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Neutrino Masses in Supersymmetric Models with *R*-Symmetry

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Abstract: In this article, we give a brief review of the origin of the neutrino mass in some interesting non-linear supersymmetric models with *R*-symmetry. These models are able to address and solve the most important problems of particle physics and provide mechanisms for neutrino mass generation and their mixing parameters in agreement with the current experimental data. Their prediction could be experimentally tested in the near future by collider experiments.

Keywords: neutrino mass; supersymmetric models; *R*-symmetry

1. Introduction

The internal symmetries of the standard model (SM) are described by the gauge group

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y, \tag{1}$$

where the subscripts C, L, and Y refer to color, left chirality, and weak hypercharge, respectively. At the weak scale, the electroweak symmetry subgroup

$$SU(2)_L \otimes U(1)_Y \tag{2}$$

is spontaneously broken to

$$SU(3)_C \otimes U(1)_{em}. \tag{3}$$

This spontaneous symmetry breakdown is driven by an $SU(2)_L$ doublet of the scalar Higgs field defined as

$$H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix} \sim (1, 2, 1), \tag{4}$$

with the following vacuum expectation value (vev):

$$< H > = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}. \tag{5}$$

The Yukawa coupling

$$Y_f H \bar{f}_R f_L + hc, \tag{6}$$

induces Dirac masses for all charged leptons except for neutrinos, which are massless at all perturbative levels due to the lack of a right-handed neutrino component.

The SM successfully describes the particle phenomenology across the energy scales probed by the large hadron collider (LHC). However, SM also has some relevant problems such as

1. The coupling constants do not meet at a single definite value.
2. The mass hierarchy problem ($M_H \ll M_{Pl}$).
3. The naturalness or fine tuning problem.
4. The CP-violation and matter anti-matter asymmetry.

In the field of particle physics, one promising class of theories that could solve the problems of the SM is formed by the supersymmetric (SUSY) extensions of the SM. More than 30 years ago, the SUSY was discovered in theoretical studies with the minimal supersymmetric standard model (MSSM) proposed by Pierre Fayet in [1] (see, for more details, e.g., [1,2]).

The arguments in favor of SUSY are based on the belief that more fundamental a theory is, the higher the internal symmetry it should have [3–5]. This structure is widely used in string theory where SUSY is promoted to a local symmetry, and the resulting supergravity demonstrates the unification of gravity with the field theory. But the SUSY can be used in particle physics without resorting to string theory because it allows for a simple and natural mechanism to cancel the quadratic divergences. Indeed, since SUSY is a symmetry between the bosonic states $|B\rangle$ and the fermionic states $|F\rangle$, there is a negative sign fermionic contribution at one-loop to the radiative correction of the scalar mass

$$\delta m_s^2 = O\left(\frac{\alpha}{2\pi}\right)\left(\Lambda^2 + m_B^2\right) - O\left(\frac{\alpha}{2\pi}\right)\left(\Lambda^2 + m_F^2\right) = O\left(\frac{\alpha}{2\pi}\right)\left(m_B^2 - m_F^2\right). \quad (7)$$

The cutoff $\Lambda \sim M_P$ no longer needs fine tuning across a 10^{32} energy scale. To stabilize the mass hierarchy, one has to set $|m_B^2 - m_F^2| \sim 1\text{TeV}^2$ (electroweak scale), which is the main reason to search for masses of superparticles in this range. This has an additional advantage as it is the mass scale predicted by some dark matter models, where the best candidates are the lighest neutralinos or sneutrinos. Other useful features of SUSY include its usefulness in solving the problem of coupling constants within the grand unified theory (GUT) framework.

As is known, the GUT models based on the gauge group $SU(5)$ and $SO(10)$ do not have an intersection of the three gauge couplings, display proton instability, and have a GUT hierarchy problem that involves the electroweak (light) and the GUT (heavy) Higgs bosons. In the supersymmetric GUT models, the free parameters are tuned to avoid predictions that are already falsified by data and the GUT hierarchy problem is completely solved.

Regarding CP violation, we know that in SM, CP violation comes from the quark sector. On the other hand, in the SUSY models, there are several new *CP* violation phases, derived from the gluino [6], neutralino [4], and Higgs fields [7,8].

Another reason to look for theories and models beyond SM is that some experimental and observational results have not found a satisfactory explanation within the SM framework, e.g.,

1. Neutrinos are massive and they oscillate.
2. The muon anomalous magnetic moment.
3. Dark matter.
4. Dark energy.

The shortcomings of the SM invite the construction extensions that can solve its problems. From another perspective, a fundamental theory must predict the unification of the gravitational interaction with the other three fundamental interactions and must solve all of the above puzzles as well as commonly asked questions in general relativity, such as the dark energy, that can be addressed and solved in SUSY models [9,10].

Recently, the SUSY models have been confronted with experimental data from the LHC. While no direct evidence of sparticles has been found yet, the weak scale SUSY is supported by virtual quantum effects (radiative corrections). Let us review the relevant data sets [11]:

1. The precision calculation of m_W versus m_t has been improved in MSSM to the level of SM prediction for m_W in the decoupling limit where the masses of all supersymmetric particles are large. Comparing the MSSM result and the SM calculation as a function of the Higgs-boson mass with the experimental values of m_W and m_t, a slight preference for non-zero SUSY contributions was found in [12].
2. The electroweak symmetry is broken correctly in MSSM at a weak scale due to the large value of the top Yukawa coupling that gives negative values to the soft term m_H^2 under the RG run [13–17].

3. As mentioned above, at the mGUT scale, the SM gauge couplings meet with each other, which does not happen in the SM.

4. The MSSM predicts the interval 115–135 GeV for the Higgs boson mass, which was detected at 125 GeV. On the other hand, the best prediction given by the SM is $m_H \leq 1$ TeV.

Therefore, SUSY is a good candidate for SM expansion because it represents a coherent framework for unifying fundamental interactions based on gauge symmetry and can explain the origin of the weak scale. Besides these arguments, it is also understood that SUSY cannot yet be rejected as a fundamental principle of nature since all data obtained at the LHC are interpreted within the framework of simplified SUSY models and for low-energy particles. The absence of sparticles at a weak scale could simply indicate that SUSY exists above the TeV scale, which is most likely correct since the masses of the sparticles depend on the supersymmetry breaking terms and can have values above the TeV scale. Models with heavy gauginos have been presented in [18,19]. For a recent discussion of exact unification at scales above TeV, see [20].

Another interesting feature of SUSY is that it can explain the nature of dark matter. One exciting avenue to explore involves the lightest and most stable supersymmetric particles that may be part of the remnants of the early universe. Stability is guaranteed in supersymmetric models by the *R*-parity, which is a conserved quantum number that can take values ± 1, and by the fact that there are no states in which the lightest particles can decay. The *R*-parity embedded into the larger *R*-symmetry, and SUSY models with *R*-symmetry contain sparticles that are viable candidates for dark matter.

