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Environmental cosmic acceleration from a phase transition in the dark sector

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Abstract: A new degravitation mechanism within the framework of scalar tensor gravity is postulated and included by prescription. The mechanism eliminates all constant contributions from the potential to the Friedmann equation, leaving only the kinematic and the dynamic terms of the potential to drive cosmic acceleration. We explore a scenario involving a density-triggered phase transition in the late-time universe, and argue that the resulting effective energy density and equation of state parameter can explain late-time cosmology when extrapolated to a region of the parameter space.

Keywords: dark energy theory, modified gravity, cosmological simulations, cosmic web

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Cosmological data indicate that the expansion of the universe is currently accelerating [1]. The simplest way to explain this is a fine-tuned cosmological constant, which is unexplained by the Standard Model of particle physics [2]. Moreover, tensions in recent observational evidence, if not due to systematics, seem to require a more phenomenologically complex ingredient than a constant [3]. Observations also suggest tensions for the dark sector at smaller scales [4], suggesting the need for more detailed modelling of the entire dark sector. This has motivated studies of extensions to the Standard Model of cosmology and general relativity.

Here we present results from a novel explanation of the late-time accelerating expansion caused by a modified version of the symmetron [5, 6]. The symmetron is a real scalar field with an environment-dependent mass term using the Anderson-Higgs mechanism; in low-density environments, its vacuum expectation value (VEV) drifts away from the origin, causing a phase of out-of-equilibrium transition for the scalar field. The environmental dependence comes from the scalar's non-minimal coupling to the matter sector, the type of which generically occurs in Horndeski models of modified gravity [7], of which the symmetron is a subset. The VEV also degenerates and takes on a non-trivial topology, leading to the appearance of domain walls at cosmological scales. The resulting model has a wide range of phenomenological consequences, which individually may seem to be able to account for the cosmological observations and alleviate tensions [6, 8–13]. Whether the symmetron, or some variant of it, can simultaneously account for the cosmological tensions remains an open question.

Phase-transitions as a cause for cosmic acceleration have been considered in the past, in e.g. [14], where a frustrated defect network forms as the result of a phase transition in the early universe, showing that the walls' curvature has to be very small. This gives an equation of state parameter $\omega \geq -2/3$. In [15], the authors agnostically consider a sudden transition in the equation of state parameter of the accelerator field, going from an initial $\omega_0 = 0$ to some freely chosen ω_f at the cosmological redshift z_t. Combining CMB data with LSS and supernova data, they indicate parameters of the phase transition of $(z_t, \omega_f) \sim (1.5, -1)$, occurring in the late-time universe. The latter is similar to the situation that we will consider, though we will be a step less agnostic and try to realise this situation in a specific variant of the symmetron model.

The classical symmetron contains a cosmological constant that is included to drive the late-time accelerating expansion [6], which in turn is fine-tuned. In the following, we add a prescription that we take as representative of some more fundamental model containing a degravitation mechanism. The prescription continuously removes constant contributions as sources to the Einstein equations (which is what we will mean by degravitation here); instead we try to account for the cosmic acceleration through the dynamic behaviour of the scalar field.

Recent advances in cosmological simulations of the symmetron [12, 16] allow the comprehensive treatment of the scalar field's energy contribution for some parameter ranges, that we present the results for here. By extrapolating our results, we indicate a region of the model parameter space that is interesting with regards to producing the late-time cosmic expansion, though we still appear to require fine-tuning of the Lagrangian parameters.

The field equations of the symmetron $\phi \in \mathbb{R}$ on a flat cosmological background is [6]

$$\ddot{\phi} - \frac{1}{a^2}\nabla^2\phi + 3H\dot{\phi} = V_{\text{eff},\phi} \equiv V_{,\phi} - \frac{A_{,\phi}(\phi)}{A(\phi)}T, \tag{1}$$

