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Large deviation-based noise mitigation in coupled quantum robotic systems

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Abstract

Quantum robotic systems hold promise for applications in molecular manipulation and high-precision sensing, but their operation is highly vulnerable to environmental noise. This work introduces a large deviation principle (LDP)-based control framework for mitigating stochastic perturbations in coupled master–slave quantum robots. The system Hamiltonian, expressed in terms of position and momentum operators, incorporates control terms alongside Gaussian and Poisson noise, capturing both gradual fluctuations and sudden jumps. By computing the large deviation rate function, we quantify the probability of rare noise-induced deviations and derive an optimal control strategy that minimizes such events in key observables. Simulations across distinct dynamical regimes demonstrate that the controlled trajectories remain close to the desired wave function, with deviations consistent with the theoretical bounds. These results validate the robustness and generality of the approach, providing a practical framework for stabilizing quantum robotic systems in noisy environments with potential applications in precision sensing, molecular chemistry, and quantum computing.

Keywords Quantum robotics · Large deviation principle · Noise cancellation · Gaussian noise · Poisson noise · Quantum control · Stochastic processes

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1 Introduction

The intersection of quantum mechanics and robotics has opened new frontiers in automation, offering revolutionary capabilities in molecular manipulation, high-precision sensing, and computation. Quantum technologies, driven by principles like superposition and entanglement, enable the development of systems surpassing classical robots' capabilities. Among these emerging technologies, quantum robotics—a fusion of quantum mechanics and robotic control—holds immense potential in diverse fields such as drug design, molecular chemistry, and high-sensitivity sensing [1, 2].

However, despite its promise, quantum robotic systems are highly vulnerable to environmental disturbances. Even weak noise sources, such as Gaussian or Poisson white noise, can lead to decoherence. This process disrupts the quantum state by causing a loss of coherence, which, in turn, compromises system reliability and performance [3, 4]. Decoherence leads to significant deviations from expected behaviour, posing severe challenges for the precise control required in quantum robots.

Given the inherent sensitivity of quantum systems to noise, classical control methods are insufficient for maintaining system stability. Traditional control strategies designed for deterministic systems are unable to address the probabilistic behaviours and the inherent uncertainties associated with quantum systems. Novel, quantum-tailored noise mitigation strategies are essential to ensure quantum robots' stable and reliable operation.

In this work, we focus on the problem of controlling coupled quantum robotic systems under noise, particularly in the context of master–slave robotic architectures. Such systems arise in scenarios including minimally invasive surgical robotics, where a master robot (controlled by a human or algorithm) directs a quantum-enabled slave system, and in molecular robotics, where precise manipulation of quantum states governs chemical processes. The stochastic noise in these systems—arising from thermal fluctuations, quantum decoherence, or control imperfections—poses a critical challenge to trajectory tracking and functional reliability. We aim to minimize such deviations through a control framework grounded in large deviation theory, allowing us to characterize and mitigate rare but impactful noise-induced events in system observables.

This research proposes a Hamiltonian-based framework for controlling noise in quantum robotic systems. The framework incorporates Gaussian and Poisson noise models, capturing the full spectrum of disturbances affecting system dynamics. These dynamics are described by the quantized angular position and momentum operators, with the Itô correction terms integrated into the Schrödinger equation to preserve unitarity and ensure quantum coherence, even in the presence of noise.

To evaluate the influence of noise, we apply the large deviation principle (LDP), a robust statistical framework well suited for estimating the probabilities of rare, noise-induced events [5]. LDP enables us to calculate the large deviation rate function, approximating the likelihood of deviations in the quantum system's trajectory due to noise. By leveraging this information, we optimize control parameters, effectively minimizing the impact of noise on the system's behaviour.

This framework is further extended by discretizing the quantum dynamics, resulting in a finite-state noisy Schrödinger equation. We introduce control inputs to optimize system responses, enabling direct comparisons between the noisy evolution and the desired, noise-free dynamics. Simulation results confirm that the proposed framework significantly enhances the reliability and precision of quantum robotic systems, even in the presence of environmental noise.

1.1 Motivations

Classical robotics relies on control strategies that track specific trajectories, ensuring that robots follow defined translational and angular paths to perform tasks. However, quantum systems introduce inherent complexities due to principles like the Heisenberg uncertainty principle, which makes exact trajectory tracking impossible. Instead, quantum systems require the tracking of desired wave functions, which correspond to the quantum moments of observables, representing the average values of physical quantities in a quantum state.

This shift from classical trajectory tracking to quantum wave function tracking carries significant implications. For example, in molecular chemistry, adjusting bond angles affects a molecule's chemical properties, which are governed by the wave functions of its constituent particles [6]. Effective wave function tracking is essential for achieving desired outcomes in drug design and molecular engineering applications.

Beyond simply tracking average observables, such as the average position or momentum of a particle, quantum control must also manage the dispersion of observables within a quantum state. This dispersion, which can be thought of as the spread or uncertainty in the measurement of these observables, is particularly important in precision-sensitive applications like drug manufacturing, where controlling molecular dynamics is critical to success. Maintaining control over both the average state and its dispersion ensures system stability and reliability in environments prone to disturbances.

However, quantum systems are highly vulnerable to environmental noise, including Gaussian and Poisson noise, which can cause significant deviations from the intended quantum state. This susceptibility underscores the need for robust control techniques to mitigate noise and ensure reliable quantum robotic operations, even in the face of unpredictable disturbances.

The LDP offers a promising framework to tackle these challenges. By estimating the probabilities of rare events, LDP allows for precise calculations of the likelihood that a quantum system will deviate from its desired trajectory due to noise. This enables the development of control strategies to minimize such deviations, thereby improving the robustness and reliability of quantum robotic systems.

This research applies LDP-based control methods to address the quantum robot tracking problem and minimize its impact on system dynamics. While noise analysis in quantum systems often involves open quantum systems theory, our approach focuses on classical noise perturbations, making it well suited for analysis through stochastic Schrödinger equations. This work provides foundational insights for developing highly reliable quantum robotic systems with potential applications in molecular chemistry, quantum computing, and other fields.

1.2 Main contributions

The key contributions of this research are:

- LDP-Based Control for Quantum Robotics: Developed a novel control framework that effectively extends traditional quantum control strategies to manage both Gaussian and Poisson noise, uniquely addressing the challenges of quantum robotic systems.
- Deterministic Noise Mitigation: Introduced a computationally efficient deterministic control algorithm, eliminating the need for real-time feedback, thus overcoming limitations in traditional noise mitigation strategies and improving robustness in noisy environments.
- Comprehensive Simulation Validation: The proposed control strategy was extensively validated through diverse simulations, demonstrating its efficacy in maintaining system stability and accurate wave function tracking across different noise intensities and system configurations.
- Advancement in Noise Control for Quantum Robotics: Pioneered the application of LDP-based control to quantum robotics, significantly advancing noise control techniques beyond classical–quantum systems by addressing stationary and non-stationary noise models.
- Literature Gap Addressed: Extended noise cancellation techniques from classical quantum systems to continuous, high-dimensional quantum robotic systems, addressing critical gaps in the literature concerning non-Gaussian noise and continuous quantum systems.

These contributions provide a significant advancement in noise control for quantum robotics, offering both theoretical insights and practical solutions to improve the reliability and performance of quantum systems under stochastic noise conditions. This research is particularly significant in the field of quantum robotics, where the management of noise is a critical factor in the development of robust and efficient systems.