In this paper, we review the relation between neutrinos and *R*-symmetry in several non-minimal SUSY models: MSSM with *R*-Parity violation, NMSSM, MSSM3RHN, μνSM, SUSYB-L, SUSYLR, and two SUSYGUTS models. (For a detailed discussion of one of these models, see [21]). With the recent experimental bounds in the minimal SUSY models from the LHC, there has been a revival of interest in non-minimal models with *R*-symmetry [22,23]. Concerning the neutrino properties and SUSY models, the bounds on SUSY and new physics calculated by using experimental constraints related to neutrino decay can be found in [24–27]. Although we do not compare calculations with experimental data, for the interested reader, we mention here that the experiments on the direct measuring of neutrino mass have been reviewed in [28], with some of new proposals presented in [29,30]. In addition to addressing neutrino problems, *R*-symmetric models also provide an effective framework for modeling dark matter with gaugino mass through *R*-symmetry breaking [31]. While our focus here is to review the relationship between the neutrino masses and *R*-symmetry, it is worth noting that other phenomenologically motivated models can be used to discuss the dark matter particles. For example, the 331-models and their supersymmetric extensions, initially develop to solve the number of families problem [32–34], have been used to calculate neutrinos masses [35,36] and, more recently, to address the dark matter at TeV scale [37].

When considering phenomenological models of neutrinos, several important questions need to be clarified. Two important aspects are numerical analysis and data matching, as well as normal and reversed hierarchical ordering. In the *R*-symmetry models discussed here, the approach to these two problems varies from analysis to analysis, and most information can be found in the original references. In general, the naturalness principle for the weak scale is implemented through the specific family dependent pattern of supersymmetry breaking masses. In the case of the 331-models mentioned above, a description of the numerical analysis of how the data are exactly matched can be found in [35,36]. Additionally, the first of these two references studies the normal hierarchy, and the second studies the inverted hierarchy.

2. Neutrinos Mass

Historically, the main phenomenological reason to introduce the neutrinos mass was the solar neutrino problem, which basically states that the ratio between the flux of electron

neutrinos detected by electron neutrinos predicted is roughly around one half but can be as low as one third [38] (for a recent review, see e.g., [39]). Several crucial experiments have supported the idea that there are three neutrino flavors of low masses at eV scale (<0.12 eV) and that they participate to weak and gravitational interactions. Since the neutrino is a spin-half electrically neutral particle, it can be described by a Majorana field with the anti-neutrino characterized by the opposed helicity. The unknown nature of the neutrino that could be a Dirac or a Majorana spinor is currently the subject of experimental investigations at CUORE [40], GERDA [41], MAJORANA [42], SNO+ [43], and EXO [44]. The experimental results suggest that the neutrinos have non-zero masses and oscillations. The best-fit values at the 1σ error level for the neutrino oscillation parameters in the three-flavor framework are [45]

$$
\begin{aligned}
\sin^2 \theta_{12} &= \sin^2 \theta_{solar} = 0.310^{+0.013}_{-0.012}, \Delta m^2_{21} = \Delta m^2_{solar} = 7.39^{+0.21}_{-0.20} \times 10^{-5} \text{ eV}^2, \\
\sin^2 \theta_{23} &= \sin^2 \theta_{atm} = 0.563^{+0.018}_{-0.024}, |\Delta m^2_{23}| = \Delta |m^2_{atm}| = 2.454^{+0.029}_{-0.031} \times 10^{-3} \text{ eV}^2, \\
\sin^2 \theta_{13} &= \sin^2 \theta_{CHOOZ} = 0.223^{+0.066}_{-0.065}.
\end{aligned}
\tag{8}
$$

3. *R*-Symmetry

We start off by recalling the important observation that SM has two accidental symmetries, $U(1)_L$ and $U(1)_B$, specifically the lepton and baryon numbers, in addition to its gauge symmetries. $U(1)_L$ and $U(1)_B$ are responsible for the stability of nucleons under decay into light leptons and for the existence of Dirac neutrinos. In general, the conservation of $U(1)_L$ implies that the neutrinos are Dirac, whereas if $U(1)_L$ is broken, the breaking pattern determines the nature of the neutrinos [46]. In the SM model, all interactions conserve both discrete symmetries. However, models beyond SM do not need to conserve $U(1)_L$ or $U(1)_B$, as is the case with several SUSY models that contain interactions that violate either $U(1)_L$ or $U(1)_B$, or both. Therefore, it is necessary to impose a discrete symmetry to obtain a model in which all interactions do not violate L and B conservation. This discrete symmetry is now called *R*-symmetry, and it was introduced independently by A. Salam and J. Strathdee [22] and P. Fayet [23].

Let us recall the *R*-symmetry. Consider the commutation relations involving the fermionic generators of the super-Poincaré algebra

$$
\begin{aligned}
\{Q_\alpha, Q_\beta\} &= \{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = 0, \\
[Q_\alpha, P_m] &= [\bar{Q}_{\dot\alpha}, P_m] = 0, \\
\{Q_\alpha, \bar{Q}_{\dot\alpha}\} &= 2\sigma^m_{\alpha\dot\alpha} P_m, \\
[M_{mn}, Q_\alpha] &= i(\sigma_{mn})_\alpha{}^\beta Q_\beta, \\
[M_{mn}, \bar{Q}^{\dot\alpha}] &= i(\bar{\sigma}_{mn})^{\dot\alpha}{}_{\dot\beta} \bar{Q}^{\dot\beta}.
\end{aligned}
\tag{9}
$$

where m, n are space-time indices and α, β are spinor indices. We denote by $M_{mn} = -M_{nm}$ the generators of Lorentz transformations and by P_m the generators of translations. In the $\mathcal{N} = 1$ version of SUSY, we have one Weyl conserved charge Q_α along with its conjugate $\bar{Q}_{\dot\alpha}$. Also, $\sigma_{mn} = \frac{i}{4}(\sigma_m \bar{\sigma}_n - \sigma_n \bar{\sigma}_m)$, with σ_m representing the Pauli sigma matrices. A pedagogical presentation of SUSY geared towards phenomenology can be found in [47].

The *R*-symmetry $U(1)_R$ is a symmetry of the super-algebra (9) defined by the following commutation relations:

$$
[P_m, R] = [M_{mn}, R] = 0,
\tag{10}
$$

$$
[Q_\alpha, R] = Q_\alpha,
\tag{11}
$$

$$
[\bar{Q}^{\dot\alpha}, R] = -\bar{Q}^{\dot\alpha}.
\tag{12}
$$

The above commutators imply that

$$
Q_\alpha \to Q'_\alpha = e^{-i\delta} Q_\alpha,
\tag{13}
$$

$$\bar{Q}_{\dot{\alpha}} \to \bar{Q}'_{\dot{\alpha}} = e^{i\delta}\bar{Q}_{\dot{\alpha}}, \tag{14}$$

which means that the R-charges of Q and $\bar{Q}$ are -1 and $+1$, respectively.

