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The optical theorem in action: radiation of an electron in a Lorentz-violating vacuum

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Abstract According to the optical theorem, the imaginary part of the one-loop radiative shift of the electron energy in an external field (IP1L) determines the total probability of photon emission by the electron. We calculate IP1L and then the probability and power of radiation of an electron in a constant background tensor field, which simulates the violation of Lorentz invariance in the framework of the Standard Model Extension. Using current experimental constraints on the background field strength, we show that the considered radiation effect can manifest itself under astrophysical conditions at ultrahigh electron energy.

1 Introduction

The Standard Model (SM) is complete [1] but it is not a complete theory due to a number of fundamental problems that cannot be solved in its framework (see, e. g., [2]): it does not include gravity; no explanation of charge quantization; too many input parameters; a huge hierarchy of particle masses and energy scales of interactions; a generation problem; no solutions to dark matter and dark energy, baryon asymmetry in the Universe; etc. These problems stimulate the development of theories generalizing the SM (see a detailed discussion of a number of them in [3]). Some of these theories include violation of Lorentz invariance, among which we single out the effective field theory, which is called the Standard Model Extension (SME) [4–6]. The SME Lagrangian is the sum of the SM Lagrangian and additional terms representing various combinations of SM fields with free Lorentzian indices (this violates Lorentz invariance), which are convoluted with constant tensors of the corresponding ranks and mass dimensions. Such a structure of the Lagrangian expands the concept of effective field theory [7], and the indicated tensors, considered as constant background fields, simulate the complex structure of the vacuum induced by the new physics beyond the SM (in particular, the effects of quantum gravity).

Various effects have been investigated within the SME framework, and we note only a few works (see also references therein), limited to the case of an electron interacting with an axial-vector background field (AVBF): production of an electron-positron pair by a photon and emission of a photon by an electron and a positron [8,9], synchrotron radiation of an electron taking into account its anomalous magnetic moment and interaction with the AVBF [10], effect of the AVBF on the radiation of a hydrogen-like atom [11], generation of a vacuum current by the AVBF [12].

In our works, we investigated the one-loop mass and vertex (at zero momentum transfer) operators in a tensor background field (of quasi-magnetic type) [13] and the emission of a photon by an electron in this field [14], using the following Lagrangian[1]:

$$\mathcal{L} = \mathcal{L}_{\text{QED}} + \mathcal{L}_{\text{T}}. \tag{1}$$

Here

$$\mathcal{L}_{\text{QED}} = \bar{\psi}\left(\gamma^\mu\left(i\partial_\mu + eA_\mu\right) - m\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\left(\partial_\mu A^\mu\right)^2 \tag{2}$$

is the Lagrangian of the standard QED in the Lorentz gauge, ψ is the electron-positron field (m and $-e < 0$ are the electron mass and charge), A^μ and $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ – 4-potential and tensor of the electromagnetic field strength;

$$\mathcal{L}_{\text{T}} = -\frac{1}{2}\bar{\psi}\sigma^{\mu\nu}H_{\mu\nu}\psi \tag{3}$$

is the Lagrangian of interaction with a tensor constant background field $H_{\mu\nu}$.

In the present paper, we generalize the results of [14] by calculating the probability and power of the electromagnetic

[1] A system of units is used in which $\hbar = c = 1$, $\alpha = e^2/4\pi \simeq 1/137$, and a pseudo-Euclidean metric with signature $(+---)$; $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, $\sigma^{\mu\nu} = i[\gamma^\mu, \gamma^\nu]/2$.

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radiation of an electron with an arbitrary orientation of its momentum relative to the vector of the background field strength. In contrast to [14], where the standard calculation method is used based on the amplitude of the radiative transition in the first order in the electromagnetic coupling, here we calculate the imaginary part of the one-loop radiative shift of the electron energy in the initial state, which, according to the optical theorem, determines the radiation probability.