2 Review of literature

The control of quantum systems, particularly noise mitigation, is central to advancing quantum computing, robotics, and sensing. Quantum systems are inherently sensitive to environmental disturbances, such as decoherence, which degrade quantum coherence and disrupt system performance. In the context of quantum robotics, these challenges are magnified due to robotic tasks' dynamic and high-precision requirements. Consequently, addressing noise and ensuring stability in quantum systems remains a critical challenge across these domains.

Early pioneering work by Rabitz et al. [7] laid the foundation for quantum feedback control by using lasers to manipulate molecular systems, introducing techniques that stabilized quantum phenomena. This framework sets the stage for subsequent advancements, such as Gradient Ascent Pulse Engineering (GRAPE) [8], which provided an efficient method for designing optimal control sequences, particularly for qubits. GRAPE has proved instrumental in mitigating Gaussian noise, which is prevalent in many quantum environments and remains a cornerstone of quantum control methodologies.

While significant progress has been made in managing Gaussian noise, the real-world complexity of quantum systems often includes non-Gaussian noise, such as Poisson or Levy processes, which present new challenges. Liu et al. [9] demonstrated the robustness of quantum state transfer under Gaussian noise using optimal control techniques. However, non-Gaussian noise models—characterized by sudden jumps or discontinuities—introduce perturbations that traditional Gaussian approaches cannot fully address.

Researchers have recently begun to focus on non-Gaussian noise and its implications for quantum control. Zhuang et al. [10] tackled this issue by introducing generalized quantum error correction schemes, incorporating non-Gaussian noise to improve stability in quantum systems affected by Levy processes. Viola et al. [11] extended these efforts by developing decoupling techniques that isolate quantum systems from non-Gaussian noise, significantly enhancing robustness against diverse noise sources.

In our prior work, we explored the behaviour of quantum robots perturbed by Levy processes [12], offering stochastic analysis and simulations that demonstrated how jump-like disturbances from Levy noise differ from the smoother perturbations of Gaussian noise. These insights were crucial for developing more effective control strategies for quantum systems operating in non-Gaussian noise environments. Additionally, another study focused on quantum robotic teleoperation in the presence of electromagnetic fields [13], investigating how environmental interactions impact stability, providing a quantum mechanical analysis of these effects.

Recent advances in quantum error mitigation have expanded the scope of noise management. Zhang et al. [14] introduced methods for mitigating quantum errors in non-Markovian noise environments, addressing noise with memory effects that influence system dynamics over time. Their work highlighted the importance of accounting for time-correlated noise in practical quantum applications, such as quantum circuits and robotic systems. In parallel, Tang and Wang [15] proposed adaptive control mechanisms for quantum circuits, where dynamic noise fluctuations necessitate real-time adaptability. These adaptive strategies offer promising potential for quantum robotics, where complex noise profiles evolve over time.

The challenge of quantum error mitigation has been further explored in quantum computing. Probabilistic error cancellation (PEC) [16] has successfully reduced noise-induced errors within quantum circuits, though these methods are primarily confined to discrete systems like qubits. Continuous quantum systems, such as quantum robots, present additional challenges, which dynamic decoupling techniques, while effective for qubits [17], cannot fully address.

Spectator qubits, which monitor environmental noise without storing quantum information, have shown potential for reducing errors in quantum processors [18]. Furthermore, MIT researchers [19] have demonstrated practical improvements in quantum coherence through device noise cancellation techniques. However, these methods are constrained by hardware complexity and real-time processing limitations, making them less effective for highly dynamic systems like quantum robotics.

In terms of quantum control strategies, Kuang et al. [20] introduced rapid Lyapunov-based control methods, which stabilize finite-dimensional quantum systems. Although effective, these techniques often rely on real-time feedback, which is challenging to scale in more complex quantum systems. For quantum robotics, where high-dimensional and continuous control is required, scalable, efficient control strategies are critical.

In classical systems, feedback-driven noise cancellation techniques, such as the Wiener filter [21], are commonly used to mitigate noise by leveraging power spectral densities. However, these methods rely heavily on continuous feedback, which may not be practical for quantum robotic systems that require rapid, deterministic solutions. Our research addresses this by proposing a deterministic control approach based on the LDP, which minimizes reliance on real-time feedback while providing an efficient solution for managing noise in quantum robotic systems.

The novelty of our approach lies in incorporating both Gaussian and Poisson noise models, a departure from the predominantly Gaussian-focused approaches in previous works. By leveraging LDP, we approximate the probability of wave function deviations and develop control parameters that minimize these deviations without needing continuous feedback. This method is particularly advantageous for quantum robotic systems, where stability under random noise conditions is critical for ensuring reliable operation.

In conclusion, our research extends the scope of quantum control and noise mitigation to handle non-Gaussian noise in continuous quantum systems. Applying LDP-based control to quantum robotics offers a novel methodology for minimizing noise effects and paves the way for future research into more complex quantum systems. By addressing a critical gap in the literature, this work opens new avenues for developing robust quantum control strategies that can be applied to various quantum technologies.

3 Problem formulation

We design controllers based on large deviation principles for tracking classical trajectories by a classical robot and by the quantum average of a quantum robot's position and velocity observables. The focus is on a quantum master–slave robot configuration, and the following Hamiltonian model describes the dynamics of this system.

3.1 Hamiltonian formulation

This subsection derives the Hamiltonian governing the dynamics of coupled quantum robots, starting from a classical Lagrangian and extending to a master–slave interaction model with stochastic control terms. The Hamiltonian of a quantum robot is given by

$$H(q, p, t) = \frac{1}{2} p^T M(q)^{-1} p - \tau(t)^T q, \tag{1}$$

where $\tau(t)$ is the applied classical torque and $M(q)$ is the robot's mass moment of inertia matrix. Here, $q \in \mathbb{R}^n$ represents the canonical position vector of the robot, encompassing both translational and rotational degrees of freedom. The canonical momentum operator vector is $p = -i\nabla_q$. This Hamiltonian is derived by applying the Legendre transformation to the Lagrangian

$$L(t, q, q') = \frac{1}{2} q'^T M(q) q' + \tau(t)^T q. \tag{2}$$

We consider a system of two robots operating in teleoperation with each other via a control interaction Hamiltonian, the coefficients of which are designed by minimizing an appropriate cost functional. The total Hamiltonian of the system is

$$\begin{aligned} H(q_m, p_m, q_s, p_s, t \mid \theta) = & \frac{1}{2} p_m^T M(q_m)^{-1} p_m \\ & + \frac{1}{2} p_s^T M(q_s)^{-1} p_s - \tau(t)^T q_m \\ & + \theta^T F(q_m, q_s, p_m, p_s) + G(t, q_s, p_s), \end{aligned} \tag{3}$$

where θ is the control parameter vector for the interaction Hamiltonian. The torque $\tau(t)$ acts only on the master robot, while an environmental force acts on the slave robot performing the surgery. This environmental interaction Hamiltonian with the slave is represented by the time-varying term $G(t, q_s, p_s)$, which plays a significant role in the system's dynamics.

The environmental force G has a classical component of randomness, and similarly, the master torque $\tau(t)$ incorporates a classical component of randomness due to tremors in the hand operator. This randomness is modelled as a superposition of a white Gaussian noise process and a white Poisson differential noise process.

