(1976) 271.
59. E. Fradkin and M. Vasiliev: Phys. Lettres **72B** (1977) 70.
60. R. Kallosh: Pisma Zh ETF **26** (1977) 575; G. Sterman, P. Townsend and P. van Nieuwenhuizen: Phys. Rev. **D17** (1978) 1501.
61. B. de Wit and M. Grisaru: Phys. Letters **74B** (1978) 57.
62. N. K. Nielsen: NORDITA preprint 78/11.
63. T. Yoneya: CCNY Report HEP-77-13.
64. S. Christenson and M. Duff: Phys. Letters **76B** (1978) 571.
65. M. Perry: Cambridge University D.A.M.T.P. preprint.
66. P. Critchley: University of Manchester preprint.
67. N. K. Nielsen, M. Grisaru, H. Römer and P. van Nieuwenhuizen: CERN preprint TH 2481.
68. N. K. Nielsen: NORDITA preprint.
69. M. Grisaru, D. Freedman and P. van Nieuwenhuizen: Phys. Letters **71B** (1977) 377.
70. S. Hawking and C. Pope, Cambridge University D.A.M.T.P. preprint

PROC. 19th INT. CONF. HIGH ENERGY PHYSICS
TOKYO, 1978

C 6 Super-Higgs Effect in Supergravity

J. SCHERK

Laboratoire de Physique Theorique de l'Ecole Normale Superieure, Paris

Supersymmetry assigns equal masses to bosons and fermions in the same multiplet. Since such a degeneracy is not observed in Nature, it is important to break supersymmetry either spontaneously or explicitly. We opt for spontaneous symmetry breaking since the introduction of non symmetrical terms would make the theory lose all predictive power. The recent advances in supergravity, namely the discovery of a minimal set of auxiliary fields, and the establishment of a tensor calculus, allow us to construct the most general coupling of supergravity (2, 3/2) to the scalar multiplet $(1/2, 0^+, 0^-)$.[1] We recover as special cases all the previously derived couplings and show that the model depends upon an arbitrary function $G(A, B)$ of the scalar (A) and pseudoscalar (B) fields.

Futher we show that for a very large class of such functions, spontaneous symmetry breaking of supersymmetry takes place. The spinor χ field of the scalar multiplet plays the role of a Goldstone fermion of supersymmetry (Goldstino). It is then absorbed by the spin 3/2 gauge field of supergravity (gravitino), just like in the Higgs model, after which banquet the gravitino becomes massive. In addition, this can occur without developing a cosmological constant, due to a cancellation between terms of opposite signs.

When supergravity[2] was first discovered, it was remarked[3] that the algebra of local supersymmetry transformations did not close unless one used the equations of motion of the spin 3/2 field, a phenomenon which occurs also in flat space supersymmetry when auxiliary (non-propagating) fields are eliminated by use of their equations of motion. This situation was cured by the discovery[4] of a very simple set of 6 auxiliary fields, consisting of an axial vector A, a scalar S and a pseudoscalar P.

This led in turn to the development of a tensor calculus[5] which generalizes to curved space the results originally obtained in flat space by Wess and Zumino. Tensor calculus applies both to scalar and vector multiplets. Since here we are interested in the coupling of supergravity to a scalar multiplet, we give a very short summary of the tensor calculus for scalar multiplets.

A scalar multiplet is a set of 5 objects $\Sigma=(A, B, \chi, F', G')$ which have well defined properties under local supersymmetry transformation. For instance $\delta A=\bar{\varepsilon}(x)\chi$, $\delta B=-i\bar{\varepsilon}(x)\gamma_5\chi$, etc. . . . The fields A, F' (B, G') are scalars (pseudoscalars) and χ is a Majorana spinor.

The tensor calculus[5] consists of two basic operations. The first is the multiplication, which to two scalar multiplets Σ_1, Σ_2 associate their product $\Sigma=\Sigma_1\otimes\Sigma_2$. In component form, one has $A=A_1A_2-B_1B_2$ and so on for the other components. This operation is commutative and does not involve any derivatives. It is purely algebraic.

The second operation is called derivation and associates to a multiplet Σ its derivative $T(\Sigma)$. The third component of $T(\Sigma)$ contains a supercovariant generalization of the Dirac operator applied on χ and the fourth and fifth components supercovariant generalizations of the d'Alembertian operator $\Box$ applied on A and B.

