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Correlation between α-particle preformation factor and α decay energy

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ABSTRACT

α-particle preformation factors, one of the most significant quantities in α decay, are systematically investigated, which are extracted from the ratios between theoretical α-decay half-lives calculated by the generalized liquid drop model and experimental data. The results indicate that both α-particle preformation factors P_α and α decay energy Q_α are important observed quantities for revealing the nuclear shell structure information. And a nice linear relationship exists between $\log_{10} P_\alpha$ and $Q_\alpha^{-1/2}$, which means the famous Geiger-Nuttall law can not only describe α decay half-lives but deal with α-particle preformation factors, as well as the nuclear shell effects play key roles in α-particle preformation. Furthermore, the results indicate that $Z = 82$ and $N = 126$ closed shells play more important roles than $Z = 50$ and $N = 82$ shell closures in which the shell effect of $N = 126$ is stronger than that of $Z = 82$ in the α decay process. Besides, the unpaired nucleons will inhibit the preformation of α-particle. After considering the above significant physical effects, a global analytic formula with only twelve parameters for α-particle preformation factors is proposed based on the direct observation quantity of the nuclear shell effect i.e., α decay energy. The outstanding precision of this formula in describing the α-particle preformation factors indicates that it can be used to perform accurate calculations for α decay half-lives as well as provide some general guidance for microscopic study on α-particle preformation factors and nuclear structure.

α decay, one of the most dominant decay modes of heavy and superheavy nuclei, receives considerable attention because it can be a probe to study the unstable nucleus and neutron-deficient nucleus, and is the only way to identify new synthesized superheavy nucleus [1–6]. Up to now, one of the important divergences between theoretical α decay half-life and experimental data is how to calculate the α-particle preformation factor, which represents the probability of an α-cluster formation on the surface of the decaying parent nucleus. However, it is still an open issue. All microscopical theories such as the R-matrix method [7–9], the Tohsaki-Horiuchi-Schuck-Röpke wave function approach [10,11], the hybrid (shell model+α-cluster) model [12], and so on [13–16] are extremely difficult for calculating the α-particle preformation factor of a nucleus that is heavier than ^{212}Po due to the complicated structure of quantum many-body systems. In the cluster model [17–20], the α-particle preformation factor is assumed as a constant, which can not reflect the microscopic information about the nuclear structure. Therefore, the important systematical deviations may occur between calculated α decay half-lives and experimental data for nuclei around shell closures.

On the other hand, there were a few works that proposed a limited number of formulas based on the valence nucleons (holes) from the viewpoint of the nuclear shell effect to calculate the α-particle preformation factors. [21–25]. Besides, some recent works showed that the α-particle preformation factor is linearly dependent on the product of valence protons (holes) and valence neutrons (holes) $N_p N_n$ for nuclei around $Z = 82$ and $N = 126$ shell closures [26,27]. Therefore, the nuclear nucleons configuration and shell structure play key roles in α-cluster preformation. At present, however, there are several formulas based on magic numbers [21–25], which lead to the calculations of α-particle preformation factors are dependent on different valence nucleons (holes) in various nuclide regions. Therefore, how to calculate the valence nucleons (holes) of nucleus whose nucleons are located in the middle of two major nuclear shells is a tricky problem. And above all,

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the magic numbers in superheavy nuclei are in suspense. Therefore, some formulas are difficult to extend to superheavy nuclei. Thus, there is an urgent need to establish a new method for accurately describing the α-particle preformation factor globally from a new viewpoint.

In our recent work [28], we proposed an analytic formula to calculate the α-particle preformation factors based on the α decay energy Q_α with the help of empirical formulas for α decay half-lives for the first time. This work revealed that the famous Geiger-Nuttall law [29] not only can describe α decay half-lives but also can deal with α-particle preformation factors. This formula can shed light on microscopic nuclear structure information such as shell and odd-even staggering effects and can be used to perform the precise calculations of α decay half-lives.

We start with the recent proposed analytic formula [28]. The calculational details for the α-particle preformation factors are given in recent work [28]. The α decay constant λ is defined as

$$\lambda = P_\alpha \nu P, \tag{1}$$

where P_α is the α-particle preformation factor. The assault frequency ν is obtained by using the classical method with the kinetic energy of the α-particle. The barrier penetrating probability P is calculated from tunneling the generalized liquid drop model (GLDM) potential barriers [30–35] with the Wentzel-Kramers-Brillouin (WKB) approximation. The experimental α decay constant λ_{Exp} can be obtained by experimental α decay half-life $T_{1/2}^{\mathrm{Exp}}$,

$$\lambda_{\mathrm{Exp}} = \frac{\ln 2}{T_{1/2}^{\mathrm{Exp}}} = P_\alpha^{\mathrm{Exp}} \nu P. \tag{2}$$