3.2 Wave function tracking and large deviation-based control

This subsection formulates a control problem for quantum robots using the Schrödinger wave function and large deviation principles. The goal is to design control parameters θ that minimize rare event deviations in the slave robot trajectory by leveraging the rate functional of the perturbed wave function.

Let $\psi(t, q_m, q_s)$ denote the joint wave function of the master and slave robots in position space. Schrödinger's equation governing its evolution is

$$i\partial_t \psi(t, q_m, q_s) = H(t, q_m, q_s, p_m, p_s|\theta)\psi(t, q_m, q_s), \tag{4}$$

where $H(t, q_m, q_s, p_m, p_s|\theta)$ is the total Hamiltonian of the system.

We assume prior knowledge of the surgical environment, specifically the trajectory $q_d(t)$ that the slave robot is expected to follow. The control vector θ must be chosen such that the average slave position

$$\langle q_s(t) \rangle = \mathbb{E} \int q_s |\psi(t, q_s, q_m)|^2 \, dq_s dq_m, \tag{5a}$$

tracks $q_d(t)$, while minimizing the expected deviation

$$\langle |q_d(t) - q_s(t)|^2 \rangle = \mathbb{E} \int |q_d(t) - q_s|^2 |\psi(t, q_s, q_m)|^2 \, dq_s dq_m. \tag{5b}$$

Here, the expectation $\mathbb{E}$ reflects classical randomness in the wave field ψ, stemming from noise in the torque τ and environmental interaction G. Assuming this randomness is small, we use large deviation principles (LDP) to quantify the probability of extreme deviations.

Letting ϵ denote the small parameter governing the noise strength, we approximate the probability of a perturbation $\delta\psi$ as $\exp(-I(\delta\psi)/\epsilon)$, where I is the associated rate functional. The integration $D\delta\psi$ in the subsequent expressions denotes a formal functional measure over the space of admissible wave function perturbations—i.e. a path integral over fluctuations $\delta\psi$ at each spatial point (q_m, q_s).

To compute I, we first solve (4) for the deterministic wave function $\psi_0(\cdot|\theta)$ at $\epsilon = 0$. Linear perturbation theory yields a PDE for the fluctuation $\delta\psi(\cdot|\theta)$, and the Dawson–Gärtner theorem [22, 23] is used to construct $I(\delta\psi|\theta)$. The statistical moments become:

$$\begin{aligned}\langle q_s(t) \rangle = &\int q_s \, |\psi_0(t, q_s, q_m|\theta) + \delta\psi(t, q_s, q_m|\theta)|^2 \\ &\times \exp(-I(\delta\psi|\theta)/\epsilon) \, D\delta\psi \, dq_s dq_m, \end{aligned} \tag{6}$$

$$\begin{aligned}\langle |q_d(t) - q_s(t)|^2 \rangle = &\int |q_d(t) - q_s|^2 \, |\psi_0(t, q_s, q_m) \\ &+\delta\psi(t, q_s, q_m|\theta)|^2 \\ &\times \exp(-I(\delta\psi|\theta)/\epsilon) \, D\delta\psi \, dq_s dq_m. \end{aligned} \tag{7}$$

The optimal control strategy minimizes the following LDP-informed objective functional:

$$A \int_0^T |\langle q_s(t) \rangle - q_d(t)|^2 dt + B \int_0^T \langle |q_d(t) - q_s(t)|^2 \rangle dt. \tag{8}$$

More generally, for an observable $X(q, p)$, we define $\langle X \rangle(t) = \int \psi^* X \psi \, dq$ and compute the rate function $I_T(X|\theta)$. The control θ should maximize the probability that the error stays below a threshold ϵ, by solving:

$$\sup_{\theta} \inf_{X} \left\{ I_T(X|\theta) : \sup_{0 \le t \le T} |X(t) - X_d(t)| > \epsilon \right\}.$$

A full matrix–vector stochastic differential equation (SDE) form of (4), capturing both Gaussian and Poissonian noise, is derived in Appendix A.1.

3.3 Rate function calculation and perturbation expansion

The goal of this subsection is to derive an approximate large deviation rate function $I(X|\theta)$ associated with the stochastic quantum state vector $X(t)$, governed by Itô–Poisson dynamics. This rate function is essential for evaluating the probability that observable deviations from desired quantum trajectories exceed a given threshold.

We use first-order perturbation theory to expand the solution $X(t) = X_0(t) + \delta X(t)$, where $\delta X(t)$ is the fluctuation caused by Gaussian and Poisson noise sources. The moment-generating functional of $\delta X(t)$ is then used to obtain the Gartner–Ellis limiting log-moment-generating function $\Lambda_T(f|\theta)$, from which the rate function is derived by a Legendre–Fenchel transform.

The resulting rate function has the form:

$$\begin{aligned} I(X|\theta) = \sup_f \Bigg(& f^T X - \tfrac{1}{2} f^T Q_1(\theta) f \\ & - \delta \sum_k \mu(k) \big(\exp(f^T b(k|\theta)) - 1 \big) \Bigg), \end{aligned} \tag{9}$$

which leads to the equation for the maximizing f:

$$X = Q_1(\theta) f + \delta \sum_k \mu(k) b(k|\theta) \exp(f^T b(k|\theta)). \tag{10}$$

This implicit equation is solved perturbatively by expanding $f = f_0 + \delta f_1 + \delta^2 f_2 + \ldots$, yielding an approximation to the rate function $I(X|\theta)$ up to $O(\delta^2)$.

The full derivation of this result, including the discretization, perturbation expansion, and Gaussian–Poisson decomposition of the log-moment-generating functional, is provided in Appendix A.2.

3.4 Perturbation theory and rate function of the wave function

Our objective in this subsection is to derive an approximate rate function for the wave function under noisy and controlled quantum evolution. To this end, we apply a combination of time-independent and time-dependent perturbation theory to model the system's evolution in the continuous position domain. The process is outlined as follows.

The Hamiltonian is given by

$$\begin{aligned} H &= \frac{1}{2} p_m^T M(q_m)^{-1} p_m + \frac{1}{2} p_s^T M(q_s)^{-1} p_s + \delta V(t|\theta) \\ &= H_0 + \delta V(t|\theta), \end{aligned} \tag{11}$$

where $V(t|\theta)$ includes both the random and control parameter terms and δ is assumed to be a small parameter. The unperturbed time-independent Hamiltonian is:

$$H_0 = \frac{1}{2}p_m^T M(q_m)^{-1} p_m + \frac{1}{2}p_s^T M(q_s)^{-1} p_s$$
$$= H_{00} + \epsilon H_{01} + O(\epsilon^2), \tag{12}$$

where

$$M(q_m) = M_0 + \epsilon M_1(q_m), \quad M(q_s) = N_0 + \epsilon N_1(q_s),$$

and

$$H_{01} = -\frac{1}{2}p_m^T M_0^{-1} M_1(q_m) M_0^{-1} p_m - \frac{1}{2}p_s^T N_0^{-1} N_1(q_s) N_0^{-1} p_s. \tag{13}$$

Here, M_0 and N_0 are the parts of the mass moment of inertia matrices that are not functions of the robot's angular or linear coordinates. This perturbative expansion leverages the matrix inversion lemma—commonly referred to as the Woodbury identity—to efficiently compute updates to the inverse inertia matrix, assuming the linear and angular displacements remain small [24]. Then,

$$H_{00} = \frac{1}{2}p_m^T M_0 p_m + \frac{1}{2}p_s^T N_0 p_s \tag{14}$$

represents a multivariate free particle Hamiltonian, which has a continuous spectrum. The evolution operator $U_{00}(t) = \exp(-itH_{00})$ can be computed easily in position space.

