SOME INVESTIGATIONS OF DOMAIN WALLS IN KALUZA-KLEIN SPACETIME

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THESIS

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Certified that Sri Sanjay Shukla has completed the research work and fulfilled all the requirements for the submission of this thesis entitled "Some Investigations of Domain Walls In Kaluza-Klein Spacetime" for the award of the degree of Doctor of **Philosophy in Physics** of V.B.S. Purvanchal University, Jaunpur (U.P.), India. The present research work has been carried on under my supervision and it is candidate's own work. The candidate has worked for the prescribed period.

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PREFACE

The present investigations have been carried out towards the fulfillment of the requirements for the award of a Ph.D. degree in Physics of V.B.S. Purvanchal University, Jaunpur (U.P.), India, under the supervision of Dr. R.K. Tripathi, Reader and Head, Department of Physics, S.D.J. P.G. College, Chandeswar, Azamgarh (U.P.), India and Co-supervision of Dr. Virendra Singh, Reader, Department of Physics, T.D.P.G. College, Jaunpur (U.P.), India.

The thesis deals with some investigations of domain walls in Kaluza-Klein spacetime. It has been divided into four chapters. The first chapter is introductory. So, we have formulated and discussed some of the techniques and results which are relevant for our subsequent investigations. Hence, we have presented, connections for a Kaluza-Klein theory with variable rest mass, de Sitter-type of model in five-dimensional theory of gravity, the anisotropic fluids generated via Kaluza-Klein spaces.

In chapter II, it is obvious from the solutions of case I and Case II spacetimes that they are reflection symmetric with respect to the wall. For a thick domain wall it is necessary that pressure and density decrease on both sides of wall away from the symmetry plane and fall off to zero as $x \to \pm\infty$.

The solutions of the Case I require that $\rho > 0$, $p > 0$ and $(\rho - p) > 0$ and may be satisfied by choosing the parameter d such that $d < -(3+\sqrt{5})/2$. For $d = -(3+\sqrt{5})/2$ then $\rho = p$. It is clear that ρ, p fall off to zero on either side of the wall.

Again for the solutions of Case II (i) and case II (ii) the fall of condition require $a > 0$. For the case II (i) this condition would conflict with $\rho > 0$, $m^2/k^2 \geq 0$. Hence, this family of solutions is not physically reliable. However, it is interesting to note that when $b = 0$, ρ and p vanish resulting into an empty spacetime given as

(1) $$ds^2 = e^{2nt} (dt^2 - dx^2) - dy^2 - dz^2$$
$$- e^{2nt} \cosh^2 (nx) d\psi^2$$

where

(2) $$n = k \gamma .$$

For the case

(3) $$b < 1/4 \quad (1-\sqrt{17})$$

i.e.

$2b^2 - b - 2 \leq -$,

the solutions of case II (ii) would satisfy the condition

(4) $a > 0$,

(5) $\rho > 0$,

(6) $m^2/k^2 > 0$.

It would have proper fall condition as well as

(7) $\rho - p \geq 0$.

Let us now discuss the dynamical behaviour of our models under different conditions imposed on various parameters occurring in the solutions. One may obtain the general relation for the three space volume as

(8) $|g_3|^{1/2} = \text{Cosh}^{a+2b}(mx)\, e^{kt(\alpha + 2\beta)}$.

Hence, the temporal behaviour reads

(9) $|g_3|^{1/2} \sim \exp[kt(\alpha + 2\beta)]$.

Hence, it is an important to note that when

(10) $\beta = -1/2$,

we recover in Case I, the Banerjee and Das solution (1998). Hence, we have taken for the Case I, β to be negative. For

(11) $$d < -(3+\sqrt{5})/2 ,$$

one obtains

(12) $$\alpha + 2\beta < 0.$$

Again if

(13) $$k > 0 ,$$

three space collapses while the extra dimension inflates. In this case, one obtains singularity because as $t\rightarrow\infty$, ρ and p diverge. If $k < 0$, the 3-space inflates while extra dimension collapses in course of time. One may also discuss the dynamical behaviour of the domain wall solutions corresponding to the case II (ii) on the similar lines.

Let us now discuss the attractive and repulsive behaviour of thick domain wall by either through the timelike geodesics in the spacetime or studying the acceleration of an observer who is taken at rest relative to the wall as presented by Wang (1992). Let us consider an observer with the four velocity

(14) $$u_i = \text{Cosh}^{\alpha} (mx)\ e^{\alpha k t}\ \delta^t_i .$$

Hence, one may obtain an acceleration A^i as

(15) $$A^i = u_{i;k}\ u^k$$

$$= ma \tan h\ (mx)\ Cosh^{-2\alpha}\ (x)\ e\text{-}2^{\alpha kt}\ \delta^{i}{}_{x}.$$

For the case I,

(16) $a = 1$,

and

if

(17) $m > 0$,

then A^x is positive. It shows that the observer is comoving with the wall having to accelerate away from the symmetry plane or it is attracted towards the wall. Similarly if

(18) $m < 0$,

then the wall represents a repulsive behaviour to the observer. Similar results may be obtained for the domain wall solutions of Case II (ii). Let us assume

(19) $T^0{}_0 = T^2{}_2 = T^3{}_3 = T^4{}_4 = \rho$,

(20) $T^1{}_1 = - p$,

(21) $T^0{}_1 = 0$,

one obtains a domain wall solution for

(22) $\rho = p$.

But this solution be the same as obtained by Banerjee and Das (1998). If one assumes e^{mx^2} in place of Cosh (x) in the separability condition, there may not be any domain wall solution. But it gives five-dimensional empty spacetime as

$$ds^2 = e^{2nt+n^2x^2}(dt^2 - dx^2) - e^{2nt/\sqrt{6}}(dy^2 + dz^2) - e^{4nt/\sqrt{6}}d\psi^2, \tag{23}$$

where n be the arbitrary constant. Hence, it represents an inhomogeneous vacuum spacetime.

Hence, we have obtained three families of exact solutions of Einstein field equations containing three parameters.

In chapter III, by considering a non-diagonal cylindrically symmetric metric in the Kaluza-Klein spacetime we have presented a number of homogeneous and inhomogeneous perfect fluid solutions including the 5-dimensional analogue of 4-dimensional non-singular stiff fluid solutions. It is observed that the dimensional reduction is admitted only in the diagonal case.

From the equation

$$R_{ik} = - 8\pi \; [(\rho = p) \; u_i \; u_k \; - \; 1/3 \; (\rho - p) \; g_{ik}],$$

One may obtain

$$R_{ik} \; u^i \; u^k \; = - \; 16\pi/3 \quad (\rho = 2p) \leq 0.$$

So in view of the Raychaudhuri (1955), equation, singularity may be avoided only when acceleration is non-zero for a vorticity free spacetime.

Hence, it is observed that a non-singular spacetime requires to be inhomogeneous. In the case of diagonal non-singular solutions, it comes out that 5-dimensional non-singular analogue exists only for the $\rho = p$. Here, we have presented the 5-dimensional analogue of non-singular non-diagonal stiff fluid ($\rho = p$) solutions. It odes not allow the dimensional reduction. Here the parameter a measures the non-diagonality of the metric and b as inhomogeneity. The perfect fluid without $p = \rho$ is allowed in the homogeneous case. We have presented a family of 5-dimensional solutions which includes 5-dimensional version of FRW flat solutions. All homogeneous solutions are expected big-bang singular.

In the last chapter, For the thick domain wall in curved spacetime one assumes the wall to have planar symmetry with two commuting Killing vectors describing translational invariance in the plane parallel to the wall and a third Killing vector related to a rotational symmetry about the X-axis perpendicular to the wall. It is symmetric about x = 0 plane. The Lagrangian for the scalar field is taken as $L = 1/2\, g^{ik}\, \varphi_i\, \varphi_k - V(\varphi)$, where φ is assumed as a function of x alone. It has been investigated by Widrow (1985) that for such a field purely static metric gives unphysical behaviour at large $|x|$, so long as $V(\varphi)$ is positive. We have presented that a thick domain wall with oplaner symmetry having $T^0_0 = T^2_2 = T^3_3 \neq 0$ and $T^1_1 = 0$ may not remain in static equilibrium in Brans-Dicke theory of gravitation. We hope if one assumes $T^1_1 \neq 0$ the result may be changed.

Every chapter has been divided in sections following decimal system: section (1.5) means fifth section of chapter first. On the same line, the equations in different chapters are also numbered i.e. Eq. (4.5) means, fifth equation of chapter four. At last references are given.

CONTENTS

Section		Page
	Preface	

CHAPTER-I
INTRODUCTION

1.1 Introduction and Motivation

1.2 Connections for a Kaluza-Klein theory with Variable Rest mass.

1.3 De Sitter-type of model in five-dimensional theory of gravity

1.4 The Anisotropic Fluids Generated via Kaluza-Klein Spaces.

CHAPTER -II
THICK DOMAIN WALLS IN FIVE DIMENSIONAL KALUZA-KLEIN SPACETIME

2.1 Introduction

2.2 The Field Equations

2.3 The Solutions

2.4 Concluding Remarks

CHAPTER-III
CYLINDRICALLY SYMMETRIC METRIC IN KALUZA-KLEIN SPACETIME

3.1 Introduction

3.2 The Metric and Field Equations

3.3 Inhomogeneous Solutions

3.4 Homogeneous Stiff-fluid Solutions

3.5 Homogeneous Perfect Fluid Solutions

3.6 Concluding Remarks

CHAPTER -IV
DOMAIN WALLS IN BRANS-DICKE THEORY

4.1 Introduction

4.2 Cosmological solutions in the Brans-Dicke Theory

4.3 Field Equations for domain wall in Brans-Dicke Theory

4.4 Concluding Remarks

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C H A P T E R - I

INTRODUCTION

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1.1 Introduction and Motivation:

There has been long-standing expectation to construct a theory of gravity that provides the same physical answers when a change or coordinates is done (covariance) and when a change of units or scales is carried out (scale invariance). There are various reasons for this desire, some of the most cogent of which have to do with cosmology as presented by Wesson (1978, 1980). It is well known that the Einstein general theory of relativity is an excellent and important theory of gravity, but it is coordinate covariant but not scale invariant. This has led several authors to suggest an alternative theories of gravity, for example, Dirac (1973), Hoyle and Narlikar (1974), Canuto et al (1977). However, these latter theories describe rather drastic departures from convention, and lack convincing observational support. Scale invariance may be obtained in less drastic fashion by extending general relativity from four dimensions to five dimensions, where the fifth dimension is the mass. The space of Einstein's theory by which is meant the 4-dimensional version of general relativity, may be regarded as embedded in this 5D space. The scale invariance is physically reasonable and a 5-dimensional variable-mass version of general relativity is mathematically elegant and agrees with observation. Whether the real Universe is best described by the 4D Einstein theory or the 5D scale-invariance theory and this question may be decided by astronomical observation and experiment.

Invariance of the testable consequences of the theory under changes of coordinates and changes of scales are separate attributes, and it is useful to recall what these kinds of invariance mean. Invariance under changes of coordinates is a statement of the fact that the physical properties of the system are not changed by a change of coordinates. A trivial example of such a change is one from Cartesians to Spherical polars. A non-change trivial example is an arbitrary change from one system of curvilinear coordinates to another, where the physical properties of the system are not affected by the device of describing them by tensors. Invariance under changes of scales is a statement of the fact that the physical properties of a system are not changed by a change in the standards of measurements or units. A trivial example of such a change is one where a measuring rod-graduated in metres is changed for one graduated in centimetres. A non-trivial example is given by the possibility that standards of measurement or scales do in fact vary from place to place in the Universe, and if the laws of physics are to be the same at all places, these latter should clearly be invariant under changes of scales. The physical properties of a system in this case may be preserved by the device of describing them by quantities sometimes called as cotensors by Dirac (1973), Canuto et al (1977). At least at present, there is no way to rule out by experiment the possibility that the sizes of the scale are coordinate-dependent. A review of other arguments for having a theory of gravity that is invariant under coordinate changes and scale changes is available as presented by Wesson (1980).

There are at least three scale invariant theories of gravity in existence. It is remarkable to examine them brtiefly to see what consequences are expected to follow from the requirement of invariance under changes of scales. The three theories are those of Dirac (1973), Hoyle and Narlikar (1974), and Canuto et al (1977). These theories share the property that in them the Newtonian gravitational parameter G and/or the masses of objects vary with time at a rate governed by the age of the Universe, which is of order of 10^{10} yr. It is not difficult to see why G and/or masses, which are constant in Einstein's theory, have to be variable in scale invariant theory. Let us consider an object like a star of mass m, surrounded by vacuum and situated in a Universe whose background metric is evolving with time as might be the case if it is expanding, as is the real Universe. In Einstein theory G and C are constants, and it is a direct consequence of the field equations that m is also constant say M_0. There are thus scales of mass M_0, length $L_0 = GM_0/c^2$ and time $T_0 = GM_0/c^3$ associated with the object, and those are constants even though the background metric is variable in time. In otherwords, the object is in some sense decoupled from the rest of the Universe. In scale invariant theory, it is obvious that scale invariance is only fully realised when there are no constant scales present as shown by Fulton et al (1962) Canuto et al (1977), Wesson (1980). This scheme is quite general, and was first appreciated in particle physics. In particle physics theories, scale invariant behaviour is fully realised in the limit of energies much larger than the energy corresponding to the rest mass or equivalently in the limit in which the constant scale corresponding to the rest mass vanishes. The scheme also holds in gravitational physics. In theories of gravity, scale invariant feature is fully realised in the limit of large masses, lengths and times, where large means of cosmological order. It means that an object like a star is described by a solution which for times small compared to 10^{10} yr contains parameters like M_o, L_o, T_o which to a good approximation are constants; while for times comparable to 10^{10} yr these constant scales vanish. In other words, the object in some sense coupled to the rest of the Universe, and this is manifested by the fact that G and/or m vary at a rate governed by the age of the Universe.

Experimentally, scale invariance is of established significance in particle physics but not in gravitational physics. This is because observations of system where gravity is the dominant force have only been carried out for a time interval $\approx 10^2$ yr much smaller than the age of the Universe $\approx 10^{10}$ yrs. In astrophysics for example, it is not known if the mass of a star like the sun is a constant over times of order 10^{10} yr, in which case it is correctly described by a scale-invariant theory. In cosmology, it is observed that systems with large sizes and ages may be described by functions which do not contain any constant scales. Two examples are follows: A typical cluster of galaxies has a radius r of order 10^{25}cm and a density ρ measured with respect to its centre which is given approximately by the scale free relation $G \rho r^2/c^2 =$ dimensionless constant. The background Universe out to the light sphere consists of many randomly-located clusters, and has a homogeneous density which is terms of the time t since the big bang is given approximately by the scale-free relation $G \rho t^2$ = dimensionless constant. The observation that cosmological systems may be described by functions which do not contain any constant scales may just be a clue to the physics to their formation in which case they may be correctly described by Einstein's theory, or may be an indication that scales vanish in the cosmological limit in which case they may be correctly described by a scale-invariant theory. Of course, it is feasible to carry out experiments to detect a possible time-variation of G and/or masses at the present epoch as shown by Wesson (1978, 1980). However, the effects involved are very small, and so far no results have emerged which are so clear cut as to allow of a choice to be made between Einstein's theory and the various scale-invariant theories.