2 Wave functions and an electron propagator in a tensor background field

The wave function of an electron in a tensor background field satisfies the Dirac equation, which follows from (1), (2), (3):

$$\left(i\gamma^{\mu}\partial_{\mu} - m - \frac{1}{2}\sigma^{\mu\nu}H_{\mu\nu}\right)\psi = 0. \tag{4}$$

As in [14], we consider the case of a background field of the quasi-magnetic type for which

$$H^{\mu\nu}H_{\mu\nu} > 0, \quad H^{\mu\nu}\widetilde{H}_{\mu\nu} = 0,$$

where $\widetilde{H}_{\mu\nu} = \varepsilon_{\mu\nu\alpha\beta}H^{\alpha\beta}/2$ ($\varepsilon_{0123} = -\varepsilon^{0123} = -1$). Then, as is known, there is a special reference frame in which, with an appropriate choice of the orientation of the axes, the nonzero components of the tensors are as follows:

$$H_{21} = -H_{12} = h, \quad \widetilde{H}_{03} = -\widetilde{H}_{30} = h, \tag{5}$$

so that the tensor field is equivalent to the axial vector (we put $h > 0$)

$$\mathbf{h} = h\mathbf{e}_z, \quad h = \left[H^{\mu\nu}H_{\mu\nu}/2\right]^{1/2}. \tag{6}$$

Let us represent Eq. (4) in this reference frame in the Hamiltonian form:

$$i\frac{\partial\psi}{\partial t} = \widehat{H}\psi, \; \widehat{H} = \boldsymbol{\alpha}\cdot\widehat{\mathbf{p}} + m\beta - \beta\Sigma_3 h, \tag{7}$$

where the momentum operator $\widehat{\mathbf{p}} = -i\nabla$ and the Dirac matrices $\boldsymbol{\alpha} = \gamma^0\boldsymbol{\gamma}$, $\beta = \gamma^0$, $\Sigma_3 = i\gamma^1\gamma^2$.

The solution of (7) has the form [14]:

$$\psi_{\mathbf{p}\zeta}(t, \mathbf{r}) = \frac{1}{\sqrt{V}}u(\mathbf{p}, \zeta)\exp\left(-iEt + i\mathbf{p}\cdot\mathbf{r}\right),$$

$$u(\mathbf{p}, \zeta) = 2^{-3/2}\begin{pmatrix} A_+\left(B_+ + \zeta B_-\right) \\ -\zeta A_-\left(B_+ - \zeta B_-\right)e^{i\phi} \\ A_+\left(B_+ - \zeta B_-\right) \\ \zeta A_-\left(B_+ + \zeta B_-\right)e^{i\phi} \end{pmatrix}. \tag{8}$$

Here V is the normalization volume;

$$A_{\pm} = \left(1 \pm \zeta\frac{m}{\varepsilon_{\perp}}\right)^{1/2}, \quad B_{\pm} = \left(1 \pm \frac{p_z}{E}\right)^{1/2}. \tag{9}$$

The wave function (8) describes the stationary state of an electron in the background field and is an eigenfunction of the Hamiltonian $\widehat{H}$, the momentum operator $\widehat{\mathbf{p}}$, and the operator of the spin projection onto the direction $\mathbf{h}$ (Oz axis),

$$\widehat{\Pi} = \gamma^5\gamma^{\mu}\widetilde{H}_{\mu\nu}p^{\nu}/h = \gamma^5\left(\gamma^0 p_z - \gamma^3 E\right), \tag{10}$$

where $p^{\nu} = (E, \mathbf{p})$; the spin quantum number $\zeta = \pm 1$ is related to the eigenvalue of the operator (10) by the relation

$$\widehat{\Pi}\psi_{\mathbf{p}\zeta} = (\zeta\varepsilon_{\perp} - h)\psi_{\mathbf{p}\zeta};$$

the electron energy

$$E = \left[(\varepsilon_{\perp} - \zeta h)^2 + p_z^2\right]^{1/2} \tag{11}$$

depends on ζ, the longitudinal p_z and transverse $p_{\perp} = \sqrt{p_x^2 + p_y^2}$ (via $\varepsilon_{\perp} = \sqrt{m^2 + p_{\perp}^2}$) components of the momentum $\mathbf{p}$; the angle ϕ in (8) specifies the direction of the transverse momentum $\mathbf{p}_{\perp} = (p_x, p_y) = p_{\perp}(\cos\phi, \sin\phi)$.

