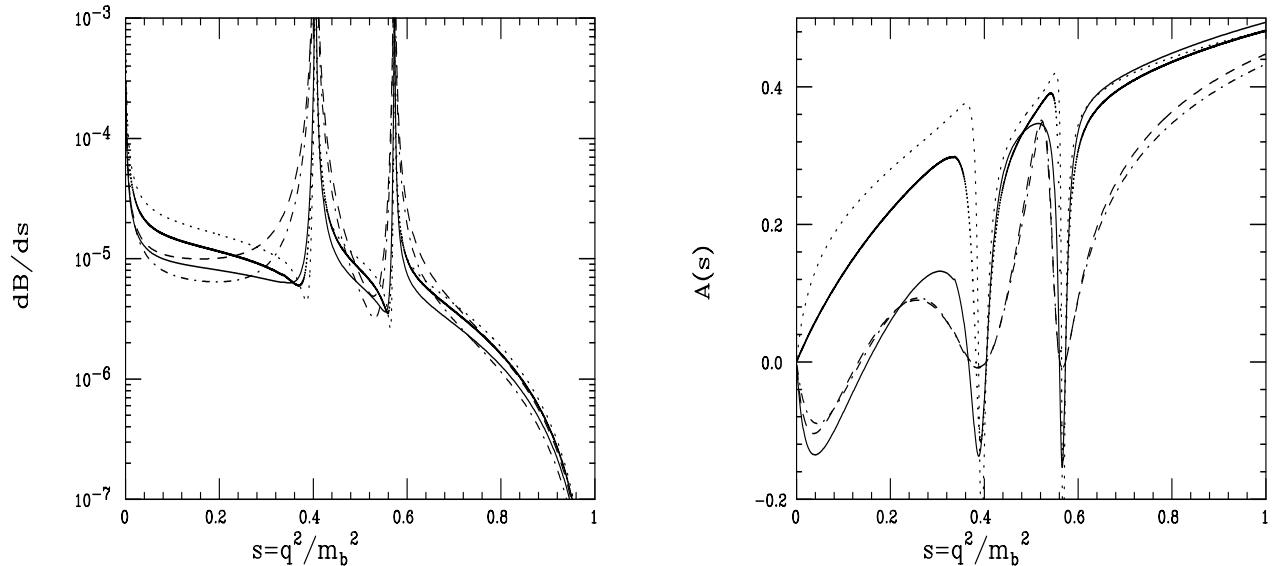


Figure 3: Differential decay distribution and lepton forward-backward asymmetry for the decay $b \rightarrow s\ell^+\ell^-$ in the SM(solid) and four representative models in the LRM parameter space which yield the SM value for the $b \rightarrow s\gamma$ branching fraction and satisfy all other existing experimental constraints: $V_L = V_R$ (square-dot), $A(1)$ (dash), $A(3)$ (dot) and $D(2)$ (dash-dot).

Other observables, such as the longitudinal polarization asymmetry of the τ 's in $b \rightarrow s\tau^+\tau^-$, can be obtained in a straightforward fashion from the expressions above and an $L \rightarrow R$ augmentation of the expressions provided by Hewett[4]. These are the only $b \rightarrow s\ell^+\ell^-$ observables we will make use of in the analysis below; the possibility of employing[13] the

transverse polarization of the τ 's has been neglected. In a similar spirit we ignore the possible information that one could gain from future photon polarization measurements in $b \rightarrow s\gamma$.

4 Analysis Results

The essential aspects of our procedure can be found in the work of Hewett[4] and we closely follow the discussion of this author. The analysis makes use of the following observables. For the process $b \rightarrow s\ell^+\ell^-$, we consider the lepton pair invariant mass distribution described by dB/ds , and the lepton pair forward-backward asymmetry, $A(s)$, for $\ell = e, \mu$, and τ , as well as the tau longitudinal polarization asymmetry $P_\tau(s)$. We will neglect the μ mass for simplicity and directly combine the e and μ samples. We also include $B(b \rightarrow s\gamma)$ to this list of observables. The lepton pair invariant mass spectrum is divided into 9 bins which are distributed as follows: 6 bins of equal size, $\Delta s = 0.05$, are taken in the low dilepton mass region below the J/ψ resonance, $0.02 \leq s \leq 0.32$, and 3 bins are in the high dilepton mass region above the ψ' pole and are taken to be $0.6 \leq s \leq 0.7$, $0.7 \leq s \leq 0.8$, and $0.8 \leq s \leq 1.0$. By using this set of bins we completely avoid the regimes where both long distance and resonance contributions are clearly important.