In order to construct models with R-symmetry, it is necessary to define the action of the $U(1)_R$ operator on the superspace and superfields. Let us denote the generator of R-symmetry acting on the superspace functions by R and the superspace coordinates by $\{x, \theta, \bar{\theta}\}$ [48]. Then, the action of R on the fermionic coordinates θ and $\bar{\theta}$ is given by the following relations:

$$R: \qquad \theta \to \theta' = e^{-i\delta}\theta, \qquad \bar{\theta} \to \bar{\theta}' = e^{i\delta}\bar{\theta}. \tag{15}$$

Hence, the R-charges of fermionic variables take discrete values $R(\theta) = -1$ and $R(\bar{\theta}) = 1$. The R-charges of various fields are summarized in the following table:

$$
\begin{array}{c|cccccc}
\text{mathematical objects} & Q & \bar{Q} & \theta & \bar{\theta} & d^2\theta & d^2\bar{\theta} \\
\hline
R - \text{charges} & -1 & +1 & +1 & -1 & -2 & +2\,.
\end{array}
\tag{16}
$$

The basic objects of the SUSY models are the superfields that live in the superspace. The operator R acts on the chiral $\Phi(x, \theta, \bar{\theta})$, anti-chiral, $\bar{\Phi}(x, \theta, \bar{\theta})$ and vector superfields $V(x, \theta, \bar{\theta})$ as follows:

$$
\begin{aligned}
R\Phi(x, \theta, \bar{\theta}) &= e^{2in_\Phi \delta}\Phi(x, e^{-i\delta}\theta, e^{i\delta}\bar{\theta}), \\
R\bar{\Phi}(x, \theta, \bar{\theta}) &= e^{-2in_\Phi \delta}\bar{\Phi}(x, e^{-i\delta}\theta, e^{i\delta}\bar{\theta}), \\
RV(x, \theta, \bar{\theta}) &= V(x, e^{-i\delta}\theta, e^{i\delta}\bar{\theta}).
\end{aligned}
\tag{17}
$$

where $2n_\Phi$ is the R-charge of the chiral superfield. The vector superfield is inert under R-symmetry. The above relations show that, in general, the terms of the superpotential have charges 2, which is two times the charge of $d\theta$. Here and in what follows, we are using the conventions and basic results from [48].

The relations (17) show that all models with discrete R-symmetries based on $\mathbf{Z}^N$ should take into account the allowed values of the R-charges of superfields, which impose constraints on their components. Consider, for example, the chiral superfield defined by the equation

$$\bar{D}_{\dot{\alpha}}\Phi = 0, \tag{18}$$

which has the following decomposition:

$$\Phi = A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y). \tag{19}$$

From the R-symmetry, we obtain the following relations between the charges $R(\Phi)$ and $R(A)$, $R(\psi)$, and $R(F)$:

$$R(A) = R(\Phi), \qquad R(\psi) = R(\Psi) - R(\theta), \qquad R(F) = R(\psi) - 2R(\theta). \tag{20}$$

Thus, the superpotential that corresponds to the $\theta\theta$ factor has a charge of either $0 \mod 2$ or $2R(\theta) \mod 2$, which prohibits the use of $\mathbb{Z}^2$ as the R-symmetry group.

The master Lagrangian of a supersymmetric model has the following general form:

$$\int d^4x \left\{ \int d^4\theta K(\bar{\Phi}_i e^{2gV}, \Phi_j) + \left[\int d^2\theta \left(W(\Phi) + \frac{1}{4}W^\alpha W_\alpha \right) + hc \right] \right\}. \tag{21}$$

where the first term is known as the *Kähler potential* and has the general form $K(\bar{\Phi}, \Phi)$, $W(\Phi)$ is the superpotential, and the last term describes the supersymmetric Yang–Mills action with

$$W_\alpha = -\frac{1}{8g}\bar{D}\bar{D}e^{-2gV}D_\alpha e^{2gV}, \tag{22}$$

$$\bar{W}_{\dot{\alpha}} = -\frac{1}{8g} DD\bar{D}_{\dot{\alpha}} e^{-2gV} e^{2gV}. \tag{23}$$

The master Lagrangian is invariant under the *R*-symmetry. By expanding the action from the Equation (21) on components, we obtain

$$\int d^4x \left[\int d^4\theta \bar{\Phi}_i \Phi_i + d^2\theta \left(\lambda_i \Phi_i + \frac{m_{ij}}{2} \Phi_i \Phi_j + \frac{g_{ijk}}{3} \Phi_i \Phi_j \Phi_k \right) + hc + \dots \right], \tag{24}$$

where m_{ij} and g_{ijk} are parameters symmetric in all their indices and ... stands for higher terms from the Yang–Mills lagrangian. The Equation (24) displays the invariance of the Kähler potential under *R*-symmetry. On the other hand, the *R*-symmetry can be broken by the superpotential term.

As an example of a supersymmetric model with *R*-symmetry, we mention here the supersymmetric quantum electrodynamics (SQED) model that contains two chiral superfields that have the following transformation properties under the *R*-symmetry

$$\begin{aligned}
\Phi'_+ &= e^{-2ie\Lambda}\Phi_+, \ R\Phi_+ = e^{2in}\Phi_+, \\
\Phi'_- &= e^{2ie\Lambda}\Phi_-, \ R\Phi_- = e^{-2in}\Phi_-.
\end{aligned} \tag{25}$$

Here, Λ is another chiral superfield and e is the $U(1)$ charge. The SQED lagrangian is given by [48]

$$\begin{aligned}
\mathcal{L}_{SQED} &= \frac{1}{4}\left(\int d^2\theta W^\alpha W_\alpha + \int d^2\bar{\theta} \bar{W}^{\dot{\alpha}} \bar{W}_{\dot{\alpha}} \right) + \int d^4\theta \bar{\Phi}_+ e^{2eV} \Phi_+ + \int d^4\theta \bar{\Phi}_- e^{2eV} \Phi_- \\
&+ M\left(\int d^2\theta \Phi_+ \Phi_- + \int d^2\bar{\theta} \bar{\Phi}_+ \bar{\Phi}_- \right).
\end{aligned} \tag{26}$$

One can easily verify that $\mathcal{L}_{SQED}$ is invariant under *R*-symmetry defined by Equations (17) and (25) above.

3.1. Continuous R-Symmetry in MSSM

MSSM is the simplest SUSY model that extends the field content of the SM by a minimal set of fields, and it has been used as a main model in the investigations of supersymmetry (see, for a recent review, e.g., [49]). The superpotential of MSSM has two terms

$$\begin{aligned}
W_2 &= \mu\epsilon\hat{H}_1\hat{H}_2 + \mu_{0a}\epsilon\hat{L}_{aL}\hat{H}_2, \\
W_3 &= f^l_{ab}\epsilon\hat{L}_{aL}\hat{H}_1\hat{E}_{bR} + f^u_{ij}\epsilon\hat{Q}_{iL}\hat{H}_2\hat{U}_{jR} + f^d_{ij}\epsilon\hat{Q}_{iL}\hat{H}_1\hat{D}_{jR} \\
&+ \lambda_{abc}\epsilon\hat{L}_{aL}\hat{L}_{bL}\hat{E}_{cR} + \lambda'_{iaj}\epsilon\hat{Q}_{iL}\hat{L}_{aL}\hat{D}_{jR} + \lambda''_{ijk}\hat{D}_{iR}\hat{U}_{jR}\hat{D}_{kR}.
\end{aligned} \tag{27}$$

The chiral supermultiplet contains three families of left-handed quarks denoted by Q_{iL}, three families of leptons L_{iL}, and the Higgs field H_1. The anti-chiral supermultiplet has three families of right-handed quarks, denoted by u_{iR} and d_{iR}; three families of right-handed leptons l_{iR}; and another Higgs field H_2.