There are several objections to the scale-invariant theories which has already been proposed by Dirac (1973), Hoyle and Narlikar (1974), Canuto et al (1977). First, these theories allow masses to be variable over cosmological times, but at the expense of introducing a gauge function which maynot be fixed except by appeal to some external criterion. The most usual way of fixing the gauge function is to use Dirac's Large Numbers Hypothesis (LNH), which leads to two possible forms of the gauge function. This hypothesis has been used by Dirac and Canuto et al, and the latter authors have also introduced a third gauge functioin consistent with the LNH. However, the LNH is an extra hypothesis not integral to the theory of Dirac or the theory of Canuto et al. An alternative way of selecting the gauge function is that of Maeder and Bouvier (1979), who have shown that the condition that the Minkowski metric be a solution of the scale-invariant field equations for empty space leads to one of the two Dirac gauges. However, while this is plausible, it maynot be taken a proof. Hence, the situation is that there are at least three possible gauge functions, with no way to select between them. It is possible in theory to constrain the gauge function in view of observational data: but this is not possible in practice because no one gauge functiion is compatible with all observations, as shown by Wesson (1978, 1980), and Dirac (1979), who has abandoned part of his original theory because of conflicts with observation. The objection just presented may be the result of the incomplete nature of the scale-invariant theories which have already been proposed, but the objection is neverthless valid. Second, extant theories are often presented in a form where there are two metrics, one of which refers to atomic physics. It is related with the fact that G is assumed to vary as measured by an atomic clock. The two-metric view is mathematically clumsy and physically obscure. This objection is related with as two metrics are related by the gauge functioin and one may argue that both objections are not too serious because they come from the nature of incomplete scale-invariant theories. However, while this might be the case as far as gravitational physics is concerned the two objections presented here represent significant obstacles to further research. Third, in theories which have already been proposed, the Newtonian gravitational parameter G and the mass of an object m are treated separately, whereas in problems where gravity is the dominant force, and therefore in the cosmological or fully scale-invariant limit as well, these two parameters always occur together in the combinatioin Gm. It is related with the fact that G is essentially plays the role of dimension-transposing constant for m, which latter is the source of the gravitational field i.e. G is on the same footing as c. In other words, it makes more sense either to regard G as the true constant and m as variable or to use the single parameter Gm as variable.

The above discussions indicate that scale-invariant theory for gravity (i) is physically sensible; (ii) implies that masses should vary slowly at a rate governed by the age of the Universe; (iii) may be relevant to real Universe; (iv) should be described by a kind of theory different to those already known. These comments provide proper motivation for further researches in these directioins. If one thinks on these comments, it becomes obvious that some new hypothesis is needed in scale-invriant theory which at the same time ensures that scale-invariance is properly taken into account and that Einstein's theory may be recovered in some appropriate limit as such. There is a hypothesis which satisfies it: There is a variable with the dimensions of a length which may be defined as Gm/c^3 where m is the mass, and this variable plays the role of a coordinate in a five-dimensional space, this latter having scale-invariant properties and containing the four-dimensional space of Einstein's theory. This hypothesis is less drastic than those on which other scale-invariant theories of gravity are based, and actually has precedents both in classical physics and gravity. In classical physics, quantities may be divided into categories depending on whether they are intensive meaning thereby definable or measurable at a point, or extensive i.e. definable or measurable with respect to an origin. Common quantities which come into the first category are pressure and density and common quantities which fall into the second category are space coordinate and time. Now mass is also a common physical quantity, and if it is questioned into which category it falls, then the answer is the second category. Hence, the mass is the same kind of quantity as a coordinate. This fact is not merely semantic, since the noted existence of two categories of two physical quantities determine to a certain extent how the equations which describe the physical behaviour are set up.

In general theory of relativity, it is the existence of constant c which underpins the logic of regarding ct or the time t as a coordinate, and allows the three-dimensional space of common experience to be extended to the four-dimensional one of conventional relativity. Likewise, it may be argued that existence of G and c underpins the logic of regarding Gm/c^2 or mass as coordinate, and allows the four-dimensional space of conventional relativity to be extended to the five-dimensional one. The above discussions indicate that introducing a fifth coordinate and a 5-D space is not such drastic step as it may appear to be on causal examination. There have been, of course, 5-D versions of relativity before, the most notable of which was the Kaluza-Klein theory, Kaluza (1921), Lein (1926), Witten (1981). However, in the Kaluza-Klein theory, the fifth dimension was introduced in order to incorporate electromagnetism into relativity; and the extra field equations which this entailed contained quantities of no physical significance, and so were not used. In the 5-dimensional theory by Wesson (1983), the fifth dimension is introduced inorder to incorporate mass into relativity in a way consistent with scale invariance: and extra dimension field equations which this entails contain the familiar quantities of physics, and are used in order to test the theory. Hence, the Kaluza-Klein theory and Wesson theory have in common the fact that they both used a 5-dimensional Riemannian space, but are otherwise different.

1.2 Connections for a Kaluza-Klein theory with Variable Rest Mass:

One an important reason for discussing theories of gravity other than that of Einstein is to try to present a better understanding of cosmology as shown by Wesson (1978, 1980). Theories of Kaluza-Klein type are currently the subject of special interest. These are 5-dimensional theories, which may give a means of extending Einstein's theory i.e. four-dimensional general relativity and of unifying gravity with other forces of physics as presented by De Sabbata and Schmutzer (1983), Lee (1984). A theory of the Kaluza-Klein type, which may be known as an embedding for general relativity with variable rest mass, has been proposed by Wesson (1983). In the theory of Wesson (1983), the conventional 4-dimensional spacetime of Einstein theory is extended to a 5D space-time-mass, in which the fifth coordinate is taken to be the rest mass. There are two reasons for identifying the mass as a coordinate. (a) It is the constant c, the velocity of light, which allows the time to be defined as coordinate $x^{o} = ct$, on the same footing as the space coordinates x^{1}, x^{2}, x^{3}. Similarly, the existence of constant G, the Newtonian constant of gravity, suggests that the rest mass m of the particle may be treated as a coordinate

$x^4 = Gm/c^2$. (b) If x^4 is a coordinate in a 5-dimensional manifold, the dimensionless velocity $w = (G/c^3)$ dm/dt describes a variation in the rest mass of a particle with time.

There is considerable interest in variable-gravity theories, and the use of x^4 and w in a 5D theory is superior to the device of allowing G to be time-variable in a 4D theory. These two reasons for identifying the mass as a coordinate lead to a theory which is mathematically straightforward. It uses a Riemannian space, and the field equations involve the 5D Einstein tensor G_{ij}, so it is in essence an extension of 4D general relativity. The theory agrees with all observations provided $w \leq 1$. In the limit $w = 0$, the fifth dimension is absent and Einstein 4D theory is recovered. In the real world, the consequences of the 5D theory may best be distinguished from those of the 4D theory in the cosmological domain. It was implied in Wesson (1983) that the relations given there could be used to investigate cosmological models. However, while those relations are correct, they are based on a 5D metric that is somewhat inconvenient, in that it does not allow of a simple comparison with the 4D metric of conventional cosmology. Hence, a more convenient metric will be taken and the relations obtained in Wesson (1983) will be rederived for the new metric.

Let us now denote the coordinates

(1.1) $$x^{o} = t ,$$

(1.2) $$x^{1} = x ,$$

(1.3) $$x^{2} = y ,$$

(1.4) $$x^{3} = z ,$$

(1.5) $$x^{4} = m ,$$

where c = G = 1. Hence, an appropriate metric for 5D i.e. homogeneous and isotropic is

(1.6) $$ds^2 = e^{v}\, dt^2 - e^{w}\, (dx^2+dy^2+dz^2) + e^{\mu}\, dm^2 ,$$

where

(1.7) $$v = v\ (t, m) ,$$

(1.8) $$w = w\ (t, m) ,$$

(1.9) $$\mu = \mu\ (t, m) .$$

This metric reduces to that of conventional cosmology, namely the Friedmann-Robertson-Walker one with zero curvature constant, when the fifth dimension is absent. The nonvanishing Christoffel symbols are:

(1.10) $\Gamma^{0}_{00} = \frac{v^*}{2}$, $\Gamma^{1}_{01} = \frac{w^*}{2}$,

(1.11) $\Gamma^{2}_{02} = \frac{w^*}{2}$, $\Gamma^{3}_{03} = \frac{w^*}{2}$,

(1.12) $\Gamma^{4}_{00} = \frac{-e^{-\mu+v} \, v^*}{2}$, $\Gamma^{0}_{04} = \frac{v^*}{2}$,

(1.13) $\Gamma^{1}_{14} = \frac{w^*}{2}$, $\Gamma^{2}_{24} = \frac{w^*}{2}$,

(1.14) $\Gamma^{3}_{34} = \frac{w^*}{2}$, $\Gamma^{4}_{04} = \frac{\dot{\mu}}{2}$,

(1.15) $\Gamma^{0}_{11} = \frac{e^{-v+w} \, \dot{w}}{2}$, $\Gamma^{4}_{11} = \frac{e^{-\mu+w} \, w^*}{2}$,

(1.16) $\Gamma^{0}_{22} = \frac{e^{-v+w} \, \dot{w}}{2}$, $\Gamma^{4}_{22} = \frac{e^{-\mu+w} \, w^*}{2}$,

(1.17) $\Gamma^{0}_{33} = \frac{e^{-v+w} \, \dot{w}}{2}$, $\Gamma^{4}_{33} = \frac{e^{-\mu+w} \, w^*}{2}$,

$$(1.18) \qquad \Gamma^{0}_{44} = -\frac{e^{-v+\mu}\,\dot{\mu}}{2}\,, \qquad \Gamma^{4}_{44} = \frac{\mu^*}{2}\,,$$

where (.) and (*) denote partial derivatives with respect to t and m respectively.

The nonvanishing components of Ricci tensor are:

$$(1.19) \qquad R_{00} = \frac{3}{2}\ddot{w} + \frac{\ddot{\mu}}{2} + \frac{3}{4}w^{*2} + \frac{\mu^{*2}}{4} - \frac{3v^*w^*}{4} - \frac{v^*\mu^*}{4}$$

$$+ e^{v-u}\left(\frac{v^{**}}{2} + \frac{v^{*2}}{4} + \frac{3v^*w^*}{4} - \frac{\mu^*v^*}{4}\right),$$

$$(1.20) \qquad R_{04} = \frac{3}{4}\dot{w}^* + \frac{3\dot{w}w^*}{4} - \frac{3\dot{w}v^*}{4} - \frac{3w^*\dot{\mu}}{4}\,,$$

$$(1.21) \qquad R_{11} = -e^{w-v}\left(\frac{\ddot{w}}{2} + \frac{3}{4}w^{*2} + \frac{\mu^*w^*}{4} - \frac{v^*w^*}{4}\right)$$

$$- e^{w-u}\left(\frac{w^{**}}{2} + \frac{3w^{*2}}{4} + \frac{v^*w^*}{4} - \frac{\mu^*w^*}{4}\right),$$

$$(1.22) \qquad R_{22} = R_{11}\,,$$

$$(1.23) \qquad R_{33} = R_{11}\,,$$

(1.24) $$R_{44} = \frac{3\overset{**}{w}}{2} + \frac{\overset{**}{v}}{2} + \frac{3\overset{*}{w}^2}{4} + \frac{\overset{*}{v}^2}{4} - \frac{3\overset{*}{w}\overset{*}{\mu}}{4} - \frac{\overset{*}{v}\overset{*}{\mu}}{4}$$

$$+ e^{\mu - v}\left(\frac{\ddot{\mu}}{2} + \frac{\dot{\mu}^2}{4} + \frac{3}{4}\overset{*}{w}\overset{*}{u} - \frac{\overset{*}{u}\overset{*}{v}}{4}\right) \quad .$$

The Ricci scalar reads

(1.25) $$R = \bar{e}^{v}\left(3\dot{w} + 3\overset{*}{w}^2 + \ddot{\mu} + \frac{\overset{*}{\mu}^2}{2} + \frac{3\overset{*}{w}\overset{*}{\mu}}{2} - \frac{3\overset{*}{v}\overset{*}{w}}{2} - \frac{\overset{*}{v}\overset{*}{\mu}}{2}\right)$$

$$+ \bar{e}^{\mu}\left(3\overset{**}{w} + 3\overset{*}{w}^2 + \overset{**}{v} + \frac{\overset{*}{v}^2}{2} + \frac{3\overset{*}{w}\overset{*}{v}}{2}\right.$$

$$\left.- \frac{3\overset{*}{\mu}\overset{*}{w}}{2} - \frac{\overset{*}{v}\overset{*}{\mu}}{2}\right) \quad .$$

The nonvanishing components of the Einstein tensor are :

(1.26) $$G_{oo} = \frac{-3\dot{w}^2}{4} - \frac{3\overset{*}{w}\overset{*}{\mu}}{4} - e^{v-w}\left(\frac{3}{2}\overset{**}{w} + \frac{3\overset{*}{w}^2}{2} - \frac{3\overset{*}{\mu}\overset{*}{w}}{4}\right) \quad ,$$

(1.27) $$G_{04} = \frac{3}{2}\dot{w}^{*} + \frac{3}{4}\dot{w}\overset{*}{w} - \frac{3\dot{w}\overset{*}{v}}{4} - \frac{3\overset{*}{w}\dot{\mu}}{4} \quad ,$$

(1.28) $$G_{11} = e^{w-v}\left(\ddot{w} + \frac{3\overset{*}{w}^2}{4} + \frac{\ddot{\mu}}{2} + \frac{\overset{*}{\mu}^2}{4} + \frac{\overset{*}{w}\overset{*}{\mu}}{2} - \frac{\overset{*}{v}\overset{*}{w}}{2} - \frac{\overset{*}{v}\overset{*}{\mu}}{4}\right)$$

$$+ e^{w-u}\left(\overset{**}{w} + \frac{3\overset{*}{w}^2}{4} + \frac{\overset{**}{v}}{2} + \frac{\overset{*}{v}^2}{4} + \frac{\overset{*}{w}\overset{*}{v}}{2} - \frac{\overset{*}{\mu}\overset{*}{w}}{2} - \frac{\overset{*}{v}\overset{*}{\mu}}{4}\right) \quad ,$$

(1.29) $$G_{22} = G_{11} \quad ,$$

(1.30) $$G_{33} = G_{11} \quad ,$$

(1.31) $$G_{44} = -\frac{3\overset{*}{w}^2}{4} - \frac{3\overset{*}{w}\overset{*}{v}}{4} - e^{\mu - v}\left(\frac{3\ddot{w}}{2} + \frac{3\overset{*}{w}^2}{2} - \frac{3\overset{*}{v}\overset{*}{w}}{4}\right) \quad .$$

In the case when the fifth dimension is absent

(1.32) $$e^{\mu} = 0 \quad ,$$

and all (*) derivatives are zero, so by using a cosmic time i.e.

(1.33) $$e^{v} = 1,$$

the five-dimensional G_{ij} reduce to four-dimensional G_{ij} of conventional cosmology. In this situation one obtains

(1.34) $$G_{oo} = -\frac{3\overset{*}{w}^2}{4} \quad ,$$

(1.35) $$G_{11} = G_{22} = G_{33} = e^{w}\left(\ddot{w} + \frac{3}{4}\overset{*}{w}^2\right) \quad .$$

When field equations are taken to be those of Einstein's theory

$$G_{ij} = - T_{ij} \quad , \tag{1.36}$$

and by taking T_{ij} as the energy momentum tensor of perfect fluid with density ρ and pressure p, then

$$8\pi\rho = \frac{3}{4} \overset{*}{w}^2 \quad , \tag{1.37}$$

$$8\pi p = - \frac{3\overset{*}{w}^2}{4} - \ddot{w} \quad . \tag{1.38}$$

These are usual Friedmann equations for the FRW models with zero curvature constant, and for

$$p = 0 \quad , \tag{1.39}$$

one obtains standard Einstein-de Sitter model.