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where a is the scale factor, H is the Hubble function, T is the trace of the stress-energy tensor of the matter sector, dotted quantities are differentiated with respect to cosmic time, $V_{\rm eff}$ is the effective scalar field potential and V is the potential

$$V = -\frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4}\phi^4, \tag{2}$$

with mass μ and self-coupling strength λ. In the non-relativistic limit, $T \sim -\rho_m$ is the energy density of matter. $A(\phi) \equiv 1 + \frac{1}{2}\left(\frac{\phi}{M}\right)^2$ is the conformal factor, with the conformal coupling strength M. Both the conformal factor $A(\phi)$ and the potential $V(\phi)$ can be viewed as Taylor expansions of a more fundamental model, in the event where their smallness parameters $\epsilon_A \sim (\phi/M)^2$ and $\epsilon_V \sim (\phi/M_{\rm pl})^2$ are small. The conformal coupling gives a term for the effective potential which provides a stable minimum at the origin as long as $\rho_m > \rho_* = \mu^2 M^2$. As ρ_m becomes smaller, the effective potential becomes the Mexican hat potential, and develops two stable minima $\tilde{v}$ (VEVs) at non-zero field values. The VEVs are found locally (at time t and position x) by minimising $V_{\rm eff}$ and are given by

$$\tilde{v}_{\pm}(t, x) = \pm\frac{\mu}{\sqrt{\lambda}}\sqrt{1 + \frac{T(t, x)}{\rho_*}}. \tag{3}$$

We define the true VEV $v_0 \equiv \tilde{v}_+(T = 0)$. The potential V, equation (2), in the true vacuum can be related to phenomenological quantities [16]

$$V_0 = \frac{\lambda}{4}v_0^4 = \frac{9\beta_*^2 L_C^2}{2a_*^6}H_0^4\Omega_{m,0}^2, \tag{4}$$

where $a_* = (1 + z_*)^{-1}$ is the scale factor of the phase transition in a homogeneous universe, β_* is the relative strength of the gravitational force in the true VEV v_0, and L_C is the Compton wavelength related to the mass μ as $L_C = \frac{1}{\sqrt{2}\mu}$. H_0 is the current time Hubble parameter and $\Omega_{m,0}$ is the current time energy fraction of matter.

For nonlinear fields such as the symmetron, that may develop large spatial gradients due to screening and topological defects, the gradients may have a significant effect on the background evolution. It has been shown in [12] that inhomogeneities in the density field cause the field to undergo phase transition much earlier than it would on the background. To include this effect into the vacuum, we define the generalised potential $V_{\rm gen}$ whose derivative $V_{{\rm gen},\phi} \equiv V_{{\rm eff},\phi} + \frac{1}{a^2}\nabla^2\phi$ includes the Laplacian term from equation (1). The generalised VEV v is found by minimising $V_{\rm gen}$, i.e. solving

$$\nabla^2 v^2 = -a^2\partial_v V_{\rm eff}(v), \tag{5}$$

and is equivalent to finding the symmetron's quasistatic limit (equation (1) with $\dot{\phi}, \ddot{\phi} \to 0$).

In the usual approach, such as for quintessence dark energy, the potential contains a constant term, $\langle V(\phi = 0)\rangle$, which contributes to the Friedmann equation, and is the driver of cosmic acceleration. To get a constant term that can account for cosmic acceleration requires a high degree of fine-tuning (this is the smallness problem of the cosmological constant). We

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therefore propose a model where the potential contribution to the Friedmann equation is[1]

$$\Omega_V(\phi, v) = \frac{\lambda}{4\rho_c}\left\langle\left(\phi^2 - v^2\right)^2\right\rangle, \tag{6}$$

which automatically removes all vacuum contributions, and eliminates the need to fine-tune the cosmological constant, if a similar prescription can be thought to act on all of the standard model fields. The full contribution to the Friedmann equation includes the kinetic and gradient terms that are expanded around the VEV,

$$\Omega_\phi = \Omega_V + \frac{1}{2\rho_c}\langle(\dot{\phi} - \dot{v})^2 + \partial_i(\phi - v)\partial^i(\phi - v)\rangle, \tag{7}$$

and we will refer to the last two terms as $\Omega_{\rm kin}$ and $\Omega_{\rm grad}$ respectively. $\rho_c(a)$ is the critical density of the Universe and the brackets mean that we take the spatial average of their argument, because the Friedmann equation is evaluated on the background.
Since v is the quasistatic solution of the field equation, Ω_V has contributions from the dynamic parts of the field. In the absence of large inhomogeneities in the scalar field (when $\nabla^2\phi \longrightarrow 0$, and $V_{{\rm gen},\phi} = V_{{\rm eff},\phi}$), then $v \to \tilde{v}_\pm$, and we recover the classical symmetron energy contribution, with an additional term proportional to $v_\pm^4$. In the late universe, large inhomogeneities in the symmetron field are sourced by the inhomogeneous matter component that it is coupled to, or caused by the formation of topological defects in the field.

