Non-Gaussianity in Cosmic Microwave Background Temperature Fluctuations from Cosmic (Super-)Strings

Atsushi Naruko[1](a), Keitaro Takahashi(b), Yuuiti Sendouda(a), Daisuke Yamauchi(a), Chul-Moon Yoo(c) and Misao Sasaki(a)

(a) *Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan*
(b) *Department of Physics and Astrophysics, Nagoya University, Nagoya 464-8602, Japan*
(c) *Asia Pacific Center for Theoretical Physics, Pohang University of Science and Technology, Pohang 790-784, Korea*

Abstract

We compute analytically the small-scale temperature fluctuations of cosmic microwave background from cosmic (super-)strings and study the dependence on the string intercommuting probability P. We develop an analytical model which describes the evolution of a string network and calculate the numbers of string segments and kinks in a horizon volume. Then we derive the probability distribution function (pdf). The resultant pdf consists of a Gaussian part due to frequent scattering by long string segments and a non-Gaussian tail due to close encounters with kinks. It contains two phenomenological parameters which are determined by comparison with the result of numerical simulations for $P = 1$ by Fraisse et al.. We predict that the non-Gaussian feature is suppressed for small P.

1 Introduction

The imprint of cosmic strings on the cosmic microwave background (CMB) has been widely studied. Although cosmic strings are excluded as a dominant source of the observed large angular scale anisotropy, they could still be observable at small scales with new arcminute CMB experiments, such as the South Pole Telescope or the Atacama Cosmology Telescope. Because the structure of a string network is highly nonlinear, it would naturally induce a non-Gaussian feature in the CMB fluctuations. In fact, Fraisse et al. [1] found that the probability distribution function (pdf) of the temperature fluctuations has a non-Gaussian tail and negative skewness. These non-Gaussian features may help us distinguish cosmic string signals from other secondary effects and hence enhance the observability.

Recently, cosmic superstrings have attracted much attention in the context of inflation in string theory [2]. Cosmic superstrings have properties different from conventional field-theoretic cosmic strings. The intercommuting probability P can be significantly smaller than unity for superstrings while $P = 1$ for field-theoretic strings. Furthermore, a superstring network can consist of more than one type of strings and may have Y-junctions. These differences may be used to distinguish superstrings from field-theoretic strings observationally.

In this talk, we derive analytically the pdf of the small-scale CMB temperature fluctuations and study its dependence on P. At small scales where the primary fluctuations are damped, only the integrated Sachs-Wolfe (ISW) effect is relevant and, because the contribution from loops was shown to be insignificant [1], we focus on the ISW effect of long string segments and kinks.

2 Temperature fluctuations due to cosmic strings

First we summarize basic formulae for the CMB temperature fluctuations due to cosmic strings, following [3]. We denote the position of a cosmic string by $\vec{r}(t, \sigma)$ where t and σ are the time and position on

[1]Email address: naruko@yukawa.kyoto-u.ac.jp

the string worldsheet. The temperature fluctuation, $\Delta \equiv \Delta T/T$, due to a segment, in the limit that the impact parameter of a ray is much smaller than the segment length, is written as

$$\Delta(\hat{n}) = 4\pi \frac{v}{\sqrt{1-v^2}} \alpha_{\rm seg} G\mu, \qquad \alpha_{\rm seg} = \hat{n} \cdot \left(\frac{\vec{r}\,'}{|\vec{r}\,'|} \times \frac{\dot{\vec{r}}}{|\dot{\vec{r}}|} \right) \tag{1}$$

where $v = |\dot{\vec{r}}|$ is the velocity of the segment and $\alpha_{\rm seg}$ is a factor which represents the configuration of the segment and the direction of line of sight, $\hat{n}$, and the dot and prime denote the derivatives with respect to t and σ, respectively.

A kink can be modeled as a nonsmooth junction of two straight strings with different directions, $\vec{r}\,'$ [3]. Then the temperature fluctuation with the impact parameter δ is

$$\Delta(\hat{n}) = -4G\mu\alpha_{\rm kink} \log \frac{\delta}{L_{\rm kink}} \Theta(L_{\rm kink} - \delta), \qquad \alpha_{\rm kink} = \hat{n} \cdot \vec{p}, \quad \vec{p} = \left[\frac{\vec{r}\,'}{|\vec{r}\,'|^2} \right]_{\sigma_{\rm kink}-0}^{\sigma_{\rm kink}+0}, \tag{2}$$

where $L_{\rm kink}$ is a distance between kinks. The step function $\Theta(L_{\rm kink} - \delta)$ represents the effect that the fluctuation becomes negligible far from the kink, $\alpha_{\rm kink}$ represents the kink configuration, $\vec{p}$ represents the amplitude of the kink and $\sigma_{\rm kink}$ is the position of the kink.