Finally an invariant action can be obtained from any scalar multiplet Σ by the formula:

$$I(\sum)=\int d^4xe\left[F'+\frac{1}{2}\bar{\phi}\cdot\gamma\chi + \frac{1}{2}\bar{\phi}_\mu\sigma^{\mu\nu}(A-iB\gamma_5)\phi_\nu+SA+PB\right]$$

In this formalism, the construction of invariant actions under supersymmetry transformations is very straightforward and is analogous to the construction of covariant action in curved space.

The most general interaction between supergravity and a scalar multiplet which involves no more than one derivative on the fermi field and 2 derivatives on the bose field can thus be written as:

$$I(\sum_{n,m} a_{nm}\sum^m\otimes T(\sum^n)+\sum b_n\sum^n)$$

a_{nm} can be taken real and symmetric as one can show that $I(\sum\otimes T(\Lambda)-\Lambda\otimes T(\sum))=0$ and we wish to have a parity conserving model. The action will thus depend a priori on two functions of the spin 0 fields.

$$\phi(z,\bar{z})=a_{nm}z^n\bar{z}^m$$
$$g(z)=\sum b_nz^n$$

where $z=A+iB$.

First one computes explicitly I as a function of $e_{a\mu}$, ϕ_μ, A_μ, S, P, A, B, χ, F', G'. The fields A_μ, S, P, F', G' are auxiliary fields and appear only quadratically in the result. Thus they can be eliminated by solving a set of linear equations and one obtains a reduced action depending only on the physical fields $e_{\mu a}$, ϕ_μ, χ, A, B.

The reduced action needs still to be put in a canonical form, as for instance, at that stage, the Einstein scalar curvature R and the Rarita–Schwinger Lagrangian appear multiplied by the function $\phi(z,\bar{z})$.

Thus one redefines the field $e_{\mu a}$ by a Weyl rescaling such that the scalar curvature R appears in the pure Einstein form with the correct normalization. Similarly one redefines the fields ϕ_μ and χ such that the kinetic terms of the spin 3/2 and spin 1/2 fields are the canonical correctly normalized terms minimally coupled to gravity.

These transformations are compatible with the reality of the vierbein and the Majorana property of the spinor fields under very general conditions, namely:

$$\phi(z,\bar{z})<0 \text{ and } G,_{z\bar{z}}<0$$

where $G(z,\bar{z})=3\ln\left[-\frac{\phi(z,\bar{z})}{3}\right]-\ln\frac{|g(z)|^2}{4}$

Remarkably enough, the final action and transformation laws involve only the function $G(z,\bar{z})$ rather than ϕ and g separately. Further the condition $G,_{z\bar{z}}<0$ also implies that the A, B fields kinetic terms have the right sign, *i.e.*, that these fields have positive metric and are not ghost-like.

The final action is given by:

$$I=I^0_{SG}+I_{MATTER}$$

where the first term is the supergravity action, and

$$I_{MATTER}=I^0_B+I^0_F+I_{INT}$$

where the bosonic action is given by:

$$I^0_B=\int d^4xe[G,_{z\bar{z}}\partial_\mu\partial z_\mu\bar{z}-V(z,\bar{z})]$$

and the potential V by:

$$V=-(\exp-G)\left[3+\frac{|G,_z|^2}{G,_{z\bar{z}}}\right]$$

The fermionic action I^0_F contains the kinetic term of the χ field and terms bilinear in the fermi fields ϕ_μ and χ, but does not contain derivatives of the scalar fields:

$$I^0_F=\int d^4xe\Big[-\frac{1}{2}\bar{\chi}D\!\!\!/\chi+\left(\exp-\frac{G}{2}\right)\Big(\bar{\phi}_\mu\sigma^{\mu\nu}\phi_\nu -(-2G,_{z\bar{z}})^{-1/2}\bar{\phi}\cdot\gamma\hat{G},_z\chi+(2G,_{z\bar{z}})^{-1}\bar{\chi} \times\left(\frac{\hat{G},_{zz\bar{z}}G,_z}{G,_{z\bar{z}}}+\hat{G}^2,_z-\hat{G},_{zz}\right)\chi\Big)\Big]$$

where by definition if $\hat{M}=\text{Re }M+i\gamma_5\text{ Im }M$.

Finally I_{INT} contains quartic terms in the fermi fields ϕ_μ and χ and bilinear of the fermi fields multiplied by derivatives of the scalar fields, for instance of the type $(G,_z\partial_\mu z-G,_{\bar{z}}\partial_\mu\bar{z})\varepsilon^{\mu\nu\rho\sigma}\bar{\phi}_\mu\gamma_\rho\phi_\sigma$.