Our analysis proceeds as follows. For a given conspiratorial choice of V_R derived above we use Monte Carlo techniques to generate binned “data” associated with the above quantities. For the $b \rightarrow s\ell^+\ell^-$ observables we assume that the errors will remain statistically dominated, while for $b \rightarrow s\gamma$ we assume a purely systematically dominated error of 7% arising from both experimental and theoretical uncertainties. These distributions are then generated for an integrated luminosity of either 5×10^7 or 5×10^8 $B\bar{B}$ pairs; these correspond to the expected total luminosity of a couple of years of running at future B -factories on the $\Upsilon(4S)$ and at the LHC, respectively. For $b \rightarrow s\ell^+\ell^-$ the number of events per bin is given

by

$$N_{\text{bin}} = \mathcal{L} \int_{s_{\text{min}}}^{s_{\text{max}}} \frac{dB}{ds} ds, \quad (6)$$

and the integrated average value of the asymmetries for each bin is then

$$\langle A \rangle_{\text{bin}} = \frac{\mathcal{L}}{N_{\text{bin}}} \int_{s_{\text{min}}}^{s_{\text{max}}} A(s) \frac{dB}{ds} ds, \quad (7)$$

where dB/ds can be obtained from the double differential expression above.

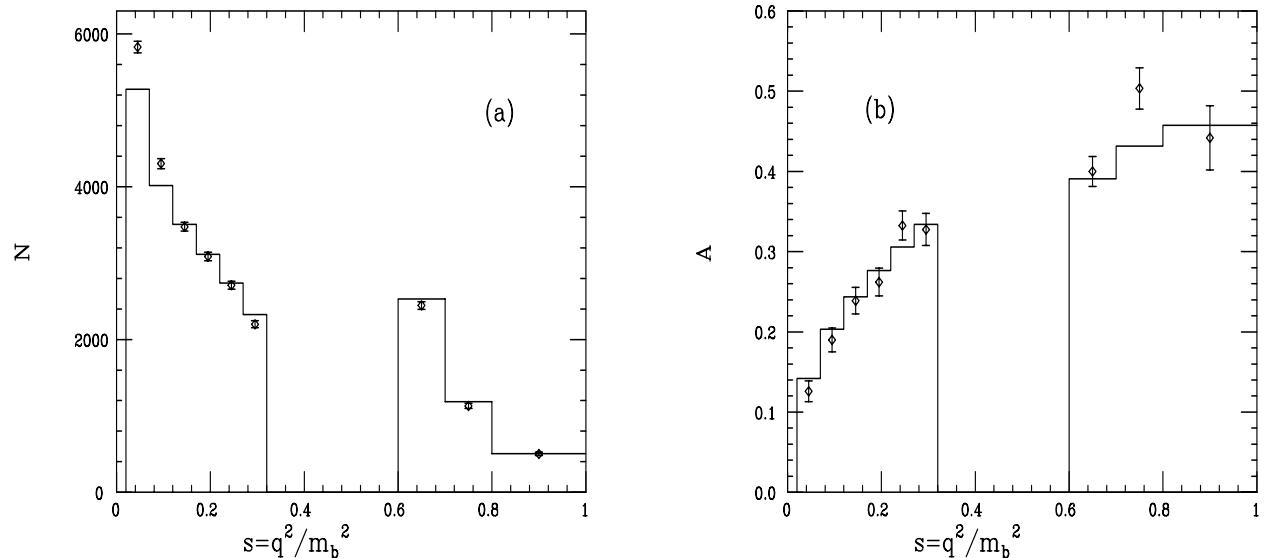


Figure 4: Comparison of a typical Monte Carlo event sample generated with the right-handed mixing matrix $B(3)$ for (a) the number of events per bin and (b) the bin averaged asymmetry. e and μ final states are summed and the histogram represents the best fit to the data. A sample of $5 \times 10^8 B\bar{B}$ pairs has been assumed.

