

Systematic dependence of asymmetric parameter and evidence of Z=64 subshell effect in rare earth region

Reetu Kaushik¹, S. Sharma^{*2} and J. B. Gupta³

¹Research Scholar, Department of Physics, Mewar University, Gangrar, Rajasthan-312901, India

²Panchwati Institute of Engineering and Technology (Uttar Pradesh Technical University),

National Highway- 58, Ghat Institutional Area, Meerut, PIN 250005, INDIA

³Ranjnas College, University of Delhi, Delhi 110007, India

email: ss110096@gmail.com

The nuclear shape-phase transition at N = 88–90, and the role of Z = 64 sub-shell closure effect therein has been a subject of study on empirical basis and in the context of the N, Z, P and NpNn scheme [1,2,3,4]. It was pointed out by Gupta [1] that the filling of neutron orbitals at N = 86, 88 & 90 plays an important role in the shape-phase transition. The filling of the proton Nilsson orbitals of varying slopes leads to the variation of nuclear structure with varying Z, which leads to the Z = 64 subshell effect, which disappears at N=90. Gupta [1] observed that the effect of n-p interaction of the $\pi h_{11/2}$ and $\nu h_{9/2}$ orbitals, along with the contribution of the $\nu i_{13/2}$ orbital leads to the shape-phase transition at N=88–90. The slopes of proton Nilsson orbitals explain the Z= 64 subshell effect.

The size of proton subshell gap between the $d_{5/2}$ and $h_{11/2}$ orbitals was reviewed by Gupta [2]. The increased gap at Z=64 was not inconsistent with experiment for occupation probabilities and the nuclear structure of N=82 isotones and ¹⁴⁶⁻¹⁵⁴Sm [2]. In IBM calculation there was no *a priori* need of the use of subshell closure [2]. In the empirical studies for this region the use of the Z=64 subshell does lead to elegant systematics in some cases. In this paper we have tested this for the asymmetry parameter (γ).

Asymmetric Rotor Model (ARM):

The Hamiltonian of ARM[5] is:

$$H = (\hbar^2/2) \sum (I_i^2/J_i) \quad (1)$$

where, I_i is the projection of angular momentum on the intrinsic axes. The moment of inertia J_i are given by the hydrodynamic relation:

$$J_i = (4/3) J_0 \sin^2(\gamma - 2\pi/3) \quad (2)$$

$$\text{where, } J_0 = 4B \beta^2. \quad (3)$$

Simple analytical expressions for the energy of two levels for $I = 2$, defined as:

$$E_{2^+ 1,2} = \frac{E[1 - (-1)^{\sigma_{1,2}}] \sqrt{1 - \frac{E}{X}}}{X} \quad (\text{in units of } \hbar^2/J_0) \quad (4)$$

where $\sigma_{1,2}=0,1$ and γ -function is written as $E_{2^+ 1,2}$.

$$\text{Here, } E_{2^+ 1} = \frac{E - \gamma \sqrt{81 - 72X}}{X} \quad \text{and } X = \sin^2 3\gamma.$$

Calculation of Asymmetric Parameter

The value of asymmetry parameter (γ) can be evaluated [6,7,8] using the experimental energies E_{2^+} and E_{2^+} of the two 2^+ states. The energy ratio $R\gamma = E_{2^+}/E_{2g}$. The asymmetry parameter is:

$$\gamma = (1/3) \sin^{-1} \left\{ \frac{9}{8} \left[1 - \frac{(R\gamma - 1)}{(R\gamma + 1)^2} \right] \right\}.$$

It can be evaluated using: (a) The energy ratio $R_4 = (E_{4g}/E_{2g})$ but only the nuclei with $2.8 \leq R_4 \leq 3.33$ will be allowed [6,8] (see fig 1 of ref.6). (b) The $B(E2)$ values also but these values are very small and available with uncertainties too. Therefore the values from energy ratio $R\gamma$ are more reliable.