We define the dynamical and topological contributions to the equation (6) as $\Omega_V = \Omega_{\rm dyn} + \Omega_{\rm top}$. If the dynamic field, ϕ, develops topological defects through a phase transition, then there is a question of how to solve the VEV v in equation (5). If we solve it using the same vacuum manifold as is obtained in the dynamical solution, by which we mean that we are solving the VEV around the same topological defects, then we remove the contribution of topological defects in equation (6). Alternatively, we can solve the VEV using a trivial vacuum manifold (i.e. all fields with the same positive sign minimum and no defects). In this case, the topological defects that develop in the field ϕ will introduce large gradients for ϕ that are absent in the VEV v and will therefore give large contributions in equation (6). Considering both solutions for the VEV allows us to separate the effect of topology $\Omega_{\rm top}$.

We simulate the matter and scalar field evolution through the phase transition from initially small amplitude perturbations $\phi/v_0 \sim 10^{-20}$ as in [12]. Now, we additionally keep track of the vacuum field v, by solving the equation (5), considering both options for the vacuum topology as mentioned in the previous paragraph. We conveniently express the model parameters (μ, M, λ) in terms of phenomenological parameters (z_*, L_C, β_*), as in [16], where z_* is the redshift of the phase transition, L_C is the interaction length scale and β_* is the relative strength of the gravitational force in the true vacuum $v_0 = \mu/\sqrt{\lambda}$. The parameters are chosen in the same range as in [12] ($z_*, L_C, \beta_* \sim 0$, $1\,{\rm Mpc/h}$, 10), where the simulations' convergence and consistency has been studied in detail, and we are resolving the relevant scales. This corresponds to Lagrangian parameters μ, M, λ of around $10^{-30}\,{\rm eV}$, $10^{-4}\,M_{\rm pl}$ and 10^{-104} respectively. For these choices, the smallness parameters $\epsilon_A, \epsilon_V \sim 10^{-5}, 10^{-12}$. $\epsilon_A \ll 1$

[1]Using $v_\pm$ instead, without a prescription for smoothing of T, we would have contributions from arbitrarily small underdense regions.

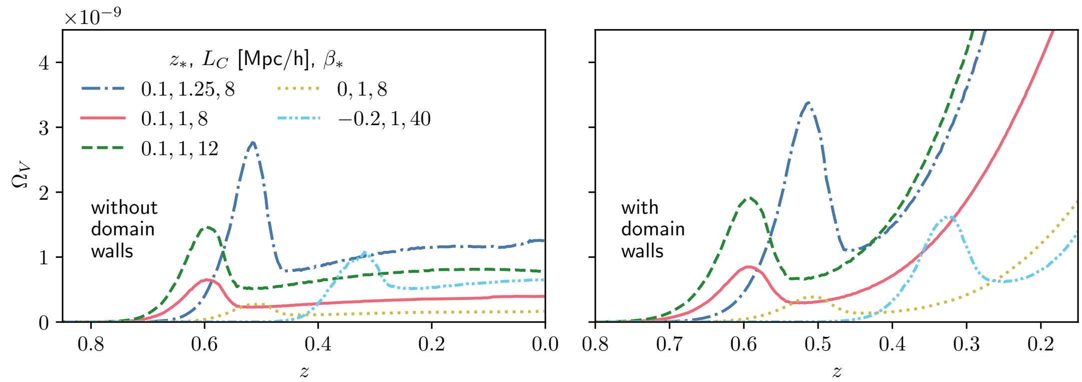

Figure 1. Energy density contribution from the scalar field potential, normalised to the critical density of the Universe ($\Omega = \rho/\rho_c$), as a function of redshift z. The left panel shows the case where the domain wall contribution is removed. Right panel includes the contribution of the domain walls.

is required by the simulation scheme, see [16]. We record the resultant volume averaged energy contributions according to equations (6) and (7).