Once the data is generated for each model we then perform a three dimensional χ^2 fit assuming only the usual three coefficients are present, *i.e.*, $C_{7L,9L,10L}(\mu)$. This is done according to the usual prescription

$$\chi_i^2 = \sum_{\text{bins}} \left(\frac{Q_i^{\text{obs}} - Q_i(C_L)}{\delta Q_i} \right)^2, \quad (8)$$

where Q_i^{obs} , $Q_i(C_L)$, δQ_i represent the “data”, the result of the expectations for a given set of $C_{7L,9L,10L}(\mu)$ values, and the error for each observable quantity Q_i . The $C_{7L,9L,10L}$ are then varied until a χ^2 minimum is obtained. Note that there are 27 bins of data in addition to $B(b \rightarrow s\gamma)$; allowing for the three free parameters $C_{7L,9L,10L}$ means that the fit has $25=27+1-3$ degrees of freedom. If the SM were realized, we would expect such a fit to yield the SM values of $C_{7L,9L,10L}$, within errors, with a good $\chi^2 \simeq 25$ as was shown by Hewett[4]. If new physics is present but the operator basis is not extended we would again expect a comparably good χ^2 fit at values of $C_{7L,9L,10L}$ which would now exclude the SM at some confidence level. In our case with an extended operator basis we would hope that the best fit obtained by varying these three coefficients alone is not very good thus demonstrating that the three parameter fit is insufficient.

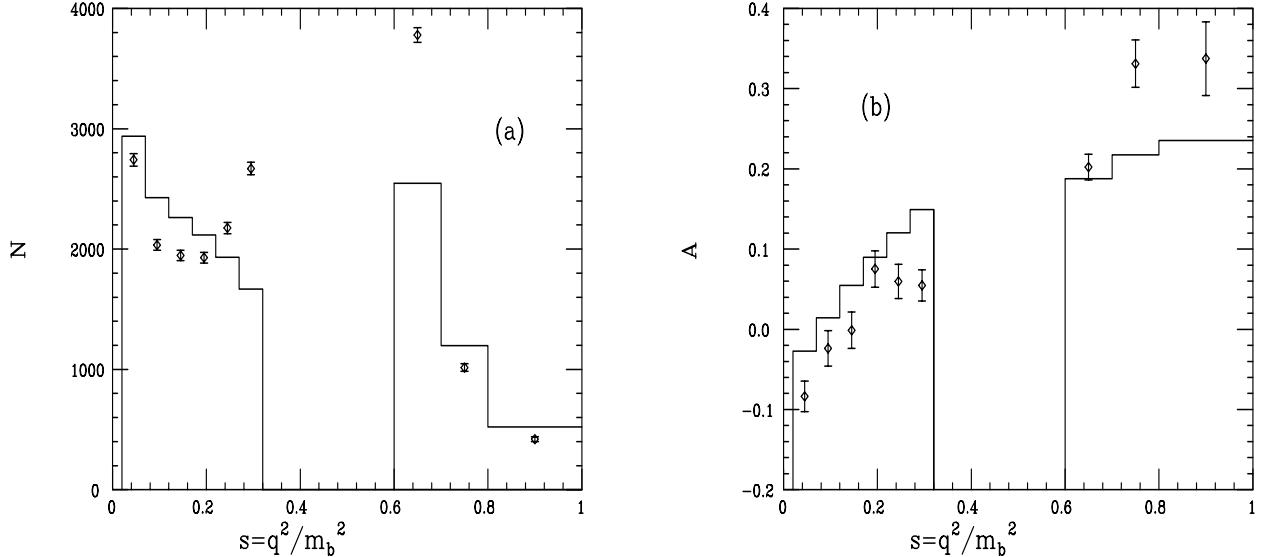


Figure 5: Same as the previous figure but now for matrix A(1).

We now give a few examples of this type of analysis. We first consider a typical Monte Carlo data sample generated for the matrix $B(3)$, assuming $5 \times 10^8 B\bar{B}$ pairs are

produced, and examine the results of the best fit. This is shown in Fig. 4 for the two observables N and A for combined e and μ data samples. Note that in the low s bins the best fit underestimates (overestimates) the number of events at the low (high) energy end and generally underestimates the asymmetry. Combining all of the observables this particular “data” set leads to a $\chi^2/d.o.f$ of 187.1/25. We note that for 25 $d.o.f.$, χ^2 values of 37.65(44.31,52.61,60.14) correspond to probabilities of a consistent fit of 5(1,0.1,0.01)%, respectively. Thus it is clear that for this particular sample the “data” are not consistent with the assumption of only three active operators and that an extended operator basis is required.