In order to analyze the *R*-symmetry of MSSM, one makes the assumption that

$$\begin{array}{ll|l}
\text{for}: & H_1, \ H_2 & n_{H_1} = n_{H_2} = 0 \\
\hline
\text{for}: & Q, \ u, \ d, \ L, \ E & n_Q = n_u = n_d = n_L = n_l = \frac{1}{2},
\end{array} \tag{28}$$

where the *R*-charges are defined in Equation (17). Then, the *R*-symmetry acts on the components of the fields as follows:

$$\begin{aligned}
H_{1,2}(x) &\xmapsto{R} H_{1,2}(x), & \tilde{H}_{1,2}(x) &\xmapsto{R} e^{-i\alpha}\tilde{H}_{1,2}(x), \\
\tilde{f}_L(x) &\xmapsto{R} e^{i\alpha}\tilde{f}_L(x), & \tilde{f}_R(x) &\xmapsto{R} e^{-i\alpha}\tilde{f}_R(x),
\end{aligned}$$

$$\Psi(x) \quad \overset{R}{\longmapsto} \quad \Psi(x). \tag{29}$$

This shows that the scalars $H_{1,2}$ and all the standard fermions are invariant under R-symmetry. One can easily see that the terms from the superpotential that conserve the R-symmetry are given by

$$W = \mu\epsilon\hat{H}_1\hat{H}_2 + f_{ab}^l\epsilon\hat{L}_{aL}\hat{H}_1\hat{E}_{bR} + f_{ij}^u\epsilon\hat{Q}_{iL}\hat{H}_2\hat{U}_{jR} + f_{ij}^d\epsilon\hat{Q}_{iL}\hat{H}_1\hat{D}_{jR}. \tag{30}$$

It is important to note here that in MSSM all charged fermions receive masses at tree-level. However, the neutrinos are massless as in the SM.

The neutrinos can be given masses in the presence of the unbroken R-symmetry in a minimal R-supersymmetric standard model (MRSSM) that generalizes MSSM. In the MRSSM, the Higgs sector is enlarged by R_u and R_d multiplets, which produce μ-terms with H_u and H_d. The R-symmetry is preserved due to the vevs of Higgs fields, which break the electroweak symmetry. More concretely, both $H_{1,2}$ have $n_H = 0$, and the particle content of the MSSM is enlarged in the following way:

$$\hat{R}_d \quad \sim \quad \left(1, 2, +\frac{1}{2}\right), \quad n_{R_d} = 2,$$
$$\hat{R}_u \quad \sim \quad \left(1, 2, -\frac{1}{2}\right), \quad n_{R_u} = 2, \tag{31}$$

The adjoint chiral superfields $\hat{O}$, $\hat{T}$, and $\hat{S}$, and their R-charge, are zero. The superpotential of this model is

$$
\begin{aligned}
W \quad = \quad & \mu_d\left(\hat{R}_d \cdot \hat{H}_1\right) + \mu_u\left(\hat{R}_u \cdot \hat{H}_2\right) + \Lambda_d\left(\hat{R}_d \cdot \hat{T}\right)\hat{H}_1 + \Lambda_u\left(\hat{R}_u \cdot \hat{T}\right)\hat{H}_2 + \lambda_d\hat{S}\left(\hat{R}_d \cdot \hat{H}_1\right) \\
+ \quad & \lambda_u\hat{S}\left(\hat{R}_u \cdot \hat{H}_2\right) - f_{ab}^l\epsilon\hat{L}_{aL}\hat{H}_1\hat{E}_{bR} - f_{ij}^u\epsilon\hat{Q}_{iL}\hat{H}_2\hat{U}_{jR} - f_{ij}^d\epsilon\hat{Q}_{iL}\hat{H}_1\hat{D}_{jR},
\end{aligned} \tag{32}
$$

where the complex triplet is defined as follows:

$$\hat{T} \quad \sim \quad \begin{pmatrix} \frac{\hat{T}_0}{\sqrt{2}} & \hat{T}_+ \\ \hat{T}_- & -\frac{\hat{T}_0}{\sqrt{2}} \end{pmatrix}. \tag{33}$$

In MRSSM, the neutrinos acquire Majorana masses [50–53]. The Majorana neutrino mass term is given by

$$H_1 H_1 L_i L_j. \tag{34}$$

Other interesting features of the MRSSM are the absence of gaugino masses, μ-terms, and trilinear A-terms, which are forbidden by R-symmetry. R-symmetry can also be used to preserve Dirac gauginos since the massless gauginos and Higgsinos are ruled out by data. For a study of lepton flavor violation, see [54].

3.2. A Problem of Continuous R-Symmetry—Discrete R-Parity

We already mentioned that the vector superfield V is invariant under R-symmetry. However, the field components of the vector superfield transform as

$$
\begin{array}{ccc}
A_m(x) & \overset{R}{\longmapsto} & A_m(x) \\
\lambda(x) & \overset{R}{\longmapsto} & e^{i\alpha}\lambda(x) \\
\bar{\lambda}(x) & \overset{R}{\longmapsto} & e^{-i\alpha}\bar{\lambda}(x) \\
D(x) & \overset{R}{\longmapsto} & D(x)
\end{array}. \tag{35}
$$

In order to break the supersymmetry, Girardello introduced a mass term for gauginos of the following form [55]:

$$m_\lambda \left(\lambda\lambda + \bar{\lambda}\bar{\lambda} \right), \tag{36}$$

that transforms under the *R*-symmetry (35) as follows:

$$m_\lambda \left(e^{2i\alpha}\lambda\lambda + e^{-2i\alpha}\bar{\lambda}\bar{\lambda} \right). \tag{37}$$

From the relations (35), one can see that the mass term given by the Equation (36) is not invariant under the *R*-symmetry.

The soft breaking of supersymmetry suggests replacing the continuous *R*-symmetry by a discrete *R*-symmetry called *R-parity*, whose operator on superfunctions is denoted by R_d. The *R*-parity solves the above problem by setting $\alpha = \pi$. In the supersymmetric models with *R*-parity, the gluinos and other gauginos become massive.

