

ON THE INVARIANCE OF SCALED FACTORIAL MOMENTS WHEN ORIGINAL DISTRIBUTION IS FOLDED WITH THE BINOMIAL

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It is shown, that the Scaled Factorial Moments of any rank do not change if the original distribution is folded with a binomial one.

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Scaled Factorial Moments (SFM) have been used for some time in the analysis of event-by-event correlations and fluctuations, as they allow for a “direct” access to the “dynamical” fluctuations of the multiplicity [1]. They were used (among others) for the intermittency analysis [1], and recently they were also suggested as a tool for testing the assumption of chemical equilibration in the nuclear collision [2]. For a selected class of particles produced with low multiplicity — ones that carry produced and conserved in the reaction charge-like quantity — the SFM would be close to 1/2 if chemical equilibrium is reached.

One example of particles, that are good candidates for such an analysis, are kaons when observed at SIS energies (beam kinetic energy up to 2 AGeV). They carry positive strangeness, and as the beam energies are close to the production threshold, their multiplicities are small.

In this case the kaons are produced either as K^+ or K^0 . However only one (usually charged) type is registered in the detector with reasonable (but still smaller than one) probability. This led to the question — how much the SFMs of the measured distributions differ from the “original” SFMs?

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Factorial Moment of rank j is defined as follows:

$$F_j = \langle I(I-1) \dots (I-j+1) \rangle = \sum_I^{\infty} I(I-1) \dots (I-j+1) P_I, \quad (1)$$

where I is a number of particles, and P_I is the probability of producing I particles in an event. P_I has a normalized distribution.

Scaled Factorial Moment of rank j is then defined as

$$SF_j = \frac{F_j}{\langle I \rangle^j}. \quad (2)$$

Capital symbols, like I and P_I denote the “original” distribution of produced kaons. Now let us assume that for each produced kaon the probability of registering it is equal to q . The resulting, “measured” distribution will then be a fold of the “original” distribution with the binomial. Denoting with small symbols i the number of registered kaons per event and p_i probability of an event with i registered kaons one obtains:

$$p_i = \sum_{I=i}^{\infty} \binom{I}{i} q^i (1-q)^{I-i} P_I. \quad (3)$$

The distribution of p_i is normalized. The following proof illustrates the technique of reordering sums, which is used in the later part of the paper. One needs to note, that for any individual case $i \leq I$, so each I contributes only to terms with $i \leq I$.

$$\begin{aligned} \sum_i p_i &= \sum_i \sum_{I=i}^{\infty} \binom{I}{i} q^i (1-q)^{I-i} P_I = \sum_{I=0}^{\infty} \sum_{i=0}^I \binom{I}{i} q^i (1-q)^{I-i} P_I \\ &= \sum_{I=0}^{\infty} P_I \sum_{i=0}^I \binom{I}{i} q^i (1-q)^{I-i} = \sum_{I=0}^{\infty} P_I = 1. \end{aligned} \quad (4)$$

The normalization of P_I and normalization of the binomial distribution were used in Eq. (4).

Factorial Moment of the measured distribution (f_j) is equal to:

$$\begin{aligned} f_j &= \sum_i i(i-1) \dots (i-j+1) p_i \\ &= \sum_i i(i-1) \dots (i-j+1) \sum_{I=i}^{\infty} \binom{I}{i} q^i (1-q)^{I-i} P_I. \end{aligned} \quad (5)$$

To calculate f_j explicitly, the same reordering of the summation as in Eq. (4) is used. One notes, that terms with $i < j$ are equal to zero and do not contribute to the sum, so in reality the summation starts not with $i = 0$, but with $i = j$. At a point k is substituted for $i - j$ and K for $I - j$. Note, that $(I - i) = (K - k)$.

$$\begin{aligned}
 f_j &= \sum_{i=0}^{\infty} i(i-1) \dots (i-j+1) \sum_{I=i}^{\infty} \binom{I}{i} q^i (1-q)^{I-i} P_I \\
 &= \sum_{I=0}^{\infty} P_I \sum_{i=j}^I i(i-1) \dots (i-j+1) \\
 &\quad \times \frac{I(I-1) \dots (I-j+1)(I-j)!}{i(i-1) \dots (i-j+1)(i-j)!(I-i)!} q^j q^{i-j} (1-q)^{I-i} \\
 &= \sum_{I=0}^{\infty} P_I I(I-1) \dots (I-j+1) q^j \sum_{k=0}^K \frac{K!}{k!(K-k)!} q^k (1-q)^{K-k} \\
 &= q^j \sum_{I=0}^{\infty} P_I I(I-1) \dots (I-j+1) = q^j F_j. \tag{6}
 \end{aligned}$$

In order to calculate the scaling denominator one needs to calculate $\langle i \rangle$. As $\langle i \rangle$ is equal to f_1 (and $\langle I \rangle = F_1$), one can use formula (6)

$$\langle i \rangle = f_1 = q^1 F_1 = q \langle I \rangle. \tag{7}$$

Finally the measured Scaled Factorial Moment, sf_j appears equal to the original one.

$$sf_j = \frac{f_j}{\langle i \rangle^j} = \frac{q^j F_j}{(q \langle I \rangle)^j} = \frac{q^j F_j}{q^j \langle I \rangle^j} = \frac{F_j}{\langle I \rangle^j} = SF_j. \tag{8}$$

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