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Template waveform and extraction of GW signal from the noisy data

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Abstract. This work reports the prerequisite information on gravitational waves and understanding of these waves. With the help of PyCBC framework, signal processing is done on the open data provided by LIGO Scientific Collaboration. As an introductory part, this chapter deals with the matched filtering technique which is considered to be the best way of extracting it in Gaussian noise. Moreover, the study of GW150914 event being the first successful detection of LIGO using two detectors of the Laser Interferometer Gravitational Wave Observatory open data simultaneously has been presented. The gravitatioanl waves from this particular event is from coalescing compact binaries (BBH merger) detected on September 14, 2015 at 09:50:45 UTC sweeping in a frequency range 50 to 250 Hz with a peak gravitational-wave strain of 1.0×10^{-21}. This work continues to report on the analysis for gravitational waves from coalescing compact binaries. We discuss the findings from the gravitational-wave searches for merging compact binaries with component masses more than 1 $M_{\odot}$ during the first and second observing runs of the advanced gravitational-wave detector network. The first observing run (O1), which ran from September 12, 2015, to January 19, 2016, and the second observing run (O2), which continued from November 30, 2016, to August 25, 2017, both saw the detection of gravitational waves from the binaryblack hole mergers. No neutron star–black hole mergers were detected.

1. Introduction

The physics of gravitational waves is presently of great interest due to its astonishing features. Moreover, the recent experimental advances have led to detect these kind of waves and its effects.General relativity [1] is the name given to Albert Einstein's theory that describes how gravity affectsthe structure of space-time. The idea, which was released in 1915, was an advancement of Einstein's special relativity theory, which he had published ten years before. Although it asserted thatspace and time are inextricably linked, the theory of special relativity rejected gravity.

Between the two publications, Einstein spent the ensuing decade learning how massive things bendspace-time, which results in the manifestation of gravity, according to NASA. General relativity isAlbert Einstein's theory that describes how gravity affects space-time. Einstein's special relativitytheory, which he had presented ten years earlier, was improved upon by the hypothesis, which was published in 1915. Although space and time are allegedly intrinsically linked, gravity is not acknowledged by the theory of special relativity. As experimental verifications for General Relativity, in the decades since Einstein's ideas were published, scientists have witnessed a plethora ofevents that meet the predictions of relativity. At that instant of time, Einstein's equations hold true.For example (as an application in astrophysics):

i) Gravitational waves [2]

ii) We have modern GPS which used Einstein theory of relativity to pinpoint our locations with impressive accuracy.

iii) Gravitational lensing in which light bends around a large object, such as a black hole, acting asa lens for what lies beyond it. Astronomers commonly employ this technique to investigate stars and galaxies hidden behind large objects.

In 1916, Einstein investigated that the catastrophic events, like as the merging of two black holes,would produce gravitational waves, which are ripples in space-time. In 2016, the Laser Interferometer Gravitational Wave Observatory (LIGO) [2] claimed for the first time that it had identified such asignal. On September 14, 2015, that discovery was made.

The restricted post-Newtonian waveform can accurately simulate a compact binary inspiral wave- form's gravitational wave signal for first-generation gravitational-wave detectors (such Initial LIGO).This is true for non-spinning bodies below around M ~ $12M_\odot$. The waveforms calculated by numerically solving the entire set of nonlinear Einstein equations [5], [6] are more accurately reproduced at higher masses than other resummation methods, such as the effective one body waveforms. In either case, as the orbital motion radiates energy and decays, the two polarisations of the gravitational wave produced by such a system show a monotonically-increasing frequency and amplitude.For $t < tc$, the waveform often referred to as a chirp waveform is given by

$$h_+(t) = -\frac{G\mathrm{M}}{2c^2 r}(1+\cos^2\iota)\left(\frac{c^3}{5G\mathrm{M}}(t_c - t)\right)^{-\frac{1}{4}}\cos 2\phi \quad (1)$$