Figures 5 and 6 show examples of generated data and the corresponding best three parameter fits for the right-handed matrices $A(3)$ and $D(2)$. In both these cases the best fits are simply incapable of obtaining the correct shape presented by the data. For these two cases the $\chi^2/d.o.f.$ from these fits are found to be 1187.2/25 and 764.1/25, respectively, for a sample of $5 \times 10^8 B\bar{B}$ pairs. Thus we again see that these particular data sets are not consistent with the three Wilson coefficient fit hypothesis.

Of course, to really ascertain if the new physics of the extended operator basis should be visible we need to generate many sets of data for each of the V_R assumptions and determine the fraction of the time that the resulting χ^2 values exceed those listed above for each of the fixed probabilities. To be specific we generate 1000 sets of data for each of the models above and perform the χ^2 fitting procedure for each data set. For very large data samples, *i.e.*, for $5 \times 10^8 B\bar{B}$ pairs, we find that for all of the above choices of V_R we obtain fit probabilities below 0.01% almost 100% of the time. This means that it would be quite clear in this case that an extended operator basis is required. For smaller data samples, *i.e.*, for $5 \times 10^7 B\bar{B}$ pairs, the results are much more model dependent and are given in Table 2. It seems that $V_L = V_R$ represents the worst case scenario. (To test the stability of our results we generated

10000 sets of data for this case and confirmed the results shown in the Table.) Obviously it is quite important to decide at what level of probability one is willing to exclude the three operator fit before drawing any conclusions about an extended operator basis. However, it is clear that even for very low probabilities, $\sim 0.01\%$, we see that the three parameter fit will fail on average a reasonable fraction, $\sim 45\%$, of the time.

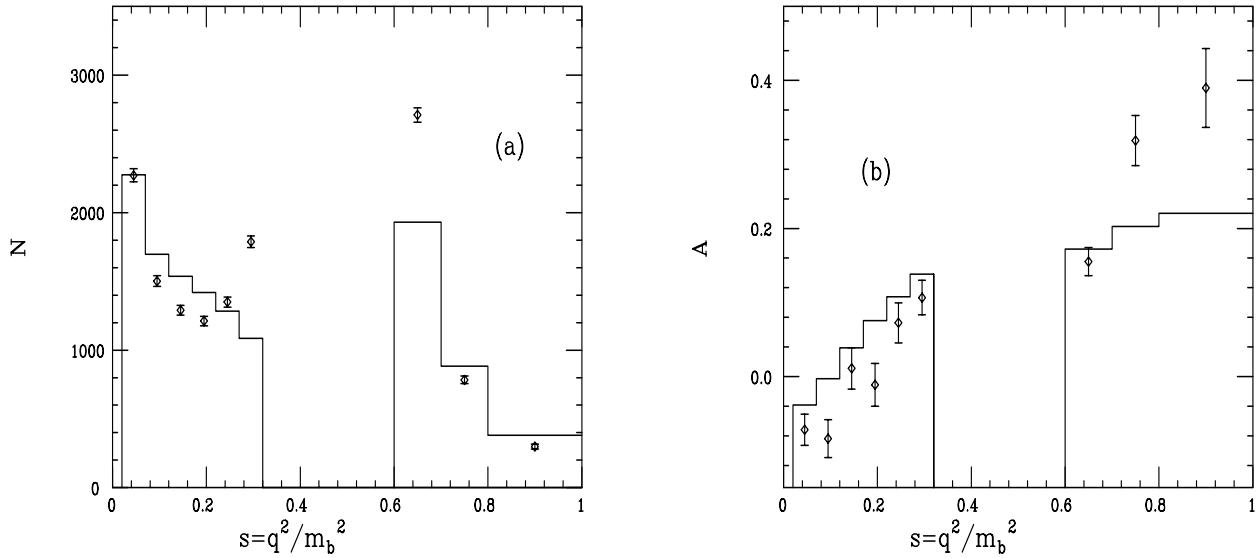


Figure 6: Same as the previous figure but now for matrix D(2).

