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MONOPOLE CONTRIBUTION TO THE WILSON LOOP IN THE $3D$ $SU(2)$ LATTICE GAUGE MODEL

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Using a plaquette formulation for lattice gauge models we describe monopoles of the $3D$ $SU(2)$ theory which appear as configurations in the complete axial gauge and violate the continuum Bianchi identity. Furthemore we derive a dual representation for the Wilson loop in arbitrary representation and calculate the form of the interaction between generated electric flux and monopoles in the region of weak coupling relevant for the continuum limit. The effective theory which controls the interaction is a generalized version of the sine-Gordon model. The mechanism of confinement is proposed on the basis of the effective model obtained.

1 Introduction

The problem of the permanent confinement of quarks inside hadrons attracts attention of the theoretical physicists for the last three decades (see [1] and refs. therein for a recent review of the problem). Two of the most popular and the most elaborated mechanisms of confinement are based on the condensation of certain topologically nontrivial configurations - the so-called center vortices or monopoles. In this paper we are interested in the second of these configurations. It was proposed in [2] in the context of continuum compact three dimensional ($3D$) electrodynamics that the string tension is nonvanishing in this theory at any positive coupling constant, and the contribution of monopoles to the Wilson loop was estimated in the semiclassical approximation. Later this consideration was extended to $U(1)$ lattice gauge theory (LGT) in $3D$ [3]. It turns out that these are precisely monopole configurations which make the string tension nonvanishing at all couplings. A rigorous proof of this property was done in [4]. While monopoles of abelian gauge models can be given a gauge invariant definition it is not the case for nonabelian models. The most popular approach consists in a partial gauge fixing such that some abelian subgroup of the full nonabelian group remains unbroken. Then, one can define monopoles in a nonabelian theory as monopoles of the unbroken abelian subgroup. Here we propose a different route to define monopoles in nonabelian models. Its main feature is complete gauge fixing. Monopoles appear as defects of smooth gauge fields which violate the Bianchi identity in the continuum limit, in the full analogy with abelian models. Our principal approach is to rewrite the compact LGT in the plaquette (continuum field-strength) representation and to find a dual form of the nonabelian theory. The Bianchi identity appears in such formulation as a condition on the admissible configurations. This allows to reveal the relevant field configurations contributing to the partition function and various observables. Such a program was accomplished for the abelian LGT in [3]. Here we are going to work out the corresponding approach for nonabelian models on the example of $3D$ $SU(2)$ LGT.

2 Plaquette formulation and monopoles

The standard and possibly the only one available now tool of an investigation of such nonperturbative phenomenon like confinement is a quantization of the gauge fields on the lattice. Originally, LGT was formulated by K. Wilson in terms of group valued matrices on links of the lattice as fundamental degrees of freedom [5]. The partition function reads

$$Z = \int DU \ \exp\{-\beta S[U_\mu(x)]\}, \tag{1}$$

where S is the standard Wilson action and the integral is calculated over the Haar measure on the group at every link of the lattice.

The plaquette representation has been invented originally in the continuum theory by M. Halpern and extended to lattice models by G. Batrouni [6]. In this representation the plaquette matrices play the role of the dynamical degrees of freedom and satisfy certain constraints expressed through Bianchi identities in every cube of the lattice. In papers [7], [8], [9] we have developed a different plaquette formulation which we outline below.

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In the complete axial gauge

$$U_3(x,y,z) = U_2(x,y,0) = U_1(x,0,0) = I \tag{2}$$

the partition function (1) can be identically rewritten on the dual lattice as

$$Z = \int \prod_l dV_l \exp[\beta \sum_l \mathrm{Re}\ \mathrm{Tr} V_l] \prod_{i=1}^{4} \prod_{x(i)} J\left(V_x^{(i)}\right). \tag{3}$$