Time-dependent perturbation theory, using the Dyson series, gives the correction to the evolution up to $O(\epsilon)$ as:

$$\exp(-itH_0) = U_0(t)$$
$$= U_{00}(t) - i\epsilon \int_0^t U_{00}(t - t_1) H_{01} U_{00}(t_1) dt_1 + O(\epsilon^2). \tag{15}$$

Another application of the Dyson series expansion (15) provides the unitary evolution up to $O(\delta)$, accounting for both the random and control terms:

$$U(t) = U_0(t) - i\delta \int_0^t U_0(t - t_1) V(t_1|\theta) U_0(t_1) dt_1 + O(\delta^2). \tag{16}$$

The formula (16) can be used to calculate the approximate rate function of the wave function. Specifically, if $\psi_0(q_m, q_s)$ is the initial wave function, then the approximate wave function at time t is given by:

$$\psi_t = U(t)\psi_0 = \psi_{0t} - i\delta \int_0^t U_0(t - t_1) V(t_1|\theta) \psi_{0t_1} dt_1 + O(\delta^2)$$
$$= \psi_{0t} + \delta\psi_t, \tag{17}$$

where

$$\delta\psi_t = -i\delta \int_0^t U_0(t - t_1) V(t_1|\theta) \psi_{0t_1} dt_1 \tag{18}$$

represents the deviation in the wave function due to the random and control terms. This formula will later be used to estimate the deviation probability of wavefunction observables under noisy dynamics.

3.5 Control design based on observable deviation and large deviations principle

We now design the control parameters to minimize the observable deviations induced by noise, using the Large Deviations Principle (LDP).

Let X be any observable on the system space. For instance, X may be specified by its position representation Hermitian kernel:

$$\langle q'_m, q'_s | X | q''_m, q''_s \rangle = X(q'_m, q'_s | q''_m, q''_s). \tag{19}$$

The average value of X in the evolving state is given, up to $O(\delta)$, by:

$$\begin{aligned} \langle X \rangle(t) &= \langle \psi_t | X | \psi_t \rangle \\ &= \langle \psi_{0t} | X | \psi_{0t} \rangle + 2\delta \operatorname{Re}(\langle \psi_{0t} | X | \delta\psi_t \rangle) \\ &= \langle X \rangle_0(t) + \delta X(t), \end{aligned} \tag{20}$$

where $\langle X \rangle_0(t) = \langle \psi_{0t} | X | \psi_{0t} \rangle$ represents the average of X in the absence of random terms (i.e. noise) and, without any control terms, tracks the desired classical trajectory $X_d(t)$. The question then is how to design the control parameters θ so that the average trajectory $\langle X \rangle(t)$ deviates by only a tiny amount from the desired trajectory $\langle X \rangle_0(t)$ over the time interval $[0, T]$. This involves minimizing:

$$P\left(\sup_{0 \le t \le T} |\delta X(t)| \ge \epsilon \right). \tag{21}$$

We observe that:

$$\delta X(t) = 2\delta \operatorname{Re}\left[\int_0^t \langle \psi_{0t} | U_0(t - t_1) V(t_1|\theta) | \psi_{0t_1} \rangle dt_1 \right]. \tag{22}$$

Assume that $V(t|\theta)$ is a Gaussian process that takes operator values in the Hilbert space $L^2(\mathbb{R}^{2d})$, where both q_m and q_s are $\mathbb{R}^d$-valued. We can write:

$$V(t|\theta) = M(t|\theta) + \sum_k y_k(t) V_k(t|\theta), \tag{23}$$

where $M(t|\theta) = \mathbb{E}(V(t|\theta))$ is the classical statistical mean of $V(t|\theta)$ and $V_k(t|\theta)$ are non-random Hermitian operator-valued processes acting in the Hilbert space $L^2(\mathbb{R}^{2d})$. The $y_k(t)$ terms are zero-mean Gaussian random processes with:

$$\mathbb{E}(y_k(t) y_m(s)) = R_{km}(t, s). \tag{24}$$

Thus, we can express $\delta X(t)$ as:

$$\delta X(t) = m(t|\theta) + \delta \sum_k y_k(t) v_k(t|\theta), \tag{25}$$

where

$$m(t|\theta) = 2\delta \,\mathrm{Re}\left[\int_0^t \langle \psi_{0t}|U_0(t-t_1)M(t_1|\theta)|\psi_{0t_1}\rangle dt_1\right], \tag{26a}$$

$$v_k(t|\theta) = 2\delta \,\mathrm{Re}\left[\int_0^t \langle \psi_{0t}|U_0(t-t_1)V_k(t_1|\theta)|\psi_{0t_1}\rangle dt_1\right]. \tag{26b}$$

For $\delta \to 0$, the scaled logarithmic moment-generating functional of the process $\delta \sum_k y_k(t) v_k(t|\theta)$, $t \in [0, T]$ is given by:

$$\delta^2 \log \mathbb{E}\left[\exp\left(\delta^{-1} \int_0^T f(t) \sum_k y_k(t) v_k(t|\theta) dt\right)\right]$$
$$= \frac{1}{2} \sum_{k,m} \int_0^T \int_0^T f(t) f(s) R_{km}(t,s) v_k(t|\theta) v_m(s|\theta) dt ds. \tag{27}$$

It follows that the rate function of $\delta X(t)$, $t \in [0, T]$ is given by:

$$I_X(\xi|\theta) = \frac{1}{2} \int_0^T \int_0^T Q_v(t,s|\theta)\xi(t)\xi(s) dt ds, \tag{28}$$

where $Q_v(t,s|\theta)$ is the inverse kernel of $R_v(t,s|\theta) = \sum_{k,m} v_k(t|\theta) v_m(s|\theta) R_{km}(t,s|\theta)$.

The large deviation principle (LDP) then gives:

$$\log\left(P\left(\sup_{0\le t\le T} |\delta X(t)| \ge \epsilon\right)\right)$$
$$\approx -\frac{1}{\delta^2} \inf\left[I_X(\xi|\theta) : |\xi| \ge \epsilon\right], \quad \delta \to 0, \tag{29}$$

where

$$|\xi| = \sup\left(|\xi(t)| : 0 \le t \le T\right). \tag{30}$$

The control design can thus be specified by maximizing the quantity:

$$\inf\left[I_X(\xi|\theta) : |\xi| \ge \epsilon\right], \tag{31}$$

with respect to θ. Alternatively, if we define:

$$|\xi|_2^2 = \int_0^T \xi(t)^2 dt, \tag{32}$$

and aim to minimize $P\left(\int_0^T |\delta X(t)|^2 dt \geq \epsilon\right)$, then we would maximize:

$$\inf\left[I_X(\xi|\theta) : |\xi|_2 > \epsilon\right], \tag{33}$$

with respect to θ. This would amount to maximizing the minimum eigenvalue of the kernel $Q_v(.,.|\theta)$ or, equivalently, minimizing the maximum eigenvalue of $R(.,.|\theta)$. The maximum eigenvalue of the kernel R can be calculated approximately using the Rayleigh variational principle.

