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LONG RANGE BEAM-BEAM TUNE SHIFTS & WIRE COMPENSATION*

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Abstract

The weak-strong model subjects the test particle in the weak beam to transverse impulses from the strong beam, resulting in betatron tune shifts. We give analytic formulae for small amplitude and asymptotic shifts for three cases: short-range, long-range, and wire compensation; and optimize the latter to minimize the non-linear tune spreads.

BEAM-BEAM MODEL

We have two counter-rotating particle beams with like charges (i.e. p-p or e-e). The electrical charges in a "weak" bunch can be equal to those in a "strong" bunch. Despite the naming *weak-strong* beam-beam (BB) model, the formulae for the short range (SR) and long range (LR) BB tune shifts are actually the "almost strong-strong" model tune shifts. The only sense in which they are not "exactly strong-strong" is that the mutual disruption within (and of) the beams is not included.

The LRBB interaction produces a closed orbit distortion (COD) for the "weak" beam in the presence of the "strong" beam. In principle, the weak-strong model can give the linear part of the mutual strong-strong COD; and Lie-algebraic methods could be used to find the strong-strong COD to higher order in displacements.

Single-Beam Tune Shifts

The space-charge tune shift of a particle within a single beam (alone) contains an electrical repulsion and a magnetic attraction – producing partial cancellation of forces.

The single within-beam tune shift becomes very small at high energy, ($\beta \rightarrow 1$, $\gamma >> 1$). But, it acts everywhere all the time. It adds up, cannot be ignored; indeed, it renormalizes the base tunes; or can be nulled by the ring quadrupoles.

Beam-Beam Tune Shifts

Beam-beam effects are well studied [1-10]. Consider a test particle in the weak beam, and forces from the strong beam. Because the witness has velocity opposite to the strong beam, the magnetic force reverses sign and acts outwards (becomes repulsive). There is another way to see this. If we think about the case of strong-strong, we realize that the SRBB interaction arises from a beam with twice the charge and zero (net) current. The electrical forces double up, and the magnetic forces are zero. The net effect is equal the SRBB value. Likewise for counter rotating particle & anti-particle beams, the electrical forces cancel (zero charge) during the SRBB interaction, and the magnetic forces double (opposite currents and opposite velocities). Again, the net effect is equal the nominal SRBB value.

In contrast to the single-beam space-charge tune shift, the BB interaction is brief and isolated to a few specific locations in the ring. But the individual shifts contributed are large ($2 >> 1/\gamma$), and are therefore important.

Roughly speaking, the width of the SRBB tune shifts is bracketed by the values at

- small/zero amplitude (largest tune shifts)
- largest amplitudes (smallest tune shifts).

Likewise, for sufficiently large beam separation, the width of the LRBB tune shifts is bracketed by the values at

- small/zero amplitude (smallest tune shifts)
- largest X amplitudes (largest tune shifts).

So, having simple expressions for zero amplitude and asymptotic large amplitude tune shifts would be useful for quick estimation. The dependence of SRBB & LRBB tune shifts on parameters has been investigated previously [3,4]; but simple formulae are not given in those sources.

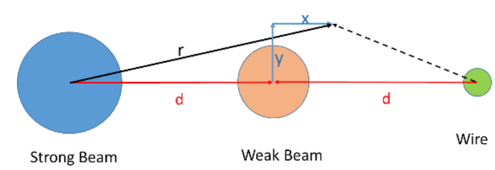

Figure 1: Geometry & coordinates of strong and weak beams.

We know field E(r) for strong beam, and hence potential $V(r)= V(r^2)$. We also know r(x,y,d); see Fig. 1. By taking derivatives w.r.t. x or y, we can find forces on the test particle in weak beam due to the strong beam. By suitable choice of x,y,d several interesting cases can be found: short-range, long-range, and wire compensation.

<u>Hamiltonian & Modulations</u>

The 2-dimensional Hamiltonian is $H=H_x + H_y$ where $H_x = T_x + U_x$ =kinetic+ (harmonic potential) = $H_x = (x')^2 + x^2$. We take action (J) and angle (q) variables: $x = \mathrm{Sqrt}[2J_x]\mathrm{Cos}[q_x]$ and $q_x' = \mu_x$; and likewise for H_y. Hence: $H = \mu_x.J_x + \mu_y.J_y$. Now add the perturbation = interaction potential $V = V[r^2] = V[r(x,y)^2] = V[r(x(Jx,qx),y(Jy,qy))^2] = V(Jx,qx;Jy,qy)$.

