Determination of $\gamma$ including BESIII inputs

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A precise measurement of the Cabibbo-Kobayashi-Maskawa unitarity triangle angle $\gamma$ is crucial for testing the Standard Model description of $CP$ violation and probing for new physics effects. The precision can be improved by exploring new $B$ and $D$ decay modes. Measurements of $\gamma$ are statistically limited and hence obtaining $D$ decay parameters from $B$ decay data leads to further loss in precision. So, external inputs are crucial, especially in multibody $D$ decays. They are measured at BESIII due to the lack of quantum correlated $D$ meson data at the $B$ decay experiments. $\gamma$ measurements at LHCb and Belle(II) which include the $D$ decay input from BESIII are presented.

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1. Introduction

A measurement of the Cabibbo-Kobayashi-Maskawa [1] unitarity triangle angle $\gamma$ tests the violation of $CP$ symmetry in the Standard Model (SM). Any discrepancy between the direct measurement and the value of $\gamma$ estimated indirectly from other parameters of the unitarity triangle would indicate possible new physics effects beyond the SM. The uncertainties need to be comparable in both cases, but currently that is not the case with indirect measurements being more precise [2]. So, it is imperative to improve the precision on direct measurements of $\gamma$.

The angle $\gamma$ is measured from the interference between the amplitudes of the colour-favored $b \rightarrow c \bar{u} s$ and colour-suppressed $b \rightarrow u \bar{c} s$ transitions. The corresponding amplitudes can be written as $A_{\text{fav}} = A$ and $A_{\text{sup}} = A_B e^{i(\delta_B - \gamma)}$, where $\delta_B$ is the strong phase difference between the two decay processes, and $r_B = |A_{\text{sup}}|/|A_{\text{fav}}|$. The statistical uncertainty on $\gamma$ scales as $1/r_B$. The value of $r_B$ is approximately equal to 0.1 for $B^\pm \rightarrow D K^\pm$ and $B^\pm \rightarrow D K^{*\pm}$ decays, 0.3 for $B^0 \rightarrow D K^{*0}$ decays and 0.005 for $B^+ \rightarrow D \pi^+$ decays. At present, the measurements are statistically limited due to the small branching fraction of the decays involved. However, an improvement in the overall precision is possible when many $B$ and $D$ decay modes are exploited.

There are different methods to determine $\gamma$ depending on the $D$ final state of interest. If it is a doubly Cabibbo suppressed decay like $D \rightarrow \pi^\pm K^\mp$ or $D \rightarrow \pi^\mp K^+ \pi^+ \pi^-$, then ADS formalism [3] is used. The corresponding favoured decays are used for normalisation purposes. For $CP$ eigenstates like $h^+ h^+$, where $h = K, \pi$, GLW formalism [4, 5] is utilised. In both the methods, charge-averaged rates and asymmetries are measured. For self-conjugate modes like $K^0_S h^+ h^+$, a model-independent binned phase space analysis is performed with BPGGSZ formalism [6, 7]. In this method, the $CP$ violation sensitive parameters $x_\pm = r_B \cos(\delta_B \pm \gamma)$ and $y_\pm = r_B \sin(\delta_B \pm \gamma)$ are measured.

2. LHCb and Belle(II)

The LHCb and Belle (and now Belle II) experiments possess competing and complementing capabilities to understand the flavour structure of elementary particles. Excellent vertex resolution, tracking and particle identification capabilities of LHCb enable precision measurements in the high background environment from proton collisions. Signal purity is good owing to larger vertex separation from high Lorentz boost. Belle(II) is very good in identifying both charged tracks as well as neutral particles. The cleaner environment from $e^+ e^-$ collisions is an added advantage. In addition, the collisions happen at $\Upsilon (4S)$ threshold and this prevents background contributions from $B_S^0$ or $\Lambda_b$ decays.

LHCb and Belle(II) have measured the angle $\gamma$ to good precision. A combination of all the measurements indicate the value to be $(63.8^{+3.5}_{-3.7})^\circ$ from LHCb [8] and $(78.6 \pm 7.3)$ from Belle and Belle II [9], as shown in Fig. 1. The dominant contribution to the precision is from multibody $D$ decays.