Note that the function (8) can be obtained from the wave function of a neutron moving in an external constant magnetic field [15] by replacing $\mu_n F_{\mu\nu} \to H_{\mu\nu}$, where μ_n is the anomalous magnetic moment of the neutron, with the corresponding change in notations.

Consider now the electron propagator in the background field

$$G(x, x') = \int \frac{d^4q}{(2\pi)^4}G(q)e^{-iq\cdot(x-x')}. \tag{12}$$

It satisfies the equation (see (4))

$$\left(i\gamma^{\mu}\partial_{\mu} - m - \frac{1}{2}\sigma^{\mu\nu}H_{\mu\nu}\right)G(x, x') = \delta^{(4)}(x - x'), \tag{13}$$

or in the momentum representation

$$\left(\gamma^{\mu}q_{\mu} - m - \frac{1}{2}\sigma^{\mu\nu}H_{\mu\nu}\right)G(q) = 1. \tag{14}$$

The explicit form of the propagator $G(q)$ (the solution of the matrix equation (14)) follows from the expression obtained in [16] for the neutrino propagator moving in a constant magnetic field by the obvious renaming:

$$\begin{aligned} G(q) &= \hat{Q}(q)R(q), \\ \widehat{Q}(q) &= \Big\{\left(q^2 - m^2\right)(\gamma\cdot q + m) \\ &\quad -h^2(\gamma\cdot q - m) - 2H_{\mu\nu}(Hq)^{\nu}\gamma^{\mu} \\ &\quad +2m(\widetilde{H}q)_{\mu}\gamma^{\mu}\gamma^5 \\ &\quad +\sigma^{\mu\nu}\left[\frac{1}{2}\left(q^2 + m^2 - h^2\right)H_{\mu\nu} - 2(Hq)_{\mu}q_{\nu}\right]\Big\}, \\ R(q) &= \Big[\left(q^2 - m^2\right)\left(q^2 - m^2 + i0\right) - 2h^2\left(q^2 + m^2\right) \\ &\quad +4(Hq)^2 + \frac{1}{2}h^2\Big]^{-1}, \end{aligned} \tag{15}$$

where $(Hq)^{\mu} = H^{\mu\nu}q_{\nu}$, $(\widetilde{H}q)^{\mu} = \widetilde{H}^{\mu\nu}q_{\nu}$.

3 Radiative shift of the electron energy and the optical theorem

The one-loop radiative energy shift of an electron in the state (8) is defined as (see, e.g., [17])

$$\Delta E = -\frac{ie^2}{T}\int d^4x d^4x' \bar{\psi}_{\mathbf{p}\zeta}(x)\gamma^\mu G(x,x')\gamma^\nu \psi_{\mathbf{p}\zeta}(x') \times D_{\mu\nu}(x,x'), \quad (16)$$

where $T(\to\infty)$ is the interaction time, the photon propagator in the Lorentz gauge

$$D_{\mu\nu}(x,x') = g_{\mu\nu}\int \frac{d^4k}{(2\pi)^4} D(k) e^{-ik\cdot(x-x')}, \quad D(k) = (k^2+i0)^{-1}. \quad (17)$$

Substituting (8), (12), and (17) into (16), after integrating over x and x' and taking into account (15), we obtain,

$$\Delta E = -\frac{ie^2}{(2\pi)^4}\int d^4q D(p-q)R(q) \times \bar{u}(\mathbf{p},\zeta)\gamma^\mu \widehat{Q}(q)\gamma_\mu u(\mathbf{p},\zeta). \quad (18)$$