V produces modulation of the angle (q) and action (J).

$$q_x' = \partial H/\partial J_x \ ; \ J_x' = -\partial H/\partial q_x = 0 \ ;$$

$$\Delta q_x' = \partial V/\partial J_x \ ; \ J_x' = -\partial V/\partial q_x \ .$$

Likewise for H_y. We use the chain rule for derivatives to find these modulations. To find the net effect, we integrate over one cycle of the unperturbed motions. Hence a double integral over q_x and q_y. For example, $\Delta\mu_x = \iint \Delta q_x' \, dq_x.dq_y$.

We take cylindrical Gaussian distributed beam density $\rho = \mathrm{Exp}[-J]$ & $J = r^2/2$ leading to field $E = (1-\mathrm{Exp}[J])/(2J)$ and potential $V = (-\mathrm{Euler}\Gamma - \Gamma[0,J] - \mathrm{Log}[J])/2$. The Γ function generalises concept of factorials to non-integers.

Short Range Beam-Beam Tune Shift & Spread

We set beam separation d=0.

Let $\chi = \mathrm{Jx}\,\mathrm{Cos}[qx]^2 + \mathrm{Jy}\ \mathrm{Cos}[qy]^2$. Due to the high symmetry, expressions for qx' & qy' are equivalent.

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$$qx' = (-1 + e^{-\chi})\mathrm{Cos}[qx]^2/(2\chi)$$

Of course, we can compute the integrals numerically. But there are useful special cases that can be found analytically.

Zero amplitude tune

$\Delta\mu x(@Jx=Jy=0) = \Delta\mu y=(@Jx=Jy=0) = -1/4$

Large amplitude tune

$\Delta\mu x(Jx=0,Jy) = -\frac{1}{4}e^{-Jy/2}\left(\mathrm{BesselI}[0,\frac{Jy}{2}] + \mathrm{BesselI}[1,\frac{Jy}{2}]\right)$

Δμx in x plane (well known result)

$\Delta\mu x(@Jx,Jy=0) = \{-1 + e^{-Jx/2}\,\mathrm{BesselI}[0,\frac{Jx}{2}]\}/(2Jx)$

Equal x & y amplitudes

$$J\Delta\mu(@Jx = Jy = J/2) = \frac{1}{2}\left(-1 + e^{-J/2}\,\mathrm{BesselI}[0,\frac{J}{4}]^2\right)$$

Sum of tune shifts

$$Jx.\Delta\mu x + Jy.\Delta\mu y = \frac{1}{2}\left(-1 + e^{-\frac{Jx}{2}-\frac{Jy}{2}}\,\mathrm{BesselI}[0,\frac{Jx}{2}]\,\mathrm{BesselI}[0,\frac{Jy}{2}]\right)$$

Small amplitude tunes and splits

$$\Delta\mu x = -\frac{1}{4}\left(1 + \frac{Jy}{4} - \frac{Jy^2}{16}\right) + \frac{Jx}{32}\left(1 - Jy + \frac{9\,Jy^2}{32}\right)$$

$$\Delta\mu x - \Delta\mu y = \frac{(Jx-Jy)}{32} + \frac{(Jx^2-Jy^2)}{64} + \frac{9\,(Jy-Jx)}{1024}$$

Note X tune shift at small Jx & large Jy is small; because the Y motion carries the particle into regions with low charge density; and, moreover, particles spend most of their time at the extremities of motion (for SHO). Like wise for small Jy & large Jx, the tune shifts are small.

LONG RANGE TUNE SHIFT & COD

Beam separation d≠0 is assumed to be horizontal. The forces acting on the test particle are no longer symmetric in "x". The residual force F_x at x=0, $Fx(0) = \left\{1 - e^{-\frac{d^2}{2}}\right\}/2$ leads to a closed orbit distortion (COD). This has to be cancelled by magnetic elements (dipoles and quads) up/down stream of the BB interaction.