3 Analytic model of cosmic string network

In this section, we develop an analytic model which describes the behavior of a cosmic string network. First, the interstring distance ξ and the rms velocity $v_{\rm rms}$ are calculated using a velocity-dependent one-scale model [4, 5]. Then, the number of kinks in a horizon volume is calculated. We assume that the scaling behavior is already realized by the recombination time.

For a universe with the scale factor $a(t) \propto t^\beta$, the evolution equations for γ and $v_{\rm rms}$ are given by [4, 5]

$$\frac{t}{\gamma}\frac{d\gamma}{dt} = 1 - \beta - \frac{1}{2}\beta\tilde{c}Pv_{\rm rms}\gamma - \beta v_{\rm rms}^2, \qquad \frac{dv_{\rm rms}}{dt} = (1 - v_{\rm rms}^2)H\left[k(v_{\rm rms})\gamma - 2v_{\rm rms}\right], \tag{3}$$

where $\tilde{c}$ is a constant which represents the efficiency of the loop formation and $k(v_{\rm rms}) \approx (2\sqrt{2}/\pi)(1 - 8v_{\rm rms}^6)/(1 + 8v_{\rm rms}^6)$ is the momentum parameter [4]. Hereafter we assume a matter-dominated universe and set $\beta = 2/3$.

It is known that a string network approaches a "scaling" regime where the characteristic scale grows with the horizon size [6]. This means that γ and $v_{\rm rms}$ are asymptotically constant in time. From (3), we obtain γ and $v_{\rm rms}$. For small $\tilde{c}P$ they can be approximately given as

$$v_{\rm rms}^2 \approx \frac{1}{2} - \frac{1}{2}\sqrt{\frac{\pi\tilde{c}P}{3\sqrt{2}}}, \quad \gamma = \frac{2v_{\rm rms}}{k(v_{\rm rms})} \approx \sqrt{\frac{\pi\sqrt{2}}{3\tilde{c}P}}. \tag{4}$$

We see that small P leads to large γ and hence large $N_{\rm seg}$ due to the inefficiency of loop formation [5].

Next, we consider the kink number evolution. Kinks are formed on string segments when they intercommute and, simultaneously, some of the existing kinks are removed through loop formation. Furthermore, kinks decay due to stretching by the cosmic expansion and the emission of gravitational waves. Here we neglect the decay due to the gravitational wave emission and focus on the decay due to cosmic expansion since it is the most efficient decay process at a matter-dominated stage [7].

According to [8], the kink amplitude, $p = |\vec{p}|$, decays with cosmic expansion as $p(t) = p_{\rm f}(t/t_{\rm f})^{-\epsilon}$, where $t_{\rm f}$ and $p_{\rm f}$ are the formation time and the amplitude at the formation respectively. We count the number of kinks with amplitude $p_{\rm min} \le p \le p_{\rm max}$. The kink number in a comoving volume $V(t) = a^3(t)V_0$ is given by the integral of the formation rate, $d\bar{N}_{\rm form}(t,p)/dtdp$. And this formation rate of kinks, which is assumed here to be independent of p, is proportional to the loop formation rate, $d\bar{N}_{\rm loop}/dt$.

$$\bar{N}_{\rm kink} = \int_{p_{\rm min}}^{p_{\rm max}} dp \int_{t_0(p)}^{t} dt\, \frac{d\bar{N}_{\rm form}(t,p)}{dtdp} = \int_{p_{\rm min}}^{p_{\rm max}} dp \int_{t_0(p)}^{t} dt\, \frac{q}{p_{\rm min}} \frac{d\bar{N}_{\rm loop}(t)}{dt} \approx \frac{2q\tilde{c}Pv_{\rm rms}\gamma^4\epsilon}{3\alpha} \left(\frac{p_{\rm max}}{p_{\rm min}}\right)^{1/\epsilon}, \tag{5}$$

where $t_0(p) = t(p/p_{\rm max})^{1/\epsilon}$, a barred quantity is a number in the comoving volume $V(t)$, q is a constant which represents the efficiency of the kink formation and α is the average loop length in units of ξ.