Likewise, the cross-polarization tensor has the following form:

$$h_{\mathrm{x}}(t) = -\frac{G\mathrm{M}}{c^2 r}\cos\iota\left(\frac{c^3}{5G\mathrm{M}}(t_c - t)\right)^{-\frac{1}{4}}\cos 2\phi \quad (2)$$

where, t_c = time of coalesence,

ι = angle of inclination,

ϕ = orbital phase of the binary,

M = Chirp mass

The *Chirp Waveform* is represented by Equations (1) and (2), and it is obvious from these equations that the GW depends on a number of factors, including chirp-mass, coalescence phase, time, and distance from the source. But, we haven't fully covered all details of a BBH Merger thatgenerates GW; for instance, we do not have a waveform with the black hole's spin as a parameter.This is due to the fact that equations (1) and (2) are calculated in a weak-field approximation, or Newtonian limit, and even the sources are travelling faster than the speed of light. To extract further information about a binary merger, we must model the whole chirp-waveform using Numerical Relativity, taking into account Post-Newtonian approximations.

The GW data analysis covers the detection of a GW signal hidden in the raw data/ noise and estimation of it's physical properties i.e. parameter estimation. To achieve this goal, GW data analysis depends on the availability of waveform templates. For the parameters defining a compact binary coalescence, Parameter estimation algorithm can require the generation of as many astemplate waveforms. Therefore, the fastest waveform generation model is required. Moreover, thetemplates must possess high degree of accuracy.

2. Amplitude Spectral density

Gravitational waves are stretched as they travel due to cosmological redshift. As a result, the observed waveform in the detector frame is different. This corresponds to a change in the parameters(e.g. mass) that has been detected . The relationship, on the other hand, is fairly straightforward:

$$M_o = M(1 + z)$$

where M_o = chirp mass in detector frame,

M = chirp mass in source frame,

z = redshift

There are different ways to access data from the LIGO and Virgo instruments using the libraries: such as Gwpy [3], PyCBC [4] for exploring the binary merger catalog. Here, we refer to the LIGO arm's strain as "data", which is $\Delta L/L$, and $L = 4km$. In many different ways, Python can read data files in the "hdf5" format. H5py module is one of the most used methods for reading this type of file. These files include strain data in the form of a python dictionary, which is read using the standard syntax for reading and extracting data from python dictionaries. While plottingthe time-strain graph, it's hard to infer the signal from such a plot. Therefore, we need to zoom orcenter it around the time of coalescence.

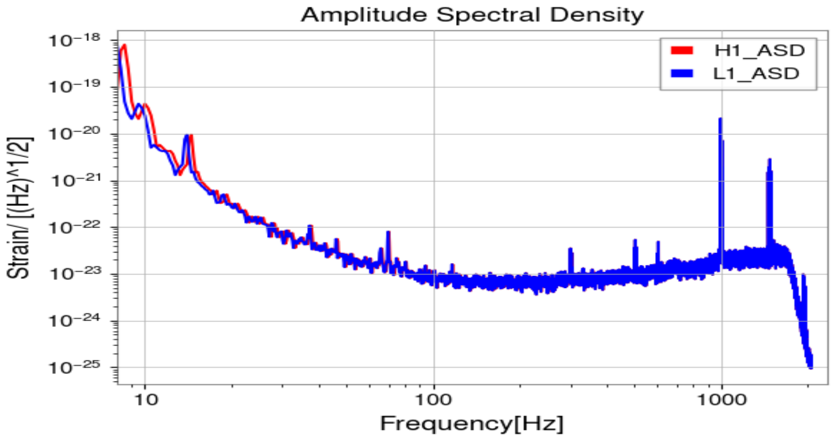

Figure 1: Amplitude Spectral Density of strain data centered around GW150914

Due to the dominance of the noise, the plot doesn't showany resemblance with a true gravitational wave signal. The GPS timing of the GW150914 event is shown in Figure 1 as the centre of the amplitude spectral density of the H1 and L1 data. By carefully examining the image, we can deduce that themeasured GW signal is dominated by coloured noise that LIGO has detected after about 1200 Hzfrequency. At low frequencies, ASD amplitude also varies greatly.

However between about the frequency ranges of 50 Hz and 500 Hz, the amplitude of ASD remains rather stable. This leads usto the conclusion that the interval contains a lot of white noise. Moreover, instrument faults are toresponsible for the higher frequency peaks.