Here, V_l is a plaquette (dual link l) matrix which satisfies constraint expressed through the group delta-function

$$J(V_x) = \sum_r d_r \chi_r\left(V_x\right), \tag{4}$$

where the sum over r is a sum over all representations of $SU(N)$, χ_r is character of r-th representation and $d_r = \chi_r(I)$. V_x is a certain product of plaquette matrices around a cube (dual site x) of the lattice taken with the corresponding connectors. Connectors provide correct parallel transport of opposite sites of a given cube for nonabelian theory. In abelain models connectors are canceled out of group delta-functions. There appear four different types of connectors in our construction. E.g., V_x for the first type is of the form

$$V_x^{(1)} = V_{l_5}^{\dagger} V_{l_1} V_{l_6}^{\dagger}\ C_{\vec{x}(1)}\ V_{l_2} V_{l_3} V_{l_4}^{\dagger}\ C_{\vec{x}(1)}^{\dagger}, \tag{5}$$

$$C_{\vec{x}(1)} = \prod_{k=z_i-1}^{1} V_{n_2}(x_i, y_i-1, k) \prod_{p=1}^{z_i-1} V_{n_1}(x_i-1, y_i, p). \tag{6}$$

In what follows we consider the $SU(2)$ gauge group. In this case it is easy to show that the constraint (4) expressed through elements of an algebra of $SU(2)$ reads

$$\left[\sum_k \omega_k^2(x)\right]^{1/2} = 2\pi m(x), \tag{7}$$

where $m(x)$ is arbitrary integers and

$$\omega_k(x) = \sum_{i=1}^{6} \theta_k(l_i) - \epsilon_{kmn} \left(\sum_{i<j}^{6} \theta_m(l_i)\theta_n(l_j) + 2\sum_{b\in C} \theta_m(b) \sum_{i=4}^{6} \theta_n(l_i) + \dots \right). \tag{8}$$

In the continuum limit the last constraint reduces to the familiar Bianchi identity if one takes $m(x) = 0$ for all x. However, when $m(x)$ differs from zero one gets violation of the continuum Bianchi identity at the point x. This is genuine feature of compact gauge models. Below we want to clarify a role of these configurations in producing the string tension. Clearly, $m(x) \neq 0$ configuration corresponds to the monopole configuration of nonabelian gauge field. Therefore, we may interpret the summation over $m(x)$, appearing below, as a summation over monopole charges which exist due to the periodicity of $SU(2)$ delta-function (in close analogy with $U(1)$ model).

Substituting (7) into (4) one can prove that the partition function (3) can be exactly rewritten to the following form [10]

$$Z_{SU(2)} = \int \prod_l \left[\frac{\sin^2 W_l}{W_l^2} \prod_k d\omega_k(l)\right] \exp\left[2\beta \sum_l \cos W_l\right] \prod_x \frac{W_x}{\sin W_x}$$
$$\prod_x \sum_{m(x)=-\infty}^{\infty} \int \prod_k d\alpha_k(x) \exp\left[-i \sum_k \alpha_k(x)\omega_k(x) + 2\pi i m(x)\alpha(x)\right], \tag{9}$$

where $\alpha(x) = (\sum_k \alpha_k^2(x))^{1/2}$.

The Wilson loop of the size $R \times T$ in some representation j gets the following form

$$W_j(C) = \mathrm{Tr} \prod_{n=R/2-1}^{0} \left(\prod_{z_1=0}^{z+T-1} V_1^{\dagger}(x, y+2n+1, z_1) \prod_{z_2=z+T-1}^{0} V_1^{\dagger}(x, y+2n, z_2) \right). \tag{10}$$

We have supposed, for simplicity that the loop contour lies in the $y-z$ plane, one side of the loop lies in the plane $z=0$ and R, T are even.

3 Effective monopole model for the Wilson loop

Here we would like to calculate the contribution of monopole configurations to the Wilson loop and estimate the string tension. We remind first the computations for the $U(1)$ compact model and then proceed to the nonabelian theory.

3.1 Monopoles in $U(1)$ LGT

The plaquette formulation of the $U(1)$ LGT on the dual lattice reads

$$Z(h) = \int_0^{2\pi} \prod_l d\omega_l \exp\left[\beta \cos\omega_l\right] \int_{-\infty}^{\infty} \prod_x dr_x \sum_{m_x=-\infty}^{\infty} \exp\left[i\sum_l \omega_l (r_x - r_{x+n}) + 2\pi i \sum_x r_x m_x + i \sum_l \omega_l h_l \right], \quad (11)$$

where the Bianchi identity has the form

$$\omega_x = \sum_{l\in x} \omega_l = 2\pi m_x. \quad (12)$$

Sources h_l have been introduced to represent the Wilson loop. Configurations with $m_x \neq 0$ violate the continuum Bianchi identity in the same way as they do for the compact $SU(2)$ model.