4 Simulation studies

In this section, we simulate a system consisting of two robots, each having two degrees of freedom for the angular positions $q_m, q_s \in \mathbb{R}^2$. Each component of these positions is assumed to take values within the range $[-A, A]$. The interval is discretized into N equal parts:

$$-A < -A + \Delta < -A + 2\Delta < \cdots < A - \Delta < A = -A + N\Delta, \tag{34}$$

where $\Delta = \frac{2A}{N}$ represents the step size.

The momentum operator components are $-i\partial/\partial q_{m1}, -i\partial/\partial q_{m2}, -i\partial/\partial q_{s1}, -i\partial/\partial q_{s2}$. Finite difference operators replace these differential operators for numerical computation. After spatial discretization, the wave function is replaced with a complex vector $\psi(t)$ of dimension N^4. The multiplication operations q_{mj}, q_{sj}, for $j = 1, 2$, are replaced by multiplications with $N^4 \times N^4$ diagonal real matrices, denoted by Q_{mj} and Q_{sj}, respectively. The actions of the momentum operators $p_{mj}, p_{sj}, j = 1, 2$ on the wave function are replaced by multiplications with purely imaginary, tridiagonal Hermitian matrices P_{mj}, P_{sj}, for $j = 1, 2$.

The system Hamiltonian is then constructed as an $N^4 \times N^4$ Hermitian matrix, expressed as:

$$H_0 = \frac{1}{2}\sum_{j,k=1}^{2} P_{mj}(M^{-1}(Q_{m1}, Q_{m2}))_{jk} P_{mk} + \frac{1}{2}\sum_{j,k=1}^{2} P_{sj}(M^{-1}(Q_{s1}, Q_{s2}))_{jk} P_{sk}. \tag{35}$$

Once this matrix is evaluated, we choose three $N^4 \times N^4$ Hermitian matrices G_0, H_1, H_2 and solve the following stochastic differential equation:

$$d\psi(t) = -\left(i(H_0 + \theta_1 H_1 + \theta_2 H_2) + \frac{G_0^2}{2}\right)\psi(t)dt - iG_0\psi(t)dB(t), \tag{36}$$

where $B(t)$ represents a standard Brownian motion process. The simulation is carried out for specific values of the control parameters θ_1 and θ_2. This stochastic differential equation is numerically solved using a temporal discretization approach:

$$\psi(t+1) = \psi(t) - \delta\left(i(H_0 + \theta_1 H_1 + \theta_2 H_2) + \frac{G_0^2}{2}\right)\psi(t) - iG_0\psi(t)\sqrt{\delta}W(t+1), \quad t = 1, 2, \ldots, \tag{37}$$

where $W(t)$ is an independent and identically distributed (iid) sequence of zero-mean normal random variables. First, we solve Eq. (37) with $G_0 = 0$, $H_1 = 0$, $H_2 = 0$, i.e. in the absence of noise and control terms. The resulting wave function trajectory is denoted by $\psi_0(t)$ for $t = 1, 2, \ldots$.

Next, we choose a Hermitian matrix X of size $N^4 \times N^4$ and compute the quantum-averaged trajectory:

$$x_d(t) = \psi_0(t)^* X \psi_0(t), \quad t = 1, 2, \ldots \tag{38}$$

Here, the superscript * denotes the Hermitian conjugate (complex conjugate transpose).

We then compute the unperturbed evolution matrices:

$$U_0(t) = \exp(-it\delta H_0), \quad t = 0, 1, 2, \ldots, \tag{39}$$

and construct the approximate solution for $\psi(t)$ as:

$$\begin{aligned}\psi(t) &= U_0(t)\psi(0) - i\delta \sum_{k=0}^{t} U_0(t-k)(\theta_1 H_1 + \theta_2 H_2 \\ &\quad + \frac{G_0^2}{2})U_0(k)\psi(0) \\ &\quad - i\sqrt{\delta}\sum_{k=0}^{t} U_0(t-k)G_0U_0(k)\psi(0)W(k) \\ &= \psi_0(t) + (v_0(t) + \theta_1 v_1(t) + \theta_2 v_2(t)) \\ &\quad + \sqrt{\delta}\sum_{k=0}^{t} v_3(t,k)W(k),\end{aligned} \tag{40}$$

where $v_0(t)$, $v_1(t)$, $v_2(t)$ represent the perturbations to the wave function due to the control terms and $v_3(t,k)$ represents the contribution from noise. The total solution is then:

$$\psi(t) = \psi_0(t) + (v_0(t) + \theta_1 v_1(t) + \theta_2 v_2(t)) + \delta\psi(t). \tag{41}$$

The approximate perturbation to $x_d(t)$ due to the control terms and noise, after quantum averaging, is given by:

$$\begin{aligned}\delta x(t) &= 2\,\mathrm{Re}\left[(\psi_0(t)+\theta_1 v_1(t)+\theta_2 v_2(t))^* X\delta\psi(t)\right]\\ &= \sum_{k=0}^{t} 2\,\mathrm{Re}\left[(\psi_0(t)+\theta_1 v_1(t)+\theta_2 v_2(t))^* X v_3(t,k)\right] W(k).\end{aligned} \tag{42}$$

Finally, we compute the autocorrelation of $\delta x(t)$:

$$\begin{aligned}R_x(t,s|\theta_1,\theta_2) &= \mathbb{E}(\delta x(t)\delta x(s))\\ &= 4\sum_{k=0}^{\min(t,s)} \mathrm{Re}\left[(\psi_0(t)+\theta_1 v_1(t)+\theta_2 v_2(t))^* X v_3(t,k)\right]\\ &\quad\times \mathrm{Re}\left[(\psi_0(s)+\theta_1 v_1(s)+\theta_2 v_2(s))^* X v_3(s,k)\right],\end{aligned} \tag{43}$$

for $t, s = 1, 2, \ldots, N$. We evaluate the maximum eigenvalues of the matrix

$$R_x(\theta_1,\theta_2) = (R_x(t,s|\theta_1,\theta_2))_{1\le t,s\le N}, \tag{44}$$

for different values of the control parameters θ_1, θ_2, and choose the values at which the maximum eigenvalue is minimized.

4.1 Simulation in the wave function domain

For computational purposes, we now work directly in the wave function domain to derive a large deviation-based algorithm for fine-tuning the parameters θ_1, θ_2 such that the effects of noise are minimized. Additionally, we derive a maximum likelihood algorithm for estimating these parameters from measurements of wave function deviations, which can be obtained from measurements of the average values of a class of observables.

We begin with the Schrödinger equation for the desired wave function ψ_d and the noisy wave function ψ, incorporating control terms. The desired wave function satisfies:

$$\frac{d\psi_d(t)}{dt} = -iH_0\psi_d(t), \tag{45}$$

whose solution is

$$\psi_d(t) = U_0(t)\psi(0), \tag{46}$$

where $U_0(t) = \exp(-itH_0)$ is the unperturbed evolution operator. The noisy wave function satisfies the stochastic differential equation (SDE):

$$d\psi(t) = -\left(i(H_0+\theta_1 H_1+\theta_2 H_2)+P\right)\psi(t)dt + V dB(t)\psi(t), \tag{47}$$

where H_0, H_1, H_2, V are 2×2 real symmetric matrices and $P = V^2/2$ is the Ito correction term. The control parameters θ_1 and θ_2 are introduced to mitigate the effects of noise. Here, $B(t)$ denotes standard Brownian motion, and $dB(t)/dt = W(t)$ represents a white noise process. The perturbed wave function dynamics are then given by:

$$d\delta\psi(t) = -iH_0\delta\psi(t)dt - i(\theta_1 H_1 + \theta_2 H_2)\psi_d(t)dt + W(t)V\psi_d(t), \tag{48}$$

where

$$\delta\psi(t) = \psi(t) - \psi_d(t) \tag{49}$$

is the wave function perturbation. We assume that the initial perturbation is zero, i.e. $\delta\psi(0) = 0$, which eliminates the homogeneous solution term from the mild formulation. The solution to this equation is:

$$\begin{aligned}\delta\psi(t) = \theta_1 &\left(-i\int_0^t U_0(t-s)H_1\psi_d(s)ds\right)\\ &+ \theta_2\left(-i\int_0^t U_0(t-s)H_2\psi_d(s)ds\right)\\ &+ \int_0^t U_0(t-s)V\psi_d(s)W(s)ds.\end{aligned} \tag{50}$$