In figure 1, we plot the volume average, equation (6), for some different parameter choices. We see that 1) In all cases, there is an initial spike in the energy density at the start of the phase transition; 2) for this choice of parameters, the energy scale is of the order of $\Omega_V(z=0) \sim 10^{-9}$. To obtain the energy scale $\mathcal{O}(1)$ required for the accelerating expansion, we have to extrapolate in the model parameters; 3) the effect of changing β_* is very well approximated[2] by the change in amplitude given by the equation (4), which means that $\langle V \rangle \mathrel{\substack{\propto \\ \sim}} \beta_*^2$. Considering all of the parameter scalings, we find $V \mathrel{\substack{\propto \\ \sim}} \frac{\beta_*^2 L_C^5}{a_*^9}$; and 4) in all cases where the domain wall contribution (topology) is included in the energy density, there is a rapidly increasing potential energy in the later stages of the phase transition.

We define the effective equation of state through the energy contribution of the scalar field [16], ρ, as

$$\omega = \frac{1}{3}\frac{\mathrm{d}\log\rho}{\mathrm{d}\log a} - 1. \tag{8}$$

Figure 2 shows the effective equation of state parameter carrying the energy of the potential in equation (6). We are neglecting the kinematic and gradient contributions $\Omega_{\rm kin}, \Omega_{\rm grad}$ for now. In all cases in figure 2 where we remove the effect of the domain walls (topology), the effective equation of state parameter is centred around -1. There is a slight drift from a phantom regime ($\omega < -1$) to larger values and some oscillation around this drift. An earlier phase transition seems to reduce the slope of the drift. In the case where the effect of domain walls (topology) is included, the topological defects seem to push the equation of state parameter towards the phantom regime. Usually in literature, having a 'phantom crossing' is considered problematic [18], but their argument does not apply in our case due to the degravitation mechanism and matter coupling.

Finally, there are the kinetic and gradient terms, $\Omega_{\rm kin}$ and $\Omega_{\rm grad}$, which we also expect to contribute to the Friedmann equation. Our results show that they both peak early in the

[2]We have generally found a very small effect of varying β on the scalar field configuration [12].

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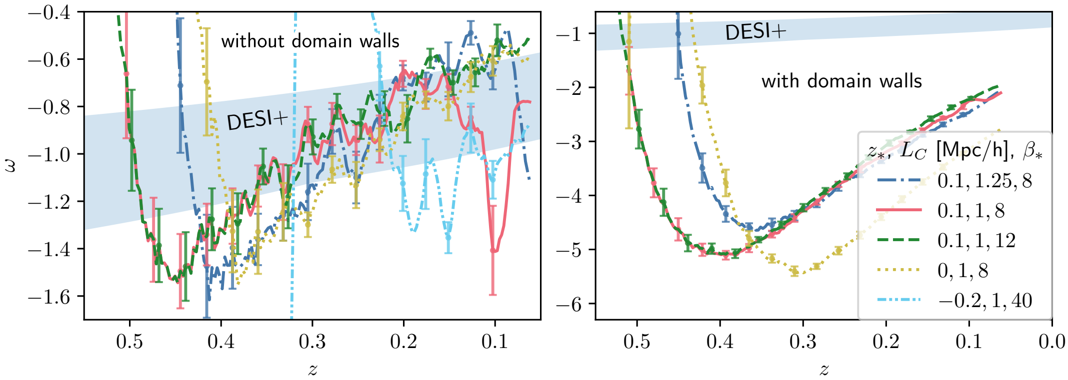

Figure 2. Effective equation of state parameters as a function of cosmological redshift z. The kinematic and gradient contributions (7) are neglected here. The left panel shows the case where the domain wall contribution is removed. The right panel includes the domain wall contribution. Error bars show 95% confidence intervals due to the intrinsic scatter in the data. The band labelled DESI+ demonstrates the 68% preferred region for the flat $\omega_0\omega_a$CDM model, presented in [17].

phase transition, after which the gradient contribution decays rapidly and $\Omega_{\rm kin}$ dominates between the two. We also find that the vacuum v generally evolves slowly $|\dot{v}| \ll |\dot{\phi}|$ and so $\Omega_{\rm kin}$ is the contribution of the scalar field kinetic energy $\dot{\phi}^2/2$ coming from the oscillations of the field. We find that the kinetic term is larger than the potential contribution, equation (6), for all of the models. The kinetic energy decays as $\rho_{\rm kin} \sim a^{-2}$.