The theoretical α decay constant λ_{Cal} is obtained by assuming the α-particle preformation factor as a constant $P_0 = 1$,

$$\lambda_{\mathrm{Cal}} = \frac{\ln 2}{T_{1/2}^{\mathrm{Cal}}} = P_0 \nu P. \tag{3}$$

Thus experimental α-particle preformation factor P_α^{Exp} can be extracted from experimental α decay half-life [35–39] and expressed as

$$P_\alpha^{\mathrm{Exp}} = \frac{\lambda_{\mathrm{Exp}}}{\lambda_{\mathrm{Cal}}} = \frac{T_{1/2}^{\mathrm{Cal}}}{T_{1/2}^{\mathrm{Exp}}}. \tag{4}$$

Taking the logarithms of both sides of Eq. (4),

$$\log_{10} P_\alpha^{\mathrm{Exp}} = \log_{10} T_{1/2}^{\mathrm{Cal}} - \log_{10} T_{1/2}^{\mathrm{Exp}}. \tag{5}$$

Two successful α decay half-life empirical formulas, namely the Royer formula [30,40,41] and the universal decay law (UDL) [42], are used to express $\log_{10} T_{1/2}^{\mathrm{Cal}}$ and $\log_{10} T_{1/2}^{\mathrm{Exp}}$. Then, in our previous work, we put forward an analytic expression for estimating α-particle preformation factor:

$$\log_{10} P_\alpha = a + bA^{1/6}\sqrt{Z} + c\frac{Z}{\sqrt{Q_\alpha}} - d\chi' - e\rho' + f\sqrt{l(l+1)}, \tag{6}$$

where $\chi' = Z_1 Z_2 \sqrt{\frac{A_1 A_2}{(A_1+A_2)Q_\alpha}}$ and $\rho' = \sqrt{\frac{A_1 A_2}{A_1+A_2} Z_1 Z_2 (A_1^{1/3} + A_2^{1/3})}$. A, Z, and Q_α express mass number, proton number, and α decay energy of the parent nucleus. A_1, Z_1, A_2, and Z_2 denote mass and proton numbers of α-particle and daughter nucleus. l is the angular momentum carried by α-particle.

Recently, we find that Eq. (6) can be further improved following these reasons:

I) Eq. (6) demonstrated that Geiger-Nuttall law not only can describe α decay half-lives but also can deal with α-particle preformation factors. Within Gamow's theory, the α decay process

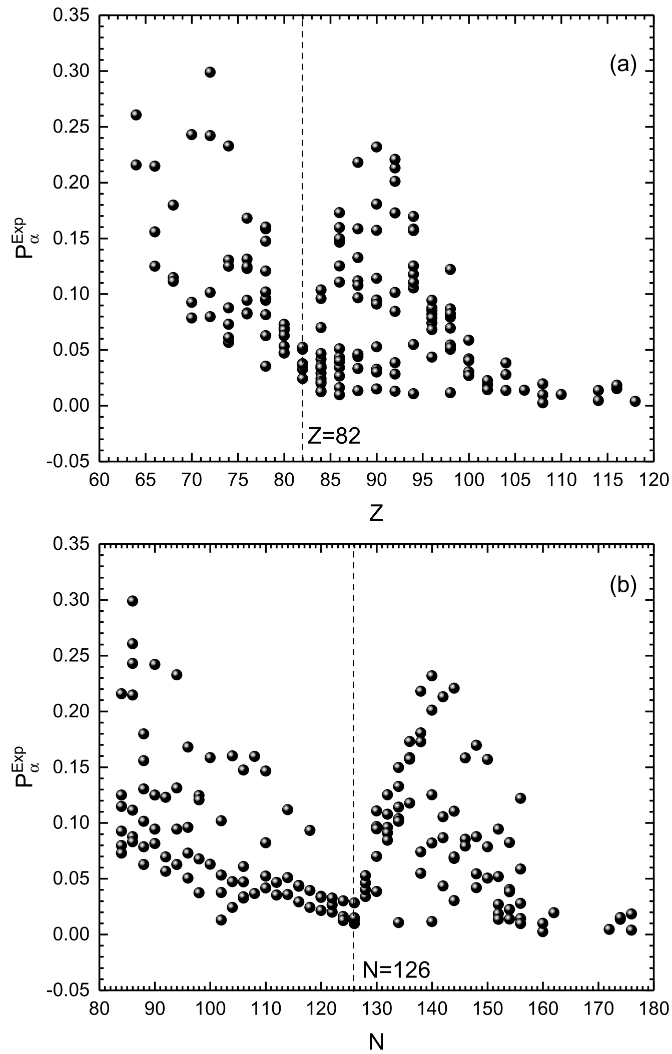

Fig. 1. The variations of extracted experimental α-particle preformation factors from Eq. (4) against proton numbers (up) and neutron numbers (bottom) for even-even nuclei.

is described as a preformed α-particle penetrating the Coulomb barrier [43,44]. However, the success of Geiger-Nuttall law [29] in describing α decay half-life is usually attributed to the quantum tunneling phenomenon. By comparably analyzing Eq. (6), we can see that the third term and fourth term dependences on Q_α are similar. Therefore, we can further develop this formula by digging out the underlying physical meanings and reducing the number of parameters to strengthen the understanding of Geiger-Nuttall law from a new viewpoint, which not only can describe quantum tunneling phenomenon but also can deal with the formation of α-particle on the nuclear surface, and to shed some new lights on topics of researching α decay and α-particle preformation factors.