This action is the most general coupling of supergravity to the scalar multiplet and depends upon one arbitrary function $G(z, \bar{z})$ of *two* real variables. It includes as special cases all previously derived couplings.[6-11]

The transformation laws of the redefined fields can also be computed. For the discussion of the super-Higgs effect, we shall only need the transformation law of χ:

$$\delta\chi_L = \tilde{\not\partial} z \varepsilon_R(-2G,_{z\bar{z}})^{1/2} + \left(\exp - \frac{G}{2}\right)\frac{G,_{\bar{z}}}{[G,_{z\bar{z}}/2]^{1/2}}\varepsilon_L + \cdots$$

where the dots indicate cubic terms in the fermi fields.

Using this very general result we can discuss the super–Higgs effect[12,13] in a model independent way. In flat space a necessary condition for spontaneous supersymmetry breaking to occur is that the auxiliary field F' (resp. D in the vector multiplet) can pick up a non-zero vacuum expectation value. It then follows that $\delta\chi$ contains a term $\delta\chi = (1/a)\varepsilon + \cdots$ where a is a constant.

The supersymmetry charge Q_α does not annihilate the vacuum, and a zero mass excitation of spin 1/2 (Goldstino) is seen to be present in the theory. In addition, a cosmological constant of fixed sign (+) is induced by the supersymmetry breaking.

In curved space, Deser and Zumino[13] used as a model for spontaneous breaking of supersymmetry the coupling to supergravity the non-linear Volkov–Akulov Lagrangian[14] which contains only one fermi field χ transforming as $\delta\chi = (1/a)\varepsilon + ia\bar{\varepsilon}\gamma^\mu\chi\partial_\mu\chi$. As $\delta\chi$ contains a $(1/a)\varepsilon$ term, χ is a candidate to represent a Goldstino field. The presence of this term implies a negative cosmological constant; however if one introduces a mass term for the spin 3/2, another cosmological constant of positive sign arises. Since experimentally the cosmological constant is very small, one imposes that the net cosmological constant vanishes. One then finds[13] that $m_\phi^2 = k^2/6a^2$ and is very small. Since $\delta\chi$ contains a constant term proportional to $\varepsilon(x)$ and we have one spinor gauge degree of freedom, χ can be gauged away completely and is absorbed by the gravitino which becomes massive. A massive gravitino indeed has 4 helicity states $\pm 3/2$, $\pm 1/2$.

In our general model we can see the same phenomenon occurring for a large class of functions $G(z, \bar{z})$.

The potential $V(z, \bar{z})$ reaches its minimum for a certain value z_0 such that:

$$V,_z|_{z_0} = V,_{\bar{z}}|_{z_0} = 0$$

If we require that the final theory does not violate parity $z_0 = \langle z \rangle$ must be real.

Further we can impose that the absolute minimum of V is reached for $V(z_0, z_0) = 0$ which implies the absence of a cosmological constant, and that $V \geq 0$ everywhere. It is indeed possible for V to be non negative as it contains two terms of opposite sign (remember that $G,_{z\bar{z}} < 0$ is a necessary condition for the A, B fields not to represent ghost particles of negative metric).

Finally the condition for spontaneous breaking of supersymmetry is that at the minimum, $\delta\chi$ contains a constant term times $\varepsilon(x)$. Looking at $\delta\chi$ we see that the requirement is that $1/a = (\exp - G/2)\{G,_{\bar{z}}/(-G,_{z\bar{z}}/2)^{1/2}\}|_{z=z_0} \neq 0$.

The condition that $1/a \neq 0$ and that $V(z_0, z_0) = 0$ are compatible as V vanishes precisely if

$$|G,_z|^2 = -3G,_{z\bar{z}}$$

in which case $1/a^2 = 6 \exp(-G)$.

Unless G becomes infinite at that point too $1/a$ is non zero.

