

MEASUREMENTS OF THE PHOTON AND ELECTRON  
STRUCTURE FUNCTIONS AT LEP\*

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The progress in the measurements of the photon and electron structure functions at LEP is discussed. The short introduction to the kinematics and theoretical framework of the structure functions measurements is given first. Then follow presentations of the recent measurements of the hadronic structure function of the photon, and in particular the charm structure function, which have been updated by using the full LEP2 statistics. The first measurement of the electron structure function is also presented.

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**1. Introduction**

The photon as the gauge boson of the quantum electrodynamics (QED) mediates the electromagnetic interactions between charged objects. In these interactions it can be regarded as a structureless object, and is called *direct*. However due to the uncertainty principle, the photon can fluctuate into a pair of fermion-antifermion system carrying the same quantum numbers as the photon. If during such a fluctuation, one of the fermions interacts with another object, then the structure of the photon is revealed, and the photon is called *resolved*. The structure of resolved photons is further subdivided into a part which is perturbatively calculable (called *point-like*) and the part where the photon fluctuates into a hadronic system (called *hadron-like*).

At LEP, the structure of the photon is studied in the interactions of electrons and positrons<sup>1</sup> proceeding via the exchange of two photons, one of them being almost real and the other virtual. Such a process is schematically shown in Fig. 1(a). It can be regarded as deep inelastic scattering

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<sup>1</sup> For conciseness, positrons are also referred to as electrons.

of the electron off the quasi-real target photon, and is experimentally identified by tagging the scattered electron in the detector. The variables in parentheses are the four-vectors of the particles shown in the diagram. The usual kinematical variables are defined in the following. The virtualities of the probe and target photons are given by  $Q^2$  and  $P^2$ , respectively:  $Q^2 \equiv -q^2 = -(k - k')^2 > 0$ , and  $P^2 \equiv -p^2 = -(l - l')^2 \approx 0$ . The center of mass energy of the  $e^+e^-$  system is given by  $s = (k + l)^2$  and the invariant mass squared of the two photon system reads  $W^2 = (q + p)^2$ . The usual Bjorken variables are:  $y_e = l \cdot q / l \cdot k$ ,  $x = Q^2 / 2q \cdot p$ ,  $z = Q^2 / 2q \cdot l$ , where  $x$  is calculated with respect to the target photon. However, the same process can be also interpreted as a deep inelastic scattering of the electron off the target electron (see Fig.1(b)). In that case the Bjorken scaling variable is called  $z$  and calculated with respect to the target electron. The kinematic

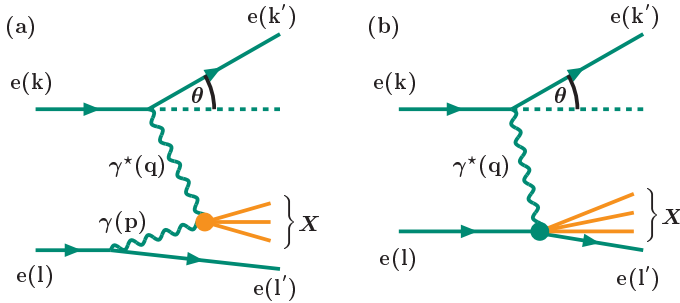


Fig. 1. Deep inelastic scattering of the electron off the quasi-real photon (a) and off the target electron (b).

variables are experimentally measured using the energy,  $E$ , and the polar angle,  $\theta$ , of the scattered electron. Only the measurement of  $x$  is based in addition on the measurement of  $W$ . As the significant part of the hadronic system is missing in the beam pipe, the measurement of  $W$  and consequently of  $x$  is much less precise than the measurement of  $z$ . Following formulas are used to calculate the kinematic variables:

$$Q^2 = 2EE_b(1 - \cos \theta), \quad y_e = 1 - \frac{E}{E_b} \cos^2 \frac{\theta}{2}, \quad x = \frac{Q^2}{Q^2 + W^2}, \quad z = \frac{Q^2}{y_e s}.$$