In principle, we should add a compensating term to the interaction potential to zero out the COD; such as

$$Vx0 = -Fx(@x=0).x = -\frac{e^{-L}(-1+e^{L})\sqrt{Jx}\,\mathrm{Cos}[qx]}{\sqrt{L}}$$

But, Vx and its derivative will integrate to zero. Hence we may omit the COD correction from the interaction potential provided that we always remember to integrate qx' over one period in the motion in qx. But if we perform tracking, the invariant of motion will wobble! (if the COD is neglected). Moreover, the COD forces should be averaged over the beam before compensation:

$$< Fx > \approx \frac{(1+d^2)}{2\,d^3}\left(2 - (2+d^2)\,e^{-\frac{d^2}{2}}\right)$$

Symmetry is broken: expressions for $q_x' \neq q_y'$.

Let $\Lambda = L + 2\sqrt{Jx}\sqrt{L}\mathrm{Cos}[qx] + \chi$.

$$q_x' = \frac{(-1+e^{-\Lambda})}{2\Lambda}\left(\frac{\sqrt{L}\mathrm{Cos}[qx]}{\sqrt{Jx}} + \mathrm{Cos}[qx]^2\right);\; q_y' = \frac{(-1+e^{-\Lambda})\mathrm{Cos}[qy]^2}{2\Lambda}$$

The double integrals $\iint dq_x dq_y$ cannot be found in closed form*; but we can compute them numerically; see Fig. 2.

For large L & J, the integrals are poorly converging if computed (summed) directly.

Numerical Examples

Typically d=9σ sigma (action L=40.5) and we study amplitudes up to x,y =6σ or J_x,J_y =18. So $J_{max} \approx L/2$.

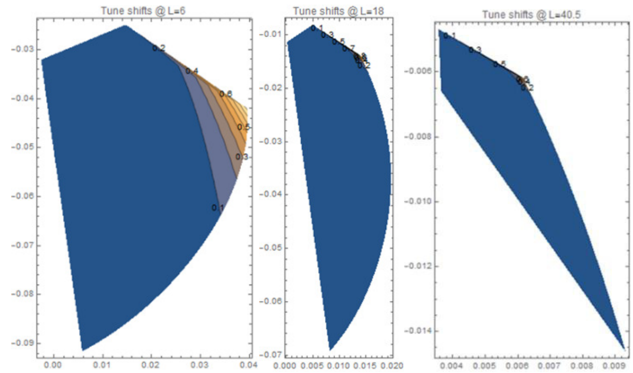

Figure 2: Beam-beam footprint in (μ_x, μ_y) tune space (horizontal, vertical). Beam core/tail is yellow/blue. Left: L=J_{max}=6; Centre L=18, J_{max}=18; Right: L=40.5, J_{max}=18.

Because of the separation *d*, all of the BB forces are reduced and the LRBB tune shifts are smaller than SRBB. However, large amplitude motion in X, will carry the Y motion closer to the strong beam, resulting in larger BB forces. Because the forces are non-linear, the average (over X) effect is to boost the Y plane BB tune shifts compared with the case of small amplitude X motion.

LRBB Tune Shift Formulae

The mathematical analysis is complicated by the fact that the μ_x limits, qx'(J_x,J_y) as Jx→0 and qx'(J_x,J_y) as J_y →0 & J_x →0, are indeterminate. The ambiguity is resolved by first performing the integration over q_x (and q_y), and then taking the limits. To do the integration, first Taylor expand qx' as a series in Cosine.

Contrastingly, the μy limits qy'(Jx,Jy) as Jx→0, qy'(Jx,Jy) as Jy→0, and qy'(Jx,Jy) Jy→0 & Jx→0 = qy'(Jx,Jy) Jx→0 & Jy→0 are all well defined.

Hence $\mu y(0,0) = (1 - e^{-L})/(4L)$.

Although the integrals over qx,qy cannot be found in closed form, we can find, by analysis, limiting cases of very small and very large amplitude; i.e. asymptotic forms are good for L>>1 and Jx≈L/2 (or less), and this happens to be the relevant regime for beams with particles at 6σ (sigma) amplitude and 9σ beam separation.