Given the following values of *R*-charges

$$
\begin{aligned}
n_{H_1} &= 0, \ n_{H_2} = 0, \ n_L = \frac{1}{2}, \ n_Q = \frac{1}{2}, \\
n_E &= -\frac{1}{2}, n_U = -\frac{1}{2}, \ n_D = -\frac{1}{2},
\end{aligned} \tag{38}
$$

the allowed terms in the superpotential are

$$W = \mu\epsilon\hat{H}_1\hat{H}_2 + f^l_{ab}\epsilon\hat{L}_a\hat{H}_1\hat{l}^c_b + f^u_{ij}\epsilon\hat{Q}_i\hat{H}_2\hat{u}^c_j + f^d_{ij}\epsilon\hat{Q}_i\hat{H}_1\hat{d}^c_j. \tag{39}$$

The above superpotential defines the known MSSM. As in the Equation (30) above, all the neutrinos are massless at all orders of perturbation theory. However, by choosing the following values for *R*-charges

$$
\begin{aligned}
n_{H_1} &= n_{H_2} = n_L = n_l = 0, \\
n_Q &= -n_u = -n_d = \frac{1}{2},
\end{aligned} \tag{40}
$$

the allowed terms that define the new superpotential are

$$
\begin{aligned}
W_H &= \mu\epsilon\hat{H}_1\hat{H}_2 + \mu_{0a}\epsilon\hat{L}_a\hat{H}_2 + f^l_{ab}\epsilon\hat{L}_a\hat{H}_1\hat{l}^c_b + f^u_{ij}\epsilon\hat{Q}_i\hat{H}_2\hat{u}^c_j + f^d_{ij}\epsilon\hat{Q}_i\hat{H}_1\hat{d}^c_j \\
&\quad + \lambda_{abc}\epsilon\hat{L}_a\hat{L}_b\hat{l}^c_c + \lambda'_{iaj}\epsilon\hat{Q}_i\hat{L}_a\hat{d}^c_j.
\end{aligned} \tag{41}
$$

As shown in [56–58], the superpotential (41) generates neutrino masses, which were calculated numerically in [36], and the values of the mixing parameters were obtained in [59].

The *R*-parity plays an important role in the baryon *B* and lepton *L* numbers violation in the MSSM. The most general superpotential of the MSSM contains interacting terms that violate the baryon and lepton numbers. In order to remove these terms, which are not allowed by the SM, the *R*-parity is invoked. The general form of the *R*-parity in terms of *B* and *L* numbers is

$$R = (-1)^{3(B-L)+2S} = (-1)^{2S}M,$$

where *M* is the matter parity and takes the following values:

$M = +1$	for Higgs and Gauge superfields
$M = -1$	for matter superfields.

$$\tag{42}$$

This remark is important for the construction of SUSYB-L models.

3.3. Nelson–Seiberg Theorem

Since the SUSY is not directly observed in nature, one has to consider that it is a broken symmetry. One important class of models that displays the dynamical SUSY breaking of

strongly coupled gauge theories at low-energies is the generalized O'Raifeartaigh models, which are weakly coupled Wess–Zumino models in which the SUSY is broken by vevs of tree-level F-term [60]. The generalized O'Raifeartaigh models are a particular case of generic calculable models that obey the following Nelson–Seiberg theorem [61]:

Theorem [Nelson–Seiberg]. *The necessary and sufficient conditions for SUSY breaking at the true vacuum in a Wess–Zumino model with a generic superpotential are:*

- *Necessary: The model must have an R-symmetry.*
- *Sufficient: The R-symmetry should be spontaneously broken.*

Here, the term *generic* refers to the property that the superpotential must contain all renormalizable terms with complex coefficients. The Nelson–Seiberg theorem has been revisited lately in [62,63], and counterexamples have been found in [64,65].

The theorem relates the global $U(1)_R$ group to the SUSY breaking by the following argument. Since the superpotential R-charge is 2, then it has the following form:

$$W = \phi_n^{\frac{2}{R(\phi_n)}} g\left(\frac{\phi_i}{\langle \phi_i \rangle^{\frac{R(\phi_i)}{R(\phi_n)}}} \right), \tag{43}$$

where $i = 1, 2, \ldots, n$ denotes the fields. However, the SUSY vacuum is a stationary point of the system

$$\frac{\partial W(\phi_i)}{\partial \phi_j} = 0, \tag{44}$$

for all i, j. Then, it is easy to see that the effective Equation (44) is reduced to a system of $n - 1$ equations with n unknowns, which does not admit a solution, in general. Therefore, one cannot determine a supersymmetric vacuum, which leads to the conclusion that in the presence of the R-symmetry, the SUSY must be spontaneously broken.

4. Next to Minimal Standard Model

The SUSY breaking term of MSSM depends on the mass μ of H_1 and H_2 fields, which, at its turn, is constrained by phenomenology to be of the order of SUSY breaking scale μ $\mu \simeq m_{SUSY}$. This is known as the μ- problem since it links the electroweak scale determined by Higgs fields vevs to m_{SUSY}. In order to solve the μ-problem, the MSSM is modified by introducing a scalar field, which couples to H_1 and H_2 through Yukawa couplings and whose vev is determined by the soft SUSY breaking terms. This model is known as the next-to-minimal supersymmetric standard model (NMSSM), and it was reviewed in detail in [66].

The scalar field S of the NMSSM is a component of a the new singlet superfield

$$\hat{S} \sim (1, 1, 0), \tag{45}$$

which can be written as a chiral superfield [21]

$$\hat{S}(y, \theta) \quad = \quad S(y) + \sqrt{2}\theta\tilde{S}(y) + \theta\theta F_S(y). \tag{46}$$

Here, the vev of the scalar field S is taken to be

$$\langle S \rangle \equiv \frac{x}{\sqrt{2}}. \tag{47}$$

The fermionic field $\tilde{S}$, defined by the Equation (46), is known as singlino.

The most general superpotential that contains the singlet extension of MSSM is given by the following relation: [49]

$$
\begin{aligned}
W_{GSEMSSM} &= W_H + \left(\xi_F M_n^2\right)\hat{S} + \lambda\left(\hat{H}_1\hat{H}_2\right)\hat{S} + \lambda_i\left(\hat{\hat{L}}_i\hat{H}_2\right)\hat{S} \\
&+ \frac{\mu_2}{2}\left(\hat{S}\right)^2 + \frac{\kappa}{3}\left(\hat{S}\right)^3,
\end{aligned}
\tag{48}
$$

where W_H is defined by the Equation (41). The parameters λ, κ are the Yukave parameters and, together with ξ_F, which determines the SUSY mass term, are dimensionless, while the parameters μ_2 and M_n have mass dimensions.