Consider the Wilson loop in the representation j. Let S^d_{xy} be some surface dual to the surface S_{xy} which is bounded by the loop C and consisting of links dual to plaquettes of the original lattice. Let b denote links from S^d_{xy}. Then, the expectation value of the Wilson loop takes the following form

$$\langle W(C) \rangle = \frac{1}{Z(0)} \exp\left[-\frac{j^2}{4\beta} \sum_{b,b'\in S^d_{xy}} G_{bb'} \right] \sum_{m_x=-\infty}^{\infty} \exp\left[-\pi^2 \beta m_x G_{x,x'} m_{x'} + i\pi j \sum_{b\in S^d_{xy}} D_b(x') m_{x'} \right], \quad (13)$$

where we have introduced the link Green functions $G_{ll'}$ and $D_l(x)$ (see [7]). Following strategy of [2], [3] one can use the dilute monopole gas approximation to perform summation over m_x. We skip all technical details which are well known. The resulting theory appears to be of the sin-Gordon type

$$\begin{aligned} \langle W(C) \rangle &= \exp\left[-\frac{j^2}{4\beta} \sum_{bb'\in S^d_{xy}} G_{bb'} \right] \int \prod_x d\phi_x \exp\left[-\frac{1}{2\beta} \sum_{x,n} (\phi_x - \phi_{x+n})^2 \right] \\ &\times \exp\left[2m^2 \sum_x \cos\left(\pi\phi_x + \pi j \sum_{b\in S^d_{xy}} D_b(x) \right) \right] \frac{1}{Z(0)}, \end{aligned} \quad (14)$$

where m^2 is a mass of the dual photons (it is exponentially small in β). To analyze this theory one can use the semiclassical approximation. The saddle-point equation is

$$\Delta\alpha(x) = 2\pi j \delta^{'}(x) - m^2 \sin\alpha(x) \, . \quad (15)$$

Far from the boundaries of the contour C the saddle-point equation (15) is essentially one dimensional and has the solution for $j = 1$

$$\alpha(z) = \begin{cases} 4\arctan(e^{-mz}), & z > 0 \\ -4\arctan(e^{mz}), & z < 0 \, . \end{cases}$$

Substituting this solution into (14) one can easily gets the area law for the Wilson loop

$$\langle W_j(C) \rangle = e^{-\sigma(j=1)S}$$

with the string tension given by

$$a^2\sigma(j=1) = \frac{8}{\sqrt{2\pi^2\beta}} \exp\left[-\frac{1}{2}\pi^2\beta G_0 \right] \, . \quad (16)$$

Here, $\beta = 1/(g^2 a)$ is dimensionless coupling constant and $G_0 \approx 0.5054$ is zero-distance Green function. A rigorous proof of permanent confinement was given in [4]. It was shown that the semiclassical expression (16) gives lower bound on the string tension.

3.2 $SU(2)$ LGT. Representation for the Wilson loop

Here we would like to extend calculations of the previous section to $SU(2)$ gauge theory. In doing this we use three approximations. First of all, we neglect connectors of the Bianchi identity because they do not contribute to the string tension in the leading orders of the strong coupling expansion. We thus assume that at weak couplings connectors produce smooth corrections to spin waves. The second approximation consists in an expansion of the Wilson loop in a power series $1/\beta$. Finally, we restrict ourselves to monopole configurations $m = 0, \pm 1$, precisely like for the $U(1)$ model. At large β, and using first of our approximations one obtains from (9) the following expression

$$Z_{SU(2)} = \sum_{m(x)=-\infty}^{\infty} \int_{-\infty}^{\infty} \prod_{l,k} d\omega_k(l) \int_{-\infty}^{\infty} \prod_{x,k} d\alpha_k(x) \times$$
$$\exp[-\frac{1}{2}\omega_k^2(l) - i\omega_k(l)\left(\alpha_k(x+e_n) - \alpha_k(x)\right) + 2\pi i\sqrt{2\beta}\sum_x \alpha(x)m(x)]. \tag{17}$$

Obviously, the last expression is an analog of the formula (11) for the $U(1)$ model. However, even in this case all integrations cannot be done exactly due to non-linear couplings of monopoles with auxiliary fields.