\footnote{also referred to as $\phi_3$}
Section 3: \(\gamma\) and BESIII inputs

Measurements of \(\gamma\) are statistically limited and hence obtaining \(D\) decay parameters from \(B\) decay data leads to further loss in precision. So, external inputs are crucial, especially in multibody \(D\) decays. The \(D\) decay parameters are measured in charm factories like CLEO-c and BESIII due to the lack of quantum correlated \(D\) meson data at the \(B\) decay experiments. The \(e^+e^-\) collisions at the \(\Psi(3770)\) threshold provide a quantum correlated dataset of \(D\) meson pairs. This is utilised to measure

- the average of cosine and sine of the strong-phase difference between \(D^0\) and \(\bar{D}^0\), \(c_i\) and \(s_i\), for self-conjugate modes,
- the strong-phase \(\delta_D\) and ratio of amplitudes with respect to the Cabibbo favoured decay, for doubly Cabibbo suppressed modes, and
- \(CP\) content \(F_+\) for quasi-\(CP\) eigenstates

This proceedings focuses on \(\gamma\) measurements from LHCb and Belle(II) where \(D\) decay inputs from BESIII are used.

3.1 Self-conjugate modes

The self-conjugate modes \(K^0_S\pi^+\pi^-\) and \(K^0_SK^+K^-\) are used to determine \(\gamma\) using the BPGGSZ formalism. The strong-phase parameters \(c_i\) and \(s_i\) are determined using the data collected by BESIII [10, 11] and are used as inputs for a model-independent measurement of \(\gamma\). The Dalitz plot is optimally binned guided by an amplitude model [12, 13]. The results are shown in Fig. 2. The uncertainty on these parameters contribute about 1° to the uncertainty on \(\gamma\).

3.1.1 \(B^+ \to D(K^0_S h^+ h^-) h^\pm\) at LHCb

A measurement of \(\gamma\) is performed with \(B^+ \to D(K^0_S h^+ h^-) h^\pm\) decays using Run 1 and Run 2 data corresponding to an integrated luminosity of 9 fb\(^{-1}\) [14]. After applying an optimised selection, a global fit is performed to the \(B\) invariant mass of the candidates in the integrated \(D\) phase space region to understand signal and background shapes, as shown in Fig. 3. Then a simultaneous fit
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Figure 2: Results of $c_i$ and $s_i$ parameters for $D \to K^0 S\pi^+\pi^-$ (left) and $D \to K^0 S K^+K^-$ (right) from BESIII. In the left plot, red dots with error bars indicate BESIII results, green open squares with error bars are CLEO-c results and black open circles are model predictions.

is performed to $B^+$ and $B^-$ candidates in phase space bins to extract the $CP$ observables $x_\pm$ and $y_\pm$. The sensitivity to $\gamma$ is almost entirely coming from $B^\pm \to DK^\pm$ decays while $B^\pm \to D\pi^\pm$ mode is used to determine the fraction of $D^0$ and $\bar{D}^0$, $F_{\pm i}$ in each bin. It also controls selection and reconstruction effects.

Figure 3: Fit projections of $B$ invariant mass distribution for $B^\pm \to D(K^0 S\pi^\pm\pi^\mp)$ candidates at LHCb.

Physics observables $\gamma$, $r_B$ and $\delta_B$ are determined from the $CP$ observables. This is the most precise single measurement of $\gamma$, $(68.7^{+5.2}_{-5.1})^\circ$. Confidence level contours in the $\gamma$ and $\gamma - r_B$ parameter space are shown in Fig. 4.