4 PDF of CMB fluctuations

A photon ray is scattered by segments many times through its way from the last scattering surface to an observer, hence the temperature fluctuation would behave like a random walk. If we treat a segment as a particle with the cross section ξ^2, the optical depth is

$$\tau = \int_0^{z_{\rm rec}} N_{\rm seg} H^3 \xi^2 \frac{dz}{H(1+z)} = \frac{N_{\rm seg}}{\gamma^2} \log(1+z_{\rm rec}), \tag{6}$$

where $z_{\rm rec} \approx 1100$ is the redshift at recombination. This is estimated as $7N_{\rm seg}\gamma^{-2} \approx 16$ for $P = 1$ and greater for smaller P. Therefore, remenbering Eq. (1), the pdf from segments can be approximated as Gaussian with the dispersion,

$$\sigma = 2\pi \frac{v}{\sqrt{1-v^2}} \alpha_{\rm seg} G\mu \sqrt{N_{\rm seg}\gamma^{-2}\log(1+z_{\rm rec})} \approx 2\pi\alpha_{\rm seg}\sqrt{\log(1+z_{\rm rec})} \left(\frac{\pi\sqrt{2}}{3\tilde{c}P}\right)^{1/4} G\mu, \tag{7}$$

where we have set $v = v_{\rm rms}$ and substituted (4) in the second equality.

Next, let us consider the contribution from kinks. The temperature fluctuation depends on the impact parameter as given by (2). Therefore the differential cross section with the temperature fluctuation Δ can be written as

$$\frac{d\sigma_{\rm kink}}{d\Delta} = \frac{L_{\rm kink}^2}{\Delta_0} e^{-|\Delta|/\Delta_0}, \quad \Delta_0 \equiv 2\alpha_{\rm kink} G\mu, \tag{8}$$

where $\alpha_{\rm kink}$ should be understood as the statistical average of the kink configuration. Then the pdf of temperature fluctuations due to kinks is

$$\frac{dP_{\rm kink}}{d\Delta} = \int_0^{z_{\rm rec}} N_{\rm kink} H^3 \frac{d\sigma_{\rm kink}}{d\Delta} \frac{dz}{H(1+z)} = \frac{\gamma^2}{K\Delta_0} e^{-|\Delta|/\Delta_0} \log(1+z_{\rm rec}). \tag{9}$$

We have a pdf of the form,

$$\frac{dP_{\rm tot}}{d\Delta} = \frac{dP_{\rm G}}{d\Delta} + \frac{dP_{\rm NG}}{d\Delta}, \quad \frac{dP_{\rm G}}{d\Delta} = \frac{1}{\sqrt{2\pi}\sigma} e^{-\Delta^2/2\sigma^2}, \quad \frac{dP_{\rm NG}}{d\Delta} = \frac{\gamma^2}{K\Delta_0} \log(1+z_{\rm rec}) e^{-|\Delta|/\Delta_0}, \tag{10}$$

with σ and Δ_0 are given by Eqs. (7) and (8), respectively. $dP_{\rm G}/d\Delta$ is the Gaussian part due to frequent scattering by string segments, and $dP_{\rm NG}/d\Delta$ is the non-Gaussian tail due to rare scattering by kinks. Here, because $dP_{\rm NG}/d\Delta \ll 1$ as we see just below, we have normalized $dP_{\rm G}/d\Delta$ as $\int_{-\infty}^{\infty} d\Delta\, dP_{\rm G}/d\Delta = 1$. In the limit $P \to 1$, we have

$$\sigma \approx 14G\mu, \quad A \approx 10\alpha_{\rm kink}^{-1} \left(\frac{p_{\rm max}}{p_{\rm min}}\right)^{-5.1} (G\mu)^{-1}, \Delta_0 = 2\alpha_{\rm kink} G\mu, \tag{11}$$

where we have set $q = 2$, $\tilde{c} = 0.23$ and $\alpha = 0.1$, as their standard values [9] and $\alpha_{\rm seg} = 1/\sqrt{2}$ for the statistical average. In addition, we have set $\alpha_{\rm kink} = 4.5$ and $p_{\rm max}/p_{\rm min} = 2.3$ as a phenomelogical parameters. On the other hand, the pdf from numerical simulations [1] can be also described as Eq. (10) with

$$\sigma_{\rm sim} \approx 12G\mu, \ A_{\rm sim} \approx 0.03(G\mu)^{-1}, \ \Delta_{0,\rm sim} \approx 9G\mu. \tag{12}$$

As we see in Fig. 1, as P decreases, the Gaussian dispersion increases and the contribution of the non-Gaussian tail is suppressed. Thus the non-Gaussianity could be a probe of the cosmic string property, P. However, future observation with typical angular resolution $5'$ will not be able to resolve kinks as the non-Gaussian feature is highly suppressed even for $P = 1$. Thus we would need observation with arcminute resolution.