5 Discussion and Conclusions

The inclusive rare decays $b \rightarrow s\gamma$ and $b \rightarrow s\ell^+\ell^-$ have been and will continue to be subjects of intense interest since they offer unique opportunities to probe for new physics beyond the Standard Model. Both these decay modes are quite clean theoretically and future B -factories will produce large event samples in both cases, allowing for in-depth studies of their associated observables. As we know, in the SM and in many of its extensions these decay observables can be expressed in terms of only three *a priori* unknown parameters,

Matrix	$P = 5\%$	$P = 1\%$	$P = 0.1\%$	$P = 0.01\%$
$V_L = V_R$	~39%	~23%	~11%	~6%
$A(1)$	~100%	~100%	~100%	~100%
$A(3)$	~90%	~76%	~51%	~30%
$B(2)$	~47%	~28%	~15%	~8%
$B(3)$	~92%	~80%	~56%	~36%
$D(2)$	~100%	~100%	~100%	~99%

Table 2: Fraction of the time the effects of the extended operator basis are observable, with different choices of the consistency of fit probabilities, for each of the V_R choices described in the text assuming a sample of $5 \times 10^7 B\bar{B}$ pairs. These results are based on 1000 Monte Carlo data samples for each of the models.

corresponding to the values of the Wilson coefficients of three operators at the scale $\mu \sim m_b$. The global fit approach provides the best model independent technique for obtaining the values of these coefficients at the low scale which can then be compared with the expectations of a given model. In this paper we have demonstrated, using the Left-Right Symmetric Model as an example, that with the statistics available at future B -factories it will be possible to observe the rather unique situation where this global fit to the canonical three coefficients fails. This result would tell us that not only does new physics exist beyond the SM but that this new physics *requires* an extended operator basis.

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References

- [1] M.S. Alam *et al.*, CLEO Collaboration, Phys. Rev. Lett. **74**, 2885 (1995).
- [2] P.G. Colrain and M.I. Williams, ALEPH Collaboration, paper contributed to the *International Europhysics Conference on High Energy Physics*, Jerusalem, August 19-26, 1997. See also P. Kluit, talk given at the *International Europhysics Conference on High Energy Physics*, Jerusalem, August 19-26, 1997.
- [3] S. Glenn *et al.*, CLEO Collaboration, hep-ex/9710003. For a general overview of the present experimental results on the electroweak penguin decays of the b quark, see T. Skwarnicki, CLEO Collaboration, hep-ph/9712253.
- [4] J.L. Hewett, Phys. Rev. **D53**, 4964 (1996); J.L. Hewett and J. Wells, Phys. Rev. **D55**, 55 (1997); See also A. Ali, G.F. Giudice and T. Mannel, Z. Phys. **C67**, 417 (1995).
- [5] M. Peskin and T. Takeuchi, Phys. Rev. Lett. **65**, 964 (1990) and Phys. Rev. **D46**, 381 (1992); W. Marciano and J. Rosner, Phys. Rev. Lett. **65**, 2963 (1990).
- [6] The author would like to thank JoAnne Hewett for suggesting this analogy.
- [7] See for example Y-B. Dai, C-S. Huang and H-W. Huang, Phys. Lett. **B390**, 257 (1997); T.M. Aliev, M. Savci, A. Ozpineci and H. Koru, hep-ph/9705222.
- [8] K.G. Chetyrkin, M. Misiak and M. Münz, Phys. Lett. **B400**, 206 (1997); C. Greub, T. Hurth and D. Wyler, Phys. Lett. **B380**, 385 (1996), Phys. Rev. **D54**, 3350 (1996); C. Greub and T. Hurth, hep-ph/9703349; A. Ali and C. Greub, Z. Phys. **C49**, 431 (1991), Phys. Lett. **B259**, 182 (1991), ZPC C60 433 1993 , Phys. Lett. **B361**, 146 (1995); K. Adel and Y.-P. Yao, Phys. Rev. **D49**, 4945 (1994); M. Ciuchini, G. Degrassi, P. Gambino and G.F. Giudice, hep-ph/9710335; N. Pott, Phys. Rev. **D54**, 938 (1996).

[9] A.F. Falk, M. Luke and M. Savage, Phys. Rev. **D49**, 3367 (1994); M.B. Voloshin, Phys. Lett. **B397**, 275 (1997); A. Khodjamirian, R. Rückl, G. Stoll and D. Wyler, Phys. Lett. **B402**, 167 (1997); Z. Ligeti, L. Randall and M.B. Wise, Phys. Lett. **B402**, 178 (1997); A.K. Grant, A.G. Morgan, S. Nussinov and R.S. Peccei, hep-ph/9702380; G. Buchalla, G. Isidori and S.J. Rey, hep-ph/9705253.