3.1.2 $B^0 \to D(K^0 S h^+ h^-) K^{*0}$ at LHCb

Neutral $B$ meson decays to $DK^{*0}$ are also utilised for $\gamma$ measurements with the self-conjugate $D \to K^0 S h^+ h^-$ decays at LHCb [15]. The charge of kaon from $K^{*0} \to K^+\pi^-$ decay indicates the $B$ meson flavour. Even though the branching fraction is less than that of $B^\pm \to DK^\pm$ decays, the
interference is larger in $B^0 \to DK^*0$ decays as $r_B^{DK^*0} \sim 3r_B^{DK^+}$. The analysis strategy is the same as that used in $B^\pm \to Dh^\pm$ analysis - a global fit to understand the signal and background shapes and then a simultaneous fit in $D$ phase space bins to extract $x_\pm$ and $y_\pm$. The $F_{\pm\pm}$ values are taken from $B^\pm \to Dh^\pm$ decays. The result obtained is $\gamma = (49^{+23}_{-18})^\circ$. Confidence level contours in the $\gamma$ and $\gamma - r_B$ parameter space are given in Fig. 5. It is useful to combine it with the result from the corresponding two- and four-body $D$ decays. This result reduces the existing tension between the measurements from $B^*$ and $B^0$ modes.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{gamma_contours}
\caption{Confidence level contours in the $\gamma$ (left) and $\gamma - r_B$ (right) parameter space for $B^\pm \to D(K_S^0h^\pm h^\mp)h^\pm$ decays at LHCb.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{gamma_contours2}
\caption{Confidence level contours in the $\gamma$ (left) and $\gamma - r_B$ (right) parameter space for $B^0 \to D(K_S^0h^+ h^-)K^*0$ decays at LHCb.}
\end{figure}

### 3.1.3 $B^\pm \to D^*h^\pm$ with $D \to K_S^0h^+h^-$ at LHCb

Two separate $\gamma$ measurements are performed using $B^\pm \to D^*h^\pm$ decays with $D \to K_S^0h^+h^-$ at LHCb using different techniques - with and without reconstructing the neutral particles from $D^*$ decay [16, 17]. In the former, the reconstruction is done through $D^* \to D\pi^0/\gamma$ decays and the additional information from $D^*$ reconstruction is exploited for signal extraction. The statistical correlation between the two analyses is negligible. Following the general strategy of BPGGSZ analyses at LHCb, a global mass fit is performed to the $B$ invariant mass and then $CP$ observables...
are extracted from a simultaneous fit to candidates in $D$ phase space bins. The invariant mass fit projections are shown in Fig. 6 and 7. The results obtained are $\gamma = (69^{+13}_{-14})^o$ and $(92^{+21}_{-17})^o$ for the fully reconstructed and partially reconstructed analyses, respectively.

**Figure 6:** Fit projections of $B$ (left) and $D^*$ (right) invariant mass distribution of $B^+ \rightarrow D^*K^+$ with $D \rightarrow K^0_S h^+ h^-$ decays at LHCb, where the reconstruction is done through $D^* \rightarrow D\gamma$ decays.

**Figure 7:** Fit projections of $B$ invariant mass distribution of $B^+ \rightarrow D^*K^+$ with $D \rightarrow K^0_S h^+ h^-$ decays at LHCb, where the neutral particles from $D^*$ decays are not reconstructed.

### 3.1.4 $B^+ \rightarrow D(K^0_S h^+ h^-)h^b$ at Belle(II)

A combined measurement of $\gamma$ is performed using the data collected by Belle and Belle II experiments corresponding to integrated luminosities of 771 fb$^{-1}$ and 128 fb$^{-1}$, respectively [18]. The signal and background shapes are determined from the variables beam energy difference, $\Delta E$ and modified output, $C'$, of the multivariate algorithm that distinguishes $B$ decays from continuum processes involving lighter quarks. The fit projections are shown in Fig. 8.

A simultaneous fit in $D$ phase space bins is performed to extract $x_b$ and $y_b$. The values of $F_{kl}$ are determined from $B^+ \rightarrow D\pi^+$ decays. The result obtained is $\gamma = (78.4 \pm 11.4 \pm 0.5 \pm 1.0)^o$, where the uncertainties are statistical, systematic and due to external $c_l$ and $s_l$ inputs, respectively. Confidence level regions for $\gamma$ are shown in Fig. 9.
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Figure 8: Fit to $\Delta E$ (left) and $C'$ (right) for $B^\pm \rightarrow D(K^0_S h^+ h^-)K^\pm$ decays at Belle (top row) and Belle II (bottom row).

Figure 9: Confidence level regions for $\gamma$ from $B^\pm \rightarrow D(K^0_S h^+ h^-)h^\pm$ decays at Belle(II).