Comparing different simulations, we see no effect of parameter variation on the relative amplitude of the kinetic and potential components. However, we do see a smaller injection of kinetic energy at the start of the phase transition in the case of stronger screening (or larger a_*). The height of the peak compares to the plateau as $\Omega_{\rm kin}^{(\rm peak)}/\Omega_{\rm kin} \underset{\sim}{\propto} \sqrt{\frac{L_C}{a_*}}$. This factor is $\Omega_{\rm kin}^{(\rm peak)}/\Omega_{\rm kin} \approx 2.7$ for the fiducial simulation (z_*, L_C, $\beta_* = 0.1, 1, 8$). While for the same parameters, today ($z = 0$), $\Omega_{\rm kin}/\Omega_V \sim 4$.

From the above, we can see that in order to dynamically generate an energy density that mimics the cosmological constant by the mechanism proposed here, the following problems must be solved by moving to an appropriate region of the parameter space

A The initial spike of energy density occurring at the start of the phase transition.

B The late rise in energy owing to a growing contribution of the domain walls to the energy budget.

C The large amplitude of the kinetic energy relative to the potential $\Omega_{\rm kin}/\Omega_V$.

D The small amplitude of the energy density Ω_V.

Therefore, we note the following part of the symmetron parameter space as interesting with regards to issues A-D: small critical energy densities ρ_* (equivalently large $a_* > 1$) and small interaction length scales L_C. Finally the force strength β_* is chosen to obtain the correct energy density amplitude.

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By having small critical energy densities ρ_* we ensure that only the least dense regions, which are mostly disjunct up to the present time undergo the phase transition. Since the domains are mostly disjunct, we expect a suppression of the final increase in energy density (B) that is caused by the formation of domain walls. $a_* > 1$ increasingly depends on nonlinearities in the density field to undergo collapse before the current time, so the scale L_C is chosen to make the field interact with these small-scale nonlinearities. In this regime, we expect an almost quasistatic transition, since the timescale of the scalar field is smaller and the spatial scale resolves the filaments. We therefore expect a suppression of the initial spike (A) that we understand to be caused by an initial period of out-of-equilibrium dynamics for the scalar.

We do not see a clear effect of varying the parameters on the relative amplitude between the kinetic and potential energies, but we have found nonlinearities in the dependence on z_* and L_C, that make the extrapolation difficult. As the initial spike (A) is the first injection of kinetic energy into the field, we expect a suppression of the relative amplitude of the kinetic energy (C). The problem (C), may therefore be resolved in the interesting limit of small L_C, z_*.

To be explicit, we make the following estimates: to remove the initial spike (A), we assume that it is sufficient that the spatial scale of the symmetron is of order $L_C \lesssim \mathcal{O}(\text{kpc})$. For such a choice, considering the inferred scaling of $a_{\text{SSB}} \mathrel{\substack{\propto\\\sim}} L_C^{0.22} a_*^{0.59}$, keeping $z_{\text{SSB}} \sim 0.6$, we set $a_* \sim 12$. This is motivated by cosmological data, where the cosmic acceleration happens around $0.4 \lesssim z \lesssim 1$ [1]. Finally, to obtain the correct energy amplitude (D), given the reported scaling of V, we set $\beta_* = 10^{18}$. Although this may seem unreasonably large compared to the values of $\beta_* \sim \mathcal{O}(1)$ typically considered in the literature, we point out that the screening is also set very large, so that the force operates only inside of the least dense voids, where the clustering constraints are at their worst and the effect of a fifth force is minimal. Moreover, this force strength is still 10^7 times weaker than the weak force and 10^{18} times weaker than electromagnetism.