The cross section for the process shown in Fig. 1(a) can be expressed in terms of photon structure functions as follows:

$$\frac{d^4\sigma_{ee}}{dx dQ^2 dz dP^2} = \frac{2\pi\alpha^2}{x^2 Q^4} [(1 + (1 - y_e)^2) F_2^\gamma(x, Q^2) - y_e^2 F_L^\gamma(x, Q^2)] \hat{f}_\gamma\left(\frac{z}{x}, P^2\right),$$

where  $\hat{f}_\gamma^e(y, P^2) = \frac{\alpha}{2\pi} \frac{1}{P^2} \left[ \frac{1 + (1 - y)^2}{y} - 2y \frac{m_e^2}{P^2} \right]$  is the flux of quasi-real photons of transverse polarisation in the electron and the  $P^2$  dependence of the

structure functions has been neglected. Treating the target photon as real is an approximation, because in fact it is always off-shell. Integrating the flux over  $P^2$  and  $z$  we arrive at the standard DIS formula for  $e\gamma$  scattering:

$$\frac{d^2\sigma_{e\gamma}}{dx dQ^2} = \frac{2\pi\alpha^2}{xQ^4} [(1 + (1 - y_e)^2) F_2^\gamma(x, Q^2) - y_e^2 F_L^\gamma(x, Q^2)] .$$

On the other hand the cross section for the process shown in Fig. 1(b) can be immediately expressed in terms of the electron structure functions [1]:

$$\frac{d^2\sigma_{ee}}{dz dQ^2} = \frac{2\pi\alpha^2}{zQ^4} [(1 + (1 - y_e)^2) F_2^e(z, Q^2) - y_e^2 F_L^e(z, Q^2)] .$$

The photon structure function,  $F_2^\gamma(x, Q^2)$ , and the electron structure function,  $F_2^e(x, Q^2)$ , are related via the following integral equation:

$$F_2^e(z, Q^2) \equiv \int_z^1 dx \int_{P_{\min}^2(\frac{z}{x})}^{P_{\max}^2} dP^2 \frac{z}{x^2} F_2^\gamma(x, Q^2, P^2) \hat{f}_\gamma\left(\frac{z}{x}, P^2\right) .$$

## 2. Measurements

A recent collection of results concerning the photon structure function measurements at LEP can be found *e.g.* in [2]. Here only a brief account of the newest results is presented.

The measurements of the hadronic photon structure function  $F_2^\gamma$  have been recently extended to higher values of  $Q^2$ . In Fig. 2 the measurements of  $F_2^\gamma/\alpha$  as a function of  $x$  performed by the ALEPH experiment [3] are shown for two average values of photon virtualities  $\langle Q^2 \rangle = 17.3 \text{ GeV}^2$  and  $\langle Q^2 \rangle = 67.2 \text{ GeV}^2$ . The data are in a good agreement with the recent parameterisations of  $F_2^\gamma$  and with the other LEP measurements. The ALEPH results in the region of  $x > 0.1$  read:

$$\begin{aligned} F_2^\gamma(0.1 < x < 0.5, \langle Q^2 \rangle = 17.3 \text{ GeV}^2) &= 0.41 \pm 0.01 \pm 0.08, \\ F_2^\gamma(0.1 < x < 0.7, \langle Q^2 \rangle = 67.2 \text{ GeV}^2) &= 0.52 \pm 0.01 \pm 0.06. \end{aligned}$$

The recent results from the DELPHI experiment [5] are shown in Fig. 3. Here the evolution of  $F_2^\gamma/\alpha$  as a function of  $Q^2$  is shown in several intervals of  $x$ . The DELPHI points have been extracted using TWOGAM model. The systematic errors due to different hadronisation models are not included. This makes difficult to compare DELPHI points with the other measurements, in which this contribution to the systematic error is often the largest.