For the case when the fifth dimension is present, the G_{ij} depend in general on both t and m, as do the R_{ij} and R. This means 5D cosmology is inherently more complicated than 4D cosmology. Also, it is by no means obvious what the field equations should be for the 5D theory. Wesson (1983) made a suggestion which involves

(1.40) $$G_{ij} = (R_{ij} - R\, g_{ij}/2) \quad ,$$

but this is not unique, and the field equations might involve some other combination of R_{ij} and R. It is possible to identify the symbol m in the above with any physically-relevant parameter.

1.3 De Sitter-type of model in five-dimensional theory of gravity:

It is argued that for any theory of cosmology to be complete it should be both covariant and scale-invariant. With this view and several other considerations, various modifications of the Einstein's theory, which is only covariant, have beeen proposed where gravitational interaction varies with time as by Wesson (1983). In this context the rest mass-varying theory of gravity was proposed by Wesson deserves serious attention, in which the 4D spacetime of Einstein has been extended to a 5D space-time-mass where the fifth coordinate is $x^4 = Gm/c^2$ with G and C retaining the status of true constants, while mass has been elevated to the status of a separate coordinate like space and time. The logical foundation of the theory is very sound in the sense that the 5D space is Reimannian in geometric properties and it also involves less drastic departures from the well known concepts than do others because just as the existence of c suggests that $x^{o} = ct$ be defined a coordinate, the existence of G suggests $x^{4} = Gm/c^{2}$ be defined as a coordinate.

We have presented exact solutions for a homogeneous, spatially isotropic cosmological models in vacuum both with or without a cosmological constant, because it is in the realm of cosmology that Wesson's theory assumes an added significance.

Let us consider the line-element for a 5D homogeneous and spatially isotropic

$$ds^2 = e^{v}\, dt^2 - e^{w}\,(dx^2 + dy^2 + dz^2) + e^{\mu}\, dm^2 \quad, \tag{1.41}$$

where v, w and μ are functions of time and mass.

Due to the extremely complicated nature of the field equations, let us make a simplifying assumption

$$e^{v} = 1 \quad, \tag{1.42}$$

in obtain explicit solutions for the metric coefficients. In this way one may obtain nonzero components of Einstein tensor

(1.43) $$G_{oo} = \frac{-3\dot{w}^2}{4} \quad \frac{-3\dot{w}\dot{\mu}}{4} - e^{-\mu}\left(\frac{3\overset{**}{w}}{2} + \frac{3\overset{*}{w}^2}{2} - \frac{3\overset{*}{w}\overset{*}{\mu}}{4}\right)$$

$$= -\Lambda \quad ,$$

(1.44) $$G_{11} = G_{22} = G_{33}$$

$$= \left(\ddot{w} + \frac{3w^2}{4} + \frac{\dot{\mu}}{2} + \frac{\dot{\mu}^2}{4} + \frac{\dot{w}\,\dot{\mu}}{2}\right)$$

$$+ e^{-\mu}\left(\overset{**}{w} + \frac{3\overset{*}{w}^2}{4} - \frac{\overset{*}{\mu}\,\overset{*}{w}}{2}\right)$$

$$= \Lambda \quad ,$$

(1.45) $$G_{04} = \frac{3}{2}\,\dot{\overset{*}{w}} + \frac{3}{4}\,\dot{w}\,\overset{*}{w} - \frac{3}{4}\,\overset{*}{w}\,\dot{\mu} = 0 \quad ,$$

(1.46) $$G_{44} = -\,3\overset{*}{w}^2 - e^{\mu}\left(\frac{3}{2}\,\ddot{w} + \frac{3\dot{w}^2}{2}\right)$$

$$= -\Lambda\, e^{\mu} \quad ,$$

where a dot and a star denote partial derivatives with respect to time and mass.

In view of eq. (1.45), one obtains

(1.47) $$\overset{*}{w}^2\, e^{w} = a^{\mu}\, A(m)$$

where A(m) be an arbitrary function of mass only i.e.

(1.48) $$e^{\mu} = \frac{\overset{*}{w}^2 e^w}{A(m)} .$$

Since $\overset{*}{w} \neq 0$, we obtain, using eqs. (1.47) and (1.48)

(1.49) $$\frac{3A}{4} + e^w \left(\frac{3}{2}\ddot{w} + \frac{3\dot{w}^2}{2} - \Lambda\right) = 0$$

Let us put

(1.50) $$e^w = D$$

and we get from eq. (1.49), on first integration

(1.51) $$\dot{D}^2 = \frac{2\Lambda}{3} D^2 - AD + B(m) ,$$

where B as the function of mass only.

Let us discuss the different cases for the eq. (1.51).

<u>Case 1:</u> $\Lambda = 0$

In this case

(1.52) $$\dot{D}^2 = -AD + B$$

or

(1.53) $$e^W = -At^2/4 + \beta t + \gamma \quad ,$$

where β and γ are arbitrary functions of mass only. In order that the remaining two field equations are satisfied one obtains after an extremely tedius but straight forward calculation that

(1.54) $$\beta = \gamma$$

and

(1.55) $$A = -(\beta + C/\beta) \quad ,$$

where C be a new constant of integration.

Case ii: $\Lambda \neq 0$

In this case, we obtain the general solution

(1.56) $$2\left(\frac{2\Delta}{3}\right)^{\frac{1}{2}}\left(\frac{2\Delta}{3}e^{2W} - Ae^W + B\right)^{\frac{1}{2}} + \frac{4\Delta}{3}e^W - A$$

$$= \exp \left[\left(\frac{2\Lambda}{3}\right)^{\frac{1}{2}} (\pm\ t + p) \right]$$

where p as an arbitrary function of mass.

For $\dot{D} = 0$, $\ddot{D} > 0$, hence, one obtains an infinite expansion analogous to the 4D de Sitter model. For the special case

(1.57) $$\mu = w \quad ,$$

we get

(1.58) $$\mu = w = \sqrt{\frac{2\Lambda}{3}}\ t \quad .$$

Hence, the mass-dependence vanishes and the solution is de Sitter like.

1.4 The Anisotropic Fluids Generated via Kaluza-Klein Spaces:

The generating algorithm of Wainwright, Ince and Marshman (1979) may only produce perfect fluid solutions obeying $\epsilon = p$. Ibanez and Verdaguer (1986) investigated a scheme to bypass this limitation, to some extent. The method is based on the paper by Belinskii and Ruffini (1980) in which the authors showed how the

Belinskii-Zakharov generation algorithm may be applied in a 5-dimensional space and used to generate solutions of the Einstein-Maxwell equations. The algorithm was later extended by Diaz, Gleiser, and Pullin (1987), to include an additional scalar field in 5-dimensional space, and its current version is as follows:

1. As presented by Belinskii and Ruffuni (1980), those solutions of the Einstein equations for which the source is of the form

(1.59) $$R_{ik} = \varphi^{-1} \varphi_{;ik} ,$$

(1.60) $$\varphi_{;l} \varphi^{;l} = 0 ,$$

are vacuum solutions in the 5-dimensional Kaluza-Klein theory, with φ being the Kaluza-Klein scalar field. There exist perfect fluid solutions for which R_{ik} has the form (1.59) and (1.60), an example is the flat FRW model with

(1.61) $$\epsilon = 3p ,$$

which is the starting point of the procedure.

2. As stated above, the Belinskii-Zakharov generation algorithm applies also in five-dimensions. Diaz, Gleiser and Pullin (1987) observed that the algorithm of Wainwright, Ince and Marshman (1979) works in 5-dimensions, too, giving solutions of the 5-dimensional Einstein equations with a scalar field source out of vacuum solutions. Applying the two algorithms successively to a 5-dimensional vacuum seed metric, a metric is obtained that obeys:

$$R_{AB} = X_{,A} X_{,B} , \qquad (1.62)$$

where X be the new scalar field and

$$A, B = 0 \ldots\ldots 4 . \qquad (1.63)$$

After reduction to 4-dimensions, the solution of eq. (1.62) will obey the Einstein equations with

$$R_{ik} = \varphi^{-1} \varphi_{;ik} + X_{,i} X_{,k} . \qquad (1.64)$$

3. The energy momentum tensor of any perfect fluid may be represented as a sum of the stiff perfect fluid with

(1.65) $$\epsilon_s = p_s \quad ,$$

and the radiative perfect fluid with

(1.66) $$\epsilon_r = 3p_r \quad ,$$

with

(1.67) $$\epsilon_r = 3\ (\epsilon - p)/2 \quad ,$$

(1.68) $$\epsilon_s = 3p/2 - \epsilon/2 \quad .$$

After performing such a decomposition, a 5-dimensional scalar field solution is obtained corresponding to the initial metric.

4. The vacuum part of the 5-dimensional metric is identified by applying the Wainwright-Ince-Marshman (WIM) algorithm in reverse.

5. The Belinskii-Ruffini generation algorithm is applied to that vacuum part.

6. The WIM algorithm is used to obtain a new scalar field solution from the vacuum solution obtained in step 5. The resulting scalar field is actually the same that was discarded in step 4.

7. The reduction from 5 to 4 dimensions by the usual Kaluza-Klein procedure gives the final solution.

Ibanez and Verdaguer (1986) applied this method in the subcase $X = 0$ to the flat FRW model with $\epsilon = 3p$, and obtained the historically earliest solutions from this family. The solutioins are solitonic perturbations travelling on the flat FRW background. They have anisotropic pressure with three different eigen values p_1, p_2 and p_3, only one of which obeys $p_1 = \epsilon$, in the direction of the soliton's propagation. One limitation of WIM algorithm is thus bypassed, but at the price of making the pressure anisotropic, and the perfect fluid limit of all solutions obtained in this way is only the FRW model with

$$\epsilon = 3p \quad . \tag{1.69}$$

The authors of all the paper described below did not provide a hydrodynamical interpretations of the source. Since all the solutions presented here have a 2-dimensional Abelian symmetry group with spacelike orbits and are orthogonally transitive, they are of extrinsic type not more general than B_3 in the scheme of Wainwright (1979). For determining the intrinsic type or delimiting the extrinsic type more exactly, not enought informatiion is obtained from these papers.

The most elaborate solution of this family was presented by Diaz, Gleiser and Pullin (1988b) as

$$ds^2 = f(t,r)\,(-dt^2+dr^2) + g_{22}dz^2 + g_{33}d\psi^2 \tag{1.70}$$

where

$$f = ct^{n-2}\, r^{-2}\, (\sigma_1-\sigma_2)^{u-2}\, (\sigma_1\,\sigma_2)^{-p+2}\, [(t+\sigma_1 r)\,(t+\sigma_2 r)^{Q} \times \tag{1.71}$$

$$\times\, [(1-\sigma_1^2)\,(1-\sigma_2^2)]^{-u/2}\, (1-\sigma_1\,\sigma_2)^2 \quad ,$$

$$g_{22} = t^n\, (\sigma_1\,\sigma_2)^{-q_r^2} \quad , \tag{1.72}$$

$$g_{33} = t^n\, (\sigma_1\,\sigma_2)^{-p} \quad , \tag{1.73}$$

$$u = 2(q^2 + p^2 + pq) \quad , \tag{1.74}$$

$$Q = -(9/2)\,(3n-2) - (p/2)\,(3n-4)\ . \tag{1.75}$$

The functions σ_1 and σ_2 are defined as

$$\sigma_k^{\pm} = -(2tr)^{-1}\{w_k^2+t^2+r^2 \pm [(w_k^2+t^2+r^2)^2-4t^2r^2]^{1/2}\} \tag{1.76}$$

(1.77) $\sigma_1 = \sigma_1^-$,

(1.78) $\sigma_2 = \sigma_2^{\pm}$,

where w_1 and w_2 are arbitrary constant and C the constant reads

(1.79) $C = (w_2^2 - w_1^2)^2 / w_1^{2Q}$,

and the two scalar fields are :

(1.80) $\varphi = t^{1-n} (\sigma_1 \sigma_2)^{(p+q)/2}$,

(1.81) $\chi = [3n (2-n)/2]^{\frac{1}{2}} \ln t.$

The source of solutions is an anisotropic fluid with energy density ϵ and the eigen values of pressure P_r, P_z and P_φ are

(1.82) $\epsilon = - a^-$,

(1.83) $P_r = a^+$,

$$(1.84) \qquad P_z = \varphi_{;zz}/(\varphi g_{22}) \quad ,$$

$$(1.85) \qquad P_\psi = \varphi_{,\psi\psi}/(\varphi g_{33})$$

where

$$(1.86) \quad a^{\pm} = (2f)^{-1} \quad (\varphi_{;rr} - \varphi_{;tt})/\varphi \pm$$

$$\{[(\varphi_{;rr} + \varphi_{;tt})/\varphi + \chi^2_{,t}]$$

$$- 4\,(\varphi_{;tr}/\varphi)^2\}^{\frac{1}{2}}\,] \quad .$$

The source could become a perfect fluid if

$$(1.87) \qquad P_r = P_z = P_\psi \quad .$$

This may happen in two cases :

$$(1.88) \qquad q = p = 0$$

or

$$(1.89) \qquad w^2{}_1 = w^2{}_2 \quad .$$

In both the cases the solution reduces to a flat FRW model. Hence, this is the only perfect fluid subcase of the solution.

Properties of the travelling solitons are discussed in considerable detail by Diaz, Gleiser and Pullin (1989). A solution of similar kind, describing 4m solitons travelling on a flat FRW background, was obtained by Cruzate, Diaz, Gleiser and Pullin (1988). It is rather complicated, and authors did not comment on its relations to other solutions. Pullin (1990) discussed the Cruzate et al (1988) with

(1.90) $$m = 1 \quad ,$$

and also another one which describes a density wave unaccompanied by a gravitational wave. The Diaz, Gleiser and Pullin (1987) solution is the subcase defined

(1.91) $$p = q = -2/3 \quad .$$

When further

(1.92) $$n = 2 \quad ,$$

i.e. when the second scalar field disappears, the Ibanez-Verdaguer (1986) solution is obtained. Another solution of Ibnez-Verdaguer (1986) results from the above as the limit

(1.93) $$n = 2$$

(1.94) $$p = 4/3, \quad q = -2/3 \quad .$$

In this family, there is one more solution by Diaz, Gleiser and Pullin (1988a) i.e.

(1.95) $$ds^2 = f(t,r)\,(-dt^2 + dr^2) + (\sigma_1\,\sigma_2)^{2(n-1)/3}\, t^n\,(dz^2 + r^2\,d\psi^2)$$

where

(1.96) $$f = C\,t^{n-2}\,r^{-2}(\sigma_1-\sigma_2)^{2/3}\,(\sigma_1\,\sigma_2)^{2(2+n)/3} \cdot [(t+\sigma_1 r)\,(t+\sigma_2\,r)]^2 \times$$
$$\times\,[(1-\sigma_1^2)\,(1-\sigma_2^2)]^{-4/3}\,(1-\sigma_1\,\sigma_2)^{-2}\ ,$$

σ_1 and σ_2 are defined as before, and c, w, and w_2 are arbitrary constants.

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C H A P T E R - II

THICK DOMAIN WALLS IN FIVE DIMENSIONAL

KALUZA-KLEIN SPACETIME

$$$

THICK DOMAIN WALLS IN FIVE DIMENSIONAL KALUZA-KLEIN SPACETIME

$$

2.1 Introduction:

Vilenkin (1985) observed that the phase transitions in the early Universe, due to spontaneous breaking of a discrete symmetry could have generated the topological defects such as domain walls, strings, and monopoles. Hill, Schramm and Fry (1989) have presented that light domain walls of large thickness may have been generated during the late time phase transitions such as those occuring after the decoupling of matter and radiation. The study of thick domain walls and associated spacetimes have attracted due to their application in the structure formation in the Universe. Vilenkin (1981), first obtained that the gravitation field of an infinite thin domain wall with planar symmetry may not be represented by a static metric Widrow (1989) showed that nor could a thick domain wall be described by a regular static metric.