As before, S^d_{xy} denotes some surface dual to the surface S_{xy} which is bounded by the loop C. Then, the expectation value of $W(C)$ at $\beta \to \infty$ we present in the form

$$\langle W_j(C)\rangle \;=\; \prod_{l\in S}\int_0^{\pi} \sin\alpha_l d\alpha_l \int_0^{2\pi} \frac{\varphi_l}{\sqrt{4\pi}}\; TR(C)\; H_j\;,$$

where

$$H_j \equiv H_j(\alpha_l, \varphi_l) \;=\; \left\langle \prod_{l\in S} Q_j(l) \right\rangle \tag{18}$$

and

$$TR(C) = \sum_{m_l=-j}^{j} \prod_{l=1}^{\nu[S]} \frac{1}{2j+1} \sum_{\lambda,k} \sqrt{2\lambda+1}\; C^{jn}_{jm\ \lambda k} Y_{\lambda k}\left(\alpha, \varphi\right)\;.$$

Here $C^{c\gamma}_{a\alpha\ b\beta}$ is the Clebsch-Gordan coefficient, $Y_{\lambda k}$ is the spherical function and $\nu(S)$ is a number of dual links that belong to the Wilson loop. Since at large β the plaquette matrix fluctuates smoothly around unit matrix $\omega(l) \approx 0$ it is allowed to use asymptotics of $Q_j(l)$ in (18) at $\omega \approx 0$ uniformly valid in j. This asymptotics is

$$Q_j(l) \;=\; \exp[-ij_k(l)\omega_k(l)]\;,$$

where

$$\omega_1 \;=\; \omega\cos\theta,\ \omega_2 \;=\; \omega\sin\theta\cos\phi,\ \omega_3 \;=\; \omega\sin\theta\sin\phi$$

and

$$j_1 \;=\; \sqrt{j(j+1)}\cos\alpha,\ j_2 \;=\; \sqrt{j(j+1)}\sin\alpha\cos\varphi,\ j_3 \;=\; \sqrt{j(j+1)}\sin\alpha\sin\varphi\;.$$

Introducing sources like

$$J_k(l) \;=\; \begin{cases} j_k(l)/\sqrt{2\beta},\ l \in S \\ 0,\ l \neq S \end{cases} \tag{19}$$

the effective monopole theory can be written down as

$$H_j = \frac{1}{Z}\sum_{m(x)=-\infty}^{\infty} \int_{-\infty}^{\infty} \prod_{l,k} d\omega_k(l) \exp\Bigg[-\frac{\omega_k^2(l)}{2} - i\omega_k(l)\left[\alpha_k(x+e_n) - \alpha_k(x)\right]$$
$$+2\pi i\sqrt{2\beta}\sum_x \alpha(x)m(x) - iJ_k(l)\omega_k(l)\Bigg]\,. \tag{20}$$

We use the following representation to perform the integration over $\alpha_k(x)$

$$\sum_{m(x)=-\infty}^{\infty} \exp[2\pi i\sqrt{2\beta}\sum_x \alpha(x)m(x)] = \sum_{m(x)=-\infty}^{\infty} \sqrt{m(x)}\left(1+\xi\frac{\partial}{\partial\xi}\right)$$
$$\times \int\prod_{k=1}^{3} d\sigma_k(x)\frac{\delta(m^2(x) - \sum_k \sigma_k^2(x))}{V(S^2)} \exp\left[i\xi\alpha_k(x)\sigma_k(x)\right], \tag{21}$$

where $\xi = 2\pi\sqrt{2\beta}$. Integration over $\omega_k(l)$ and $\alpha_k(x)$ gives

$$H_j = H_j^{gl}\ H_j^{mon}\ , \tag{22}$$

$$H_j^{gl} \ = \ \exp[-\frac{1}{4}J_k(l)G_{ll'}J_k(l')]\ ,$$

$$H_j^{mon} = \frac{1}{Z}\sum_{m(x)=-\infty}^{\infty}\sqrt{m_x}\left(1+\xi\frac{\partial}{\partial\xi}\right)\int\prod_{k=1}^{3}d\sigma_k(x)\frac{\delta\left(\sum_k\sigma_k^2(x)-1\right)}{V(S^2)}\exp\left[S_{eff}\right]$$

where the effective action S_{eff} is of the form

$$S_{eff} = -\frac{1}{4}\xi_x m(x)\sigma_k(x)G_{xx'}\sigma_k(x')m(x')\xi_{x'} + \frac{i}{2}D_l(x)\xi_x m(x)\sigma_k(x)J_k(l)\ .$$