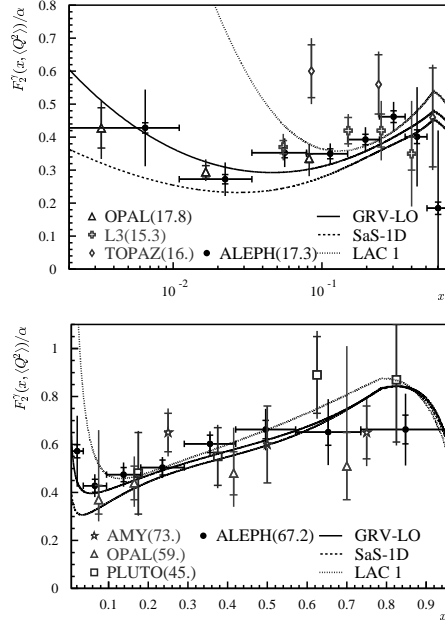


Fig. 2. The ALEPH measurement of  $F_2^\gamma/\alpha$  shown as a function of  $x$  for two average photon virtualities:  $\langle Q^2 \rangle = 17.3 \text{ GeV}^2$  and  $\langle Q^2 \rangle = 67.2 \text{ GeV}^2$ .

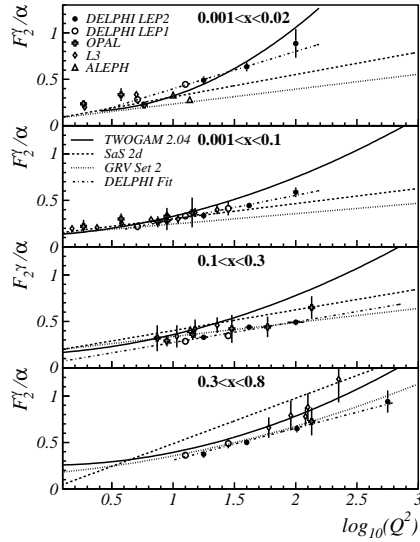


Fig. 3. The DELPHI measurement of  $F_2^\gamma/\alpha$  shown as a function of  $Q^2$  in different intervals of  $x$ :  $0.001 < x < 0.02$ ,  $0.001 < x < 0.1$ ,  $0.1 < x < 0.3$  and  $0.3 < x < 0.8$ .

The recent OPAL measurement [4] at highest  $Q^2 = 780 \text{ GeV}^2$  is shown in Fig. 4(a). This result together with the measurements at lower values of  $Q^2$  enables the study of  $Q^2$  evolution in a broad range of photon virtualities (Fig. 4(b)). As predicted by QCD, the data show positive scaling violations in  $F_2^\gamma$  with  $F_2^\gamma/\alpha = (0.08 \pm 0.02^{+0.05}_{-0.03}) + (0.13 \pm 0.01^{+0.01}_{-0.01}) \ln Q^2$ , where  $Q^2$  is in  $\text{GeV}^2$  and for the central  $x$  region 0.1–0.6. The QCD inspired parameterisations qualitatively follow the data, whereas the QPM model gives a bad description of the data, especially at low  $x$ .

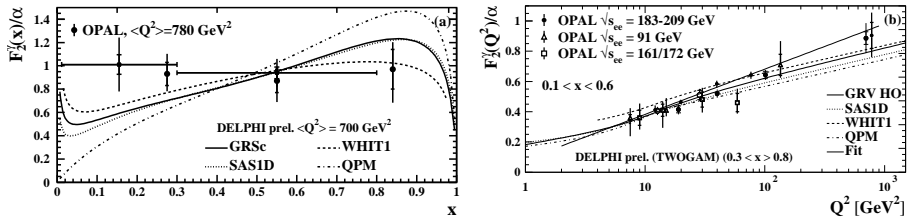


Fig. 4. (a) The measured  $F_2^\gamma/\alpha$  as a function of  $x$  for an average  $\langle Q^2 \rangle = 780 \text{ GeV}^2$  compared to LO predictions; (b) The evolution of  $F_2^\gamma/\alpha$  as a function of  $Q^2$  for the central region  $0.1 < x < 0.6$ . For comparison also DELPHI results are shown.