This assumption of zero initial perturbation is maintained in the discrete-time formulation as well, allowing us to omit any homogeneous evolution term in $\delta\psi[0]$. In the discrete time domain, using a time discretization step size δ, Eq. (50) is expressed as:

$$\begin{aligned}\delta\psi[t] = \theta_1 &\left(-i\delta\sum_{k=0}^{t} U_0[t-k]H_1\psi_d[k]\right)\\ &+ \theta_2\left(-i\delta\sum_{k=0}^{t} U_0[t-k]H_2\psi_d[k]\right)\\ &+ \sqrt{\delta}\sum_{k=0}^{t} U_0[t-k]V\psi_d[k]W[k],\end{aligned} \tag{51}$$

where $W[t], t = 0, 1, 2, \ldots$ is an iid $N(0, 1)$ process. Collecting all K time samples, Eq. (51) is expressed in vector notation as:

$$\delta\psi = \theta_1 f_1 + \theta_2 f_2 + \sqrt{\delta}GW, \tag{52}$$

where f_1 is a $2M$ vector with components $-i\delta\sum_{k=0}^{t} U_0[t-k]H_1\psi_d[k]$ and f_2 is a $2M$ vector with components $-i\delta\sum_{k=0}^{t} U_0[t-k]H_2\psi_d[k]$. The matrix G is a $2M \times 2M$ matrix with 2×2 block components $g(t, k) = U_0[t-k]V\psi_d[k]$.

By measuring the wave function deviation, we can estimate the control parameters using the maximum likelihood method:

$$\hat{\theta} = (\hat{\theta}_1, \hat{\theta}_2) = \arg\min_{\theta} \parallel G^{-1}(\delta\psi - F\theta) \parallel^2, \tag{53}$$

where

$$F = [f_1, f_2] \in \mathbb{C}^{2M \times 2}, \tag{54a}$$

$$\theta = [\theta_1, \theta_2]^T. \tag{54b}$$

This gives the parameter estimate solution as follows:

$$\hat{\theta} = (F^*(GG^*)^{-1}F)^{-1}(GG^*)^{-1}\delta\psi. \tag{55}$$

Here, the superscript * denotes the Hermitian conjugate (complex conjugate transpose), consistent with earlier usage.

Alternatively, the control problem can be posed as determining the parameters θ such that for a given threshold ϵ, the probability that the time-averaged mean square wave function deviation $\sum_{t=1}^{T} \parallel \delta\psi(t) \parallel^2 = \parallel \delta\psi \parallel^2 = \delta\psi^*\delta\psi$ exceeds ϵ is minimized. From the large deviation principle, this probability for small δ is approximately $\exp(-Q(\theta)/\delta)$, where

$$Q(\theta) = \frac{1}{2}\inf\left\{\parallel G^{-1}(\delta\psi - F\theta) \parallel^2 : \delta\psi^*\delta\psi > \epsilon\right\}. \tag{56}$$

The optimal θ is obtained by maximizing $Q(\theta)$. This optimization problem can be solved using Lagrange multipliers by setting $\delta\psi = a\eta$, where $a = \sqrt{\epsilon}$ and $\eta^*\eta = 1$. We then minimize $\parallel G^{-1}(a\eta - F\theta) \parallel^2$ subject to the constraint $\eta^*\eta = 1$.

The function to be optimized is given by:

$$S(\psi, \lambda|\theta) = (\delta\psi - F\theta)^* M(\delta\psi - F\theta) - \lambda(\delta\psi^*\delta\psi - \epsilon), \tag{57}$$

where $M = (GG^*)^{-1}$. Setting the gradient of S with respect to ψ^* to zero gives:

$$M(\delta\psi - F\theta) - \lambda\delta\psi = 0, \tag{58}$$

leading to the solution:

$$\delta\psi = (M - \lambda I)^{-1}MF\theta. \tag{59}$$

The Lagrange multiplier λ is obtained from the equation:

$$\epsilon = \delta\psi^*\delta\psi = \theta^T(MF)^*(M - \lambda I)^{*-1}(M - \lambda I)^{-1}(MF)\theta. \tag{60}$$

This gives λ as a function of θ and ϵ, denoted as $\lambda(\theta, \epsilon)$. The optimal value of $\delta\psi$ is then:

$$\delta\psi(\theta) = (M - \lambda(\theta, \epsilon)I)^{-1}MF\theta. \tag{61}$$

Finally, the optimal value of θ is obtained by maximizing:

$$E(\theta) = S(\psi(\theta), \lambda(\theta, \epsilon)|\theta) = (\delta\psi(\theta) - F\theta)^* M(\delta\psi(\theta) - F\theta). \tag{62}$$

Rather than solving this problem in its entirety, we can directly formulate a joint optimization problem to optimize $S(\delta\psi, \lambda|\theta)$ concerning $\delta\psi$, θ, and λ:

$$S(\psi, \lambda|\theta) = (\delta\psi - F\theta)^* M(\delta\psi - F\theta) - \lambda(\delta\psi^*\delta\psi - \epsilon). \tag{63}$$

The simultaneous equations to be solved are:

$$\frac{\delta S}{\delta\delta\psi^*} = 0, \quad \frac{\delta S}{\delta\theta} = 0, \quad \frac{\delta S}{\delta\lambda} = 0, \tag{64}$$

which yield, after noting that θ is a real parameter vector, the following system of equations:

$$\delta\psi = (M - \lambda I)^{-1} M F\theta, \tag{65a}$$

$$2\,\mathrm{Re}(F^* M(\delta\psi - F\theta)) = 0, \tag{65b}$$

$$\delta\psi^*\delta\psi = \epsilon. \tag{65c}$$

The second equation can be rewritten as:

$$\mathrm{Re}(F^* M F)\theta = \mathrm{Re}(F^* M\delta\psi). \tag{66}$$

This system allows us to solve for the optimal control parameters θ that minimize the effect of noise on the wave function deviation while maximizing the likelihood of tracking the desired trajectory under noise perturbations.

By solving this joint optimization problem, we obtain both the control parameters θ_1, θ_2 and the wave function deviation $\delta\psi$ that minimize the probability of exceeding the threshold ϵ, thereby ensuring robust performance in noisy environments.

4.2 Summary of simulation studies

The simulation studies demonstrate that the large deviation-based control design effectively mitigates the effects of noise in the quantum robotic system. By discretizing the time and solving the stochastic differential equations for the wave function perturbations, we can calculate the optimal control parameters θ_1, θ_2 that minimize the wave function deviation. Furthermore, the maximum likelihood estimation method provides an efficient way to estimate these parameters from observable measurements.