The all above considerations show that non-static metrics are more suitable to describe the field of a thick domain wall. Many workers have described non-static solutions of the Einstein scalar field equations for thick domain walls such as Widrow (1989), Goetz (1990), Mukherjee (1993).

But these solutions presented peculiar behaviour. It is observed that in these solutions the energy scalar is independent of time whereas the metric tensor depends on both space and time. Letelier and Wang (1993) have described exact solutions to the Einstein field equations representing the collision of plane thin walls. Wang (1994) obtained a two parameter family of solutioins of the Einstein field equations describing gravitational collapse of a thick domain wall. Thick domain walls are described by the energy momentum tensor

(2.1) $$T_{ik} = \rho\,(g_{ik} + w_i\, w_k) + p\, w_i\, w_k$$

(2.2) $$w_i\, w^i = -1 \;,$$

where ρ be the energy density of wall, p as the pressure in the direction normal to the plane of the wall and w_i be a unit spacelike vector in the same direction. In the case of Wang solution, the energy scalar and the metric tensor both are functions of space and time coordinates.

There are two methods to study the thick domain walls. In first method one studies the field equations as well as the equations of domain wall as the self-interacting scalar field. In the second method one uses the energy momentum tensor in the form (2.1) - (2.2) and then field equations are solved.

In this work we have adopted the second method which seems easier and applied it to a more general five dimensional Kaluza-Klein spacetime. The researches in supergravity in 11-dimensions and superstring in 10-dimensions show that higher dimensionality of space is apparently a good reflection of the dynamics of interactions over the distances where all forces unify. Chodos-Detweiler (1980) were first presented the implications of higher dimensions and they have discussed Kasner-type vacuum solutions in a five dimensional spacetime. Their solutioins show the feature of dimensional reduction. Banerjee and Das (1998) have obtained thick domain walls in higher dimensions. So, it is worthwhile to investigate the time dependent thick domain walls in a five dimensional Kaluza-Klein spacetime. Here we have investigated some other new exact solutions of the Einstein field equations describing gravitational field of thick domain walls in five dimensional spacetime.

2.2 The Field Equations:

Let us consider the general five-dimensional plane symmetric metric with ione additional Killing vector

$$ds^2 = A^2\,(dt^2 - dx^2) - D^2(dy^2 + dz^2) - E^2 d\psi^2 \tag{2.3}$$

where

$$A = A\,(x,t)\quad, \tag{2.4}$$

$$D = D\,(x,t)\quad, \tag{2.5}$$

$$E = E\,(x,t)\quad, \tag{2.6}$$

and ψ be the fifth coordinate.

Let us introduce the pentad

$$\theta^{o} = A\, dt \quad , \tag{2.7}$$

$$\theta^{1} = A\, dx \quad , \tag{2.8}$$

$$\theta^{2} = D\, dy \quad , \tag{2.9}$$

$$\theta^{3} = D\, dz \quad , \tag{2.10}$$

$$\theta^{4} = E\, d\psi \quad . \tag{2.11}$$

Hence, the metric assumes the form

$$ds^2 = (\theta^{o})^2 - (\theta^{1})^2 - (\theta^{2})^2 - (\theta^{3})^2 - (\theta^{4})^2 . \tag{2.12}$$

One may easily obtain the nonvanishing components of Ricci tensor R_{ab}.

$$A^2 R_{oo} = \frac{\ddot{A}}{A} + \frac{2\ddot{D}}{D} + \frac{\ddot{E}}{E} - \frac{\dot{A}}{A}\left(\frac{\dot{A}}{A} + \frac{2\dot{D}}{D} + \frac{\dot{E}}{E}\right) - \left[\frac{A''}{A} + \frac{A'}{A}\left(\frac{2D'}{D} + \frac{E'}{E} - \frac{A'}{A}\right)\right] \quad , \tag{2.13}$$

$$A^2 R_{11} = \frac{A''}{A} + \frac{2D''}{D} + \frac{E''}{E} - \frac{A'}{A}\left(\frac{A'}{A} + \frac{2D'}{D} + \frac{E'}{E}\right) - \left[\frac{\ddot{A}}{A} + \frac{\dot{A}}{A}\left(\frac{2\dot{D}}{D} + \frac{\dot{E}}{E} - \frac{\dot{A}}{A}\right)\right] \; , \tag{2.14}$$

(2.15) $$A^2R_{22} = A^2R_{33}$$

$$= \frac{D''}{D} + \frac{D'}{D}\left(\frac{D'}{D} + \frac{E'}{E}\right) - \left[\frac{\ddot{D}}{D} + \frac{\dot{D}}{D}\left(\frac{\dot{D}}{D} + \frac{\dot{E}}{E}\right)\right]$$

(2.16) $$A^2\, R_{44} = \frac{E''}{E} + \frac{2E'D'}{ED} - \left(\frac{\ddot{E}}{E} + \frac{2\dot{D}\dot{E}}{DE}\right) ,$$

(2.17) $$A^2\, R_{01} = \frac{2\dot{D}'}{D} + \frac{\dot{E}'}{E} - \frac{\dot{A}}{A}\left(\frac{2D'}{D} + \frac{E'}{E}\right)$$

$$- \frac{A'}{A}\left(\frac{2\dot{D}}{D} + \frac{\dot{E}}{E}\right) ,$$

where prime denotes derivatives with respect to x and dot for derivatives with respect to t.

Now the Einstein field equations are

(2.18) $$R_{ik} = - 8\ [T_{ik} - \frac{1}{3}\, Tg_{ik}\ ] .$$

This domain walls are characterised by the energy momentum tensor

(2.19) $$T_{ik} = \rho\,(g_{ik} + w_i w_k) + p\, w_i\, w_k ,$$

(2.20) $$w_i \, w^i = -1 \quad ,$$

For the thick domain wall the energy stress components in the comoving coordinates are

(2.21) $$T^o_o = T^2_2 = T^3_3 = \rho \quad ,$$

(2.22) $$T^1_1 = -p \quad ,$$

(2.23) $$T^0_1 = 0 \quad ,$$

(2.24) $$T^4_4 = 0 \quad ,$$

where ρ denotes the energy density of wall which is also equal to the tension along y and z directions in the plane of the wall, p reads the pressure along x-direction. T^4_4 being the stress component corresponding to extra dimension, which is taken as zero. In view of the eqs. (2.21) - (2.24), the equations (2.18) read

(2.25) $$R_{01} = 0 \quad ,$$

(2.26) $$R_{22} = -R_{00} \quad ,$$

$$(2.27) \qquad 3R_{22} + R_{11} - R_{44} = 0 ,$$

$$(2.28) \qquad 8\pi p = 3R_{22} ,$$

$$(2.29) \qquad 8\pi \rho = -(R_{11} + 2R_{22}) .$$

2.3 The Solutions:

It is quite difficult to obtain the general solutions of the above system of equations. So, one has to make the following separability assumptions for the metric potentials

$$(2.30) \qquad A = \text{Cosh}^a (mx)\, e^{\alpha kt} ,$$

$$(2.31) \qquad D = \text{Cosh}^b (mx)\, e^{\beta kt} ,$$

$$(2.32) \qquad E = \text{Cosh}^d (mx)\, e^{\gamma kt} ,$$

where a, b, d, α, β, γ, m and k are real constants.

In view of the above relations eq. (2.25) gives

$$(2.33) \qquad 2\beta(b-a) - \alpha(d+2b) + \gamma(d-a) = 0 .$$

The eq. (2.26) leads to

$$(2.34)\quad (a-b)\, m^2 \,[\, (2b + d-1)\, \mathrm{sech}^2 (mx) - (2b+d) \,]$$

$$+ k^2 \,[\, \gamma^2 - \gamma (\alpha + \beta) - 2\alpha\beta \,] = 0.$$

The eq. (2.27) gives on simplification

$$(2.35)\quad m^2 \,[8b^2-2ab-ad + bd] + m^2 \,\mathrm{sech}^2 (mx)$$

$$[a+5b - 8b^2+ 2ab + ad-bd \,]$$

$$= k^2 (2\beta + \gamma)(\alpha + 3\beta - \gamma) \quad .$$

The eq. (2.28) gives the physical parameter p

$$(2.36)\quad \frac{8\pi}{3} pA^2 = bm^2 \,[(2b+d) + (1-2b-d)\, \mathrm{sech}^2 (mx)]$$

$$- \beta (2\beta + \gamma) k^2 \quad ,$$

and the eq. (2.29) also reads the physical parameter

$$(2.37)\quad -8\pi\rho A^2 = m^2 (6b^2+d^2-2ab + bd-ad)$$

$$+ m^2 \text{sech}^2 (mx) [-6b^2-d^2+2ab$$

$$+ ad-2bd + a+d+4b]$$

$$- (\alpha + 2\beta)(2\beta + \gamma) k^2 .$$

Now from eq. (2.33), we obtain

$$(2.38)\quad \gamma(d-a) = \alpha(d+2b)-2\beta (b-a)$$

It is obvious from eq. (2.34) that

$$(2.39)\quad (a-b)(2b+d-1) = 0 ,$$

which will imply either

$$(2.40)\quad a = b$$

or

$$(2.41)\quad 2b + d = 1 .$$

Hence, correspondingly we have the following two cases.

Case I: $a = b$

In view of the above, one obtains from equations (2.33) - (2.35)

(2.42) $$a = b = 1 ,$$

(2.43) $$\beta^2 = \frac{m^2}{k^2} ,$$

(2.44) $$\alpha = \frac{d(d-1)}{(d+2)} \beta ,$$

(2.45) $$\gamma = \beta d ,$$

where d, β, and k are arbitrary parameters.

In view of the above, the pressure p and the energy density read

(2.46) $$8\pi p = - 3m^2 (d+1) \operatorname{sech}^4 (mx) e^{-2\alpha kt} ,$$

(2.47) $$8\pi\rho = m^2 (d^2-2) \operatorname{sech}^4 (mx) e^{-2\alpha kt} .$$

Hence,

(2.48) $$\rho = - \frac{(d^2-2)}{3(d+1)} p .$$

Hence, we have obtained a three parameter family of solutions representing thick domain walls. For

$$\beta = -\tfrac{1}{2} = \sqrt{m/k} \quad , \tag{2.49}$$

it reduces to the two parameter family of solutions as presented by Banerjee and Das (1998).

<u>Case II:</u>

$$2b + d = 1 \tag{2.50}$$

or

$$2b = 1-d \tag{2.51}$$

In case we obtain

$$d = 1-2b \quad , \tag{2.52}$$

$$a = b\,(3b-2) \quad , \tag{2.53}$$

$$\alpha = 6b\,(1-b)\,\beta + (1-3b^2)\,\gamma \quad , \tag{2.54}$$

$$-3b(1-b)\,m^2 = k^2\,(\gamma^2 - 2\alpha\beta - \alpha\gamma - \beta\gamma), \tag{2.55}$$

(2.56) $$-3b(1+b)\, m^2 = k^2\, (\gamma - \alpha - 3\beta)\,(2\beta + \gamma)\,.$$

In view of eqs. (2.55) - (2.56), we get

(2.57) $$\frac{m^2}{k^2} = -\,\frac{(\gamma^2 - 2\alpha\beta - \alpha\gamma - \beta\gamma)}{3b(1-b)}$$

$$= -\,\frac{(\gamma - \alpha - 3\beta)\,(2\beta + \gamma)}{3b(1+b)}\,,$$

which gives

(2.58) $$(b-1)\,(1-4b^2)\,\frac{\beta^2}{\gamma^2} + b\,(1+2b-4b^2)\,\frac{\beta}{\gamma} - b^2 = 0.$$

The eq. (2.58) gives the following two roots

(2.59) $$\frac{\beta}{\gamma} = \frac{b}{1-2b},\quad \frac{b^2}{(1-b)\,(2b+1)}\,.$$

In view of the above two roots, we have two separate cases.

<u>Case II (i)</u>

In this case, one obtains

(2.60) $$\alpha = \frac{(1-2b + 3b^2)\gamma}{(1-2b)} ,$$

(2.61) $$d = 1 - 2b ,$$

(2.62) $$\beta = \frac{b\gamma}{1-2b} ,$$

(2.63) $$a = b(3b-2) ,$$

(2.64) $$\frac{m^2}{k^2} = \frac{(1-b+4b^2)}{(1+b)\ (1-2b)}\gamma^2 .$$

In view of the above, one obtains the pressure and density as

(2.65) $$8\pi p = \frac{6k^2b^2\ \gamma^2(3b-4b^2-2)}{(1-2b)^2\ (1+b)} \operatorname{sech}^{2a}(mx)e^{-2k\alpha t},$$

(2.66) $$8\pi\rho = \frac{b\ \gamma^2k^2\ (1+3b^2)(8b^2-12b+7)}{(1-2b)^2\ (1+b)} \operatorname{sech}^{2a}(mx)e^{-2k\alpha t},$$

which gives

(2.67) $$\rho/p = \frac{(1+3b^2)\ (8b^2-12b+7)}{6b\ (3b-4b^2 - 2)} ,$$

and it is a constant.

Case II (ii):

In this case one obtains

(2.68) $$\alpha = \frac{(1-b)\ (1+3b)}{(1+2b)} ,$$

(2.69) $$\beta = \frac{-\ b^2 \gamma}{(b-1)\ (2b+1)},$$

(2.70) $$d = (1-2b) ,$$

(2.71) $$a = b(3b-2) ,$$

(2.72) $$\frac{m^2}{k^2} = \frac{(b(2b^2-b-2)}{(b-1)^2\ (2b+1)} \gamma^2 .$$

In this case the pressure and density read

(2.73) $$8\pi p = \frac{3b^2k^2\gamma^2(2b^2-2b-3)}{(b-1)^2\ (2b+1)^2} \operatorname{sech}^{2a}(mx)e^{-2\alpha kt} ,$$

(2.74) $$8\pi\rho = \gamma^2k^2 \frac{(-db^5+6b^4+4b^3-b^2+4b+1)}{(b-1)^2\ (2b+1)^2} \operatorname{sech}^{2a}(mx)e^{-2\alpha kt} .$$

Again the ratio of /p is a constant

$$(2.75) \qquad \rho/p = \frac{(-6b^5+6b^4+4b^3-b^2+4b+1)}{3b^2(\ 2b^2 - 2b - 3)} \ .$$

Hence, we have obtained two and three parameter families of solutions for domain walls, the parameters are b, γ , and k.

2.4 Concluding Remarks:

It is obvious from the solutions of Case 1 and Case II spacetimes that they are reflection symmetric with respect to the wall. For a thick domain wall it is necessary that pressure and density decrease on both sides of wall away from the symmetry plane and fall off to zero as $x \longrightarrow \pm \infty$.

The solutions of the Case 1 require that $\rho > 0$, $p > 0$ and $(\rho - p) > 0$ and may be satisfied by choosing the parameter d such that $d < -(3+\sqrt{5})/2$. For $d = -(3+\sqrt{5})/2$ then $\rho = p$. It is clkear that ρ , p fall off to zero on either side of the wall.

Again for the solutioins of Case II (i) and Case II (ii) the fall of condition require $a > 0$. For the case II (i) this condition would conflict with $\rho > 0$, $m^2/k^2 \geq 0$. Hence, this family of solutions is not physically reliable. However, it is interesting to note that when $b = 0$, ρ and p vanish resulting into an empty spacetime given as

$$(2.76) \qquad ds^2 = e^{2nt}(dt^2 - dx^2) - dy^2 - dz^2 - e^{2nt}\cosh^2(nx)\, d\psi^2$$

where

$$(2.77) \qquad n = k\gamma \quad .$$

For the case

$$(2.78) \qquad b < \frac{1}{4}(1-\sqrt{17})$$

i.e.

$$2b^2 - b - 2 \leq 0 \quad ,$$

the solutions of Case II (ii) would satisfy the condition

$$(2.79) \qquad a > 0 \quad ,$$

(2.80) $\rho > 0$,

(2.81) $m^2/k^2 > 0$.