Derivatives are calculated at $\xi_x = \xi = 2\pi\sqrt{2\beta}$. One proves that at large j this leads to the representation

$$\langle W_j(C)\rangle \ = \ \int_0^{\pi}\sin\alpha\, d\alpha\int_0^{2\pi}\frac{d\varphi}{\sqrt{4\pi}}\ H_j(\alpha,\varphi).$$

As is known the dual photon contribution H_j^{gl} produces only the perimeter law. In the next subsection we evaluate in the semiclassical approximation contribution of H_j^{mon} to the Wilson loop.

3.3 $SU(2)$ LGT. Sine-Gordon type model

In order to perform the summation over monopole configurations $m_x = 0, \pm 1$ we follow the strategy of Refs.[4], [11]. Using decomposition

$$G_{xx'} = B_{xx'} + G_{xx'}(M)\ ,$$

where

$$G_{xx'}(M) = \frac{1}{L^3}\sum_{k_n}\frac{e^{\frac{2\pi}{L}k_n(x-x')_n}}{3-\sum_n\cos[\frac{2\pi}{L}k_n]+\frac{1}{2}M^2}\ ,$$

$$G_{xx'} = G_{xx'}(M=0)\ ,\ B_{xx'} = G_{xx'} - G_{xx'}(M)$$

we rewrite the effective action in the form ($\eta_k(x) = \xi_x m(x)\sigma_k(x)$)

$$\begin{aligned} S_{eff} \ &= \ -\frac{1}{4}\eta_k(x)B_{xx'}\eta_k(x') - \frac{1}{4}G_0\sum_x\xi_x^2 m_x^2 \\ &- \ \frac{1}{4}\sum_{x\neq x'}\eta_k(x)G_{xx'}(M)\eta_k(x') + \frac{i}{2}D_l(x)\eta_x J_k(l). \end{aligned} \tag{23}$$

The first term in (23) is presented as

$$\begin{aligned} \exp\left[-\frac{1}{4}\eta_k(x)B_{xx'}\eta_k(x')\right] &= (\det B_{xx'}^{-1})^{3/2} \\ \times\int_{-\infty}^{\infty}\prod_{x,k}d\alpha_k(x)&\exp\left[-\alpha_k(x)B_{xx'}^{-1}\alpha_k(x') + i\alpha_k(x)\eta_k(x)\right]. \end{aligned} \tag{24}$$

The behaviour of $G_{xx'}(M)$ in the thermodynamic and continuum limits is well known

$$G_{xx'}(M) = \frac{2}{\pi R}e^{-MR/2}, \quad R = \left[\sum_k(x_k - x'_k)^2\right]^{1/2}. \tag{25}$$

This behaviour allows us to keep only self-energy of monopoles if $MR \gg 1$, i.e. the term

$$S_{eff}^{SE} = -\frac{1}{4}G_0(M)\xi^2\sum_x m_x^2\ . \tag{26}$$

Inserting (24) into (23) and taking into account (26) one can integrate out $\sigma_k(x)$. After taking all derivatives we keep in the sums over monopoles only configurations $m = 0, \pm 1$. This finally gives the effective model which appears to be of the sine-Gordon type

$$H_j^{mon} = \int_{-\infty}^{\infty}\prod_{x,k}d\alpha_k(x)\exp\left[-\alpha_k(x)B_{xx'}^{-1}\alpha_k(x') + \gamma\sum_x V[\alpha(x)]\right],$$

where

$$V[\alpha(x)] = \cos\xi\mu(x) - \frac{1}{2}G_0(M)\xi\,\frac{\sin(\xi\mu(x))}{\xi\mu(x)},$$

$$\gamma = 2\exp[-2\pi^2\beta G_0(M)], \quad \mu(x) = \left(\sum_k \mu_k^2(x)\right)^{1/2},$$

$$\mu_k(x) = \alpha_k(x) + \frac{1}{2}D_l(x)J_k(l).$$

Collecting all formulae together we get for the Wilson loop

$$\langle W_j(C)\rangle = \exp\left[-\frac{1}{4}J_k(l)G_{ll'}J_k(l')\right]\int_0^{\pi}\sin\alpha d\alpha\int_0^{2\pi}\frac{d\varphi}{\sqrt{4\pi}}\;H_j^{mon}\;. \tag{27}$$