II) By analyzing extracted experimental α-particle preformation factors, we find that the unpaired nucleons will inhibit the α-particle preformation on the surface of its decaying parent nucleus.

Therefore, Eq. (6) should be further improved by revealing its deeper physical meaning behind this form and considering the blocking effect of unpaired nucleons, and reducing the parameters in the formula. This is the motivation of this letter.

The variations of extracted experimental α-particle preformation factors P_α^{Exp} from Eq. (4) against proton numbers Z (up) and neutron numbers N (bottom) for even-even nuclei are shown in Fig. 1. From Fig. 1 (a), it is found that the closer proton numbers approach the proton closed shell $Z = 82$, the smaller P_α^{Exp} are. And when proton numbers cross $Z = 82$ shell closure, P_α^{Exp} increase rapidly, until proton numbers are close to the next proton shell closure. P_α^{Exp} have the maximum values in the middle of the two major shells. A similar variation tendency occurs in Fig. 1 (b) reflecting the neutron shell effect on α-particle preformation factors. Fig. 1 indicates that the closer proton or neutron number is to the full shell, the more difficult it is to form an α-cluster on the surface of the nucleus during α decay. Therefore, the nuclear nucleons configuration and shell structure play key roles in α-cluster preformation. In addition, when proton and neutron numbers evolve into superheavy nuclei regions, P_α^{Exp} still decrease as the numbers of protons and neutrons approaching $Z = 82$ and $N = 126$ closed shells, which indicates that extracted experimental α-particle preformation factors provide a positive signal for the presence of the island of stability for superheavy nuclei. Moreover, one can find that change trends of P_α^{Exp} on both sides of the $Z = 82$ closed shell are different. When proton numbers cross the $Z = 50$ closed shell and evolve to the $Z = 82$ shell closure, P_α^{Exp} keep decreasing. But after Z crossing $Z = 82$ shell closure, P_α^{Exp} increase at first and then drop down with Z approaching the next proton full shell. A similar situation also appears on both sides of the $N = 126$ closed shell. P_α^{Exp} continue to drop when neutron numbers N cross the $N = 82$ closed shell and approach $N = 126$ shell closure. After crossing $N = 126$, P_α^{Exp} rapidly increase and reach the maximum value in N reaching the middle position of the two major shells. Then when N evolve to the next neutron closed shell, P_α^{Exp} continue to decrease. It shows that for α decay process, the shell effects of $Z = 50$ and $N = 82$ closed shells are relatively weaker than that of $Z = 82$ and $N = 126$ shell closures. In addition, if the $N = 82$ neutron closed shell plays a leading role, the matching proton magic number should be $Z = 50$. However, the dominant decay mode of nuclei in this area is β^- decay. Therefore, when the number of protons and neutrons are away from $Z = 50$ and $N = 82$ shell closures and close to the $Z = 82$ and $N = 126$ closed shells, the α-particle preformation factor keeps decreasing indicating that $Z = 82$ and $N = 126$ closed shells play key roles in the α decay process.

The experimental α decay energies Q_α for even-even nuclei are shown as functions of proton numbers Z (up) and neutron numbers N (bottom) in Fig. 2. Before $N = 126$, Q_α increase slowly with increasing neutron numbers N. When neutron numbers cross $N = 126$ closed shell, Q_α decrease dramatically until neutron numbers approaching the next neutron shell closure, indicating that α decay energy Q_α can also reflect the nuclear shell structure. In addition, one can see that $Z = 82$, a well-known magic number, does not have an obvious stability excess from the Q_α values, indicating that proton magic number $Z = 82$ provides a smaller shell effect than neutron number $N = 126$ for α decay. This phenomenon can also be explained from another aspect: all α decay nuclei exist on the neutron-deficient side away from the β stability line [6]. Thus the influence of a neutron closed shell is stronger than that of a proton shell closure because an increased neutron will enhance the stability of the nucleus against α decay.