Using well-known relations from the algebra of Dirac matrices [17] and taking into account (15) and (5), we transform (18):

$$\Delta E = -\frac{i\alpha}{2\pi^3}\int d^4q D(p-q)R(q)F(q), \quad (19)$$

where

$$F(q) = (q^2-m^2)\langle 2m - \gamma\cdot q\rangle + h^2\langle 2m+\gamma\cdot q\rangle + 2h^2\langle \gamma^1 q_x + \gamma^2 q_y\rangle + 2hm\langle(\gamma^0 q_z - \gamma^3 q_0)\gamma^5\rangle \quad (20)$$

with $\langle\cdots\rangle = \bar{u}(\mathbf{p},\zeta)(\cdots)u(\mathbf{p},\zeta)$.

In further calculations, taking into account the axial symmetry of the background field (6), without loss of generality we set $p_y = 0$ and then the angle $\phi = 0$ in (8). Using the explicit form of the bispinor $u(\mathbf{p},\zeta)$, we find (for $\zeta = -1$, see a comment below):

$$\begin{aligned}\langle 1\rangle &= \left(1+\frac{h}{\varepsilon_\perp}\right)\frac{m}{E},\\ \langle\gamma\cdot q\rangle &= q_0 - \left(1+\frac{h}{\varepsilon_\perp}\right)\frac{q_x p_x}{E} - \frac{q_z p_z}{E},\\ \left\langle\gamma^1 q_x + \gamma^2 q_y\right\rangle &= \left(1+\frac{h}{\varepsilon_\perp}\right)\frac{q_x p_x}{E},\\ \left\langle(\gamma^0 q_z - \gamma^3 q_0)\gamma^5\right\rangle &= \frac{m}{\varepsilon_\perp}\left(q_0 - \frac{q_z p_z}{E}\right).\end{aligned} \quad (21)$$

According to the optical theorem [17], which follows from the unitarity of the S-matrix, the imaginary part of the electron energy shift determines the total photon emission probability as

$$w = -2\mathrm{Im}\Delta E. \quad (22)$$

In turn, this imaginary part is determined from (19) according to the Cutkosky rules (see, e. g., [17]) by the following replacement in the integrand of the right-hand side (19):

$$2i\mathrm{Im}\Delta E = \Delta E\Big(D(p-q)\to -2\pi i\delta(D^{-1}(p-q)), \quad R(q)\to +2\pi i\delta(R^{-1}(q))\Big). \quad (23)$$

Note that the "+" sign in front of the second delta function is due to the additional factor q^2-m^2 in the denominator of the electron propagator (15) and the well-known relation $\delta(x)/a = \mathrm{sgn}(a)\delta(ax)$.

From (19), (22) and (23), taking into account (5), (15) and (17), we obtain the representation of the total radiation probability in the form

$$\begin{aligned} w &= \frac{2\alpha}{\pi}\int d^4q\,\delta(X_\gamma)\delta(X_e)F(q),\\ X_\gamma &= D^{-1} = (p-q)^2,\\ X_e &= R^{-1} = (q^2-m^2-h^2)^2 - 4h^2(q_\perp^2+m^2).\end{aligned} \quad (24)$$

Note that the energy spectrum (the eigenvalues of the Hamiltonian (7)) is determined by the poles of the electron propagator (15) with respect to the variable q_0 [17], i.e., by the roots of the equation $X_e(q_0) = 0$:

$$q_0 = \pm\left[\mathbf{q}^2 + m^2 + h^2 \pm 2h\sqrt{q_\perp^2+m^2}\right]^{1/2}, \quad (25)$$

which agrees with (11), and negative values of q_0 correspond to the positron.