X-plane Tune Shifts & Spreads

Small amplitude

$\Delta\mu x(@L>Jx<1\&Jy=0) =$

$$e^{-L}(-1 + e^{L} - 2L)/(4L) + e^{-L}\,Jx\,(-3 + 3\,e^{L} - 3L - 4\,L^3)/(16L^2)$$

Limit of zero amplitudes

$\Delta\mu x(@Jx=Jy=0) = e^{-L}(-1 + e^{L} - 2L)/(4L)$

$\Delta\mu x(@L=Jy=0) = (-8 + 3Jx)/32$

Asymptotic Limit of very large amplitudes

$\Delta\mu x(@L>>1\&Jx\approx L/2\&Jy=0) =$

* Lopez [4,6] writes them as 1-D integrals over "u" summed over an infinite series of Bessel functions.

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$$(1-\mathrm{e}^{-L})\left(-1+\frac{\sqrt{L}}{\sqrt{-\mathrm{Jx}+L}}\right)/(2\mathrm{Jx})$$

Δμx(@L>>1&Jy≈L/2&Jx=0) =

$$-\mathrm{e}^{-\frac{\mathrm{Jy}}{2}-L}L^2\left(1-\mathrm{e}^{\frac{\mathrm{Jy}}{2}+L}+2\mathrm{Jy}+2L\right)/\{4\left(L(\mathrm{Jy}+L)\right)^{3/2}\}$$

Y-plane Tune Shifts & Spreads

Small amplitude

Δμy(@Jy<1&Jx=0) =

$$(-1+\mathrm{e}^{-L})/(4L)+3\mathrm{e}^{-L}\mathrm{Jy}(-1+\mathrm{e}^{L}-L)/(16L^2)$$

Limit of zero amplitudes

Δμy(@Jx=Jy=0) = $(-1+\mathrm{e}^{-L})/(4L)$

Δμy(@L=Jx=0) = $(-8+3\mathrm{Jy})/(32)$

Asymptotic Limit of very large amplitudes

Δμy(@L>>1&Jy≈L/2&Jx=0) =

$$(1-\mathrm{e}^{-L})\left(-1+\frac{\sqrt{L}}{\sqrt{\mathrm{Jy}+L}}\right)/(2\,\mathrm{Jy})$$

Δμy(@L>>1&Jx≈L/2&Jy=0) = $\frac{(-1+\mathrm{e}^{-L})\sqrt{L}}{4\,(-\mathrm{Jx}+L)^{3/2}}$

WIRE COMPENSATION OF TUNE SHIFT

Compensation of the LRBB tune shift by a current carrying wire is a relatively new idea [11-14]. To model this we add the potential of the wire, with strength parameter Ws: wire = Log $[(d-x)^2/2]$ Ws/2.

Pitfall: the wire must not be inside the weak beam, else the 1/r type singularity at $x=d$ will kick particles to large amplitudes and produce very large tune shifts. Some of this effect is artificial, as the wire has thickness. The effect can be eliminated, for example, by making the current distribution inside the wire equal to a narrow Gaussian. But here we shall simply take J_{max} < L/2.

There are several options for how to choose Ws.

Taking Ws $\rightarrow -1+\mathrm{Exp}[-d^2/2] = 1-\mathrm{e}^{-L}$ has the effect of setting Fx(x=0)=0. This removes the COD, but doubles the tune shifts and non-linear tune spreads in both planes. In such case, the currents in the strong beam and in the wire are in same direction.

Or we may choose Ws to zero out the linear tune shift $\Delta\mu_x$:

$$\mathrm{Ws}\rightarrow 1-\mathrm{e}^{-\frac{d^2}{2}}-d^2\,\mathrm{e}^{-\frac{d^2}{2}}=1-e^{-L}-2e^{-L}L$$

In such case, currents in strong beam and wire are oppositely directed. The COD forces are doubled, but they can be compensated by dipoles.