The study confirms that the proposed control framework can reduce the impact of noise, as evidenced by the reduction in the maximum eigenvalue of the autocorrelation matrix $R_x(\theta_1, \theta_2)$ across different simulation scenarios. Optimization ensures that the control parameters are fine-tuned to achieve robust performance, even in significant noise disturbances.

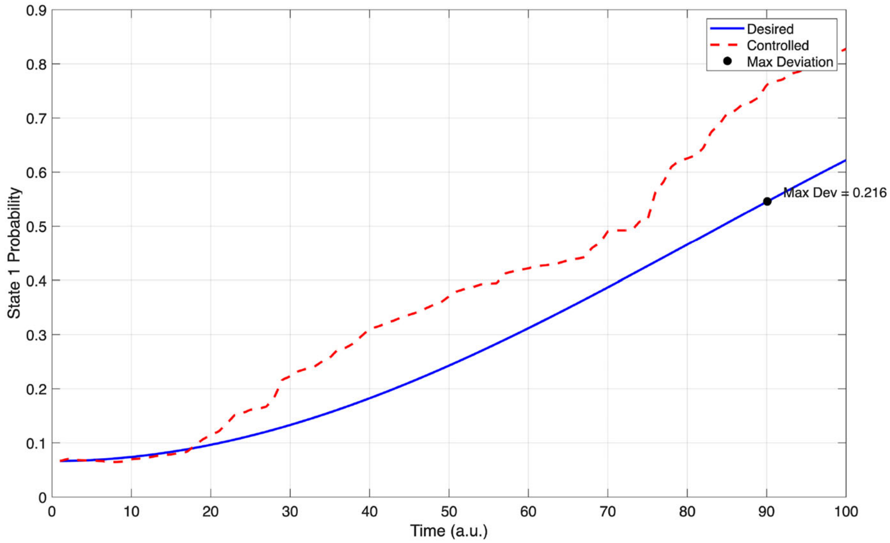

Fig. 1 State 1 probability under monotonic growth (Max Dev = 0.216)

5 Results and discussion

The objective of these simulations is to evaluate the effectiveness of our LDP-based control strategy in minimizing noise-induced deviations in a two-dimensional Hilbert space governed by Schrödinger wave function dynamics. The Hamiltonian matrix defines the system's energy dynamics, while the noise matrices introduce stochastic fluctuations. Control parameters are applied to mitigate these fluctuations and stabilize the wave function.

We begin by simulating the desired wave function by removing noise components from the Schrödinger equation and setting control parameters to zero. This simulation establishes the baseline probability trajectory for State 1, which refers to the ground state of the quantum system and provides insight into the system's stability under ideal conditions.

Next, white Gaussian noise is introduced via a 2×2 coupling Hermitian matrix. We apply control parameters calculated through a large deviation theory-based algorithm to counteract the noise. Large deviation theory is employed for its ability to estimate the probability of rare, noise-induced events, making it well suited for optimizing control parameters in noisy environments. This optimization is achieved using a gradient search method, similar to Newton's, until the deviation probability falls below a predefined threshold.

Once the control parameters converge, we simulate the noisy wave function with control applied. Figures 1, 2, 3 and 4 present overlays of the desired (noise–free) and controlled (noisy) probability trajectories on a common time axis, with the empirical maximum absolute deviation ("Max Dev") annotated in each case. The first three scenarios for State 1 represent canonical evolution patterns—monotonic growth,

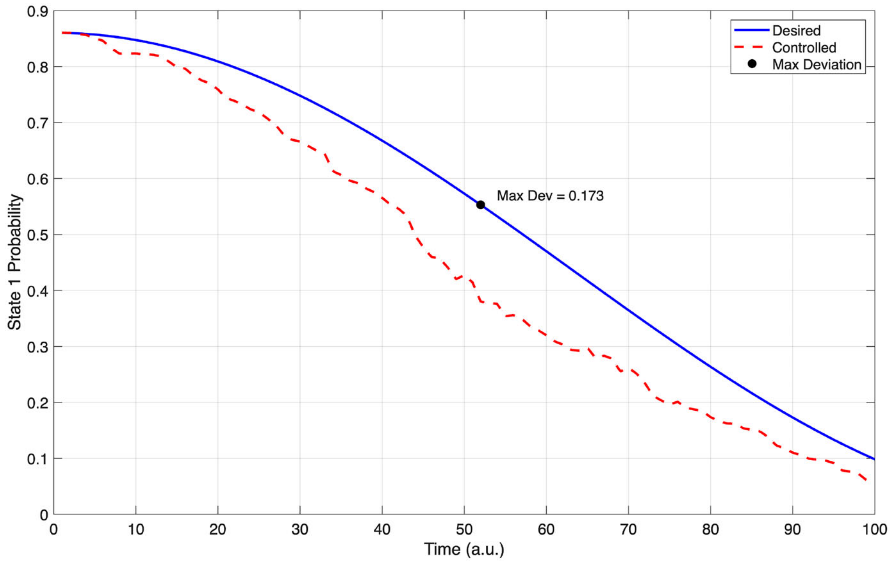

Fig. 2 State 1 probability under monotonic decay (Max Dev = 0.173)

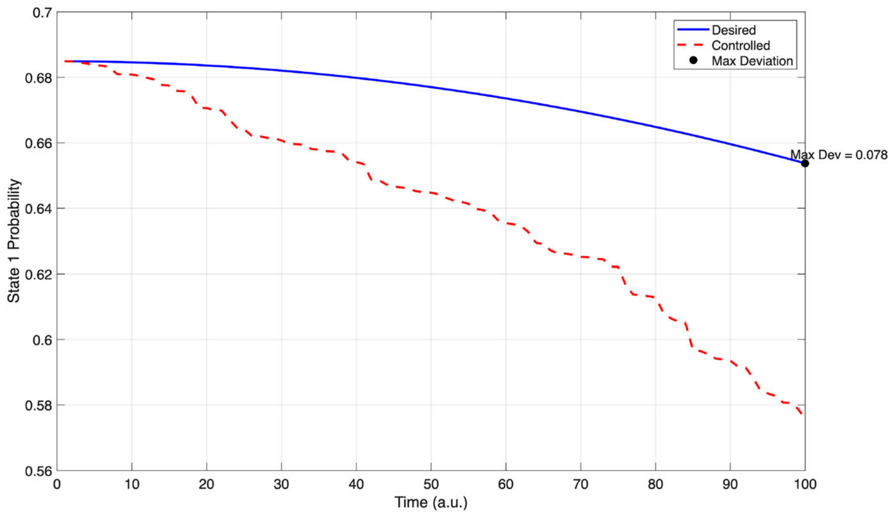

Fig. 3 State 1 probability under slow relaxation (Max Dev = 0.078)

monotonic decay, and slow relaxation—that arise naturally in quantum robotic subsystems, such as master–slave surgical platforms, where quantum states must be stabilized or modulated under stochastic perturbations. These representative cases demonstrate that the controller maintains bounded deviations of 0.216, 0.173, and 0.078, respectively, even across qualitatively different dynamical regimes. Figure 4 shows results for State 2, where the maximum deviation (0.091) is comparable to State 1, confirming that the approach generalizes consistently across coupled states. Importantly, the