It would have proper fall condition as well as

(2.82) $\rho - p \geq 0$.

Let us now discuss the dynamical behaviour of our models under different conditions imposed on various parameters occuring in the solutioins. One may obtain the general relation for the three space volume as

(2.83) $|g_3|^{\frac{1}{2}} = \text{Cosh}^{a+2b}(mx)\, e^{kt(\alpha+2\beta)}$.

Hence, the temporal behaviour reads

(2.84) $|g_3|^{\frac{1}{2}} \sim \exp\,[kt\,(\alpha+2\beta)]$.

Here, it is an important to note that when

(2.85) $\beta = -\frac{1}{2}$,

we recover in Case I, the Banerjee and Das solution (1998). Hence, we have taken for the Case I, β to be negative. For

(2.86) $$d < -(3+\sqrt{5})/2 \quad ,$$

one obtains

(2.87) $$\alpha + 2\beta < 0 \; .$$

Again if

(2.88) $$k > 0 \quad ,$$

three space collapses while the extra dimension inflates. In this case, one obtains singularity because as $t \longrightarrow \infty$, ρ and p diverge. If $k < 0$, the 3-space inflates while extra dimension collapses in course of time. One may also discuss the dynamical behaviour of the domain wall solutions corresponding to the Case II (ii) on the similar lines.

Let us now disucss the attractive and repulsive behaviour of thick domain wall by either through the timelike geodesics in the spacetime or studying the acceleration of an observer who is taken at rest relative to the wall as presented by Wang (1992). Let us consider an observer with the four velocity

$$u_i = \cosh^{\alpha} (mx)\, e^{\alpha kt}\, \delta_i^t \quad . \tag{2.89}$$

Hence, one may obtain an acceleration A^i as

$$A^i = u^i{}_{;k}\, u^k \tag{2.90}$$

$$= ma \tanh (mx) \cosh^{-2\alpha} (mx)\, e^{-2\alpha kt}\, \delta_x^i \quad .$$

For the case I,

$$a = 1 \quad , \tag{2.91}$$

and

if

$$m > 0 \quad , \tag{2.92}$$

then A^x is positive. It shows that the observer is comoving with the wall having to accelerate away from the symmetry plane or it is attracted towards the wall. Similarly if

(2.93) $m < 0$,

then the wall represents a repulsive behaviour to the observer. Similar results may be obtained for the domain wall solutions of Case II (ii). Let us assume

$$T^0_0 = T^2_2 = T^3_3 = T^4_4 = \rho \quad , \tag{2.94}$$

$$T^1_1 = - p \quad , \tag{2.95}$$

$$T^0_1 = 0 \quad , \tag{2.96}$$

one obtains a domain wall solution for

$$\rho = p \quad . \tag{2.97}$$

But this solution be the same as obtained by Banerjee and Das (1998). If one assumes e^{mx^2} in place of Cosh (mx) in the separability condition, there may not be any domain wall solution. But it gives five-dimensional empty spacetime as

$$ds^2 = e^{2nt+n^2x^2} (dt^2-dx^2) - e^{2nt/\sqrt{6}} (dy^2 + dz^2) - e^{4nt/\sqrt{6}} d\psi^2 \quad , \tag{2.98}$$

where n be the arbitrary constant. Hence, it represents an inhomogeneous vacuum spacetime.

Hence, we have obtained three families of exact solutions of Einstein field equations containing three parameters.

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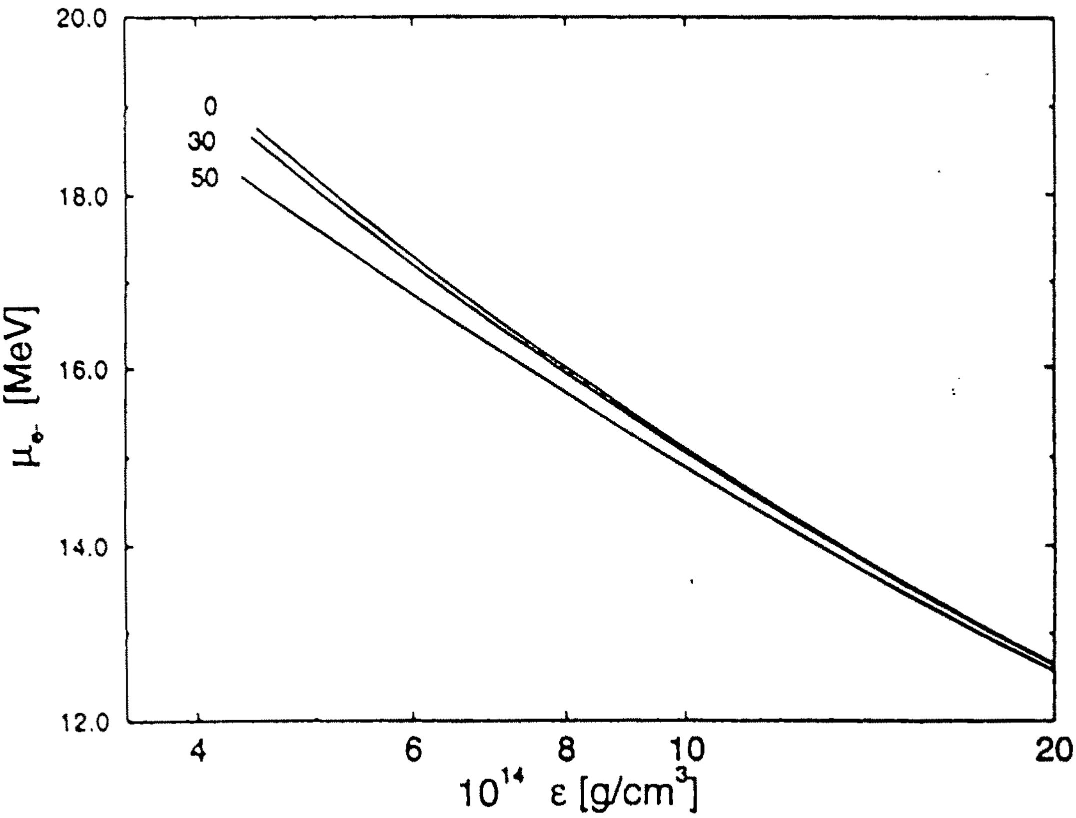
20.0
18.0
16.0
14.0
12.0
0
30
50
μ_{e^-} [MeV]
4
6
8
10
20
10^{14} ε [g/cm^3]

C H A P T E R - III

CYLINDRICALLY SYMMETRIC METRIC IN KALUZA-KLEIN SPACETIME

CYLINDRICALLY SYMMETRIC METRIC IN KALUZA-KLEIN SPACETIME

3.1 Introduction:

Several efforts have been made to obtain solutions of Einstein's equatioin in higher dimensions in the context of the early universe both from cosmological considerations as well as particle physics points of view. It is discussed that the extra dimensions are not observable at the present time, due to their size being assumed of the order of the Planck length, perhaps they may be relevant for the very early Universe as presented by Appelquist (1987). It is expected that as t increases extra dimensions shrink rapidly to leave us with 4-dimensional Universe. Chodos and Detweiler (1980) presented the 5-dimensional Kasner vacuum solutions in which the extra dimension shrank while the usual 3-space expanded with time t. Chatterjee et al (1993), Chatterjee et al (1994), Banerjee et al (1994) have studied 5-dimensional spacetimes in the context of inhomogeneous cosmologies. Inhomogeneous cosmologies are important for several reasons, basically to have general generic initial conditions and to facilitate formation of large scale structures in the Universe. Sahdev (1984), Ishiibara (1984), Chatterjee and Bhui (1990), presented several Kaluza-Klein extensions of the Friedman-Robertson-Walker (FRW) models in higher dimensions, but they are all big bang singular.

They important property of inhomogeneous spacetime is that they permit nonsingular family of models satisfying the strong energy condition

$$\rho + 3p \geq 0 , \tag{3.1}$$

with equation of state

$$\rho = p , \tag{3.2}$$

as presented by Ruiz and Senovilla (1992), Dadhich, Patel and Tikekar (1995). Banerjee, Das and Panigrahi

(1995) obtained 5-dimensional generalisation of nonsingular family in Kaluza-Klein (KK) spacetime. There has been attempt by Dadhich (1995) to study the nonsingular models in general diagonal metric which is separable in space and time in the comoving coordinates. Mars (1995) investigated a non-diagonal singularity free model with equation of state $\rho = p$. The metric reads

$$(3.3) \qquad ds^2 = e^{ca^2r^2} \text{Cosh}\,(2at)\,(dt^2-dr^2) - r^2 \text{Cosh}\,(2at)\,d\theta^2 - \text{Cosh}^{-1}\,(2at)\,(dz + ar^2\,d\theta)^2 ,$$

where a and c are constants.

3.2 The Metric and Field Equations:

Let us consider the metric in non diagonal form (3.3) keeping time dependence free to be evaluated. The 5-dimensional analogue of the Mars (1995) metric reads

$$(3.4) \qquad ds^2 = T^{2\alpha}\, e^{2br^2}\,(dt^2-dr^2) - r^2T^{2\beta}\, d\varphi^2 - T^{2\beta}\,(dz+ar^2d\varphi)^2 - T^{2\delta}\, d\psi^2 ,$$

where T(t) and all other parameters are consand and ψ be the Kaluza-Klein parameter with

$$(3.5) \qquad 0 \leq \psi \leq 2\pi R_5$$

where R_5 be the radius of Kaluza-Klein circle. The metric is globally regular for the whole range of the other four coordinates i.e.

$$(3.6) \qquad 0 \leq r < \infty \quad ,$$

$$(3.7) \qquad -\infty < t \quad ,$$

$$(3.8) \qquad z < \infty \quad ,$$

$$(3.9) \qquad 0 \leq \varphi \leq 2\pi \quad .$$

Let us now introduce the orthonormal tetrad

$$(3.10) \qquad \theta^1 = T^{\alpha} e^{br^2} dr \quad ,$$

$$(3.11) \qquad \theta^2 = T^{\beta} (dz + ar^2 d\varphi)$$

$$(3.12) \qquad \theta^3 = rT^{\gamma} d\varphi \quad ,$$

(3.13) $$\theta^4 = T^{\delta} \; d\psi \quad ,$$

(3.14) $$\theta^5 = T^{\alpha} \; e^{br^2} \; dt \quad ,$$

which gives

(3.15) $$g_{ik} = (-1, -1-, -1, -1, 1) \quad ,$$

and hence forth all the quantities are taken to this frame.

In the orthonormal tetrad frame the nonvanishing R_{ik} read

(3.16) $$B^2 R_{15} = - \mu \; [2br \; (\beta + \gamma + \delta) + \frac{\alpha - \gamma}{r} \;]$$

(3.17) $$B^2 R_{11} = - \alpha \dot{\mu} - \alpha (\beta + \gamma + \delta) \; \mu^2 + 2a^2 T^{2(\beta - \gamma)} \quad ,$$

(3.18) $$B^2 R_{22} = - \beta \dot{\mu} - \beta (\beta + \gamma + \delta) \; \mu^2 - 2a^2 T^{2(\beta - \gamma)} \quad ,$$

(3.19) $$B^2 R_{33} = - \gamma \dot{\mu} - \gamma (\beta + \gamma + \delta)$$

$$\mu^2 + 2a^2 T^{2(\beta - \gamma)} \quad ,$$

$$(3.20) \qquad B^2 R_{44} = -\delta\,[\dot{\mu} + (\beta + \gamma + \delta)\,\mu^2] \quad ,$$

$$(3.21) \qquad B^2 R_{55} = -4b + (\alpha + \beta\gamma + \delta)\,\dot{\mu}$$

$$+ [\beta(\beta-\alpha) + \gamma(\gamma-\alpha) + \delta(\delta-\alpha)]\,\mu^2 \quad ,$$

where

$$(3.22) \qquad B^2 = T^{2\alpha}\, e^{2br^2}$$

$$(3.23) \qquad \mu = \dot{T}/T \quad ,$$

$$(3.24) \qquad \dot{T} = dT/dt \quad .$$

Let us consider the matter distribution as the perfect fluid with energy momentum tensor

$$(3.25) \qquad T_{ik} = (\rho + p)\, u_i\, u_k - p\, g_{ik} \quad .$$

Hence, the Einstein equations read

$$(3.26) \qquad R_{ik} = -8\pi\, [\,(\rho + p)\, u_i\, u_k - \tfrac{1}{3}(\rho - p)\, g_{ik}].$$

Let us consider the comoving coordinates to obtain

$$u_i = (0, 0, 0, 0, 1) \quad . \tag{3.27}$$

In view of eq. (3.27), one obtains from equation (3.26)

$$R_{15} = 0 \quad , \tag{3.28}$$

$$R_{11} = R_{22} = R_{33} = R_{44} \quad , \tag{3.29}$$

$$8\pi\rho = -\frac{1}{2}(R_{55} + 4R_{22}) \quad , \tag{3.30}$$

$$8\pi p = \frac{1}{2}(-R_{55} + 2R_{22}) \quad . \tag{3.31}$$

In view of the field equations and eqs. (3.28) - (3.31) it is obvious that for $b \neq 0$ only $\rho = p$, stiff fluid is allowed, showing that inhomogeneous perfect fluid with equation of state different from $\rho = p$ may not be sustained by the metric (3.4). However, for $b = 0$ i.e. spacetime is homogeneous, perfect fluid $\rho = p$ as well as for $\rho \neq p$ is admitted. The situation $\rho = p$ implies that

$$R_{22} = 0 \quad . \tag{3.32}$$

One may now obtain from field equations

$$(3.33) \qquad \alpha = \gamma \quad ,$$

$$(3.34) \qquad b = 0 \quad \text{or} \quad \gamma + \beta + \delta = 0,$$

$$(3.35) \qquad \alpha \dot{\mu} + \alpha(\beta + \gamma + \delta)\mu^2 - 2a^2T^{2(\beta-\gamma)} = 0$$

$$(3.36) \qquad \beta \dot{\mu} + \beta(\beta + \gamma + \delta)\mu^2 + 2a^2 T^{2(\beta-\gamma)} = 0$$

$$(3.37) \qquad \delta[\dot{\mu} + (\beta + \gamma + \delta)\mu^2] = 0$$

Again in view of

$$(3.38) \qquad \alpha = \gamma \quad ,$$

$$(3.39) \qquad R_{11} = R_{33} \quad .$$

Now let us consider the two cases $b \neq 0$ and $b = 0$ separately and they will give inhomogeneous and homogeneous solutions respectively. The expansion scalar θ associated with four velocity u_i reads

(3.40) $$B\theta = \mu \ (\alpha + \beta + \gamma + \delta) ,$$

(3.41) $$B = T^{\alpha} \ e^{br^2}$$

3.3 Inhomogeneous Solutions:

Let us consider $b \neq 0$, then

(3.42) $$\gamma + \beta + \delta = 0 ,$$

from eq. (3.34) .

The other eqs. (3.35) - (3.37) lead to

(3.43) $$\alpha \dot{\mu} = 2a^2 \ T^2 \ (\beta - \gamma)$$

(3.44) $$\beta \dot{\mu} = -2a^2 \ T^2 \ (\beta - \gamma) ,$$

(3.45) $$\delta \dot{\mu} = 0 ,$$

(3.46) $$(\alpha + \beta) \dot{\mu} = 0 .$$

In view of the above we have two following cases:

(3.47) (i) $\alpha = -\beta = \frac{1}{2}$, $\delta = 0$ and $T = \text{Cosh } 2at$, (3.48) (ii) $\dot{\mu} = 0, \quad a = 0$ and $T = e^{kt}$

<u>Case (i):</u> In this case the density and pressure read

$$(3.49) \qquad 8\pi\rho = 8\pi p = (2b-a^2)e^{-abr^2} \operatorname{Cosh}^{-1}(2at) \quad .$$

being positive for

$$(3.50) \qquad 2b \geq a^2 \quad ,$$

and one obtains the 5-dimensional singularity free non-diagonal spacetime.