4 Probability and power of radiation

Let us consider the angular distribution of the radiation probability. Having made in (24) the change of integration variables, $k = p - q$ (it is the photon 4-momentum),

$$\begin{aligned} d^4q &= dk_0 d^3k,\\ \delta\left(X_\gamma\right) &\to \frac{1}{2\omega}\delta\left(k_0-\omega\right),\\ \omega &= |\mathbf{k}|,\ \mathbf{k} = \omega\mathbf{n},\ |\mathbf{n}| = 1,\end{aligned}$$

and after trivial integration over k_0 we obtain

$$\frac{dw}{d\Omega} = \frac{\alpha}{\pi}\int d\omega\omega\delta\left(X_e\right)F(q), \quad q = (E-\omega, \mathbf{p}-\omega\mathbf{n}), \quad (26)$$

where $d\Omega$ is the solid angle element in the $\mathbf{n}$ direction. Next, we transform the argument of the delta function in (26) with use of (24):

$$X_e = 4\left(E_{\mathbf{n}}^2 - h^2 n_\perp^2\right)\omega(\omega - \omega_{\mathbf{n}}),\, n_\perp^2 = 1 - n_z^2,$$
$$E_{\mathbf{n}} = E - \mathbf{n}\cdot\mathbf{p},$$
$$\omega_{\mathbf{n}} = \frac{2h(\varepsilon_\perp E_{\mathbf{n}} - h n_x p_x)}{E_{\mathbf{n}}^2 - h^2 n_\perp^2}. \quad (27)$$

Using (27), we integrate over ω in (26) and obtain the angular distribution of the radiation probability:

$$\frac{dw}{d\Omega} = \frac{\alpha F(q)}{4\pi(E_{\mathbf{n}}^2 - h^2 n_\perp^2)},$$
$$q = (E - \omega_{\mathbf{n}}, \mathbf{p} - \omega_{\mathbf{n}}\mathbf{n}), \quad (28)$$

where $F(q)$ is defined by (20) and (21).

As follows from (27), the radiation frequency $\omega_{\mathbf{n}}$ is determined by the radiation direction $\mathbf{n}$, and the radiative transition is due to the electron spin flip (see (11) and (25)): $\zeta = -1 \to \zeta' = +1$.

Expression (28) is exact in terms of the background field strength h, the value of which is strictly limited from above [6]:

$$h \lesssim 10^{-17}\ \text{eV}. \quad (29)$$

Therefore, in what follows, we restrict ourselves to taking into account only the leading terms in the expansion with respect to the parameter h.

In this approximation, for the function F in (28), taking into account (20) and (21), we obtain

$$F = 4h^3\sqrt{1 - v_z^2}\frac{f(\mathbf{v}, \mathbf{n})}{(1 - \mathbf{v}\cdot\mathbf{n})^2},$$
$$f(\mathbf{v}, \mathbf{n}) = (1 - n_z v_z)^2\left(1 + \frac{1 - v^2}{1 - v_z^2}\right)$$
$$-(1 - v^2)(1 - n_z^2) - v_x^2 n_x^2, \quad (30)$$

where $\mathbf{v} = \mathbf{p}/\varepsilon = (v_x, 0, v_z)$ is the velocity of a free electron ($\varepsilon = E(h = 0) = \sqrt{m^2 + \mathbf{p}^2}$). Substituting (30) into (28), we obtain (in the leading approximation with respect to h)

$$\frac{dw}{d\Omega} = w_0\sqrt{1 - v_z^2}\frac{1 - v^2}{(1 - \mathbf{v}\cdot\mathbf{n})^4} f(\mathbf{v}, \mathbf{n}),$$
$$w_0 = \frac{\alpha h^3}{\pi m^2}. \quad (31)$$