The logarithmic values of extracted experimental α-particle preformation factors $\log_{10} P_\alpha^{\mathrm{Exp}}$ (top) and the reciprocal of the square root of experimental α-decay energies $Q_\alpha^{-1/2}$ (bottom) for even-even nuclei are plotted as a function of neutron numbers N in Fig. 3. In this figure, one can see that as neutron numbers changing, the variation trend of $\log_{10} P_\alpha^{\mathrm{Exp}}$ shows good agreement with that of $Q_\alpha^{-1/2}$. When neutron number N is close to $N = 126$ shell closure, both $\log_{10} P_\alpha^{\mathrm{Exp}}$ and $Q_\alpha^{-1/2}$ decrease rapidly. After N cross $N = 126$ closed shell, both $\log_{10} P_\alpha^{\mathrm{Exp}}$, and $Q_\alpha^{-1/2}$ increase until N approaching the next neutron closed shell. The consistent

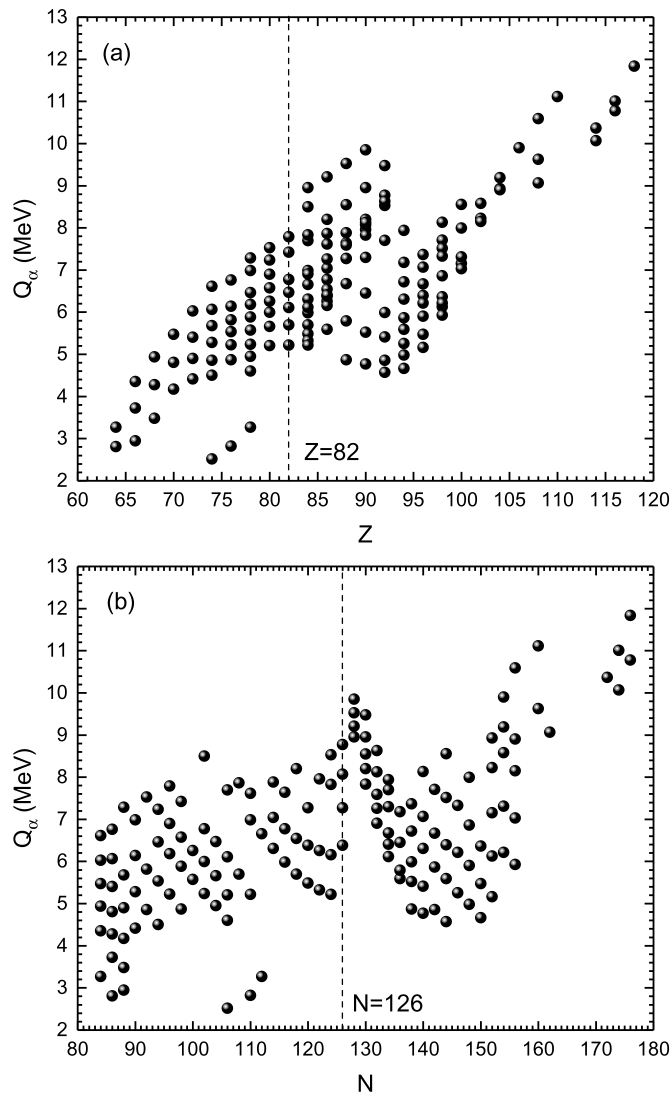

Fig. 2. The variations of experimental α decay energies against proton numbers (up) and neutron numbers (bottom) for even-even nuclei.

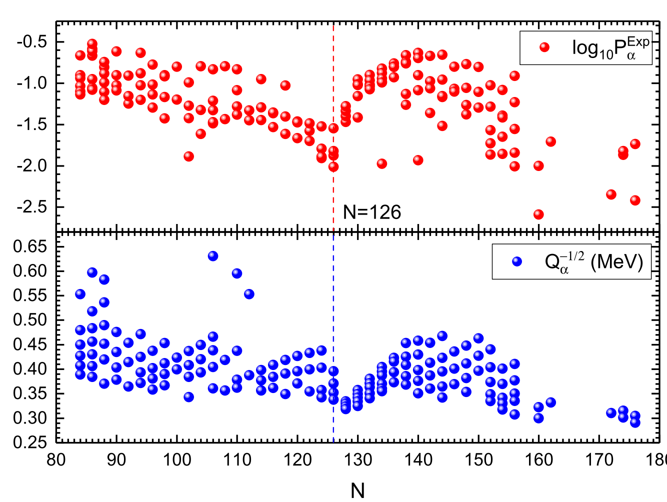

Fig. 3. The variations of logarithmic values of extracted experimental α-particle preformation factors $\log_{10} P_\alpha^{\mathrm{Exp}}$ (top) and the reciprocal of the square root of α decay energy $Q_\alpha^{-1/2}$ (bottom) against neutron numbers for even-even nuclei.