The NMSSM can be obtained either from super-GUT models or from superstring $E(6)$ models [21,49]. To obtain NMSSM, one needs to set

$$
\mu = \mu_{0i} = \xi_F = \mu_2 = \lambda''_{ijk} = 0.
\tag{49}
$$

Then, the superpotential takes the following form:

$$
\begin{aligned}
W_{NMSSM} &= \sum_{i,j=1}^{3}\left[f_{ij}^l(\hat{H}_1\hat{L}_i)\hat{l}_j^c + f_{ij}^d(\hat{H}_1\hat{Q}_i)\hat{d}_j^c + f_{ij}^u(\hat{H}_2\hat{Q}_i)\hat{u}_j^c \right] \\
&+ \lambda\left(\hat{H}_1\hat{H}_2\right)\hat{S} + \lambda_i\left(\hat{\hat{L}}_i\hat{H}_2\right)\hat{S} + \frac{\kappa}{3}\left(\hat{S}\right)^3 + \sum_{i,j,k=1}^{3}\left[\lambda_{ijk}(\hat{L}_i\hat{L}_j)\hat{l}_k^c + \lambda'_{ijk}(\hat{L}_i\hat{Q}_j)\hat{d}_k^c \right].
\end{aligned}
\tag{50}
$$

The superpotential that generates neutrino masses in NMSSM was given in [67], where the following function was proposed:

$$
\begin{aligned}
W_{NMSSM} &= \sum_{i,j=1}^{3}\left[f_{ij}^l(\hat{H}_1\hat{L}_i)\hat{l}_j^c + f_{ij}^d(\hat{H}_1\hat{Q}_i)\hat{d}_j^c + f_{ij}^u(\hat{H}_2\hat{Q}_i)\hat{u}_j^c \right] \\
&+ \lambda\left(\hat{H}_1\hat{H}_2\right)\hat{S} + \mu_i\left(\hat{\hat{L}}_i\hat{H}_2\right) + \frac{\kappa}{3}\left(\hat{S}\right)^3 \\
&+ \sum_{i,j,k=1}^{3}\left[\lambda_{ijk}(\hat{L}_i\hat{L}_j)\hat{l}_k^c + \lambda'_{ijk}(\hat{L}_i\hat{Q}_j)\hat{d}_k^c \right].
\end{aligned}
\tag{51}
$$

From W_{NMSSM}, just one massive neutrino is generated as in MSSM with R-parity violation terms. All of the conclusions presented after Equation (41) still hold in this case.

An important conclusion is that, if we want to generate masses for all neutrinos at tree level, we need to introduce three right-handed neutrinos. We discuss this case in some detail below.

5. Minimal Supersymmetric Standard Model with Three Right-Handed Neutrinos

The simplest supersymmetric model to explain the masses of the left-handed neutrinos is known as the minimal supersymmetric standard model with third generation right-handed neutrinos (MSSMRH). The particle content of this model is the same as MSSM with the addition of the right-handed neutrinos $\hat{N}_i \sim (1,0)$ (see, e.g., [4,68–70]). The superpotential of the MSSMRH is given by

$$
W_{MSSMRH} = W_H + \frac{1}{2}M_{ij}^n\hat{N}_i\hat{N}_j + \frac{1}{3}f_{ij}^n\left(\epsilon\hat{H}_2\hat{L}_i\right)\hat{N}_j + hc,
\tag{52}
$$

where W_H is defined by the Equation (41). For this model, the R-charges are given by Equation (40) together with the condition $n_N = 0$.

In this MSSMRH model, the masses of neutrinos are obtained from the following terms:

$$
-\left\{ \frac{1}{2}\left[\mu_{0i}(\epsilon H_2 L_i) + M_{ij}^n N_i N_j \right] + \frac{1}{3}f_{ij}^n(\epsilon H_2 L_i)N_j \right\} + \text{h.c..}
\tag{53}
$$

Here, M_{ij}^n are the symmetric Majorana mass matrices and f_{ij}^n are the sources of the Dirac masses. The MSSMRH model is interesting because it can generate mass to all neutrinos at the tree level and explain all mixing data about neutrinos, as shown in [71].

If we consider $\mathcal{Z}_3$-symmetry that acts on the chiral superfields by multiplying them with a real phase factor

$$\Phi \to \exp\left(\frac{2\pi\omega}{3}\right)\Phi, \tag{54}$$

where ω is an entire number, then the Majorana masses can be avoided and only the Dirac mass term survives

$$\frac{1}{3} f_{ij}^n (\epsilon H_2 L_i) n_j^c. \tag{55}$$

In order to explain the lightness of the neutrino masses, one must have $f \leq 10^{-12}$ [72].

6. μ from *ν* Supersymmetric Standard Model (μ*ν*SSM)

The MSSM as well as MSSMRH models suffer from the μ-problem [73], which can be formulated as the generation of a μ coupling in the $\mu\hat{H}_1\hat{H}_2$ term of the order of the electro-weak scale. The μ-problem is solved by NMSSM, as discussed above. From the point of view of neutrino oscillations, NMSSM is not interesting because its neutrinos are massless [4]. Nevertheless, one can construct a non-minimal SUSY model that solves the μ-problem and at same time gives masses to all neutrinos. This is called μ from ν supersymmetric standard model (μ*ν*SSM), and it was proposed in [74].

The superpotential of μ*ν*SSM can be obtained by requiring that it be invariant under $\mathcal{Z}_3$-symmetry; see Equation (54) above. This leads to the following formula:

$$\begin{aligned} W_{\mu\nu suppot} &= f_{ab}^l \epsilon \hat{L}_a \hat{H}_1 \hat{E}_b + f_{ij}^u \epsilon \hat{Q}_i \hat{H}_2 \hat{U}_j + f_{ij}^d \epsilon \hat{Q}_i \hat{H}_1 \hat{D}_j \\ &+ \frac{1}{3} \sum_{i,j=1}^{3} \left[f_{ij}^n (\epsilon \hat{H}_2 \hat{L}_i) \hat{N}_j + h_i^n (\epsilon \hat{H}_2 \hat{H}_1) \hat{N}_i + \kappa_{ijk}^n \hat{N}_i \hat{N}_j \hat{N}_k \right]. \end{aligned} \tag{56}$$

This superpotential is consistent with the phenomenological models derived from the superstring theory, and it provides the following neutralino mass matrices

$$\begin{bmatrix} \mathrm{M}_{7\times7} & \mathrm{m}_{3\times7} \\ \mathrm{m}_{3\times7}^T & 0_{3\times3} \end{bmatrix} \tag{57}$$

The μ*ν*SSM model has Majorana neutrinos. Therefore, several processes as the double beta decay can take place without neutrinos. More details can be found in [21].

7. Supersymmetric *B-L* Model

Right-hand neutrinos at the TeV scale can be naturally obtained in the supersymmetric B-L(SUSYB-L) extension of the SM, which is one of the simplest non-linear class of models beyond SM that provide a viable and testable solution to the neutrino mass. The SUSYB-L model can also account for the experimental results of the light neutrino masses and their large mixing angles.

The emph *B-L* number defined as the baryon minus lepton numbers can be obtained from a gauge symmetry that is broken at the TeV scale [75,76], which is the SUSY breaking scale necessary to explain the hierarchy problem in MSSM. In the SUSYB-L, as in MSSM, the Higgs potential receives large radiative corrections that induce spontaneous *B-L* symmetry breaking at the TeV scale, in analogy to the electroweak symmetry breaking in MSSM [75].

The SUSYB-L model has new complex phases in the leptonic sector that can generate lepton asymmetry that is converted to baryon asymmetry [76]. The relevant terms from the Lagrangian that are responsible for this process are the soft SUSY-breaking terms [77,78]

$$\mathcal{L}_{\text{soft}} = \frac{\tilde{m}_n^2}{2}\left(\tilde{N}\right)^\dagger \tilde{N} + \frac{B_n^2}{2}\tilde{N}\tilde{N} + A_n Y_n\left(\epsilon \tilde{L} H_2\right)\tilde{N} + h.c. \tag{58}$$

The above Lagrangian describes the mixing between the sneutrino $\tilde{N}$ and the anti-sneutrino $\tilde{N}^\dagger$. The CP violation phase in this mixing generates lepton asymmetry in the final states of the $\tilde{N}$-decay. This is converted into baryon asymmetry through the sphaleron process [79].