When these very general conditions for supersymmetry breaking to occur are met, one can explicitly show by a further redefinition of the fermi fields that χ can be completely eliminated from the action and that the spin 3/2 acquires a mass given by $m_\phi^2 = 1/6a^2 = e^{-G}$ (in $k=1$ units), recovering the result obtained by Deser and Zumino.[13] It is easier however to write the full action in the $\chi = 0$ gauge where the Goldstino has been "eaten up" by the gravitino:

$$I = \int d^4x \Big[-\frac{e}{2}R - \frac{1}{2e}\varepsilon^{\mu\nu\rho\sigma}\bar{\phi}_\mu\gamma_5\gamma_\nu D_\rho\phi_\sigma + \left(\exp - \frac{G}{2}\right)\bar{\phi}_\mu\sigma^{\mu\nu}\phi_\nu + G,_{z\bar{z}}[(\partial_\mu A)^2 + (\partial_\mu B)^2] + (\exp - G)\left(3 + \frac{|G,_z|^2}{G,_{z\bar{z}}}\right) - \frac{1}{8}(G,_z\partial_\mu z - G,_{\bar{z}}\partial_\mu\bar{z}) \times \frac{1}{e}\varepsilon^{\mu\nu\rho\sigma}\bar{\phi}_\nu\gamma_\rho\phi_\sigma \Big]$$

In general A and B acquire different masses. In the simple case where $G,_{z\bar{z}} = -1/2$ (canonical kinetic term for A, B) one can establish the general mass formula:

$$m_A^2 + m_B^2 = 4m_\phi^2$$

which is independent upon the remaining arbitrary function of one variable $g(z)$. Particular examples of functions exhibiting the super-Higgs effect are easily found.

Our investigation shows thus that the super Higgs effect can take place in a large class of models, and that realistic models where supersymmetry is spontaneously broken even in curved space can be constructed, without having huge cosmological constants. It would be interesting to extend these result to the 0(N) supergravity theories which once they are gauged, create their own potential for $N \geq 4$ since they contain spin 0 fields. Unfortunately the potentials found in the O(4) and SU(4) theories[15] do not lead to spontaneous symmetry breaking, being unbounded below, which is a disappointing result. However, the discovery of auxiliary fields for 0(N) supergravity theories may reveal more flexibility in this construction than originally thought, as similarily it was believed that the coupling of supergravity to the scalar multiplet was unique.

References

1. E. Cremmer, B. Julia, J. Scherk, P. van Nieuwenhuizen, S. Ferrara and L. Girardello: LPTENS 78/17 preprint (to be published) and CERN preprint (in preparation).
2. D. Z. Freedman, P. van Nieuwenhuizen and S. Ferrara: Phys. Rev. **D13** (1976) 3214; S. Deser and B. Zumino: Phys. Letters **62B** (1976) 335.
3. D. Z. Freedman and P. van Nieuwenhuizen: Phys. Rev. **D.14** (1976) 912.
4. S. Ferrara and P. van Nieuwenhuizen: Phys. Letters **74B** (1977) 333; K. Stelle and P. C. West: Phys. Letters **74B** (1977) 330.
5. S. Ferrara and P. van Nieuwenhuizen: LPTENS preprint 78/14, to be published in Phys. Letters **B**; S. Ferrara and P. van Nieuwenhuizen: Phys. Letters **76B** (1978) 404; P. C. West and K. S. Stelle: ICTP report /77–78/15 and ICTP/77–78/24.
6. S. Ferrara, F. Gliozzi, J. Scherk and P. van Nieuwenhuizen: Nucl. Phys. **B117** (1976) 133. S. Ferrara, D. Z. Freedman, P. van Nieuwenhuigen, F. Gliozzi, J. Scherk and P. Breitenlohner: Phys. Rev. **D15** (1977) 1013.
7. E. Cremmer and J. Scherk: Phys. Letters **69B** (1977) 97.
8. J. Polonyi: Budapest preprint KFKI–1977–93.
9. A. Das, M. Fischler and M. Rocek: Phys. Letters **69B** (1977) 186; and Phys. Rev. **D16** (1977) 3427.
10. B. de Wit and D. Z. Freedman: Nucl. Phys. **B130** (1977) 105.
11. M. Kaku and P. K. Townsend: Phys. Letters **76B** (1078) 54; A. Das, M. Kaku and P. K. Townsend: Phys. Rev. Letters **40** (1978) 1215.
12. D. V. Volkov and V. A. Soroka: JETP Letters **18** (1973) 312.
13. S. Deser and B. Zumino: Phys. Rev. Letters **38** (1977) 1433.
14. D. V. Volkov and V. P. Akulov: Phys. Letters **46B** (1973) 109.
15. D. Z. Freedman and J. H. Schwarz: Nucl. Phys. **B137** (1978) 133.