The metric reads

$$(3.51) \quad ds^2 = e^{2br^2} \operatorname{Cosh}(2at)\,(dt^2-dr^2) - r^2 \operatorname{Cosh}(2at)\,d\varphi^2$$

$$- \operatorname{Cosh}^{-1}(2at)\,(dz+ar^2 d\varphi)^2 - d\psi^2 \quad .$$

It is 5-dimensional generalisation of the 4-dimensional Mars (1995) stiff fluid solutions (3.3) for ψ = constant. The behaviour of the model is typical than non singular models of Ruiz and Senovilla (1992) and Dadhich et al (1995). It shows

$$(3.52) \qquad \rho \longrightarrow 0 \quad \text{as} \quad t \longrightarrow \pm\infty$$

or

$$(3.53) \qquad r \longrightarrow \infty ,$$

as t increases ρ increases with the contraction, at $t = 0$ contraction turns to expansion and $\rho \longrightarrow 0$ as $t \longrightarrow \infty$.

Again for $a = 0$, the metric becomes diagonal and static describing a static stiff fluid solution with ρ being maximum at $r = 0$. IOt is 5-dimensional analogue of the 4-dimensional stiff fluid solution of Bronikov (1979) and Raychaudhuri (1955) and the metric reads

$$(3.54) \quad ds^2 = e^{2br^2}(dt^2-dr^2) - r^2 d\phi^2 - dz^2 - d\psi^2 .$$

<u>Case (ii):</u> For $a - 0$, $T = e^{kt}$.

In such case one obtains

$$(3.55) \quad 8\pi\rho = 8\pi p = \tfrac{1}{2}\,[4b-k^2(\alpha^2+\delta^2+\alpha\delta)]e^{-2\alpha kt}e^{-2br^2}$$

In order that $\rho \geq 0$, one has

$$(3.56) \qquad 4b \geq k^2 (\alpha^2 + \delta^2 + \alpha\delta) \geq 0 .$$

In this case the metric reads

$$(3.57) \qquad ds^2 = e^{2br^2} e^{2\alpha kt} (dt^2 - dr^2) - r^2 e^{2\alpha kt} d\varphi^2$$

$$- e^{-2(\alpha+\delta)kt} dz^2 - e^{2\delta kt} d\psi^2 .$$

We again obtain 4 parameters, k, b, α and δ free. If one selects $k\delta < 0$, ψ-dimension will reduce exponentially leaving 4-dimensional spacetime.

We obtain a very interesting case $\alpha = 0$, which gives pressure and density as

$$(3.58) \qquad 8\pi\rho = 8\pi p = \tfrac{1}{2} (4b - k^2 \delta^2) e^{-2br^2} ,$$

and metric (3.57) reduces to

$$(3.59) \quad ds^2 = e^{2br^2} (dt^2 - dr^2) - r^2 d\varphi^2 - e^{-2\delta kt} dz^2$$

$$- e^{2\delta kt} d\psi^2 .$$

It is observed that though the metric time dependent yet ρ is independent of time. It is the non-static generalisation of the metric (3.54), resulting for

$$\delta = 0 \quad . \tag{3.60}$$

If we put

$$4b = k^2 \delta^2 \quad . \tag{3.61}$$

then we get the matter-free limit of (3.59) as 5-dimensional empty space and the metric is given by

$$ds^2 = e^{\frac{1}{2}\lambda^2 r^2}(dt^2 - dr^2) - r^2 d\varphi^2 - e^{-2\lambda t}dz^2 - e^{2\lambda t}d\psi^2 \quad , \tag{3.62}$$

where

$$\lambda = \delta k \quad . \tag{3.63}$$

It is an inhomogeneous and anisotropic vacuum spacetime which is everywhere regular. The dimensional reduction is possible for $\lambda < 0$, $\lambda = 0$ implies that

$$ds^2 = (dt^2 - dr^2) - r^2 d\varphi^2 - dz^2 - d\psi^2 \tag{3.64}$$

which 5-dimensional flat spacetime.

3.4 **Homogeneous Stiff-fluid Solutions:**

In this case

(3.65) $$\alpha = \gamma$$

and

(3.66) $$b = 0 \; .$$

In view of the above one obtains

(3.67) $$\delta = 0 \quad ,$$

(3.68) $$\dot{\mu} \quad (\alpha + \beta + \delta) \mu^2 = 0.$$

Hence, we get two cases:

Case (i) $\delta = 0$. For this case one obtains

(3.69) $$(\alpha + \beta) \dot{\mu} + (\alpha + \beta)^2 \mu^2 = 0.$$

If $(\alpha + \beta) \neq 0$ showing that a = 0 which makes the metric diagonal. As $\mu = \dot{T}/T$, so by solving eq. (3.69), we get

(3.70) $$T = t^{1/\alpha+\beta}$$

and the corresponding density and pressure read

(3.71) $$8\pi\rho = 8\pi p = \frac{2\alpha}{\alpha+\beta} t^{-2(2\alpha+\beta)} ,$$

and the metric is given by

(3.72) $$ds^2 = t^{2\alpha/\alpha+\beta}(dt^2-dr^2) - r^2 + t^{2\alpha/\alpha+\beta} d\phi^2$$

$$- t^{2\beta/\alpha+\beta}(dz-ar^2d\phi)^2 - d\psi^2 .$$

Again for $\alpha = 0$, one obtains 5-dimensional non-diagonal vacuum metric

(3.73) $$ds^2 = (dt^2-dr^2) - r^2d\phi^2 - t^2(dz-ar^2d\phi)^2$$

$$- d\psi^2 .$$

The metric (3.73) has big-bang singularity at $t = 0$. On the otherhand if

(3.74) $$\alpha + \beta = 0 ,$$

then

(3.75) $$T = \cosh^{\frac{1}{2}\alpha}(2at) ,$$

but

(3.76) $$\rho \sim - a^2 ,$$

Hence, it is ruled out.

Case (ii) For $\delta \neq 0$, we get two following subcases:

Subcase (i) $\alpha + \beta \neq 0$,

(3.77) $$T^{\alpha + \beta + \gamma} = t ,$$

$$a = 0$$

In this case the metric reads

(3.78) $$ds^2 = t^{2\alpha/n}(dt^2-dr^2-r^2d\varphi^2) - t^{2\beta/n}\, dz^2 - t^{2\delta/n}\, d\psi^2$$

where

(3.79) $$n = \alpha + \beta + \delta .$$

The pressure and density are given by

$$(3.80) \qquad 8\pi p = 8\pi\rho = \frac{\alpha n+2\alpha(n-\alpha)+2\beta(n-\alpha-\beta)}{n^2t^2\,(1+\alpha/n)} .$$

In this case it is possible to select α, β, δ such that $\rho \geq 0$. For $\delta = 0$, the metric (3.78) reduces to (3.72). It allows coordinate reduction for $\delta/n < 0$. The vacuum spacetime is obtained for

$$(3.81) \qquad \alpha(\alpha+3\beta+3\delta)+2\beta\delta = 0.$$

For

$$(3.82) \qquad \alpha+\beta = 0 ,$$

we get

$$(3.83) \qquad a = 0$$

and

$$(3.84) \qquad T = t^{1/\delta} ,$$

so the metric

$$(3.85) \qquad ds^2 = t^{2\alpha/\delta}(dt^2-dr^2-r^2d\varphi^2) - t^{-\frac{2\alpha}{\delta}}dz^2 - t^2\,d\psi^2 ,$$

and the corresponding pressure and density read

$$(3.86) \qquad 8\pi\rho = 8\pi p = \frac{\alpha(\alpha+\delta)}{\delta^2 t^{2(1+\alpha)}} \ .$$

Again for $\alpha = -1$, ρ becomes constant > 0 provided $1 \geq \delta$. Here, it is interesting to note that the metric (3.85) satisfies one of the Kasnerian constants, $p_1 + p_2 + p_3 + p_4 - p_5 = 1$ but not the other. Vacuum spacetimes are obtained for $\alpha = 0$ or $\alpha + \delta = 0$. When $\alpha = 0$, 4-dimensional spacetime is flat.

3.5 Homogemeous Perfect Fluid Solutions:

In this case b = 0 and we obtain

$$(3.87) \qquad R_{15} = 0 \quad ,$$

$$(3.88) \qquad R_{11} = R_{22} = R_{33} = R_{44}$$, so, one obtains

$$(3.89) \quad (\beta-\alpha)\dot{\mu} + (\beta-\alpha)(\beta+\alpha+\delta)\mu^2 + 4a^2T^{2(\beta-\alpha)} = 0,$$

$$(3.90) \quad (\beta-\delta)\dot{\mu} + (\beta-\delta)(\beta+\alpha+\delta)\mu^2 + 2a^2T^{2(\beta-\alpha)} = 0,$$

and

$$(3.91) \qquad \alpha = \gamma \quad ,$$

(3.92) $\mu = \dot{T}/T$.

The above equations lead following two cases:

(3.93) (i) $\dot{\mu} + (\beta + \alpha + \delta)\mu^2 = 0, \quad 2\delta \neq \alpha + \beta$,

(3.94) (ii) $(\beta - \alpha)(\dot{\mu} + 3\delta\mu^2) + 4a^2 T^{2(\beta - \alpha)} = 0$,

(3.95) $2\delta = \alpha + \beta$.

Case (i) For this, we get

(3.96) $T = t$,

(3.97) $\alpha = \dfrac{8a^2+5}{6}$,

(3.98) $\beta = \dfrac{8a^2-1}{6}$,

(3.99) $\delta = \dfrac{4a^2+1}{3}$.

The pressure and density read as

(3.100) $8\pi p = \frac{1}{12}(4a^2+1)(11-8a^2)\, t^{-\frac{8a^2+11}{3}}$, (3.101) $8\pi\rho = \frac{1}{12}\ (4a^2+1)\ (32a^2+11)\ t^{-\frac{8a^2+11}{3}}$.

Now p and ρ are positive for $a^2 \leq 11/8$. Again for $a^2 \neq 11/8$, one obtains

(3.102) $$\rho = \frac{11+32a^2}{11-8a^2}\ p\ .$$

For $a^2 = 11/28$, we get

(3.103) $$\rho = 3p\ ,$$

and for a = 0

(3.104) $$\rho = p\ .$$

<u>Case (ii)</u> If $\alpha = \beta$, a = 0, one obtains pressure and density as

(3.105) $$8\pi p = -\ 3\ \ddot{S}/s^3\ ,$$

(3.105) $$8\pi\rho = \frac{(3\alpha -1)}{\alpha}\ \frac{\dot{S}^2}{S^4}$$

where

(3.107) $T\ (t) = S(t)$.

It is now obvious that

(3.108) $$\ddot{S} < 0 \quad ,$$

and

(1.109) $$\alpha \geq 1/3 \quad .$$

or

(1.110) $$\alpha < 0 \quad .$$

Again for $S = t$, we get dust distribution.

3.6 Concluding Remarks:

By considering a non-diagonal cylindrically symmetric metric in the Kaluza-Klein spacetime we have presented a number of homogeneous and inhomogeneous perfect fluid solutions including the 5-dimensional analogue of 4-dimensional non-singular stiff fluid solutions. It is observed that the dimensional reduction is admitted only in the diagonal case.

From the equation

$$R_{ik} = - 8\pi \; [(\rho + p)\, u_i u_k - \frac{1}{3} \; (\rho - p)\; g_{ik}] \; ,$$

One may obtain

$$R_{ik} \; u^i u^k = - \frac{16\pi}{3} \; (\rho + 2p) \leq 0.$$

So in view of the Raychaudhuri (1955), equation, singularity may be avoided only when acceleration is non-zero for a vorticity free spacetime.

Hence, it is observed that a non-singular spacetime requires to be inhomogeneous. In the case of diagonal non-singular solutions, it comes out that 5-dimensional non-singular analogue exists only for the $\rho = p$. Here, we have presented the 5-dimensional analogue of non-singular non-diagonal stiff fluid ($\rho = p$) solutions. It does not allow the dimensional reduction. Here the parameter a measures the non-diagonality of the metric and b as inhomogeneity. The perfect fluid without $p = \rho$ is allowed in the homogeneous case. We have presented a family of 5-dimensional solutions which includes 5-dimensional version of FRW flat solutions. All homogeneous solutions are expected big-bang singular.

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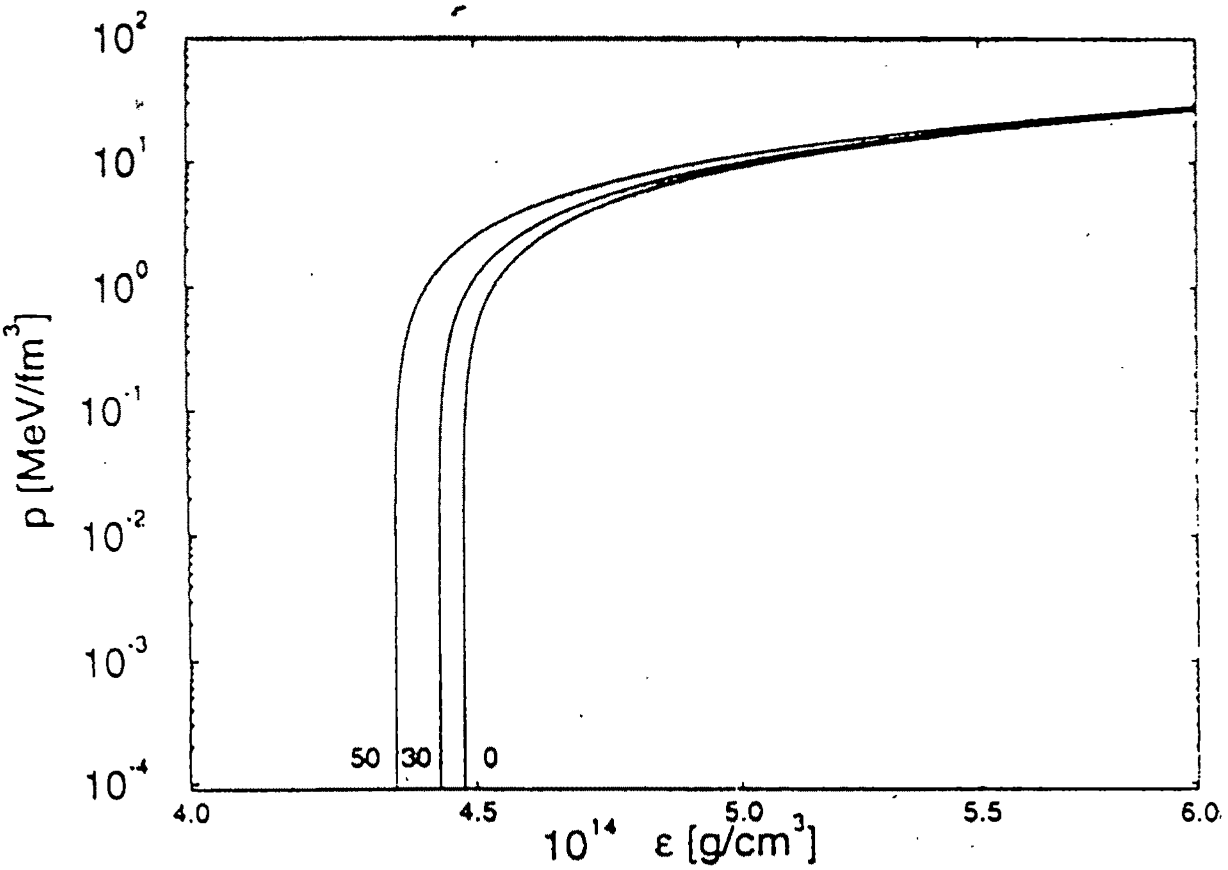

p [MeV/fm^3]
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
50
30
0
4.0
4.5
5.0
5.5
6.0
10^{14} ε [g/cm^3]

C H A P T E R - IV

DOMAIN WALLS IN BRANS-DICKE THEORY

DOMAIN WALLS IN BRANS–DICKE THEORY

4.1 Introduction:

Brans, C. and Dicke, R.H. (1961) presented an interesting alternative to general theory of relativity based on Mach's principle. To understand the reasons leading to their field equations, we first note that the concept of a variable inertial mass itself leads to a problem of interpretation. We need an independent unit of mass against which increase or decrease of a particle mass may be measured Such unit is given by gravity i.e. the so-called Planck mass :

$$(4.1) \qquad \left(\frac{\hbar c}{G}\right)^{\frac{1}{2}} \simeq 2.16 \times 10^{-5} \text{ g}.$$

Hence, the dimensionless quantity

$$(4.2) \qquad X = m \left(\frac{G}{\hbar c}\right)^{\frac{1}{2}} ,$$

measured at different spacetime points may tell us whether masses are changing. Or, alternatively, if one insists on using mass units that are the same everywhere, a change of X would tell us that G is chaning. We could of course assume that $\hbar$ and c also change. However, by keeping $\hbar$ and c constant one follows the principle of least modification of existing theories. hence, special relativity and quantum theory are unaffected if one keeps $\hbar$ and c fixed. This is the conclusion Brans and Dicke arrived at in their approach to mach's principle. They observed for a framework in which the gravitational constant G arises from the structure of the Universe, so that a changing G could be looked upon as the Machian consequence of a changing Universe.