The Minimal B-L Supersymmetric Model

The light neutrino masses can be obtained from the minimal gauged $U(1)_{B-L}$ that is invariant under the following gauge group:

$$SU(2)_L \otimes U(1)_Y \otimes U(1)_{B-L}. \tag{59}$$

This model violates the R-parity [80]. The matter chiral supermultiplets for leptons and their $SU(2)_L$, $U(1)_Y$, and $U(1)_{B-L}$ quantum numbers are given by

$$\hat{L}_i \sim (2,-1,-1), \ \ \hat{E}_i \sim (1,2,1), \ \ \hat{N}_i \sim (1,0,1). \tag{60}$$

Given the above matter content, the superpotential takes the following form:

$$W_{BL} = f_{ab}^l \epsilon \hat{L}_a \hat{H}_1 \hat{E}_b + f_{ij}^u \epsilon \hat{Q}_i \hat{H}_2 \hat{U}_j + f_{ij}^d \epsilon \hat{Q}_i \hat{H}_1 \hat{D}_j + \frac{1}{3}\sum_{i,j=1}^{3} f_{ij}^n\left(\epsilon \hat{H}_2 \hat{L}_i\right)\hat{N}_j. \tag{61}$$

Once the R-parity is broken, the neutralinos and neutrinos can mix as in MSSM. Due to this mixing, all neutrinos masses are generated via the see-saw mechanism. The main arguments for the generation of light neutrino masses are given in [80]. Let us briefly recall them. By breaking the R-parity, the neutralinos and neutrinos mix. The relevant fields are

$$\left(\nu, \nu^c, \tilde{B}, \tilde{B}', \tilde{W}_L^0, \tilde{H}_d^0, \tilde{H}_u^0\right).$$

One particular simple case is given by the vev $v_L \to 0$ and $Y_\nu^D << 1$. Then,

$$M_\nu = M_\nu^I + M_\nu^R,$$

where the vevs of the Higgs doublets are

$$\left\langle H_u^0\right\rangle = \frac{v_u}{\sqrt{2}}. \qquad \left\langle H_d^0\right\rangle = \frac{v_d}{\sqrt{2}},$$

Here, M_ν^I is the see-saw contribution and M_ν^R is obtained from the R-parity violation term. These two contributions have the form

$$M_\nu^I = \frac{1}{2}Y_\nu^D M_{\nu^C}^{-1}\left(Y_\nu^D\right)^T v_u^2, \qquad M_\nu^R = m M_{\tilde{\chi}^0}^{-1} m^T$$

where

$$M_{\nu^C} \approx \left(M_{BL} + \sqrt{4M_{Z'}^2 + M_{BL}^2}\right)/2, \qquad m = \text{diag}\left(0,0,0,0,Y_\nu^D v_R/\sqrt{2}\right).$$

In the above formula, $M_{\tilde{\chi}^0}$ is the neutralino mass matrix, which is calculated as in the MSSM. The main conclusion is that the light neutrino masses are obtained from the

R-symmetry breaking in the minimal $SUSY B - L$ model. For more details, we refer to the original paper [80].

8. Supersymmetric Left–Right Model

The minimal SUSYB-L model discussed in the above section is not the only one that can explain the light neutrinos. The SUSYLR model represents a different proposal, which can also solve the strong CP problem [81]. In the SUSYLR model, the gauge symmetry group is

$$SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}. \tag{62}$$

The lepton content of SUSYLR is different from the lepton content of MSSM, where the left-handed fermions belong to the doublet representation, while the right-handed fermions are singlets of $SU(2)_L$. Here, both left-handed and right-handed leptons are doublets in the corresponding $SU(2)$ gauge groups

$$\hat{L}_i \sim (2,1,-1), \ \hat{L}_i^c \sim (1,2,1). \tag{63}$$

In the literature, two different SUSYLR models have been discussed: the first one uses $SU(2)_R$ triplets (SUSYLRT) [82], and the second one has $SU(2)_R$ doublets (SUSYLRD) [83]. Since the neutrinos in SUSYLRD are massless, we will not discuss the details of this model here.

The scalars of the triplet model (SUSYLRT) are

$$
\begin{aligned}
\hat{\Delta}_L &\sim (3,1,2), \ \hat{\Delta}_L' \sim (3,1,-2), \\
\hat{\delta}_L^c &\sim (1,3,-2), \ \hat{\delta}_L'^c \sim (1,3,2), \\
\hat{\Phi} &\sim (2,2,0), \ \hat{\Phi}' \sim (2,2,0).
\end{aligned}
\tag{64}
$$

The most general superpotential *W* is given by [82]

$$
\begin{aligned}
W \ = \ & M_\Delta Tr(\hat{\Delta}_L \hat{\Delta}_L') + M_{\delta^c} Tr(\hat{\delta}_L^c \hat{\delta}_L'^c) + \mu_1 Tr\big[(i\sigma_2)\hat{\Phi}(i\sigma_2)\hat{\Phi}\big] + \mu_2 Tr\big[(i\sigma_2)\hat{\Phi}'(i\sigma_2)\hat{\Phi}'\big] \\
+ \ & \mu_3 Tr\big[(i\sigma_2)\hat{\Phi}(i\sigma_2)\hat{\Phi}'\big] + f_{ab} Tr\big[\hat{L}_a(i\sigma_2)\hat{\Delta}_L\hat{L}_b\big] + f_{ab}^c Tr\big[\hat{L}_a^c(i\sigma_2)\hat{\delta}_L^c\hat{L}_b^c\big] \\
+ \ & h_{ab}^l Tr\big[\hat{L}_a\hat{\Phi}(i\sigma_2)\hat{L}_b^c\big] + \tilde{h}_{ab}^l Tr\big[\hat{L}_a\hat{\Phi}'(i\sigma_2)\hat{L}_b^c\big],
\end{aligned}
\tag{65}
$$

where $h^l, \tilde{h}^l$ are the Yukawa couplings for the leptons. This model can be embedded in a supersymmetric grand unified theory (SUSYGUT) with $SO(10)$, some of which will be briefly discussed in the Section 9.

The masses of neutrinos in the SUSYLRT are given by the following matrix [84]:

$$
\begin{aligned}
M_{ab}^\nu \ = \ & \frac{1}{\sqrt{2}}\Big[k_1 h_{ab}^l + k_2' \tilde{h}_{ab}^l\Big](\nu_a \nu_b^c + hc) + \frac{\nu_R}{\sqrt{2}} f_{ab}^c (\nu_a^c \nu_b^c + hc) \\
- \ & \frac{\nu_L}{\sqrt{2}} f_{ab}(\nu_a \nu_b + h.c.).
\end{aligned}
\tag{66}
$$

This result is in agreement with the one given in [85] for $\nu_L = 0$.

