ISBN: 978-3-95450-231-8 ISSN: 2673-5490 doi: 10.18429/JACoW-IPAC2023-WEPL084

BENCHMARKING FOR CODAL BEAM DYNAMICS CODE: LASER-PLASMA ACCELERATOR CASE STUDY

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Abstract

Laser-plasma electron beams are known for their large divergence and energy spread while having ultra-short bunches, which differentiate them from standard Radio-Frequency (RF) accelerated beams. To study the laser-plasma beam dynamics and to design a transport line, simulations with CODAL, a code developed by SOLEIL[1] in collaboration with IJCLab, have been used. CODAL is a 6D *kick* tracking code based on the symplectic integration of the local Hamiltonian for each element of the lattice. CODAL also includes collective effects simulations such as space charge, wakefield and coherent synchrotron radiation.

To validate the studies in the framework of Laser-Plasma accelerator development, results from CODAL have been compared to TraceWin, a well-known tracking code developed by CEA. The comparison has been made using the outcome of Laser WakeField Acceleration (LWFA) particle-in-cell simulations as initial start particle coordinates from a case study of PALLAS project[2], a Laser-Plasma Accelerator test facility at IJCLab.

INTRODUCTION

Numerical simulation codes for transport and beam dynamics [1–3] allow the determination of the trajectories of charged particles. They are widely used to design accelerators. Depending on the complexity of the projects and the quality of the required particle beam, numerical simulations can handle the particle trajectories independently or by taking into account collective effects such as space charge or coherent synchrotron radiation.

At first, we are interested in solving the equations of motion in 6 dimensions, in order to determine the particle's coordinates along the accelerator, according to the initial conditions. Even in the case where the trajectories are considered independent, some precautions should be taken regarding the approximations used. All non-linear effects for off-axis and off-momentum particles must be included. This is particularly true in the case of divergent beams with large energy spread. This is the case, for example, for beams produced by wake-field acceleration in a very high gradient with a short accelerating wave period that we will treat here.

For tracking codes, there are two major approaches depending on the method chosen to solve the equations of motion for external forces. The first method is a direct numerical integration of differential equations, using classical iterative methods such as Runge-Kutta. The second is based on analytical expressions which includes higher order effects. These analytical formula are often called *transfer functions* and are applied for each particle. This second approach is sometimes also referenced as *kick* method (where kicks are applied to all 6 coordinates of each particle).

In this paper, we will compare two codes, CODAL [4] and TraceWin [5], in the framework of LWFA beams in order to validate such a use for CODAL. TraceWin is a well-known code widely used for accelerator design. CODAL being an homemade code, a brief presentation will first be made. Then the concordance of the CODAL results with TraceWin will be examined,for a case relevant to laser wakefield acceleration (LFWA) beams, via the evolution of global statistic values as well as some phase-spaces.

CODAL: A CODE APPROPRIATE FOR ULTRA-SHORT BUNCHES

CODAL is a 6D tracking code capable of simulating single-bunch collective effects, such as space charge, wakefield impedance or coherent synchrotron radiation. It is mainly applicable to ultra-short relativistic electron bunches accelerators, but not only. It was first developed specifically for the design and studies of the damping-free ring (ThomX [6]). Since then, CODAL has recently been adapted with a complete longitudinal and transverse analytic model of standing wave RF cavities in the context of an Energy Recovery Linac project (PERLE [7]). Futhermore, this paper presents the validation for Laser-PLasma accelerators among CODAL's applications.

In CODAL, the free particle tracking in drifts and magnets is done using transfer functions. These transfer functions are analytical expressions derived from the resolution of the local differential expressions. As each type of magnets has a specific field shape, each type of magnet can be associated with a corresponding local Hamiltonian expression. This allows for the resolution of the local motion equations from which result the analytic expressions for the evolution between the entry and the exit of the magnet of the particle's 6 coordinates. In CODAL, the magnets and drift transfer function all come from the work of L. Nadolski [8] which

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[1] Courtesy of A. Loulergue
[2] This work is supported by ANR/PIA3-PACIFICS, CNRS/IN2P3, CPER Université Paris-Saclay

14th International Particle Accelerator Conference,Venice, Italy JACoW Publishing
ISBN: 978-3-95450-231-8 ISSN: 2673-5490 doi: 10.18429/JACoW-IPAC2023-WEPL084

details the symplectic integrator used to solve the differential equations as well as the local Hamiltonian expressions.

LWFA BEAM SPECIFICITY

Beams resulting from LWFA have large divergence and energy spread, with values of the order of a few mrad and % instead of the typical sub mrad divergences and sub % energy spread associated to the RF standards. To fully reproduce an LWFA beam, the input distributions has been generated by using Smilei [9] to run fast particle-in-cell simulations. All the results presented here use the same initial beam input (see Table 1). The generated beam results from a PIC simulation. The characteristics, though not representative of good performances, has been chosen as it provides a good framework to test the simulation validity of the CODAL results in situations with strong off-axis and off-momentum effects.