Sciama (1953) presented general arguments leading to a relationship between G and the large-scale structure of the Universe.

$$(4.3) \qquad \rho_o = \frac{3H_o^2}{4\pi G} q_o .$$

where

(4.4) $$\frac{\dot{a}}{a}\Big|_{t_\circ} = H_\circ ,$$

the present value of Hubble's constant, and

(4.5) $$\frac{\ddot{a}}{a} = - q(t)\ [H(t)]^2 ,$$

(4.6) $$H(t) = \dot{a}/a ,$$

with their present values by $q_\circ$ and $H_\circ$, the $q_\circ$ is called the deceleration parameter. If one put

(4.7) $$R_\circ = c/H_\circ ,$$

as the characteristic length of the Universe and

(4.8) $$M_\circ = 4\pi \rho_\circ \quad R_\circ^3/3 ,$$

as the characteristic mass of the Universe.then one obtains

(4.9) $$\frac{1}{G} = \frac{M_\circ}{R_\circ C^2}\ q_\circ^{\ -1} \sim \frac{M_\circ}{R_\circ C^2} \sim \sum \frac{m}{rc^2} .$$

Brans and Dicke used this relatioin as one that determines G^{-1} from a linear superposition of inertial contribution m/rc^2 being from m at a distance r from the point where G is determined. Since m/r is a solution of a scalar wave equation with a point source of strength m, Brans and Dicke postulated that G behaves as the reciprocal of a scalar field ϕ:

$$G \sim \phi^{-1} , \tag{4.10}$$

where ϕ is expected to satisfy a scalar wave equation whose source is all the matter in the Universe.

The Brans-Dicke action principle reads

$$A = \frac{c^3}{16} \int_V (\phi R + w\phi^{-1}\phi^k \phi_k)(-g)^{\frac{1}{2}} d^4x + \Lambda . \tag{4.11}$$

In the above equation the coefficient of R is $c^3\phi/16\pi$, instead of $c^3/16\pi$ G as in the Einstein-Hilbert action. The reason for this lies in the anticipated behaviour of G as given in (4.10). The second term, with $\phi_k = \partial\phi/\partial x^k$, ensures that ϕ will satisfy a wave equation, while third term includes, through a Lagrangian density L, all the matter and energy present in the spacetime region V. The energy momentum tensor T^{ik} is related to Λ,

$$\delta \Lambda = \frac{1}{2c} \int \underset{(\Lambda)}{T^{ik}} (-g)^{\frac{1}{2}} \, \delta g_{ik} \, d^4x \quad , \tag{4.12}$$

and w being a coupling constant.

The variation of A for small changes of g^{ik} leads to the field equations

$$R_{ik} - \tfrac{1}{2} g_{ik} R = - \frac{8\pi}{c^4 \varphi} T_{ik} \tag{4.13}$$

$$\frac{-w}{\varphi^2} (\varphi_i \varphi_k - \tfrac{1}{2} g_{ik} \varphi^{\ell} \varphi_{\ell})$$

$$- \frac{1}{\varphi} (\varphi_{;ik} - g_{ik} \Box \varphi) .$$

Similarly, the variation φ leads to

$$2\varphi \Box \varphi - \varphi_k \varphi^k = \frac{R}{w} \varphi^2 \quad , \tag{4.14}$$

and finally one obtains

$$\Box \varphi = \frac{8\pi}{(2w+3)c^4} T \quad , \tag{4.15}$$

where T be the trace of T^i_k . Hence, eq. (4.15) leads to the anticipated scalar wave equation for φ with sources in matter, $\Box$ being the wave operator. As it contains a scalar field ϕ in addition to the metric tensor g_{ik}, the Brans-Dicke theory is often known as the scalar-tensor theory of gravitation. It is obvious from these field equations that as $w \longrightarrow \infty$ the Brans-Dicke tends to general relativity. For w = 0 (1) the theory gives significantly different results from general relativity in a number of solar system tests.

The topological defects come in picture through a series of phase transition in the early Universe. One such defect is a domain wall, which is formed when a discrete symmetry is spontaneously broken. Windrow (1985), Goetz (1990), Mukherjee (1990) studied the domain walls with finite thickness due to the proposal for a new scenario of galaxy formation due to Hill, Schramm and Fry (1989). The gravitational field of infinitely thin walls has been computed by Vilenkin (1981), Tpser and Sikivi (1984). For a thick domain wall in curved spacetime, one assumes the wall to have planar symmetry with two commuting Killing vectors describing translational invariance in the plane parallel to the wall and a third Killing vector related to a rotational symmetry about the x-axis perpendicular to the wall. It is symmetric about x = 0 plane. The Lagrangian for the scalar field reads

$$L = \tfrac{1}{2} g^{ik} \varphi_i \varphi_k - V(\varphi) \quad , \tag{4.17}$$

where φ is assumed to be a function of x alone. It has been observed by Widrow that for such a field purely static metric exhibits unphysical behaviour at large $|x|$, so long as $V(\varphi)$ is positive. One may wonder if it is possible to obtain such a static thick domain wall in the Brans-Dicke theory. It is considered in the first step $T_o^o = T_2^2 = T_3^3 \neq 0$ and $T_1^1 = 0$. Such form of energy momentum tensor has been used by Raychaudhuri and Mukherjee (1987), who have given in a general way that such walls in Einstein theory maynot remain in static equilibrium.

4.2 Cosmological solutions in the Brans-Dicke Theory:

Let us consider the homogeneous and isotropic cosmological solutions in the Brans-Dicke theory. let us also consider the Robertson-Walker line element and the energy momentum tensor for a perfect fluid. The line element reads

$$ds^2 = c^2 dt^2 - a^2(t) \left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \right]. \tag{4.18}$$

Let us set

(4.19) $$x^0 = ct$$

(4.20) $$x^1 = r$$

(4.21) $$x^2 = \theta$$

(4.22) $$x^3 = \varphi$$

The non-zero comonents of g_{ik} and g^{ik} are

(4.23) $$g_{00} = 1, \quad g_{11} = - \frac{a^2}{1-kr^2} ,$$

(4.24) $$g_{22} = - a^2r^2 , \quad g_{33} = - a^2r^2 \sin^2\theta ,$$

(4.25) $$g^{00} = 1, \quad g^{11} = - \frac{1-kr^2}{a^2} ,$$

(4.26) $$g^{22} = - \frac{1}{a^2r^2} , \quad g^{33} = - \frac{1}{a^2r^2\sin^2\theta} .$$

(4.27) $$(-g)^{\frac{1}{2}} = \frac{a^2r^2\sin^2\theta}{(1-kr^2)^{1/2}} .$$

The non-zero components of Γ^{i}_{kl} are

(4.28) $$\Gamma^{1}_{01} = \Gamma^{2}_{02} = \Gamma^{3}_{03} = \frac{\dot{a}}{ca} ,$$

(4.29) $\Gamma^{0}_{11} = \frac{a\dot{a}}{c(1-kr^2)}$, $\Gamma^{0}_{22} = \frac{a\dot{a}r^2}{c}$, $\Gamma^{0}_{33} =$

$\frac{a\dot{a}r^2\sin^2\theta}{c}$

(4.30) $\Gamma^{1}_{11} = \frac{kr}{1-kr^2}$, $\Gamma^{2}_{12} = \Gamma^{3}_{13} = 1/r$,

(4.31) $\Gamma^{1}_{22} = - r(1-kr^2)$, $\Gamma^{1}_{33} = - r\,(1-kr^2)\,\sin^2\theta$

(4.32) $\Gamma^{2}_{33} = - \sin\theta\,\mathrm{Cos}\theta$, $\Gamma^{3}_{33} = \mathrm{Cot}\,\theta$.

The non-zero components of R^i_k are

(4.33) $R^0_0 = \frac{3}{c^2}\frac{\ddot{a}}{a}$,

(4.34) $R^1_1 = R^2_2 = R^3_3$

$= \frac{1}{c^2}\left(\frac{\ddot{a}}{a} + \frac{2\dot{a}^2+2kc^2}{a^2}\right)$.

From these, one obtains

(4.35) $$R = \frac{6}{c^2}\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2 + kc^2}{a^2}\right.,$$

and hence

(4.36) $$G_1^1 = G_2^2 = G_3^3 = R_1^1 - \tfrac{1}{2} R$$

$$= -\frac{1}{c^2}\left(\frac{2\ddot{a}}{a} + \frac{\dot{a}^2+kc^2}{a^2}\right),$$

(4.38) $$G_0^0 = R_0^0 - \tfrac{1}{2} R = -\frac{3}{c^2}\left(\frac{\dot{a}^2+kc^2}{a^2}\right)$$

In view of the we obtain nontrivial equations

(4.39) $$\frac{2\ddot{a}}{a} + \frac{\dot{a}^2+kc^2}{a^2} = \frac{8\pi G}{c^2} T_1^1 = \frac{8\pi G}{c^2} T_2^2 = \frac{8\pi G}{C^2} T_3^3,$$

(4.40) $$\frac{\dot{a}^2+kc^2}{a^2} = \frac{8\pi G}{3c^2} T_0^0 .$$

In view of (4.39), one obtains

(4.41) $$T_1^1 = T_2^2 = T_3^3 = -p$$

(4.42) $$T_0^0 = \epsilon \quad .$$

The scalar field ϕ is a function of time only. Hence, the field equations assume the form

(4.43) $$\frac{2\ddot{a}}{a} + \frac{\dot{a}^2+kc^2}{a^2} = -\frac{8\pi p}{\phi c^2} - \frac{2\dot{\phi}\dot{a}}{\phi a} - \frac{w\dot{\phi}^2}{2\phi^2} - \frac{\ddot{\phi}}{\phi} \quad ,$$

(4.44) $$\frac{\dot{a}^2+kc^2}{a^2} = \frac{8\pi\epsilon}{3\phi c^2} - \frac{\dot{\phi}\dot{a}}{\phi a} + \frac{w\dot{\phi}^2}{6\phi^2} \quad .$$

The conservation equation reads

(4.45) $$\frac{d}{da}(\epsilon a^3) + 3pa^2 = 0 \quad .$$

In additioin, one obtains the field equation for ϕ

(4.46) $$\frac{1}{a^3}\frac{d}{dt}(\dot{\phi}a^3) = \frac{8\pi}{(2w+3)c^2}(\epsilon - 3p) \quad .$$

One anticipates that big bang solutions will emerge from these equations and keeping the big bang epoch at t = 0. The eq. (4.66) gives

$$(4.47) \quad \dot{\rho} a^3 = \frac{8\pi}{(2w+3)c^2} \int_0^t (\epsilon - 3p)\, a^3\, dt + D$$

where D is constant. Two types of solutions are obtained depending on $D = 0$ or $D \neq 0$.

Case (i) $D = 0$.

Let us consider the simplest case with $k = 0$, $p = 0$ and $\epsilon = \rho c^2$. Hence, this solution is analogous to the Einstein-de Sitter of general relativity. Let us put

$$(4.48) \quad a = a_o \left(\frac{t}{t_o}\right)^A$$

$$(4.49) \quad \rho = \rho_o (t/t_o)^B \quad ,$$

so that

$$(4.50) \quad \rho \propto t^{-3A} \quad ,$$

and the field equations provide

$$(4.51) \quad A = \frac{2w+2}{3w+4} \quad ,$$

(4.52) $$B = \frac{2}{3w+4} ,$$

and

(4.53) $$\rho_o = \frac{(2w+3)\ B\varphi_o}{8\pi\ t_o^2} .$$

It may be shown that as $w \longrightarrow \infty$ this solution tends to the Einstein-de Sitter model.

Case (ii) $D \neq 0$.

In this case one obtains

(4.54) $$\frac{8\pi}{(2w+3)c^2} \int_o^t (\epsilon - 3p)\ a^3\ dt \ll |D| ,$$

for the cases both of dust and of radiation. For $p = 0$, we get

(4.55) $$3A + B = 1$$

(4.56) $$t_o = \frac{a_o^3\ \varphi_o B}{D} .$$

Again for $p = \frac{1}{3}\epsilon$, we obtain

$$(4.57) \qquad A^2 = AB + \frac{wB^2}{6} \quad .$$

Hence,

$$(4.58) \qquad A = \frac{w+1 + [(2w/3) + 1]^{\frac{1}{2}}}{(3w+4)} \quad ,$$

$$(4.59) \qquad B = \frac{1 \pm 3\ [(2w/3)+1]^{\frac{1}{2}}}{(3w+4)} \quad .$$

The plus sign holds when $D > 0$ and minus sign when $D < 0$. For $D > 0$, $\phi \longrightarrow 0$ when $a \longrightarrow 0$, while for $D < 0$, $\phi \longrightarrow \infty$ for $a \longrightarrow 0$. These results hold irrespective of the values of k or the equation of state. Since $G \propto \phi^{-1}$, a time-dependent ϕ means a time-dependent gravitational constant . For $D = 0$

$$(4.60) \qquad \frac{\dot{G}}{G} = -\frac{2}{(3w+4)}\frac{1}{t} = -\frac{H}{(w+1)} \quad .$$

4.3 Field Equations for domain wall in Brans-Dicke Theory:

Let us consider the general static metric with planar symmetry of the form

$$ds^2 = e^{2\alpha}\ dt^2 - e^{2\beta}\ dx^2 - e^{(\beta-\alpha)}\ (dy^2 + dz^2) \tag{4.61}$$

where

$$\alpha = \alpha(x) \quad , \tag{4.62}$$

$$\beta = \beta(x) \quad . \tag{4.63}$$

The energy momentum tensor for a static scalar field reads

$$T^0_0 = T^2_2 = T^3_3 = \rho = \tfrac{1}{2}\, e^{-2\beta}\ \varphi'^2 + V(\varphi) \quad , \tag{4.64}$$

and

$$T^1_1 = -\tfrac{1}{2}\, e^{-2\beta}\ \varphi'^2 + V(\varphi) \quad . \tag{4.65}$$

The field equations in Brans-Dicke theory assumes the form

$$(4.66) \quad \alpha'' = \frac{8\pi}{\psi}\left[T^0_0 - \frac{w+1}{2w+3}\,T\right] e^{2\beta} - \frac{\alpha'\psi'}{\psi} \quad ,$$

$$(4.67) \quad \beta'' + \tfrac{1}{2}(\alpha' + \beta')(\alpha' - \beta')$$

$$= \frac{8\pi}{\psi}\left[T^1_1 - \frac{w+1}{2w+3}\,T\right] e^{2\beta}$$

$$- \frac{w\psi'^2}{\psi^2} + \frac{\beta'\psi'}{\psi} - \frac{\psi''}{\psi} \quad ,$$

$$(4.68) \quad \frac{1}{2}(\alpha'' - \beta'') = -\frac{8\pi}{\psi}\left[T^2_2 - \frac{w+1}{2w+3}\,T\right] e^{2\beta}$$

$$+ \frac{1}{2}(\beta' - \alpha')\frac{\psi'}{\psi} \quad ,$$

where ψ denotes the Brans-Dicke scalar field.