3. GW Signal Search in the Strain Data

If we know what signal we're looking for in the data, matched filtering is known to be the best way for extracting it in Gaussian noise. Let's assume that s(t) is the strain signal that was detectedby the LIGO detector during its observations. Knowing that s(t) is the sum of noise n(t) and the GW signal h(t), or entirely noise n(t), depending on whether or not there is a signal in the data at agiven time interval.

$$s(t) = h(t) + n(t) \tag{3}$$

The goal is to minimise the data's dynamic range and suppress low-frequency behavior that can cause numerical aberrations. Since, high frequency content isn't important, we might want to resample the data. As an example, for GW150914 event, let us get the data from the desired detectors say H1 & L1 (see Figure 2)and then remove the low frequency content. Downsampling the data to 2048 Hz(see Figure 3)

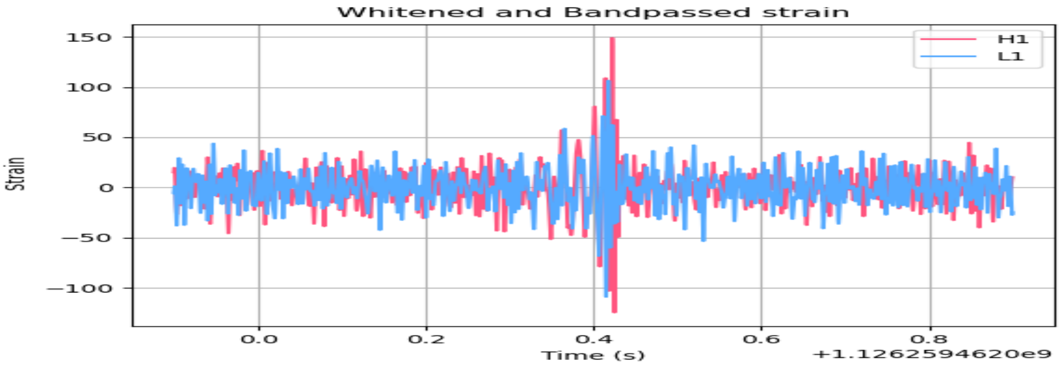

Figure 2: Bandpassing the strain data centered around GW150914

Now, removing 2 seconds of data from both the beginning and end.

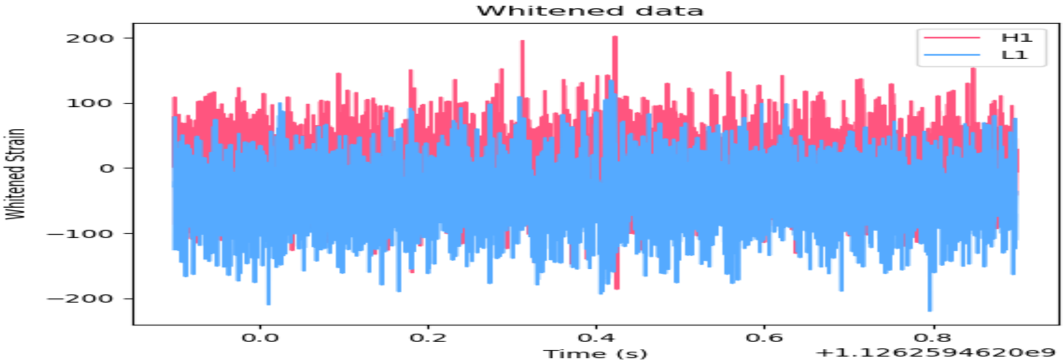

Figure 3: Cropped ends in the bandpassed strain data

So, to whiten the data, we first do a Fourier transform on the signal s(t) in the frequency domain, then divide it by the noise amplitude spectrum before performing a Fourier transform on thesignal in the time domain. The desired signal, as we previously noted while speculating on amplitude spectral densities, can be found between the frequencies of 50 Hz and 500 Hz because this region is dominated by white noise. As a result, we will bandpass the data from the whitened strainand eliminate all the high frequency components. We will then be able to see any GW signals thatmay be present in the noisy data as a bump on the graph. After whitening the data, we need to bandpass, a crucial step. The bandpassed strain measurements are shown in Figure 4. As we don't need to focus on the signal's higher frequency content above 500 Hz or lower frequency content below 10 Hz, we bandpass the data in the frequency range of 50 Hz to 250 Hz. After that, we eliminate frequencieswhile keeping in mind our knowledge of the predefined instrumental noise included in the data. In this instance, we eliminated the data's 60Hz frequency, which corresponds to the power mainsline, from the data. In order to execute signal processing, we must eliminate these characteristic frequencies.