[10] A.J. Buras, A. Kwiatkowski and N. Pott, hep-ph/9707482 and 9710336. For a recent review of the present situation, see A. Ali, hep-ph/9709507.

[11] M. Misiak, Nucl. Phys. **B393**, 23 (1993) and *erratum* Nucl. Phys. **B439**, 461 (1995); A. J. Buras and Münz, Phys. Rev. **D52**, 186 (1995).

[12] See Falk, Luke and Savage as well as Buchalla, Isidori and Rey in Ref. [9] and also A. Ali, G. Hiller, L.T. Handoko and T. Morozumi, Phys. Rev. **D55**, 4105 (1997).

[13] See for example F. Krüger and L.M. Sehgal, Phys. Lett. **B380**, 199 (1996).

[14] For example, the NLO matching conditions for the Two Higgs Doublet model have only recently been performed: M. Ciuchini, G. Degrassi, P. Gambino and G.F. Giudice, hep-ph/9710335 and P. Ciafaloni, A. Romanino and A. Strumia, hep-ph/9710312.

[15] For a review and original references, see R.N. Mohapatra, *Unification and Supersymmetry*, (Springer, New York, 1986). See also P. Langacker and S.U. Sankar, Phys. Rev. **D40**, 1569 (1989) and T. Hayashi, hep-ph/9705214.

[16] The possibility of right-handed currents playing an important role in tree level B decays has been recently revived by M.B. Voloshin, Mod. Phys. Lett. **A12**, 1823 (1997).

[17] A. Jodiddo *et al.*, Phys. Rev. **D34**, 1967 (1986) and Phys. Rev. **D37**, 237 (1988); J. Imazoto *et al.*, Phys. Rev. Lett. **69**, 877 (1992).

- [18] See, for example, G. Beall, M. Bander, and A. Soni, Phys. Rev. Lett. **48**, 848 (1982).
- [19] F. Abe *et al.*, CDF Collaboration, Phys. Rev. Lett. **74**, 2900 (1995) and Phys. Rev. **D55**, 5263 (1997); S. Abachi *et al.*, D0 Collaboration, Phys. Rev. Lett. **76**, 3271 (1996); B. Abbott *et al.*, D0 Collaboration, paper submitted to the *XVIII International Symposium on Lepton Photon Interactions*, Hamburg, Germany, July 28-August 1, 1997.
- [20] T.G. Rizzo, Phys. Rev. **D50**, 325 (1994).
- [21] For a recent reviews see A. Buras, hep-ph/9711217; P.S. Drell, hep-ex/9711020; P. Paganini, F. Parodi, R. Roudeau and A. Stocchi, hep-ph/9711261.
- [22] M. Gronau and S. Wakaizumi, Phys. Rev. Lett. **68**, 1814 (1992).
- [23] T.G. Rizzo, in preparation.
- [24] M. Acciarri *et al.*, L3 Collaboration, Phys. Lett. **B351**, 375 (1995).
- [25] D. Buskulic *et al.*, ALEPH Collaboration, Phys. Lett. **B365**, 367 (1996) and Phys. Lett. **B365**, 437 (1996).
- [26] J.E. Duboscq *et al.*, CLEO Collaboration, Phys. Rev. Lett. **76**, 3898 (1996); S. Sanghera *et al.*, CLEO Collaboration, Phys. Rev. **D47**, 791 (1993); see also H. Albrecht *et al.*, ARGUS Collaboration, Z. Phys. **C57**, 533 (1993).
- [27] P. Langacker and U. Sankar, Phys. Rev. **D40**, 1569 (1989).
- [28] R.M. Barnett, *et al.*, (Particle Data Group), Phys. Rev. **D54**, 1 (1996).
- [29] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).

[30] T.G. Rizzo, Phys. Rev. **D50**, 3303 (1994) and hep-ph/9705209; D. Cocolicchio *et al.*, Phys. Rev. **D40**, 1477 (1989); G.M. Asatryan and A.N. Ioannisyan, Yad. Fiz. **51**, 1350 (1990); K.S. Babu, K. Fujikawa and A. Yamada, Phys. Lett. **B333**, 196 (1994); P. Cho and M. Misiak, Phys. Rev. **D49**, 5894 (1994); G. Bhattacharyya and A. Raychaudhuri, Phys. Lett. **B357**, 119 (1995).

[31] See A. Kagan, hep-ph/9701300 and references therein.