9. Supersymmetric Grand Unification Theory

In the standard minimal SUSYGUT scenario, the models have both the supersymmetry and the unified gauge symmetry at the unification scale. The main properties of a finite SUSYGUT are [86]

1. The number of generations is fixed by the requirement of finiteness.
2. There are various realistic possibilities given by the gauge groups $SU(5)$, $SU(6)$, $SO(10)$, and $E(6)$.

The last point above leads to a series of models that are compatible with SUSYGUT. Below, we will give a very brief overview of the two supersymmetric grand unification models that provide masses for neutrinos $SU(5)_{RN}$ and $SO(10)_M$.

9.1. SU(5) Grand Unified Model with Right-Handed Neutrinos $SU(5)_{RN}$

There are some very good reviews of the $SU(5)$ SUSY model on [87,88]. The SUSYGUT model with the $SU(5)$ gauge group solves observational difficulties arising from the proton decay channel [89]

$$p \longrightarrow K^+ \nu^c.$$

Additionally, neutrinos are massless, and the lepton region must be modified to provide a mechanism capable of giving mass to neutrinos. For a discussion of the challenges posed by the $SU(5)$ model, see [90].

In $SU(5)$ grand unification model with right-handed neutrinos $SU(5)_{RN}$, we can introduce a charged lepton similarly to the minimal $SU(5)$ model, in the form $\Phi_i \sim \bar{5}$ (5^* representation of $SU(5)$, and the indices i, j, as usual, represent generations, and A is the $SU(5)$ index that runs from 1 to 5). Additionally, we need to introduce right-handed neutrinos in the representation $\eta_i \sim 1$. The scalars of the model are given by $H \sim 5$ and $\overline{H} \sim \bar{5}$. Then, the superpotential is given by [91,92]:

$$W_{SU(5)_{RN}} = f_{ij}^n \eta_i \Phi_{jA} H^A + \frac{1}{2} M_{ij} \eta_i \eta_j. \tag{67}$$

Here, we only explicitly give terms that generate the neutrino mass. Also, M_{ij} is the Majorana mass matrix for right-handed neutrinos, and f^n is the Dirac mass. Both masses are generated via the see-saw mechanism.

9.2. Minimal SO(10) Supersymmetric Model $SO(10)_M$

Supersymmetric $SO(10)$ models are discussed in detail in [93,94]. Since our goal is to present the generation of neutrino mass, we can illustrate the $SO(10)$ group using the model presented in [95,96]. In this case, the lepton is in $\psi_i \sim 16$ representation of $SO(10)$ and the Higgs fields are in the representations $H_{10} \sim 10$ and $\Delta \sim 126^*$ of $SO(10)$. The terms of the superpotential that are responsible for the lepton masses are given by

$$W_{SO(10)_M} = h_{ij} \psi_i \psi_j H_{10} + f_{ij} \psi_i \psi_j \Delta, \tag{68}$$

where h and f are symmetric matrices in (i, j) that denote generation indices. The neutrino masses can be calculated in this model from

$$M_{\nu^D} = \left(h^* \cos \alpha_u - 3 f^* e^{i \gamma_u} \sin \alpha_u \right) \sin \beta, \tag{69}$$

where α_u is a new parameter of the model responsible for non-null CKM mixing angles [95].

$SO(10)$ supersymmetric model has a second mass generating mechanism that produces the neutrino masses from the scalar of the 126 representations of $SO(10)$. A large mass mixing in the $\nu_\mu - \nu_\tau$ channel gives the following form to the low energy Majorana neutrino mass matrix [97]

$$M_{\nu_{LL}} \sim \begin{pmatrix} 0 & 0 & t \\ 0 & 1 & 1 \\ t & 1 & 1 \end{pmatrix} \Lambda, \tag{70}$$

with eigenvalues in Λ units

$$|m_1| \simeq \frac{t}{\sqrt{2}} - \frac{t^2}{8} - \frac{3t^3}{64\sqrt{2}}, \quad |m_2| \simeq \frac{t}{\sqrt{2}} + \frac{t^2}{8} - \frac{3t^3}{64\sqrt{2}}, \quad |m_3| \simeq 2 + \frac{t^2}{4}. \tag{71}$$

By making assumptions about the texture of the Dirac mass matrix (similar form to the up-quark mass matrix), the other Majorana mass matrices take the form

$$M_{\nu_{LR}} = \begin{pmatrix} 0 & 0 & \alpha \\ 0 & \beta & \gamma \\ \alpha & \gamma & 1 \end{pmatrix} \eta$$

with $\alpha \simeq \beta \ll \gamma \ll 1$, and

$$M_{\nu_{RR}} = \begin{pmatrix} 0 & 0 & \delta_1 \\ 0 & \delta_2 & \delta_3 \\ \delta_1 & \delta_3 & 1 \end{pmatrix} M_R$$

with

$$\delta_1, \delta_2, \delta_3 \ll 1 \,.$$

The following relations hold:

$$\delta_1 = \frac{\alpha^2}{2\alpha - 2\alpha\gamma + \gamma^2 t} \,, \quad \delta_2 = \frac{\beta^2 t}{2\alpha - 2\alpha\gamma + \gamma^2 t} \,, \quad \delta_3 \simeq \frac{\alpha(\gamma - \beta) + \beta\gamma t}{2\alpha - 2\alpha\gamma + \gamma^2 t} \,.$$

The see-saw mechanism imposes the following constraint on the Majorana matrices

$$M_{\nu_{LL}} = -M_{\nu_{LR}}^T M_{\nu_{RR}}^{-1} M_{\nu_{LR}} \,.$$

R-symmetry can be used to solve the μ problem since the latter can be connected to supersymmetry breaking and allows one to construct viable models with neutrinos. The fermion mass hierarchy can be explained by constructing models with pseudo-anomalous $U(1)_R$ symmetry. Discrete non-abelian symmetries based on $\mathbb{Z}^N$ can be used to obtain neutrino mixing. These models solve fermion mass hierarchies and the hierarchy problem without fine-tuning while modeling neutrino mixing using non-abelian flavor symmetry. For more details on these models, see [98].

10. Conclusions

In this paper, we examined the connection between neutrino mass and R-symmetry in several interesting nonlinear SUSY models. Interest in these ideas, some older and some new, has been revived by results obtained in recent collider experiments and atmospheric, astronomical, and cosmological observations. More importantly, all the models presented here can be tested by high-energy experiments.

It is important to note that data related to neutrino oscillations alone are not sufficient to impose many restrictions on the extended parameters of the SM. For example, in the case of the R-parity-violating MSSM, the mixture of regular fermions with Higgsinos allows a neutrino to acquire a mass at tree level, via a type I see-saw mechanism, and we use it to explain atmospheric neutrinos. To produce the mass of another neutrino, we do calculations at 1-loop and with a type III see-saw mechanism and explain the solar data. Therefore, the comparison between calculated neutrino masses and experimental data takes into account different model-dependent mechanisms.

In summary, nonlinear extensions of SUSY with R-symmetry can solve key problems in particle physics beyond SM and are viable theories for solving other fundamental problems in quantum field theory and gravity, such as dark matter and dark energy. These models can be tested experimentally, making them attractive for both theoretical and experimental explorations.

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