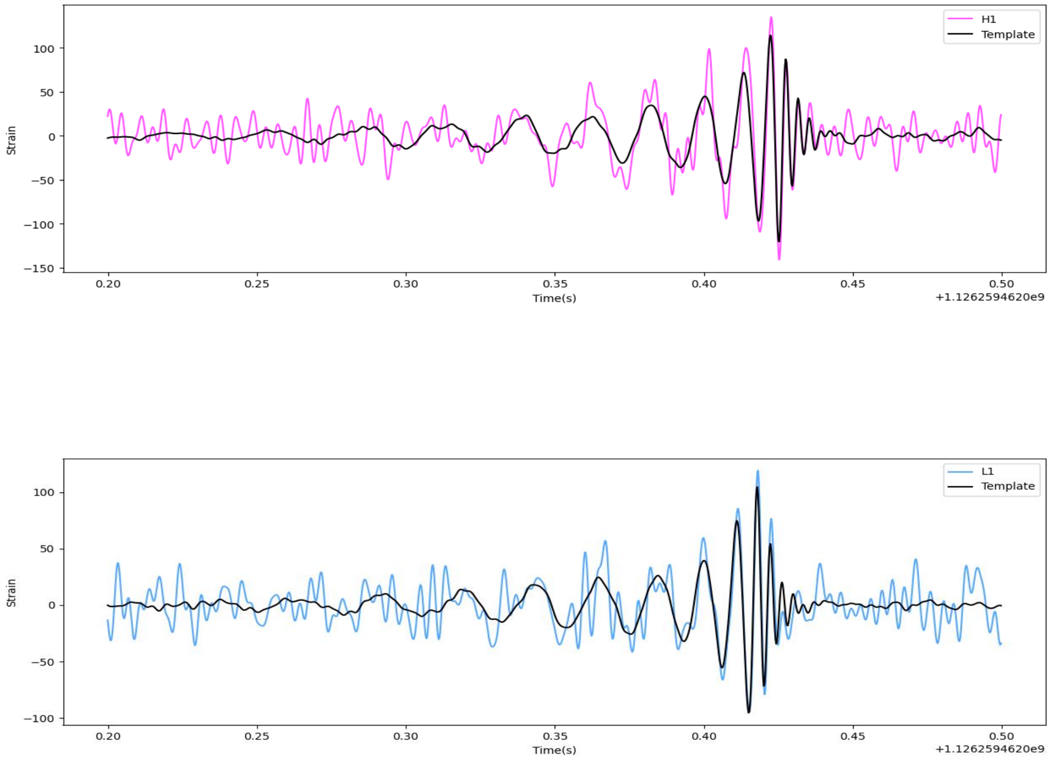

Figure 4: Whitened and bandpassed Strain Data

Examine the signal's rise at time 1126259462.40 s on the x-axis. Even after much of the noise content in the data has been removed, the amplitude of the time-domain strain data at this specificperiod is significant, suggesting that this could be the GW signal we are looking for.

4. The template waveform

The LIGO-Virgo Collaboration uses approximate analytical models because it is currently very difficult to solve all of the equations computationally. Three families can be broadly distinguishedfrom these: Effective-one-body (EOB) [9] waveform models, phenomenological models, and surrogates based on numerical relativity (NR) are among the options.

EOB models are the best option for training our model because: They can be performed rather quickly, and even though they are only checked (and NR informed) for the small portion of the parameter space covered by NR simulations, they can often create waveforms for a large numberof parameters that are not covered by NR information (e.g., large mass ratios and spins). The LIGO-Virgo Collaboration uses SEOBNRv4 [10] as its primary EOB-based model. Here, in our modelwe are going to use SEOBNRv4 approximant for the template waveform generation.

We are now employing PyCBC to create waveform models and look for the best data fit. The primary and secondary masses of black holes will be intrinsic parameters for our time - domain waveform, but luminosity distance will be extrinsic.