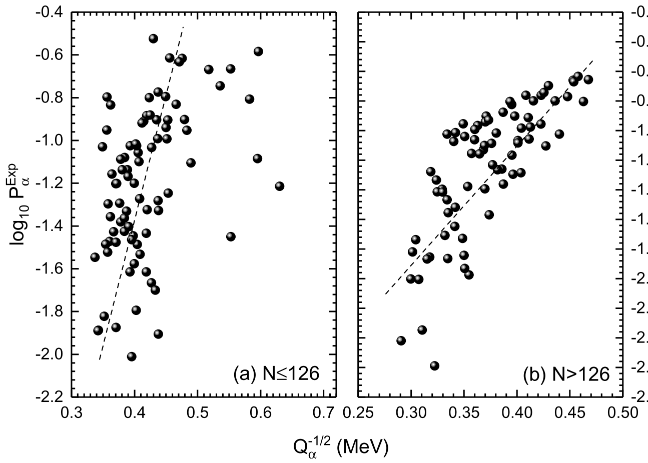

Fig. 4. The logarithmic values of extracted experimental α-particle preformation factors $\log_{10} P_\alpha^{Exp}$ as a function of the reciprocal of the square root of α decay energy $Q_\alpha^{-1/2}$ for even-even nuclei.

changing trends between $\log_{10} P_\alpha^{Exp}$ and $Q_\alpha^{-1/2}$ imply that there may be a linear relationship between them. Fig. 4 shows the evolution of $\log_{10} P_\alpha^{Exp}$ as a function of $Q_\alpha^{-1/2}$ for even-even nuclei. In this figure, one can find that $\log_{10} P_\alpha^{Exp}$ is linearly dependent on $Q_\alpha^{-1/2}$ indicating that both α-particle preformation factors and α-decay energy can be two well-observed quantities to reveal the nuclear shell effect.

Fig. 5 is the variations of extracted experimental α-particle preformation factors P_α^{Exp} for Th and Pa isotopes. In Th isotopes, one can see that the P_α^{Exp} show the odd-even staggering effect with the changes of neutron number, and this effect becomes more significant after neutron number crossing $N = 126$ shell closure. Moreover, for Th isotopes, the P_α^{Exp} of odd-A nuclei are less than that of neighboring even-even nuclei, for instances, the P_α^{Exp} decreased by 53.8% from 0.091 (^{222}Th) to 0.042 (^{223}Th) and it dropped by 58.6% from 0.232 (^{230}Th) to 0.096 (^{231}Th). This implies that compared to even-even nuclei, even-Z odd-N nuclei have an unpaired neutron, which makes it more difficult to form an α-cluster on its surface, and the α-particle preformation factor is significantly reduced. A similar situation can be seen in Pa isotopes. For odd-Z even-N Pa isotopes that have one more unpaired proton than corresponding even-even Th isotopes, their P_α^{Exp} are less than those of corresponding even-even Th isotopes. For example, the P_α^{Exp} dropped by 74.7% from 0.091 (^{222}Th) to 0.023 (^{223}Pa) revealing that an unpaired proton also inhibits the α-cluster preformation. For doubly odd Pa isotopes, which have one more unpaired neutron and one more unpaired proton than corresponding even-even Th isotopes, their P_α^{Exp} are less than those of corresponding odd-A Pa isotope and less than those of corresponding even-even Th isotopes. For example, P_α^{Exp} values drop from 0.157 (^{226}Th) to 0.072 (^{227}Pa) to 0.004 (^{228}Pa). Thus the unpaired nucleons will inhibit the α-particle preformation and lead to the α-particle preformation factors showing the odd-even staggering effect with the changes of neutron number for an isotope chain. Similar to the odd-even staggering effect in the binding energy of nuclei, the odd-even staggering effect of the α-particle preformation factor also is a signal of the existence of pairing correlation in the nucleus. Because nucleons tend to pair, even-even nuclei are more likely to form an α-cluster on its surface than odd-A and doubly odd nuclei. So the extracted experimental α-particle preformation factors P_α^{Exp} can correctly reflect the microscopic nuclear structure information such as shell effect, odd-even staggering effect, and

Table 1
The parameters of Eq. (7). In the second column, "E-E, O-A, and O-O" denote "even-even nuclei, odd-A nuclei, and doubly odd nuclei", respectively.

Region	Nuclei	a	b	c	d	h
$N \le 126$	E-E O-A O-O	21.232	−6.210	0.003	−0.049	– −0.150 −0.652
$N > 126$	E-E O-A O-O	37.421	−10.900	0.040	−0.088	– −0.323 −0.851

pairing correlation. In addition, P_α^{Exp} of ^{220}Th is a little bigger than that of ^{219}Pa. The extracted experimental data on these two nuclei should probably be examined again.