The Bianchi identity reads

$$(4.69) \quad T^k_{1;k} = 0 \quad ,$$

which gives

$$(4.70) \quad \beta' = 0 \quad ,$$

for a nontrivial case.

Let us consider the case

(4.71) $$T^1_1 = 0$$

i.e.

(4.72) $$V(\varphi) = \tfrac{1}{2} e^{-2\beta} \varphi'^2$$

In view of the above the eq. (4.65) reads

(4.73) $$\alpha'' + \frac{\alpha'\Psi'}{\Psi} = \frac{8\pi\rho}{\Psi} \left[1 - \frac{3(w+1)}{(2w+3)}\right] e^{2\beta},$$

and the eq. (4.67) gives

(4.74) $$\frac{1}{2}\left(\alpha'' + \frac{\alpha'\Psi'}{\Psi}\right) = -\frac{8\pi\rho}{\Psi} \left[1 - \frac{3(w+1)}{(2w+3)}\right] e^{2\beta}.$$

Now, it is observed that equations (4.73) and (4.74) are clearly inconsistent showing that a static domain wall with planar symmetry and

$$(4.75) \qquad T_1^1 = 0 ,$$

does not exist in Brans-Dicke theory. The result will be changed if one assumes $T_1^1 \neq 0$. We hope $T_1^1 \neq 0$ will provide some new result.

4.4 Concluding Remarks:

For the thick domain wall in curved spacetime one assumes the wall to have planar symmetry with two commuting Killing vectors describing translational invariance in the plane parallel to the wall and a third Killing vector related to a rotational symmetry about the X-axis perpendicular to the wall. It is symmetric about x = 0 plane. The Lagrangian for the scalar field is taken as $L = \frac{1}{2} g^{ik} \phi_i \phi_k - V(\phi)$, where ϕ is assumed as a function of x alone. It has been investigated by Widrow (1985) that for such a field purely static metric gives unphysical behaviour at large $|x|$, so long as $V(\phi)$ is positive. We have presented that a thick domain wall with oplanar symmetry having $T_0^0 = T_2^2 = T_3^3 \neq 0$ and $T_1^1 = 0$ may not remain in static equilibrium in Brans-Dicke theory of gravitation. We hope if one assumes $T_1^1 \neq 0$ the result may be changed.

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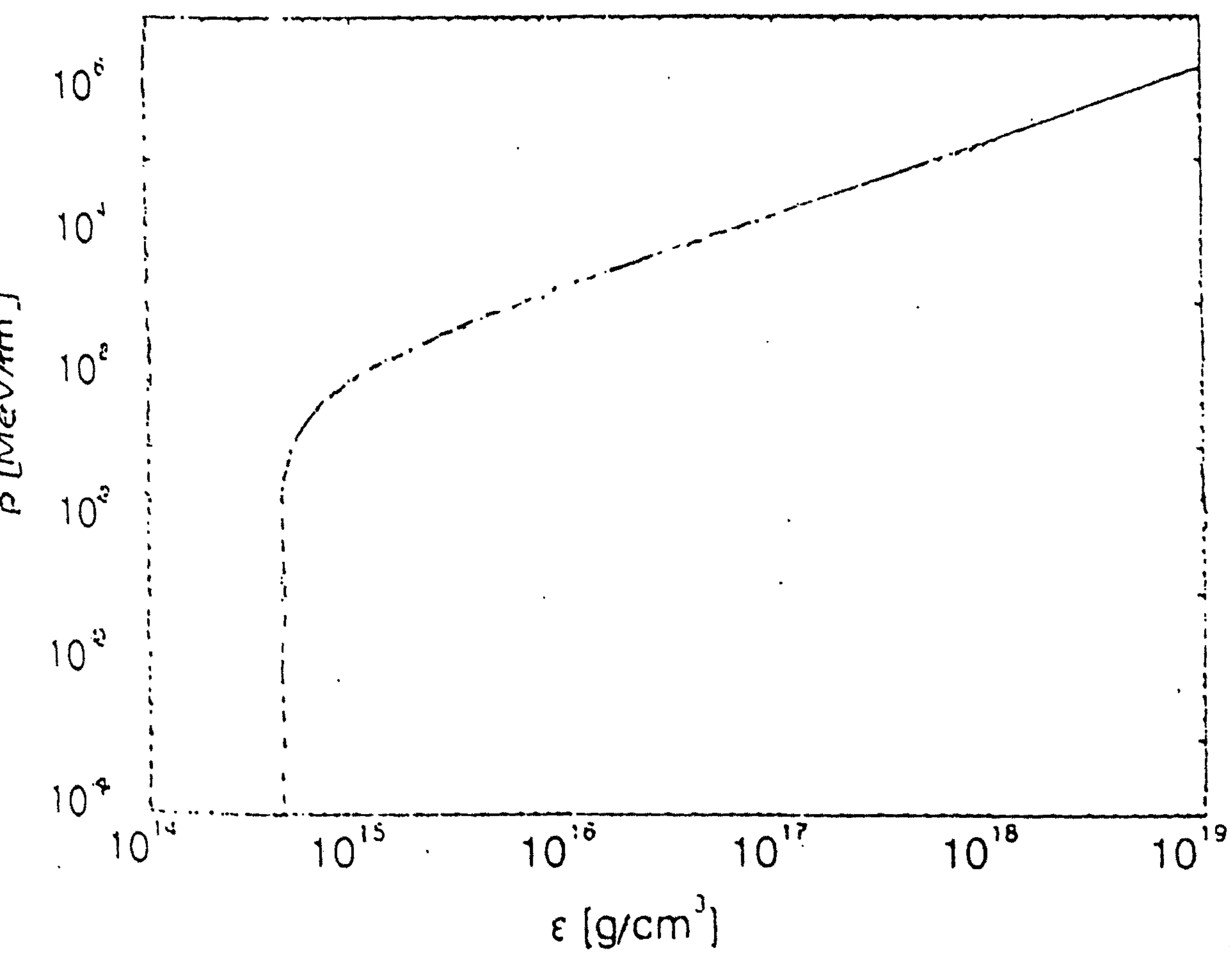
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10
10
10
10
10
10
10
10
10
10
10
10
ε [g/cm]

R E F E R E N C E S

$$$

R E F E R E N C E S

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Wainwright, J., Ince, W.C.W., and Marshman, B.J., (1979), Gen., Re. Grav., 10, 259.

Ibanez, J., Verdaguer, E. (1986), Astrophys. J. 306, 401.

Belenskii, V., and Ruffini, R., (1980), Phys. Lett. B89, 195.

Verdaguer, E. (1993), Phys. Rep. 229 No.1, 1.

Diaz, M.C., Gleiser, R.J. and Pullin, J.A. (1987), Class. Q. Grav. 4, L23.

Wainwright, J,. (1979), J. Phys. A12, 2015.

Diaz, M.C., Gleiser, R.J. and Pullin J.A. (1988a), Class. Q. Grav. 5, 641.

Diaz, M.C. Gleiser, R.J. and Pullin, J.A. (1988b), J. Math. Phys. 29, 169.

Diaz, M.C., Gleiser, R.J. and Pullin, J.a. (1989), Astrophys. J. 339, 1.

Pullin, J. (1990), Astrophys. Space Sci. 164, 309.

Cruzate, J., Diaz, M., Gleiser, R. and Pullin, J.A. (1988), Class. Q. Grav. 5, 883.

Vilenkin, A. (1985), Phys. Rep. 121, 263.

Hill, C.T., Schramm, D.N. and Fry, J.N. (1989), Nucl. Part. Phys. 19, 25.

Vilenkin, A. (1981), Phys. Rev. D23, 852.

Widrow, L.M. (1989), Phys. Rev. D39, 3571.

Goetz, G. (1990), J. Math. Phys. 31, 2683.

Mukherjee, M. (1993), Class. Q. Grav. 10, 131.

Letelier, P.S. and Wang, A. (1993), Class. Q. Grav. 10, L-29.

Wang, A., (1994), Mod. Phys. Lett. A90, 3605.

Chodos, A. and Detweider, S. (1980), Phys. Rev. D21, 2167.

Banerjee, a. and Das, A. (1998), Int. J. Mod. Phys. D7, 81.

Wang, A. (1992), Phys. Rev. D 45, 3534.

Appelquist, T., Chodos, A. and Freund, P.G.O. (1987), eds. Modern Kaluza-Klein Theories (Addison-Wesley, Reading, M.A.)

Chatterjee, S., Panigrahi, D. and Banerjee, A. (1993), Class. Q. Grav. 10, L1.

Chatterjee, S., Panigrahi, D. and Banerjee, A. (1994), Class. Q. Grav. 11, 371.

Banerjee, A., Panigrahi, D. and Chatterjee, S. (1994), Class.Q. Grav. 11, 1405.

Chatterjee, S., Bhui, B., Basu, M.B. and Banerjee, A. (1994), Phys. Rev. D50, 2924.

Sahdev, D. (1984), Phys. Lett. 137B, 155.

Ishiibara, H. (1984), Prog. Theor. Phys. 72, 376.

Chatterjee, S. and Bhui, B. (1990), Mon. Not. R. Astron. Soc. 240, 317.

Ruiz, E. and Senovilla, J.M.M. (1992), Phys. Rev. D45, 1995.

Dhadhich, N., Patel, L.K. and Tikekar, R. (1995), Parmana. (J. Phys.) 44, 303.

Banerjee, A., Das, A. and Panigrahi, D. (1995), Phys. Rev. D51, 6816.

Dadhich, N. (1995), in Inhomogeneous Cosmological Models eds. A Molina and J.M.M. Senovilla (World Scientific, Singapore).

Mars, M. (1995), Phys. Rev. D51, R 3989.

Bronikov, K.A. (1979), J. Phys. A12, 201.

Raychaudhuri, A.K. (1955), Phys. Rev. 90, 1123.

Dadhich, N. (1999), Mod. Phys. Lett,. A14, 337.

Dadhich, N. (1999), Mod. Phys. Lett. A14, 79.

Dadhich, N. (1999), in Black Holes, Gravitational Radiation and the Universe, eds. B.R. Iyer and B. Bhawal (Kluwer).

Dadhich, N. (1999), Dual Spacetimes, Mach's principle and topological defects, qr-qc/9902066.

Barriola, M. and Vilenkin, A. (1989), Phys. Rev. Lett. 63, 341.

Nouri-Zonoz, M., Dadhich, N. and Lynden-Bell, D. (1999), Clas. Q. Grav. 16, 1021.

Banerjee, A., Chatterjee, S. and Sen, A.A. (1996), Class. Q. Grav. 13, 3141.

Gross, J.D. and Perry, M.J. (1983), Nucl. Phys. B226, 29.

Dadhich, N., Patel, L.K. and Tikekar, R. (1998), Class. Q. Grav. 15, L27.

Goetz, G. (1990), J. Math. Phys. 31, 2683.

Ipser, J. and Sikivi, P. (1984), Phys. Rev. D30, 712.

Brans, C. and Dicke, R.H. (1961), Phys. Rev. 124, 925.

La, D. and Steinhardt, P.J. (1987), Phys. Rev. Lett. 62, 376.

Raychaudhuri, A.K. and Mukherjee, G. (1987), Phys. Rev. Lett. 59, 1504.

Einstein, A. and Rosen, N. (1937), J. Franklin Inst. 223, 43.

Rindler, W. (1966), Am. J. Phys. 34, 1174.

Wesson, P.S. (1983), Astron. Astrophys., 119, 145.

Wesson, P.S. (1985), Astron. Astrophys. 143, 233.

Chatterjee, S. (1986), Gen. Rel. Grav. 18, 1073.

Chatterjee, S. (1987), Astron. Astrophys., 179, 1.

Sabatta, De, Schmutzer, V. (1983), Unified Field Theories of More than 4 Dimensions Including Exact Solutions, World Scientific Singapore.

Fukui, T. (1987), Gen. Rel. Grav. 19, 43.

Hoyle, F. (1975), Astrophys. J. 196, 661.

Jackson, J.D. (1962), Classical Electrodynamics, Wiley, New York.

McVittie, G.C. (1965), General Relativity and Cosmology, University of Illinois Press, Urbana, Illinois.

Wesson, P.S. (1980), Gravity, Particles, and Astrophysics, Reidel Dordreht.

Wesson, P.S. (1984), Gen. Rel. Grav. 16, 193.

Wesson, P.S. (1985), Astron. Astrophys. 143, 233.

Wesson, P.S. (1986), Astron-Astrophys. 166, 1.

Blake, G.M. (1978), MNRAS, 185, 399.

Maeder, A. (1977a), Astron. Astrophys. 56, 359.

Maeder, A. (1977b), Astron. Astrophys. 57, 125.

Robertson, H.P. and Noonan, T.W. (1968), Relativity and Cosmology, Saunders, Philadelphia.

VandenBerg, D.A. (1977), MNRAS, 181, 695.

Canuto, V., Adams, P.J., Hsieh, S.H. and Tsiang, E. (1977), Phys. Rev. (Ser-3) D16, 1643.

Canuto, V. and Hsieh, S.H. (1978), Astrophys. J. 224, 302.

Dirac, P.A.M. (1973), Proc. Roy. Soc. London A333, 403.

Dirac, P.A.M. (1979), Proc. Roy. Soc. London A365, 19.

Fluton, T., Rohrlich, F. and Witten, L. (1962), Rev. Mod. Phys. 34, 442.

Kaluza, T. (1921), Sitz Preuss. Akad. Wiss., 36, 966.

Klein, O. (1926), Z. Phys. 37, 895.

Maeder, a. and Bouvier, P. (1979), Astron. Astrophys. 73, 82.

McCrea, W.H. (1978), Observatory, 98, 52.

Ritter, R.C., Gillies, G.T., Rood, R.T. and Beams, J.W. (1978), nature. 271, 228.

Wesson, P.S. (1978), Cosmology and Geophysics, Hilger, Bristol, U.K. and Oxford University Press, New York, U.S.A.

Witten, E. (1981), Nucl. Phys. B186, 412.

Lee, H.C. (1984), An Introduction to Kluza-Klein Theories, Proc. Conf. August 11-16, 1983, Chalk River/Deep River, Ontario Canada, World, Scientific Singapore.

Borde, A., Ford, L.H. and Roman, T.A. (2002), Phys. Rev. D65, 084002.

Ford, L.H. et al (2002), Phys. Rev. D66, 124012.

Dadhich, N. et al (2002), Phys. Rev. D65, 064004.

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