Table 1: Initial Beam Parameters

Parameter	Value	Unit
Nominal Energy E_n	245.0	MeV
Normalized Energy Spread σ_δ	2.9	%
Longitudinal Size σ_s	1.1	μm
Transverse Size $\sigma_{x/y}$	0.9 / 1.6	μm
Divergence $\sigma_{x'/y'}$	1.6 / 2.8	mrad
Transverse Normalized Emittance $\epsilon_{n,x/y}$	0.6 / 1.9	mm·mrad
Charge	26	pC
Twiss Parameter β	5.7e^{-4} / 6.15e^{-4}	m
Twiss Parameter α	0.49 / 0.48	

BENCHMARKING

Statistical Evolution

The overall behavior of a beam is first seen through the evolution of statistical global quantities, such as the normalized emittance or standard deviation values for size or divergence. The evolution of such quantities, computed from the particle coordinates' evolution, has been compared for the two codes.

Figure 1a illustrates an example of transverse size evolutions *(left axis)* for a beam focused by four quadrupoles. Simulation results from both the CODAL *(solid)* and TraceWin *(dashed)* are represented. In Fig. 1b, is plotted the corresponding normalized transverse emittances. The emittance growth is considerable and originates from off-momentum particles amplified by their off-axis position. Being off-momentum a difference in the force seen by the particle for the same gradient is created. With a large energy spread, some particles will then be over-focused while others will be under-focused. This causes an increase in the emittance. The blow-up, in each direction, mainly happens when the beam is being focused through a quadrupole when the beam

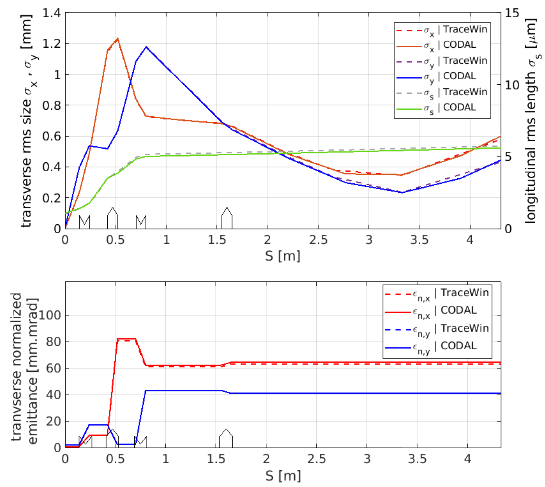

Figure 1: a) CODAL-simulated *(solid)* transverse σ_x, σ_y and longitudinal σ_s beam sizes [mm] evolution along the direct line of PALLAS [10] and the corresponding TraceWin results *(dashed)*,
b) the corresponding transverse normalized emittance $\epsilon_{n,x}$, $\epsilon_{n,y}$ [mm·mrad] evolution.

is also at its largest, as seen in Fig. 1a. The off-axis effect amplifies the emittance growth to the extent of a blow-up. The phenomenon is visible in both TraceWin and CODAL's results.

Figure 1a also shows, the evolution of the bunch length *(left axis)*. A clear lengthening, with a more than doubling size, occurs in the first meter of propagation, before the reduction of the divergence of the beam by the first three quadrupoles.

Table 2: Final Beam Parameters

Parameter	Value	Unit
Longitudinal Size σ_s	5.6	μm
Transverse Size $\sigma_{x/y}$	0.44 / 0.61	μm
Divergence $\sigma_{x'/y'}$	0.40 / 0.37	mrad
Transverse Normalized Emittance $\epsilon_{n,x/y}$	64.3 / 40.8	mm·mrad

Overall, the global behavior and the statistical values are in agreement between TraceWin and CODAL, with a maximum variation of a few percents on the beam sizes. However, with such a large energy spread, the response to the focusing field is far from uniform from a particle to another, adding to an already large range of transverse positions and divergence from the beam particles. This greatly limits the significance of the global parameters and a more detailed observation of the particle behavior is required.

Beam Distributions

The different representations of the beam phase space at various positions allow for distribution and correlation visualizations. Their comparison is a necessity to ensure

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a good agreement between the two simulation codes at a level more accurate than the statistical values. Two relevant phase-space comparisons are presented here.

Since the quadrupole forces vary strongly for some off-momentum and off-axis particles, it is interesting to examine the phase-spaces after the beam has seen all four quadrupoles. The results presented here illustrate the phase-space at the focal point.