To sum up, we further clarify the correlation between extracted experimental α-particle preformation factors and α decay energies. The results show that α-particle preformation factors and the α decay energies are two important observed quantities for revealing the nuclear shell structure information. By analyzing the calculations, it is found that the logarithmic value of extracted experimental α-particle preformation factor $\log_{10} P_\alpha^{Exp}$ is linearly dependent on the reciprocal of the square root of experimental α-decay energy $Q_\alpha^{-1/2}$. And unpaired nucleons will inhibit the α-cluster formation on the surface of its decaying parent nucleus. Therefore, we further develop the original formula and propose a new formula by considering these important physical effects, which is expressed as

$$\log_{10} P_\alpha^{Eq} = a + b(A_1^{1/6} + A_2^{1/6}) + c\frac{N}{\sqrt{Q_\alpha}} + d\sqrt{l(l+1)} + h, \quad (7)$$

where A_1 and A_2 are the mass numbers of α-particle and daughter. N and Q_α denote the neutron number and α decay energy of the parent nucleus. The third term is dependent on the neutron number because the shell effect of $Z = 82$ is weaker than that of $N = 126$ in the α decay process according to the preceding analysis. The fourth term reflects the hindrance effect of the centrifugal potential and is independent of mass. l is the angular momentum carried by the α-particle, which satisfies the conservation of angular momentum and parity. The last item represents the blocking effect of unpaired nucleons since the nucleons tend to pair, and the α-particle tends to form on the surface of even-even nuclei. For odd-A and doubly odd nuclei, the α-particle preformation factors will decrease due to the presence of unpaired nucleons. The values of adjustable parameters a, b, c, d, and h are obtained by fitting the extracted experimental α-particle preformation factors and listed in Table 1. In this table, the values of b are negative. It is because that firstly, as the increase of mass number A, the general trend of α-particle preformation factors is to decrease. Secondly, for an isotope, the increase of neutron number will enhance the stability of the nucleus against α decay, because α decay occurs mainly in the neutron-deficient nucleus [6]. The values of d are also negative indicating that centrifugal potential plays a hindrance effect on α-cluster preformation. For even-even nucleus, $h = 0$. For odd-A and doubly odd nuclei, the signs of h are also negative reflecting the blocking effect of unpaired nucleons on α-cluster preformation. And $|h|$ values of the doubly odd nucleus are bigger than those of the odd-A nucleus because the doubly odd nucleus has two unpaired nucleons, which are consistent with the results in Fig. 5. Associating with Eq. (7), P_α^{Eq} can be obtained by

$$\begin{aligned} P_\alpha^{Eq} &= 10^{a+b(A_1^{1/6}+A_2^{1/6})+c\frac{N}{\sqrt{Q_\alpha}}+d\sqrt{l(l+1)}+h} \\ &= 10^{a+b(A_1^{1/6}+A_2^{1/6})+c\frac{N}{\sqrt{Q_\alpha}}+d\sqrt{l(l+1)}} \times 10^h. \end{aligned} \quad (8)$$

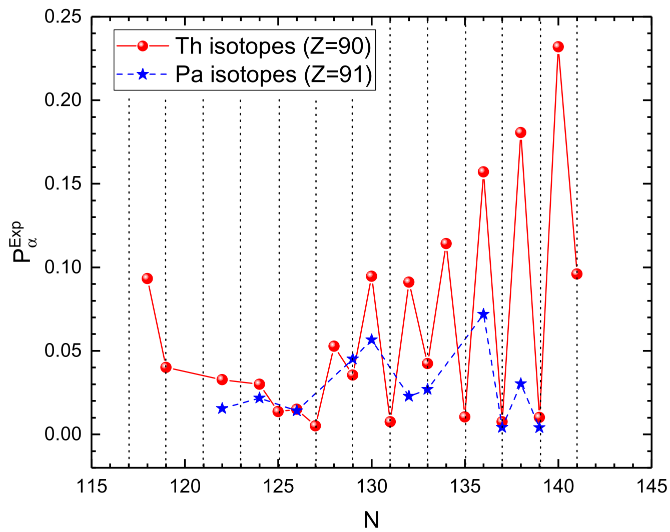

Fig. 5. The extracted experimental α-particle preformation factors from Eq. (4) for Th (denoted as red circle) and Pa (denoted as blue star) isotopes.

Therefore, for $N \leq 126$, the α-particle preformation factors of odd-A nuclei and doubly odd nuclei will be reduced $1-10^{-0.150} = 29.2\%$ and $1-10^{-0.652} = 77.7\%$ compared to the corresponding even-even nucleus due to the existence of unpaired nucleons. And for $N > 126$, the α-particle preformation factors of odd-A nuclei and doubly odd nuclei will be reduced $1-10^{-0.323} = 52.5\%$ and $1-10^{-0.851} = 85.9\%$ compared to the corresponding even-even nucleus because of the existence of unpaired nucleons. In addition, the strengths of centrifugal potential and blocking effect terms show that the sequence of nuclei in the order of decreasing α-particle preformation factors is the even-even nucleus, odd-A nucleus, and doubly odd nuclei, which satisfy the variation tendencies of α-particle preformation factors obtained by different models [17–20,26,27,45,46]. The strengths of centrifugal potential and blocking effect terms also indicate that α-particle is easier to form on the surface of an even-even nucleus.