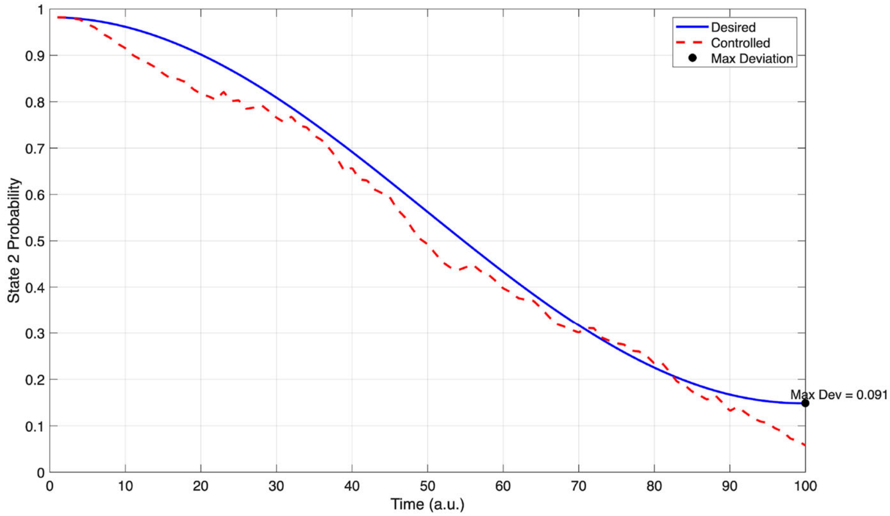

Fig. 4 State 2 probability trajectory (Max Dev = 0.091)

observed deviations lie within the rare event bounds predicted by the rate function analysis of Secs. 3–4, confirming that the LDP-based synthesis translates effectively into practical noise suppression.

These findings align with stochastic process theory, demonstrating that deterministic control strategies can effectively mitigate noise when noise statistics are known. This extends traditional models, showing that our deterministic approach offers a viable alternative when continuous feedback control methods are impractical. Thus, the simulations provide not only a validation of the proposed controller but also illustrate its potential as a general tool for stabilizing quantum robotic subsystems under mixed Gaussian–Poisson noise, with broader implications for quantum computing and molecular simulations.

6 Conclusion and future scope

This work has presented a control framework for quantum robotic systems that mitigates the effects of stochastic noise while preserving stable and precise system behaviour. Incorporating Itô correction into the system Hamiltonian was essential for maintaining unitarity and probability conservation under noisy dynamics. Control terms were introduced and optimized using large deviation principles (LDP), which provided a statistical framework for quantifying the likelihood of rare noise-induced deviations. Theoretical predictions were validated by simulations, which demonstrated that the empirical deviations between desired and controlled trajectories remain bounded and consistent with the rate function analysis.

The present framework assumes stationary Gaussian noise and therefore does not yet capture more complex noise processes. Future work will extend the approach to non-Gaussian models, such as Lévy processes, which introduce sudden jump-like perturbations that better reflect realistic environments. Incorporating these processes will enhance robustness and further align the method with experimental quantum systems. Another promising direction is the application to molecular quantum systems, where precise control of bond angles—formulated here as a quantum robotic task—can be used to tune molecular properties with implications for drug design and materials science.

By advancing toward these extensions, this research contributes a foundation for more resilient quantum systems capable of operating effectively in noisy, real-world environments. The framework not only supports reliable control of coupled quantum robots but also provides tools relevant to broader domains such as quantum computing, quantum chemistry, and advanced materials engineering.

Appendix

A.1 Stochastic Schrödinger Equation in Matrix–Vector Form

To explicitly model classical stochastic noise in the torque and environmental terms, we discretize the spatial variables and write the wave field $\psi(t, q_m, q_s)$ as a real-valued vector $X(t)$, stacking the sampled values of $\mathrm{Re}(\psi)$ and $\mathrm{Im}(\psi)$. The resulting matrix–vector stochastic differential equation becomes:

$$dX(t) = \left(H_0(t) + \sum_k \theta_k H_k(t)\right) X(t)\,dt + \left(\sqrt{\epsilon} \sum_k G_k(t)\,dB_k(t) + \epsilon \sum_k P_k(t)\,dN_k(t/\epsilon)\right) X(t), \tag{67}$$

where

- $H_0(t)$, $H_k(t)$, $G_k(t)$, $P_k(t)$ are time-dependent square matrices;
- $B_k(t)$ are independent standard Brownian motions;
- $N_k(t/\epsilon)$ are time-rescaled Poisson processes of rate λ_k;
- ϵ is a small parameter controlling the noise magnitude.

This formulation allows the application of large deviation theory for matrix-valued stochastic dynamics and is essential for deriving the rate functional $I(\delta\psi|\theta)$ in a finite-dimensional setting.

A.2 Derivation of the rate function

To evaluate the rate function for the deviation $\delta X(t)$, we begin by expressing:

$$X(t) = X_0(t) + \delta X(t), \tag{68}$$

where $X_0(t)$ solves the deterministic part of the stochastic differential equation and $\delta X(t)$ incorporates both Gaussian and Poisson noise components.

The moment-generating functional is:

$$M_{T,\epsilon}(f) = \mathbb{E}\left[\exp\left(\int_0^T f(t)^T \delta X(t) dt\right)\right], \tag{69}$$

which decomposes into products over k of Gaussian and Poisson expectations. This leads to the logarithmic moment-generating functional:

$$\begin{aligned}\Lambda_{T,\epsilon}(f) = \frac{\epsilon}{2}\sum_k \int_0^T g_k(\tau|\theta)^2 d\tau \\ + \frac{1}{\epsilon}\sum_k \lambda_k \int_0^T \left(\exp(\epsilon h_k(\tau|\theta)) - 1\right) d\tau,\end{aligned} \tag{70}$$

where g_k and h_k are defined as convolutions over transition matrices with initial conditions.

Taking the Gartner–Ellis limit,

$$\Lambda_T(f|\theta) = \lim_{\epsilon\to 0} \epsilon \Lambda_{T,\epsilon}(\epsilon^{-1} f), \tag{71}$$

yields

$$\begin{aligned}\Lambda_T(f|\theta) = \frac{1}{2}\sum_k \int_0^T g_k^2(\tau|\theta) d\tau \\ + \sum_k \lambda_k \int_0^T \left(\exp(h_k(\tau|\theta)) - 1\right) d\tau.\end{aligned} \tag{72}$$

After discretizing time and introducing perturbation parameter δ, the rate function becomes:

$$\begin{aligned}I(X|\theta) = \sup_f \Bigg[f^T X - \frac{1}{2} f^T Q_1(\theta) f \\ - \delta \sum_k \mu(k)\Big(\exp(f^T b(k|\theta)) - 1\Big)\Bigg],\end{aligned} \tag{73}$$

To compute this supremum, we solve:

$$X = Q_1(\theta) f + \delta H(f|\theta), \tag{74}$$

where

$$H(f|\theta) = \sum_k \mu(k) b(k|\theta) \exp(f^T b(k|\theta)). \tag{75}$$

Expanding $f = f_0 + \delta f_1 + \delta^2 f_2 + \ldots$, we solve iteratively:

$$f_0 = Q_1(\theta)^{-1} X,$$
$$f_1 = -Q_1(\theta)^{-1} H(f_0|\theta),$$
$$f_2 = -Q_1(\theta)^{-1} H'(f_0|\theta) f_1.$$

Substituting back gives the approximation for $I(X|\theta)$ up to $O(\delta^2)$, which is used in the stochastic control design objective.

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Declarations

Conflict of interest The author confirms there is no conflict of interest.

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