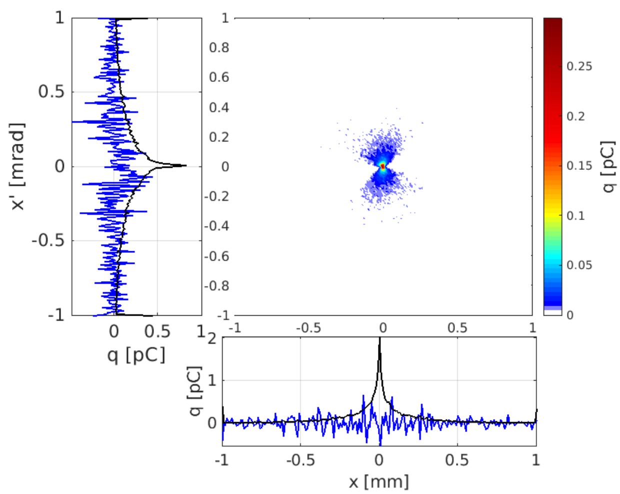

Figure 2: Simulated Transverse phase-space (x',x) [mrad, mm] at the focal point from CODAL and associated projections *(black)* as well as 10 times the projection differences between TraceWin and CODAL *(blue)*.

Figure 2 shows, as simulated by CODAL, the transverse phase space (x',x) [mrad.mm], at the focal point, with the associated projected charge distributions *(black)*, as well as 10 times the difference between the projection from TraceWin and CODAL *(blue)*. The area has a specific signature of a '*butterfly*' shape, which is explained by the chromatic behavior induced by the quadrupoles. If one could slice the beam according to the energies of the particles, each slice would be spread as an opening fan in the transverse phase space. This explains the large total emittance increase seen in Fig. 1b. This shape and the amplitude of the phase-space, as well the projected distributions are in agreement between TraceWin and CODAL as the plot of 10 the discrepancy (*blue plot* Fig. 2) shows.

Off-axis and off-momentum particles have a significant impact not only on the transverse motion but also on the longitudinal dynamics. The longitudinal position is affected by an off-momentum contribution which depends on a factor $\frac{\delta}{\gamma_L^2}$ where $\delta = \frac{p-p_n}{p_n}$ is the normalized energy spread and $\gamma_L = 1/\sqrt{1-\frac{v}{c}}$ is the Lorentz Factor. Even if LWFA beam are relativistic beams, the large energy spread needs to include the difference in behavior caused by the momentum difference in the longitudinal position dynamics. This off-momentum effect contributes to the longitudinal phase-space seen in Fig. 3. However, in the example illustrated here, this is not the dominant effect on the longitudinal position. This position also depends on the longitudinal distance that the particle travels. This distance is influenced by the transverse characteristics of the particle along the accelerator. If the differences between these transverse quantities from one particle to another can be substantial, as it is the case for LWFA beams, then off-axis effects are seen on the longitudinal beam distribution. It can be hinted in Fig. 3 where the lengthening is not specifically discriminated by the particle energy and in Fig. 1, for which the bunch lengthening happens mainly before the divergence is reduced. Here again, the phase-space is strongly impacted by off-axis particles as well as some influence of the momentum differences. The simulations for both CODAL and TraceWin provide similar results with a few % of the variation between the projections at the focal point.

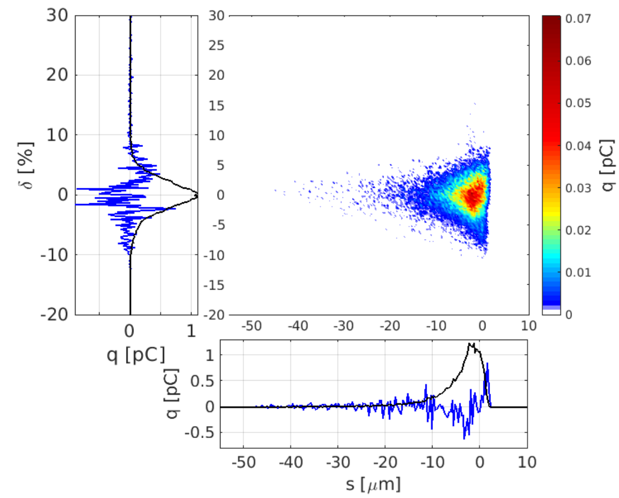

Figure 3: Simulated Transverse phase-space (δ,s) [%, μm] at the focal point from CODAL and associated projections *(black)* as well as 10 times the projection differences between TraceWin and Codal *(blue)*.

CONCLUSION

Compared to the RF accelerated electron bunches, the ones resulting from LWFA have their (x',x) , (y',y) and (δ,s) phase-spaces rotated: the longitudinal and transverse sizes are very narrow, but at the expense of the energy spread and divergences. These are generally of the order of a few % and a few mrad respectively, which is at least an order of magnitude higher than typical RF performances.

These characteristics lead to a necessity to properly model the strong off-momentum and off-axis effects on the dynamics.To ensure that this applies to CODAL, simulation results have been compared to TraceWin, for both global statistical values and phase-space distributions. A good agreement between the codes has been observed. This allows to confirm the suitability of CODAL for LWFA beams and to validate its use for LWFA electron line studies.

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