Eq. (7) is based on the direct observation quantity of the nuclear shell effect i.e., α decay energy, to calculate the α-particle preformation factor, which is different from a few existing formulas based on the idea of valence nucleons (holes) scheme [21–25]. Therefore, this formula has the following advantages over other formulas. The first key advantage is that this formula reflects the shell effect from a new perspective and establishes a connection between α decay energy and α-particle preformation factor. If α decay energy is available, one can easily estimate the α-particle preformation factor. The second major superiority is that this formula is independent of magic numbers. For nuclei whose nucleons are located between the two major shells, this formula is more convenient to calculate their α-particle preformation factor. The third important strength is that this formula also can easily extend to the superheavy nuclei region, which is useful for future experiments in synthesizing new superheavy elements and isotopes. The fourth important advantage is that this formula has only 12 parameters, and 22 parameters less than the original formula that is proposed in recent work [28]. The fifth significant benefit is that this formula introduces the hindrance effect of centrifugal potential and blocking effect of unpaired nucleons. Thus this formula can uniformly describe the favored decay and unfavored α decay of even-even, odd-A, and doubly odd nuclei.

Both extracted experimental α-particle preformation factors P_α^{Exp} by Eq. (4) and calculated one P_α^{Eq} from Eq. (7) are shown in Fig. 6 for even-even nuclei (up), odd-A nuclei (middle), and doubly odd nuclei (bottom), respectively. One can see that this figure

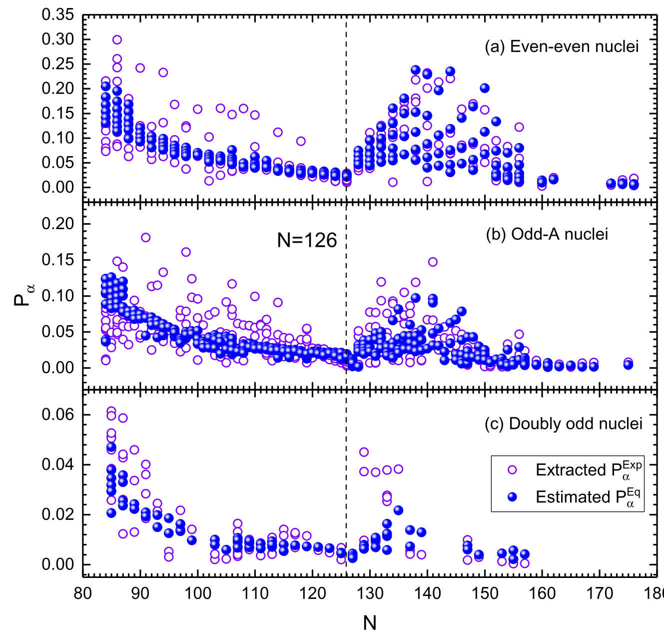

Fig. 6. The extracted experimental α-particle preformation factors P_α^{Exp} (denoted as purple open circle) from Eq. (4) and the estimated ones P_α^{Eq} (denoted as blue solid circle) from Eq. (7) of even-even nuclei (up), odd-A nuclei (middle) and doubly odd nuclei (bottom), respectively.

shows excellent consistencies of values and variation tendencies between estimated P_α^{Eq} by Eq. (7) and extracted P_α^{Exp} from Eq. (4). Before $N \leq 126$, both P_α^{Eq} and P_α^{Exp} decrease with neutron numbers increasing. After neutron numbers cross $N = 126$ closed shell, P_α^{Eq} and P_α^{Exp} rapidly increase until neutron numbers are close to the next neutron shell closure.

In order to examine the agreements of estimated α-particle preformation factors P_α^{Eq} by Eq. (7) with extracted experimental ones P_α^{Exp} from Eq. (4), the standard deviations for all 535 nuclei, including 159 even-even nuclei, 295 odd-A nuclei, and 81 doubly odd nuclei, are calculated by

$$\sigma = \sqrt{\frac{1}{n}\sum(\log_{10} P_\alpha^{Eq} - \log_{10} P_\alpha^{Exp})^2}. \tag{9}$$

The standard deviation values for even-even nuclei, odd-A nuclei, doubly odd nuclei, and all 535 nuclei are 0.218, 0.314, 0.339, and 0.293, respectively. For even-even nuclei, odd-A nuclei, doubly odd nuclei, and all 535 nuclei, the P_α^{Eq} can well reproduce P_α^{Exp} within factors of $10^{0.218} = 1.65$, $10^{0.314} = 2.06$, $10^{0.339} = 2.18$, and $10^{0.293} = 1.96$, respectively. When P_α^{Eq} are used to calculate α decay half-lives, the calculations can well reproduce experimental α decay half-lives data with the same precision as α-particle preformation factors for corresponding nuclei types. The satisfactory standard deviation values also demonstrate that a nice linear relationship exists between $\log_{10} P_\alpha$ and $Q_\alpha^{-1/2}$ as well as Geiger-Nuttall law can not only describe α decay half-lives but deal with α-particle preformation factors.