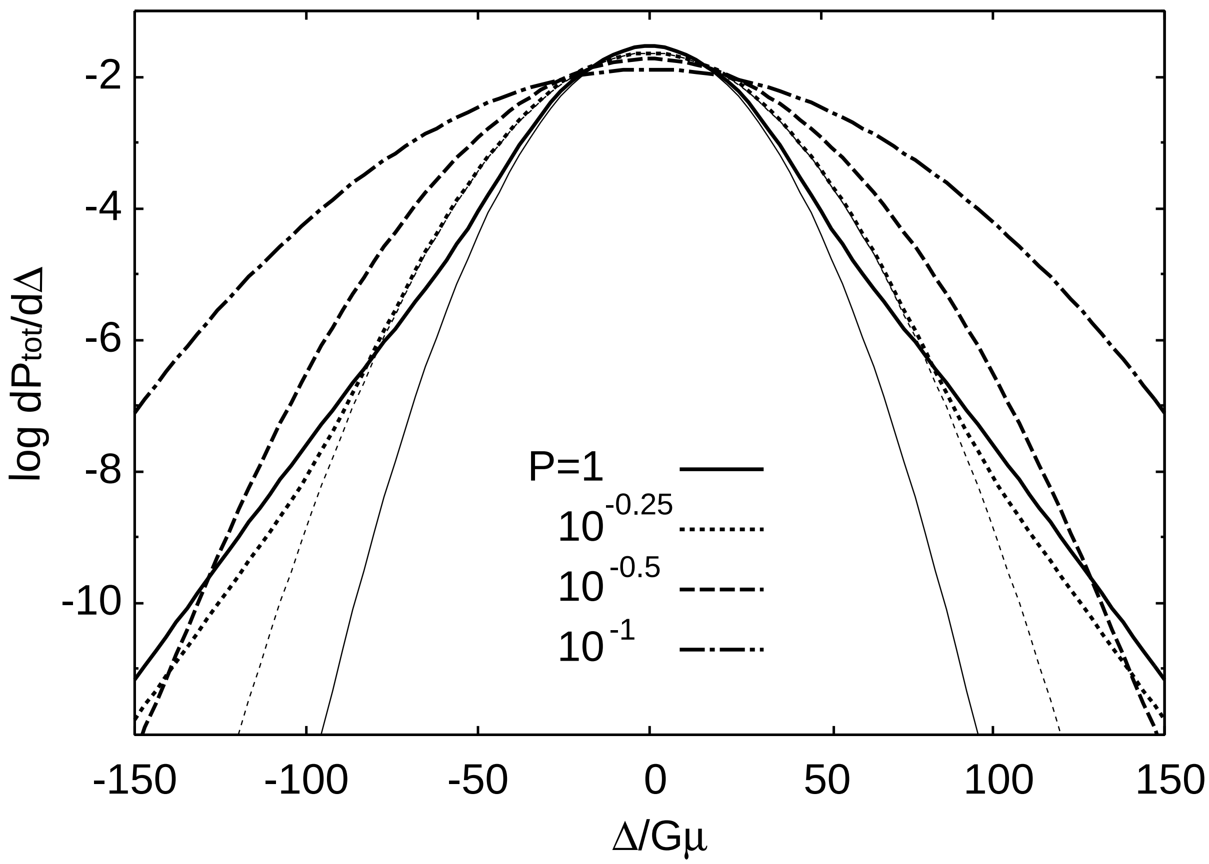

Figure 1: Dependence of the pdf on the intercommuting probability P (thick lines). The respective Gaussian parts are plotted with thin lines for comparison. For $P = 1$ and $10^{-0.25}$, the pdfs deviate significantly from the Gaussian distribution. For $P \lesssim 10^{-0.5}$, pdfs are almost Gaussian.

References

[1] A. A. Fraisse, C. Ringeval, D. N. Spergel and F. R. Bouchet, Phys. Rev. D **78**, 043535 (2008).

[2] M. Majumdar, arXiv:hep-th/0512062.

[3] A. Stebbins, Astrophys. J. **327**, 584 (1988); A. Stebbins and S. Veeraraghavan, Phys. Rev. D **51**, 1465 (1995).

[4] C. J. A. P. Martins and E. P. S. Shellard, Phys. Rev D **54**, 2535 (1996); C. J. A. P. Martins and E. P. S. Shellard, Phys. Rev D **65**, 043514 (2002).

[5] A. Avgoustidis and E. P. S. Shellard, Phys. Rev. D **73**, 041301 (2006).

[6] T. W. B. Kibble, Nucl. Phys. B **252**, 277 (1985).

[7] B. Allen and R. R. Caldwell, Phys. Rev. Lett. **65**, 1705 (1990); B. Allen and R. R. Caldwell, Phys. Rev. D **43**, R2457 (1991); B. Allen and R. R. Caldwell, Phys. Rev. D **43**, 3173 (1991).

[8] D. P. Bennett and F. R. Bouchet, Phys. Rev. D **41**, 2408 (1990).

[9] V. Vanchurin, K. Olum and A. Vilenkin, Phys. Rev. D **74** 063527 (2006); C. Ringeval and M. Sakellariadou, JCAP 0702 023 (2007); C. J. A. P. Martins and E. P. S. Shellard, Phys. Rev. D **73**, 043515 (2006).