The charm production cross section  $\sigma(e^+e^- \rightarrow e^+e^-c\bar{c}X)$  shown in Fig. 5(a) and the charm structure function of the photon  $F_{2,c}^\gamma$  shown in Fig. 5(b) have been measured from the cross section for  $D^*$  production [6]. The data are compared to the LO and NLO calculations. The band for the NLO calculation indicates the theoretical error from uncertainties in the

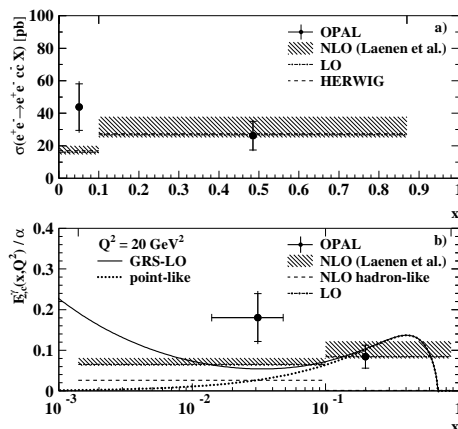


Fig. 5. (a) The cross section for  $\sigma(e^+e^- \rightarrow e^+e^-c\bar{c}X)$  with  $5 < Q^2 < 100 \text{ GeV}^2$ ; (b) Charm structure function of the photon  $F_{2,c}^\gamma(x, Q^2)/\alpha$  at  $Q^2 = 20 \text{ GeV}^2$ . The data are compared to several theoretical predictions discussed in the text.

charm quark mass and renormalisation and factorisation scales. For  $x > 0.1$  the perturbative QCD calculation at NLO order agrees perfectly with the measurement. For  $x < 0.1$  the point-like component lies below the data. Subtracting the NLO point-like prediction a measured value for the hadron-like part of  $0.154 \pm 0.059 \pm 0.029$  is obtained.

The first measurement of the electron structure function  $F_2^e$  has been presented by the DELPHI experiment [7]. The preliminary results are shown in Fig. 6.

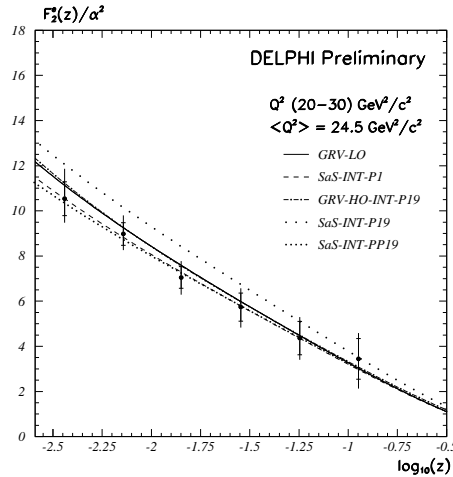


Fig. 6.  $F_2^e/\alpha^2$  is shown as a function of  $z$  in the range of  $30 < Q^2 < 50 \text{ GeV}^2$ . The points represent the DELPHI LEP2 data. The theoretical predictions are obtained from the convolution of  $F_2^\gamma$  and the photon flux  $\hat{f}_{\gamma^*/e}(y, P^2)$ . The integration over  $P^2$  extends to  $1 \text{ GeV}^2$  (INT-P1) or to  $19 \text{ GeV}^2$  (INT-P19) in case when  $P^2$  dependence is taken into account only in the photon flux, and to  $19 \text{ GeV}^2$  (INT-PP19) in case of SaS1D parameterisation with  $P^2$  dependence taken into account in both photon flux and  $F_2^\gamma$ .

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