Multiplying (31) by the photon energy (see (27))

$$\omega_{\mathbf{n}} = \frac{2h\sqrt{1 - v_z^2}}{1 - \mathbf{v}\cdot\mathbf{n}}, \quad (32)$$

we obtain the angular distribution of the radiation power

$$\frac{dW}{d\Omega} = \omega_{\mathbf{n}}\frac{dw}{d\Omega} = W_0(1 - v_z^2)\frac{1 - v^2}{(1 - \mathbf{v}\cdot\mathbf{n})^5} f(\mathbf{v}, \mathbf{n}),$$
$$W_0 = \frac{2\alpha h^4}{\pi m^2}. \quad (33)$$

In a spherical coordinate system with a polar axis Oz, the components of the unit vector $\mathbf{n}$ in (30), (31) and (33) are as follows:

$$n_x = \sin\theta\cos\varphi, \quad n_y = \sin\theta\sin\varphi, \quad n_z = \cos\theta, \quad (34)$$

and $d\Omega = \sin\theta d\theta d\varphi$. To calculate the total probability and power of radiation, it is convenient to express the angles in (34) in terms of the angles (marked with the index 0) in the reference frame moving with the velocity v_z along the axis Oz (as in the theory of synchrotron radiation [18]), using the corresponding boost, which does not change the configuration of the quasi-magnetic background field (6):

$$n_z = \frac{n_{0z} + v_z}{1 + v_z n_{0z}}, \quad n_x = \frac{\sqrt{1 - v_z^2}\, n_{0x}}{1 + v_z n_{0z}},$$
$$v_x = v_{0x}\sqrt{1 - v_z^2}, \quad d\Omega = \frac{1 - v_z^2}{(1 + v_z n_{0z})^2} d\Omega_0. \quad (35)$$

Using (35), we represent (31) and (33) as

$$\frac{dw}{d\Omega_0} = \sqrt{1 - v_z^2}\frac{dw^{(0)}}{d\Omega_0} = w_0\sqrt{1 - v_z^2}\frac{1 - v_0^2}{(1 - v_0 n_{0x})^4} f_0,$$
$$\frac{dW}{d\Omega_0} = (1 + v_z n_{0z})\frac{dW^{(0)}}{d\Omega_0} = W_0\frac{(1 + v_z n_{0z})(1 - v_0^2)}{(1 - v_0 n_{0x})^5} f_0,$$
$$f_0 = 1 - v_0^2 n_{0x}^2 + (1 - v_0^2) n_{0z}^2. \quad (36)$$

Here $v_0 \equiv v_{0x}$ is invariant under boosts along the axis Oz:

$$v_0 = \frac{p_\perp}{\varepsilon_\perp} = \frac{v_\perp}{\sqrt{1 - v_z^2}} \quad (37)$$

with $v_\perp = \sqrt{v^2 - v_z^2}$.

From (36) we get relations for total probability and power of radiation

$$w = \sqrt{1 - v_z^2}\, w^{(0)}, \quad W = W^{(0)} \quad (38)$$

in agreement with the special relativity. We emphasize that the total radiation power is a Lorentz invariant (see, e.g., [19]).

The integration of the angular distributions (36) is greatly simplified if we choose Ox as the polar axis (in the reference frame, where $v_z = 0$). Then $n_{0x} = \cos\alpha$, $n_{0z} = \sin\alpha\sin\beta$, which allows independent integration over α and β. As a result, we obtain explicit expressions for (38):

$$w = \frac{8\alpha h^3}{3m^2}\sqrt{1 - v_z^2}\frac{2 + v_0^2}{1 - v_0^2},$$
$$W = \frac{32\alpha h^4}{3m^2}\frac{1 + v_0^2}{(1 - v_0^2)^2}. \quad (39)$$

For an unpolarized electron, it is necessary to introduce an additional factor $1/2$ into the right-hand sides of (39).

5 Discussion

Expressions (39) are valid for an arbitrary angle between the electron momentum $\mathbf{p}$ and the direction of the background field $\mathbf{h}$. Let's consider two special cases.