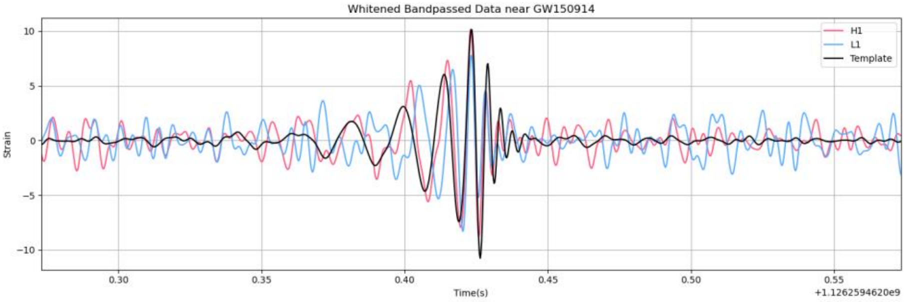

Figure 5: The “signal” (GW150914)

The GW150914 signal can be seen in Figure 5. The graph makes it obvious that our model was a good one because it evenly suited the data. Yet, the reason the template model differs from reality may be that we take non-spinning black holes into account, which precludes the BBH system fromprecessing because our model is already so simple.

5. Signal to noise ratio (SNR)

When we are certain that a GW signal might be discovered within a certain window of time, we specifically deal with strain arrays that might carry a GW signal. The SNR of the data is now calculated using the matched filtering technique to ensure that there's a GW signal in the strain data. When we can't extract the signal's noise and find the signal hidden in it because we lack sufficient knowledge about the signal's noise, matched-filtering is used. Instead, we employ a template, which is essentially a gravitational waveform, in order to evaluate SNR [7]. Figure 6 displaysthe waveform of GW signal utilizing our approach.

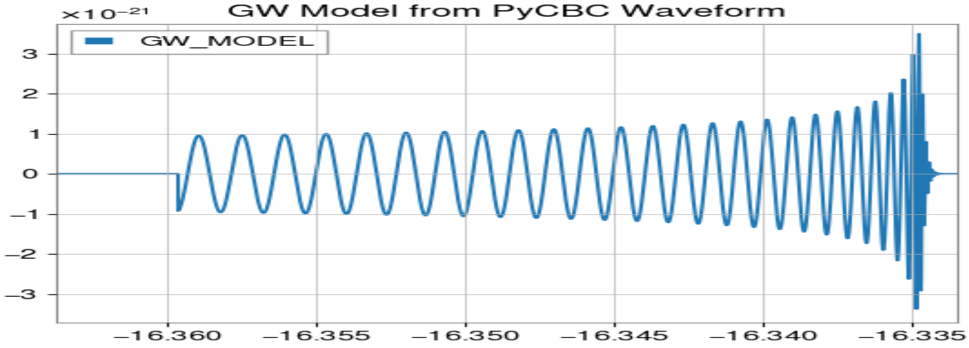

Figure 6: The template

We analyse thousands of templates to determine SNR and come to the conclusion that a particular template matched the signal in the datasetfor which SNR is highest [8]. To make calculations quickly, this calls for a lot of simulations and a number of computer cores. The Signal to noise ratio of the GW150914 event as calculated by our GW model is shown in Figure 7.