Recently, Xu et al. presented a microscopic calculation of α-cluster formation in heavy nuclei ^{104}Te (α+^{100}Sn), ^{212}Po (α+^{208}Pb), and their neighbors by using the quartetting wave function approach [11,47]. Table 2 lists the extracted α-particle preformation factors P_α^{Exp} and calculated α decay half-lives $\lg T_{1/2}^{Cal1}$ within the GLDM and ones calculated by using the quartetting wave function approach [11,47] for nuclei around the doubly magic nuclei. In this table, we can see that the P_α^{Exp} can well reproduce the results by using the quartetting wave function approach [11,47] and show

Table 2
The extracted α-particle preformation factors P_α^{Exp} and calculated half-lives $\lg T_{1/2}^{\mathrm{Cal1}}$ within the GLDM and ones calculated by using the quartetting wave function approach [11,47]. For ^{210}Pb, ^{210}Po, and ^{212}Po, experimental α decay half-lives and α decay energy are taken from the Ref. [49] and Refs. [50,51], respectively. The experimental data of ^{104}Te are taken from the Ref. [52]. The α-decay energy and half-lives are in the unit of MeV and s, respectively. The "$\lg T_{1/2}^{\mathrm{Cal}}$" denote the logarithmic values of α-decay half-lives "$\log_{10} T_{1/2}^{\mathrm{Cal}}$".

Nuclei	Q_α^{Exp}	$\lg T_{1/2}^{\mathrm{Exp}}$	P_α^{Exp}	$\lg T_{1/2}^{\mathrm{Cal1}}$	P_α^{Cal2} [47]	$\lg T_{1/2}^{\mathrm{Cal2}}$ [47]	P_α^{Cal3} [11]	$\lg T_{1/2}^{\mathrm{Cal3}}$ [11]
^{104}Te	5.10	< -7.74	0.815	−7.93	0.724	−7.83		
^{210}Pb	3.79	16.57	0.028	15.18	0.018	16.25		
^{210}Po	5.41	7.08	0.007	6.42	0.014	7.03		
^{212}Po	8.95	−6.53	0.034	−6.87	0.105	−6.47	0.142	−6.52

the same change trends. Both our calculated α decay half-lives $\lg T_{1/2}^{\mathrm{Cal1}}$ and the calculations in Refs. [11,47] are in good agreement with the experimental data. For ^{210}Pb, both P_α^{Exp} and the results in Refs. [11,47] are less than those of ^{212}Po due to the proton number of ^{210}Pb is magic number $Z = 82$. And for ^{210}Po, both P_α^{Exp} and the results in Refs. [11,47] are also less than those of ^{212}Po since the neutron number of ^{210}Po is magic number $N = 126$. It indicates that the closer proton or neutron number is to the full shell, the more difficult it is to form an α-cluster on the surface of the nucleus during α decay. This is consistent with our previous discussion, i.e., the nuclear nucleons configuration and shell structure play key roles in α-cluster preformation. In addition, for ^{104}Te, both P_α^{Exp} and results in Refs. [11,47] are enhanced significantly implying that the interactions between protons and neutrons occupying similar single-particle orbitals could enhance the α-particle preformation factors and result in the superallowed α decay. A detailed study on this issue has also been submitted [48].

In summary, we systematically study the α-particle preformation factors, which are extracted from the ratios between calculated α-decay half-lives and experimental data. The results show that α-particle preformation factors P_α and α decay energy Q_α are two important observed quantities for revealing the nuclear shell structure information. And there is a remarkable linear relationship between $\log_{10} P_\alpha$ and $Q_\alpha^{-1/2}$ indicating that the famous Geiger-Nuttall law can not only describe α decay half-lives but deal with α-particle preformation factors, as well as the nuclear shell effects play key roles in α-particle preformation. Furthermore, the results indicate that $Z = 82$ and $N = 126$ closed shells play more important roles than $Z = 50$ and $N = 82$ shell closures, and the shell effect of $Z = 82$ is weaker than that of $N = 126$ in the α decay process. In addition, the results also show that the unpaired nucleons will inhibit the preformation of the α-cluster on the surface of its α decaying parent nucleus. Based on these important physical effects, we propose a global analytic formula for estimating α-particle preformation factors with only 12 parameters, which has 22 parameters less than the original formula. After considering the α-particle preformation factors obtained by this formula, for 535 nuclei, calculated α decay half-lives can reproduce experimental data varying from 6.90×10^{-8} to 6.34×10^{26} s within a factor of 1.96. It is well known that the physics of the atomic nucleus constitutes a true many-body problem, where the number of constituents is too large for exact calculations but too small for applying the methods of statistical physics, and the incomplete knowledge of nuclear force, so it is difficult to obtain α-particle preformation factor microscopically. The new formula of α-particle preformation factor is proposed associated with Geiger-Nuttall law in the letter, which will shed some light on α decay of nuclear physics in the future.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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