3.2 ADS and GLW modes

External inputs are necessary for $\gamma$ measurements using ADS and GLW formalism, especially with multibody $D$ decays. For $D \rightarrow K^- \pi^+ \pi^- \pi^+$ and $D \rightarrow K^- \pi^+ \pi^0$ modes, strong-phase $\delta_D$ and coherence factor $R$ are taken as inputs. These are measured using the quantum correlated data collected at BESIII [19]. Coherence factor treats these modes like a two-body decay with a single effective strong-phase in the ADS method. It is demonstrated that dividing the $D \rightarrow K^- \pi^+ \pi^- \pi^+$ phase space into bins enhances the sensitivity to $\gamma$ [20].

For quasi-$CP$ eigenstate modes like $D \rightarrow 4\pi$, $CP$ content $F_+ \equiv$ is measured at BESIII. This allows for a $\gamma$ measurement using GLW formalism with interference effects that is diluted by $(2F_+ - 1)$. The result obtained is $F_+^{4\pi} = 0.735 \pm 0.016$ [21].
3.2.1 $B^\pm \to D(K^\mp \pi^\pm \pi^\mp)h^\pm$ at LHCb

The decays $D \to K^- \pi^+ \pi^- \pi^-$ have an all-charged-particle final state and a high branching fraction. This makes it an excellent addition to the set of $\gamma$ measurements at LHCb. A measurement is done using $B^\pm \to D h^\pm$ decays in bins of $D$ phase space since the asymmetry could be larger in bins where $R$ is larger [22]. The direct $CP$ asymmetry is visible from the invariant mass fit projections for $B^-$ and $B^+$ candidates shown in Fig. 10. The result, $\gamma = (54.8^{+6.0}_{-5.8} \pm 0.6 + 6.7)\degree$ is comparable in precision to that from $D \to K_S^0 h^+ h^-$ decays, except for the larger uncertainty from the $D$ decay inputs.

Figure 10: Fit to $B$ invariant mass distribution of $B^- \to D(K^+ \pi^- \pi^- \pi^-)K^-$ (left) and $B^+ \to D(K^- \pi^+ \pi^- \pi^-)K^+$ (right) candidates at LHCb.

3.2.2 $B^\pm \to D(h^0 h^\mp \pi^\mp \pi^\mp)h^\pm$ at LHCb

Three-body modes involving a $\pi^0$ are utilised to perform ADS and GLW measurements of $\gamma$ at LHCb [23]. $D$ decays to $K\pi\pi^0$ are used in ADS formalism with $\delta_D$ and $R$ as inputs, whereas $\pi^+ \pi^- \pi^0$ and $K^+ K^- \pi^0$ are used in GLW formalism with $F_\pi$ as input. There are more than one solution to $\gamma$ due to trigonometric ambiguities as seen in Fig. 11. The solution consistent with the LHCb combination is $\gamma = (56^{+24}_{-19})\degree$.

3.2.3 $B^0 \to D(h^0 h^\mp (\pi^\pm \pi^\mp))K^{*0}$ at LHCb

Two- and four-body $D$ decays with charged tracks are used for $\gamma$ measurements involving $B^0 \to DK^{*0}$ decays using ADS and GLW methods [24]. The $D$ decay parameters $\delta_D$, $R$ and $F_\pi$ are taken as external inputs from BESIII measurements. The resultant parameter space for $\gamma$ and $r_D^{DK^{*0}}$ is shown in Fig. 12. The degeneracy can be broken when combined with the corresponding result using BPGGSZ formalism discussed in Sec. 3.1.2.

4. Conclusions

Precise measurements of $\gamma$ are essential for testing the description of $CP$ violation in SM. Results from LHCb and Belle(II) are presented for a variety of $D$ decay modes that requires different analysis formalisms. Multibody $D$ decays drive the precision on $\gamma$ and $D$ decay parameter inputs from BESIII are important in this context. With Run 3 at LHCb and new data at Belle II, the
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Figure 11: Confidence level contours in the $\gamma - \delta_B$ parameter space for $B^+ \rightarrow D(h^+ h^0 \pi^0) h^+$ decays at LHCb.

Figure 12: Confidence level contours in the $\gamma - r_B$ parameter space for $B^0 \rightarrow D(h^+ h^0 (\pi^+ \pi^-)) K^{*0}$ decays at LHCb.

The $B$ decay dataset will increase in multi folds, causing the results to become systematically dominated. Hence, larger dataset from BESIII is crucial for more precise $D$ decay measurements.

References


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