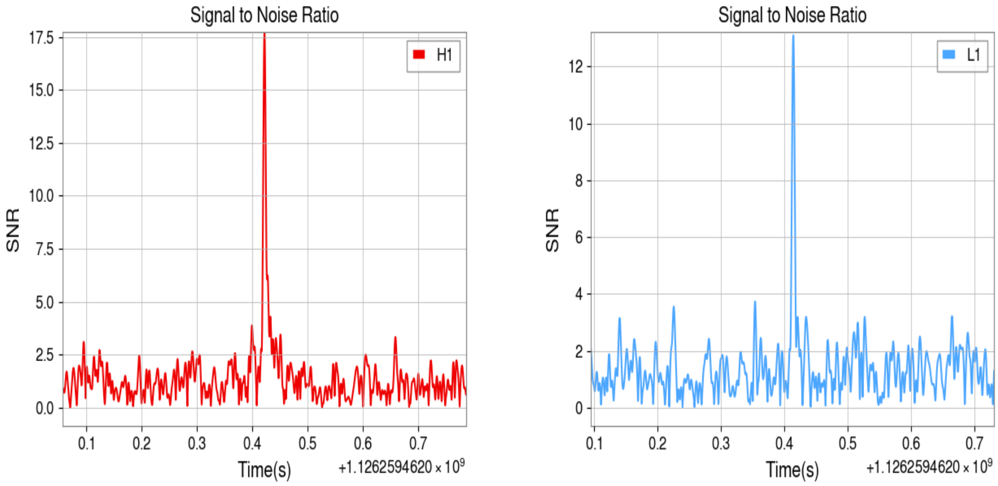

Figure 7: SNR for GW150914 of H1 and L1 detector

6. Observations of GW events

The signal, which originated from a location of around 1.4 billion light-years away, was an illustration of a compact binary coalescence, which occurs when two extremely dense objects merge. Oneof the many gravitational wave sources that the LIGO detectors are looking for are binary systemslike this one. The two objects in such a binary system spiral toward one another as they circle dueto gravitational waves, which are ripples in space-time itself and transport energy away from them.The objects are drawn closer and closer together until they eventually join in this spiral. As they propagate throughout the cosmos, the gravitational waves generated by the binary compress and distort space-time. Observatories like Advanced LIGO can identify this stretching and compressing, which can be utilised to learn more about the sources of the gravitational waves. Here are theten BBH [11] occurences' measurements:

GW event	UTC time	Luminosity distance (MPc)	Primary mass ($M\odot$)	Secondary mass ($M\odot$)	Type	Time of coalescence (s) [estimated]	Time of coalescence (s) [actual]
GW150914	09:50:45	430^{+150}_{-170}	$35.6^{+4.8}_{-3}$	$30.6^{+3.0}_{-4.4}$	BBH	1126259462.4	1126259462.4
GW151012	09:54:43	1060^{+540}_{-480}	$23.3^{+14.0}_{-5.5}$	$13.6^{+4.1}_{-4.8}$	BBH	1128678900.4	1128678900.4
GW151226	03:38:53	440^{+180}_{-190}	$13.7^{+8.8}_{-3.2}$	$7.7^{+2.2}_{-2.6}$	BBH	1135136350.6	1135136339.1
GW170104	10:11:58	960^{+430}_{-410}	$31.0^{+7.2}_{-5.6}$	$20.1^{+4.9}_{-4.5}$	BBH	1167559948.99	1167559936.6
GW170608	02:01:16	320^{+120}_{-110}	$10.9^{+5.3}_{-1.7}$	$7.6^{+1.3}_{-2.1}$	BBH	1180922482.9	1180922494.5
GW170729	18:56:29	2750^{+1350}_{-1320}	$50.6^{+16.6}_{-10.2}$	$34.3^{+9.1}_{-10.1}$	BBH	1185389806.5	1185389807.3
GW170809	08:28:21	990^{+320}_{-380}	$35.2^{+8.3}_{-6.0}$	$23.8^{+5.2}_{-5.1}$	BBH	1186302531.9	1186302519.8
GW170814	10:30:43	580^{+160}_{-210}	$30.7^{+5.7}_{-3.0}$	$25.3^{+2.9}_{-4.1}$	BBH	1186741873.9	1186741861.5
GW170818	02:25:09	1020^{+430}_{-360}	$35.5^{+7.5}_{-4.7}$	$26.8^{+4.3}_{-5.2}$	BBH	1187058314.62	1187058327.1
GW170823	13:13:58	1850^{+840}_{-840}	$39.6^{+10.0}_{-6.6}$	$29.4^{+6.3}_{-7.1}$	BBH	1187529256.5	1187529256.5
GW190814	21:11:18	280^{+110}_{-110}	$23.3^{+1.4}_{-1.4}$	$2.59^{+0.08}_{-0.09}$	!	1249852264.14	1249852257.0

Table 1: GW events with Model Parameters

Conclusion

Gravitational wave is the purest form of information. Heretofore, a wide range of experiments on capturing gravitational waves have been carried out using the available detectors with the helpof the concepts of theories of relativity. Theoretically, with the help of simulations, we conclude our signal processing of LIGO data. This work covers the data analysis of 10 gravitaional waveevents which are purely BBH merger and one event caused by the inspiral and merger of two compact objects—one a black hole and the other of unknown nature. We need a model, which werefer to as a template, to calculate the signal's signal-to-noise ratio. The higher order PN and NR solutions are the ideal way to model a binary merger because this framework relies on the solutionsto Einstein Field Equations. But nonetheless, the fact that these are approximate solutions and wehave no precise models presents a challenge. As a result, we started with a basic model with nospin and then gradually add complexity.

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