The resultant model is one where the nearly quasistatic field's displacement from the drifting and environmentally dependent vacuum, is the cause of cosmic acceleration. Despite the field being strongly screened, we expect observables such as a modification of the density profile within large cosmic voids, which is expected to give clear signatures in observables such as the Integrated Sachs-Wolfe (ISW) effect [12, 19–21]. We expect upcoming constraints on the equation of state parameter from among other the continuation of the DESI survey [17]. And the oscillating energy density and topological defects can leave observable imprints in gravitational waves [13, 22].

The appeal of this mechanism is manifold:

1. The nearly quasistatic evolution of the field as a generator of cosmic acceleration addresses the smallness problem of the cosmological constant by suppressing the energy density contribution to the Friedmann equation relative to that of the original potential (2).

2. Since the phase transition is triggered by nonlinearities in the overdensity field, the mechanism partially solves the coincidence problem by making 'now', when nonlinearities grow large, (i.e. $z \lesssim 1$) a generic time for the accelerated expansion.

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3. The addition of a fifth force at a strength scale midway between electromagnetism and gravity improves the hierarchy problem of why gravity is uniquely weak, and provides a more continuous range of force scales among the five forces.

4. In terms of minimality, the symmetron has been considered as a dark matter candidate for similar parameter choices for the interaction scale $L_C \sim$kpc [8, 9, 11], in which case it behaves as a fuzzy dark matter with an extended mass spectrum that depends on the environment.

5. The relatively large energy scale of the degravitated potential V_0 and the phantom behaviour caused by the formation of topological defects may allow the connection of the low energy, late-time acceleration with the higher energy scale expansion during inflation [23] in a cyclic universe scenario. In this scenario, the symmetron has the additional advantage of being symmetry restored during reheating. Looking at the degravitated potential V_0 from the equation (4), we see that it has a scale $V_0 \sim 10^{29} \cdot \rho_{c,0} \sim 10^{39}\,\mathrm{GeV/m^3} = 10^{19}\,(\mathrm{eV})^4$, which can be made available in the topological defects or upon symmetry restoration.

However, in spite of these optimistic remarks, a change of perspective from the consideration of the phenomenological parameters (L_C, a_*, β), to the Lagrangian ones (μ, M, λ), reveals caveats. While $L_C \sim$kpc can make the symmetron an interesting fuzzy dark matter candidate, the mass μ is then of order $10^{-27}\,\mathrm{eV}$, lighter than the typical standard model mass scale by 27 orders of magnitude. Meanwhile $a_* = 12$ corresponds to conformal coupling parameters $M \sim 10^{20}\,\mathrm{eV}$, or in terms of the Planck mass $M/M_{\mathrm{pl}} \sim 10^{-8}$. Finally, the force strength $\beta_* = 10^{18}$, together with the other two parameter choices, results in the self-coupling strength $\lambda \sim 10^{-114}$. From naturalness, we expect dimensionless numbers $\lambda \sim \mathcal{O}(1)$. While, the nature of the fine-tuning issue here is different from the cosmological constant, where the prior for a 120 orders of magnitude larger cosmological constant value comes from particle physics predictions, the discrepancy is of similar magnitude. A first-principles understanding to explain the apparent fine tuning remains to be found. Furthermore, the resultant smallness parameters $\epsilon_A, \epsilon_V \sim 10^{20},\ 10^4$ both indicate the relevance of higher order operators within this part of the parameter space, a study into which is beyond the scope of the current work.

In conclusion, the large discrepancy between the cosmological constant expected from quantum field theory and that required to explain the cosmological acceleration motivates: 1) the introduction of a degravitation mechanism of constant term contributions to the Friedmann equation and 2) dynamical drivers of the cosmological acceleration. In this work we have introduced a model where there is a late-time, environment-dependent phase transition in the dark sector whose dynamic energy component drives the late-time accelerated expansion. We have studied the evolution and parameter scaling of the energy components within the parameter space computationally accessible to our simulations. By extrapolation, we point to a specific part of the symmetron parameter space, $(L_C, a_*, \beta_*) \sim (\mathrm{kpc}, 12, 10^{18})$, which can account for the late-time cosmic acceleration.

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