The linear tune can be corrected by quadrupoles. So a better option is to cancel the sextupole-like component of the 1/r force from the strong beam; leading to the choice:

$$\mathrm{Ws}\rightarrow 1-\mathrm{e}^{-\frac{d^2}{2}}-\frac{1}{2}d^2\mathrm{e}^{-\frac{d^2}{2}}-\frac{1}{6}d^6\mathrm{e}^{-\frac{d^2}{2}}$$
$$=\mathrm{e}^{-L}\,(-1+\mathrm{e}^{L}-L-4L^3/3)$$

This happens also to almost zero out the linear tune shift.

But, the wire has also an effect on the Y plane tune spread. Fortunately, neither X nor Y is particularly sensitive to the choice of Ws; and X is less sensitive than Y. Therefore, we can jointly optimize X and Y to minimize the non-linear tune shifts; leading to wire strength:

$$\mathrm{Ws}\rightarrow 1-\mathrm{e}^{-\frac{d^2}{2}}-\frac{1}{2}d^2\mathrm{e}^{-\frac{d^2}{2}}=1-\mathrm{e}^{-L}-\mathrm{e}^{-L}L$$

Numerical Examples

Figure 2 gave examples of tune footprint before wire compensation. Now examples with compensation. Figure 3 for small beam separation. By varying J_{max} we can see the shifts of the core ($J_{max}\leq 2$) versus the full beam with tails ($J_{max}\leq 6$).

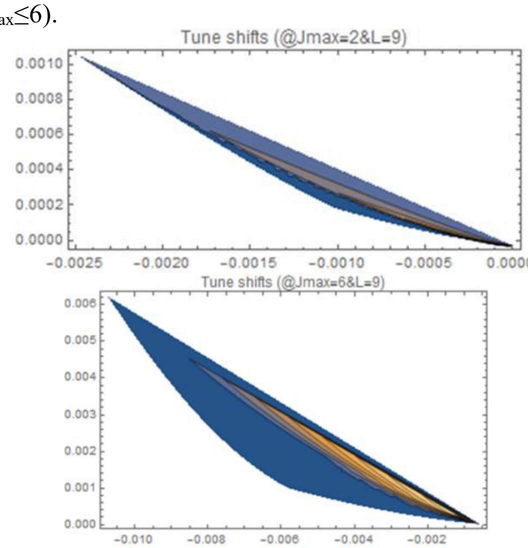

Figure 3: Beam-beam tune footprint in (μ_x, μ_y) tune space for separation d=4.24σ (L=9) and wire compensation. Upper: beam core; lower full beam with tails.

Figure 4 is for large beam separation d=9σ. After optimized wire compensation, the tune spread of the beam core is reduced almost to zero: few 10^{-16} horizontal and few 10^{-8} vertical. Figure 4 shows the tune spread out to the tails ($J_{max}\leq 18$).

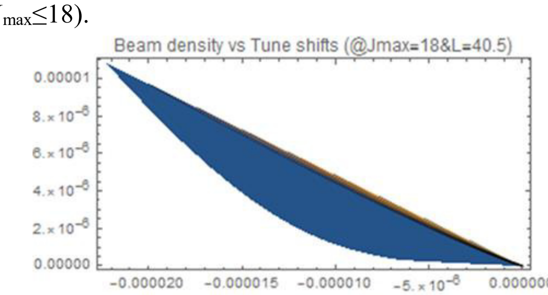

Figure 4: Beam-beam tune footprint in (μ_x, μ_y) tune space for separation d=9σ (L=40.5) and wire compensation.

Tune Shifts & Spreads After Wire Compensation

We use the optimised wire strength, leading to estimates.

Small Amplitude Tune Spread (Jy=0,Jx<<1)

$$\Delta\mu x=\ -\frac{\mathrm{e}^{-L}}{4}(1+Jx\,L)-\frac{e^{-L}\,Jx^2\,(15+10L-20L^2+8\,L^3)}{192L}$$

Small Amplitude Tune Spread (Jx=0,Jy<<1)

$$\Delta\mu y=\frac{\mathrm{e}^{-L}(-1+\mathrm{e}^{L}-2L)}{4L}++\mathrm{Jy}\frac{3}{16\,L^2}(1-\mathrm{e}^{-L}-L\mathrm{e}^{-L})$$
$$+\mathrm{Jy}^2\frac{5}{32L^3}\left(-1+\mathrm{e}^{-L}(1+L+\frac{L^2}{2})\right)$$

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