Result and Discussions:

The variation of $E_{2\gamma}$ state (in MeV) versus neutron number (N) is shown in Fig. 1 for N=86-122. The data points are joined for same element so the N dependence of $E_{2\gamma}$ is visible. The value of $E_{2\gamma}$ is having maximum scattering (0.7 to 1.6 MeV) at N=104 for Yb to Pt isotopes corresponding to β hard core structure of these nuclei[9] The fig. 1 is reproduced from [9].

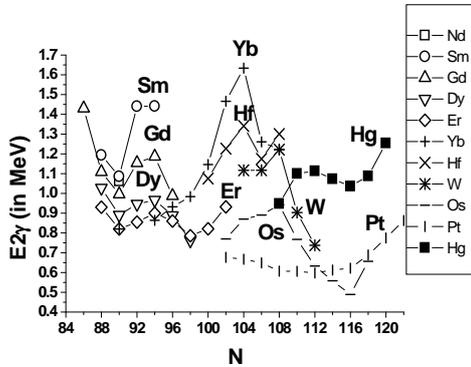


Fig. 1. The variation of energy of 2γ state (in MeV) versus neutron number (N).

The variation of $E_{2\gamma}$ versus proton number (Z) is shown in Fig. 2 for $Z = 60 - 80$. The data points are joined for each isotones for $N = 86 - 116$. The value of $E_{2\gamma}$ is suddenly increasing from 0.8 to 1.6 MeV for a fixed value of $Z = 70$ when N is changing from 90 to 104. The $E_{2\gamma}$ decreases sharply on increasing Z from 60 to 68 for each isotones i.e. $N = 88 - 98$ indication shape phase transition from Vibration to Rotation i.e. $SU(5)$ to $SU(3)$ limits of IBM. The slope for $N = 88$ and 90 are same and there is no indication for subshell effect in this fig.

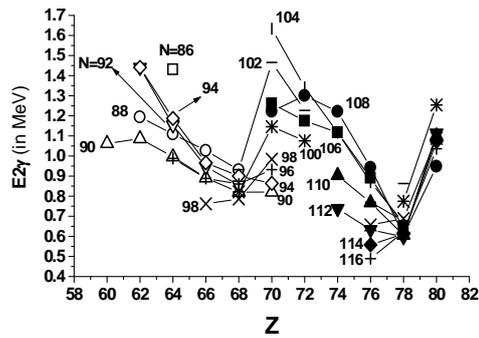


Fig. 2. The variation of energy of 2γ state (in MeV) versus proton number (Z) for $Z = 60 - 80$.

The variation of asymmetric parameter (γ) versus proton number (Z) for $N = 82 - 96$ isotones for $Z = 58 - 72$ region is shown in Fig.3. The gap is maximum i.e. 7.6 at $Z = 64$ when N changes

from 88 to 90 indication the subshell effect at $Z = 64$ for $N < 90$. Since the γ is evaluated from E_{2g} and $E_{2\gamma}$. However the $Z = 64$ subshell effect is not evident in $E_{2\gamma}$ (see fig. 2) and in $E_{0\beta}$ (see fig.4 ref. [9]). It is evident only in E_{2g} [4] and R_4 [4 and see fig. 12 of ref. 10].

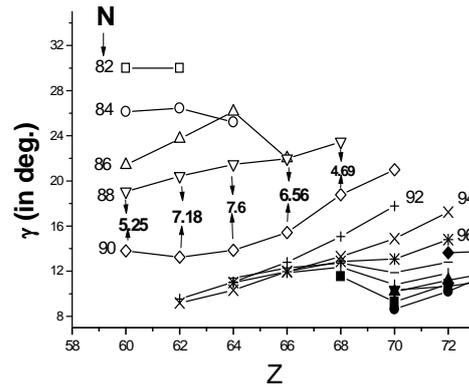


Fig. 3. The variation of asymmetric parameter (γ) versus proton number (Z) for $N = 82 - 96$ isotones for $Z = 58 - 72$ region.

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