Making use of the fact that $B^{-1}_{xx'}(M) \approx G^{-1}_{xx'}$ for M sufficiently large, one obtains after a shift

$$\alpha_k(x) \to \alpha_k(x) - \frac{1}{2}\sum_l D_l(x)J_k(l) \equiv \alpha_k(x) - h_k(x) \tag{28}$$

the following expression for the monopole contribution

$$H_j^{mon} = \int_{-\infty}^{\infty}\prod_{x,k} d\alpha_k(x)\exp[-S^m_{eff}], \tag{29}$$

where

$$S^{mon}_{eff}[\alpha_k(x)] = -\left[\alpha_k(x) - h_k(x)\right]G^{-1}_{xx'}\left[\alpha_k(x') - h_k(x')\right] + \gamma\sum_x V[\alpha(x)]\;, \tag{30}$$

$$V[\alpha(x)] = \cos\xi\alpha(x) - \frac{1}{2}G_0(M)\xi\,\frac{\sin(\xi\alpha(x))}{\alpha(x)}\;. \tag{31}$$

To make semiclassical calculations we take the continuum limit. In this limit the saddle-point equation reads

$$\Delta\alpha_k(x) = 2\pi\mu_k(x) - m^2\frac{\alpha_k(x)}{\alpha(x)}W[\alpha(x)]\;, \tag{32}$$

$$\mu_k(x) = \sum_{l\in S} j_k(l)\theta(l) = \sum_{y\in S} j_k(y,n)\theta_n(x-y)\;,$$

where n is fixed and orthogonal to S and

$$\theta_n(x-y) = \begin{cases} -1, & x = y \\ +1, & x = y+n \\ 0, & \text{otherwise}\;. \end{cases}$$

Here we have introduced the Debye mass

$$m^2 = 16\pi^2\beta\exp\left[-2\pi^2\beta G_0(M)\right]. \tag{33}$$

In the continuum limit one has

$$W[\alpha(x)] = \sin\alpha(x) + 4\pi^2\beta G_0(M)\left[\frac{\cos\alpha(x)}{\alpha(x)} - \frac{\sin\alpha(x)}{\alpha^2(x)}\right],$$

$$\mu_k(x) = \int_{y\in S} dy\; j_k(y, n=3)\;\delta_3'(x-y).$$

To find a solution of the saddle-point equation we insert the anzatz

$$\alpha_k(x) = j_k(z, n=3)\alpha(z).$$

This gives

$$\Delta\tilde{\alpha}(z) = \pi(2j+1)\delta_z'(z) - m^2\left(\sin\tilde{\alpha}(z) + 4\pi^2\beta G_0(M)\left[\frac{\cos\tilde{\alpha}(z)}{\tilde{\alpha}(z)} - \frac{\sin\tilde{\alpha}(z)}{\tilde{\alpha}^2(z)}\right]\right),$$

where

$$\tilde{\alpha}(z) = (2j+1)\alpha(z) \ .$$

One can easily construct the approximate solution if one takes $\beta G_0(M) \approx 0$. Then for $j = 1/2$ the saddle-point equation reduces to the form (15). It leads to the desirable area law

$$\langle W_j(C) \rangle = e^{-\sigma(j=1/2)S}$$

with the string tension

$$\sigma = \frac{4m}{\pi^2 \beta} \ .$$

The mass of dual photons are given in (33). This result coincides with that qouted in [11].

4 Conclusion

In this paper we calculated nontrivial $2D$ theory for the expectation value of the Wilson loop at large values of β valid for all values of representations j and which takes into account both the dual photon and the monopole contributions. For the fundamental representation in the semiclassical approximation we have found that the Wilson loop obeys the area law and $\sigma(j = 1/2) \sim m$. The most important conclusion is that the monopole contribution is sufficient to produce the area law and thus to explain confinement in $3D$ nonabelian models. It remains unclear at the moment if this contribution is also necessary condition of confinement. Another open problem is to compute the Wilson loop in the adjoint representation. It is well known that the adjoint string tension vanishes at large distances therefore it is important to understand if the proposed mechanism of confinement is able to reproduce this essential feature of the theory.

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