For the case of transverse motion of an electron, putting in (39) $v_z = 0$, $v_0 = v$, we obtain

$$w_\perp = \frac{8\alpha h^3}{3m^2}(2+v^2)\gamma^2, \quad W_\perp = \frac{32\alpha h^4}{3m^2}(1+v^2)\gamma^4, \quad (40)$$

where $\gamma = \varepsilon/m = (1-v^2)^{-1/2}$ is the Lorentz factor.

For the case of longitudinal motion ($v_z = v$, $v_0 = 0$), we have

$$w_\parallel = \frac{16\alpha h^3}{3m^2}\gamma^{-1}, \quad W_\parallel = \frac{32\alpha h^4}{3m^2}. \quad (41)$$

Results (40) and (41) are consistent with those obtained earlier in our paper [14]. Note that due to the Lorentz invariance of the radiation power (see (38)) $W_\parallel = W_\perp(v=0)$, so that an electron at rest also radiates.

Consider the average emitted energy of an electron, i.e., the average photon energy

$$\langle\omega\rangle = \frac{\int \omega dw}{\int dw} = \frac{W}{w}. \quad (42)$$

According to (42) and the remark after (28), over a time interval

$$\tau_R = 1/w \quad (43)$$

an electron emits a photon, having made a spin-flip transition to a state that is radiatively stable: a radiative transition from it is forbidden (see (11)). Consequently, if the electron beam is initially unpolarized, then as a result of radiation it becomes completely polarized along the direction of the background field $\mathbf{h}$, and the characteristic polarization time is equal to (43). A similar effect of polarization due to a radiative transition with spin flip was noted for neutrons moving in a magnetic field [15], as well as for neutrinos in a magnetic field and matter (neutrino spin light) [20,21].

Substituting (39) into (42), we obtain

$$\langle\omega\rangle = \frac{4h(1+v_0^2)}{\sqrt{1-v_z^2}(1-v_0^2)(2+v_0^2)},$$

$$\langle\omega\rangle_\perp = 4h\gamma^2\frac{1+v^2}{2+v^2}, \ \langle\omega\rangle_\parallel = 2h\gamma. \quad (44)$$

It follows from (40)–(44) that the effects of Lorentz violation increase with increasing electron energy, and are much more noticeable for the transverse motion. For this case, for $\gamma \gg 1$ and $h\gamma \ll 1$, we find the radiative polarization length $L_R = v\tau_R$ (see (43)) in ordinary units [14]

$$L_R \simeq \frac{c}{w_\perp} \simeq \frac{\lambda\!\!\!^{-}_e}{8\alpha}\left(\frac{m}{h}\right)^3\gamma^{-2}, \quad (45)$$

where $\lambda\!\!\!^{-}_e$ is the Compton wavelength of the electron.

For a numerical estimation, we set $h = 10^{-17}$ eV (see (29)) and $\varepsilon = 10^{16}$ GeV (the energy scale of the Grand Unification of the three fundamental interactions [2,3]). Then from (44) and (45) we obtain $\langle\omega\rangle_\perp \simeq 10^{13}$ GeV that two orders of magnitude greater than the maximum registered energy of particles in cosmic rays $\simeq 10^{11}$ GeV (see, e.g., the review [22]) and $L_R \simeq 2.3\times10^{20}$ cm (for comparison, the distance from the Sun to the nearest star $\simeq 4\times10^{18}$ cm [23], and from the Sun to the center of the Galaxy $\simeq 2.5\times10^{22}$ cm [1]).

6 Conclusion

Using the optical theorem, we calculated the probability and power of electromagnetic radiation by an electron in a constant background field of the quasi-magnetic type simulating a Lorentz-violating vacuum. It is shown that the radiative transition due to spin flip leads to complete polarization of the initially unpolarized electron beam along the direction of the background field. The results obtained generalize the results of our previous work to the case of an arbitrary angle between the electron momentum and the background field strength vector. We have shown that the considered radiative effect can be noticeable under astrophysical conditions for ultrahigh-energy electrons.

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