CDF/ANAL/EWK/CDFR/1422
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COMPARISON OF ELECTROWEAK PARAMETERS

INTRODUCTION

sin2(theta_bar), the square of the Weinberg angle at the Z-pole, has recently been measured both at LEP and by CDF. This note is intended to address, given a measurement of sin2(theta_bar), how one can compare to other measurements of Electroweak parameters, MW and 1-(MW/MZ)**2 in particular.

There is currently considerable disagreement within CDF about how to make such a comparison (not about the sin2(theta_bar) measurement itself). My hope is that this note will help clarify matters. It is a longer version of a mail message on this subject sent to interested parties on January 25, 1991.

MINIMAL STANDARD MODEL, BUT IGNORING RADIATIVE CORRECTIONS

When radiative corrections are ignored, the minimal Standard Model has just 3 independent parameters. If these are chosen to be alpha, GF and sin2(theta), one can express the masses MW and MZ as

MW**2 = (pi*alpha)/(SQRT(2)*GF*sin2(theta))=(37.281 GeV)**2/sin2(theta)

MZ**2 = MW**2/cos2(theta) = (37.281 GeV)**2/(sin2(theta)*cos2(theta))

It is easy to use a measurement of sin2(theta) to predict MW using these relations. If, for instance, sin2(theta) = 0.228 +- 0.016, one can deduce that MW = 78.1 +- 2.7 GeV/c**2. Note that this is indeed what was done some years ago, before the W and the Z were found, with an error bar on sin2(theta) not much different from the one used in this example (for the error bar, I am just using dMW = -(MW/2)*(dsin2(theta)/sin2(theta)) here).

As we will see below, including radiative corrections does not make much difference.

MINIMUM STANDARD MODEL, INCLUDING RADIATIVE CORRECTIONS

A particularly nice way to look at electroweak radiative corrections was presented by Chris Hill in his January 1991 Academic Training Lectures, see Fermilab-Pub-91/Draft-T, version 2.0, upon which these comments are based. I am grateful to Chris for several useful discussions on this subject. I wish to stress, however, that I (not he) am responsible for any and all misunderstandings and errors that may exist in this CDF note.

Ignoring the weak Higgs mass dependence, there are only 4 parameters needed to calculate all others in the minimal Standard Model, given the fact that the running of the coupling constants is specified by the model, and that the effective masses of all the Fermions, except for the top quark, are known. Let alpha(M) denote the value of the running alpha at mass M.

It is natural to pick alpha, GF and MZ as three of the four parameters since all three are now known very accurately. The choice of the fourth is a matter of convenience. In this case, where we have measured sin2(theta_bar) and wish to predict MW, it is clearly expedient to use sin2(theta_bar) as the fourth. There are then known functions that give

MW = MW(alpha(MW),GF,MZ,sin2(theta_bar))

MTOP = MTOP(alpha(MW),GF,MZ,sin2(theta_bar))

1 - (MW/MZ)**2 = function of (alpha(MW),GF,MZ,sin2(theta_bar)) , etc.

So we don't need MTOP to get MW from sin2(theta_bar)!!

Using relation 3.21 from Chris Hill, it is easy to work out the details:

MW**2 = (PI*alpha(MW))/(SQRT(2)*GF*sin2(theta_bar)

which is very similar to the relation without radiative corrections. This leads to the following table:

Source	sin2(theta_bar)	MW	1 - (MW/MZ)**2
LEP	0.2302+- 0.0021	80.13+-0.37	0.2276+-0.0070
CDF	0.228 +- 0.016	80.52+-2.83	0.220 +-0.055
CDF	----------	79.91+-0.39	0.2319+-0.0075

I have used alpha(MW) = 1/128.85 here, ignoring any error in alpha. This is certainly OK as far as the new CDF measurement is concerned, since the error on alpha is probably less than 0.4%. The LEP numbers, also taken from Chris Hill's note, are shown for comparison to our own. The last column is obtained using MZ = 91.177 GeV/c**2

So if one wanted to use our new sin2(theta_bar) = 0.228+-0.016 measurement to derive MW for comparison with our MW measurement, the correct statement would be:

The measurement of sin2(theta_bar) = 0.228+-0.016 implies a W mass of 80.5+-2.8 GeV/c**2, to be compared with our previous measurement MW = 79.91+-0.39 GeV/c**2. The measurements are clearly consistent.

I have two comments (opinions) about this:

1) Given the size of the error bar on MW, this comparison seems rather uninteresting: It belongs to the time before accurate W and Z masses were known.
2) The comparison, if done, should be in terms of MW, not in terms of 1 - (MW/MZ)**2, since this quantity, especially when labeled sin2(theta_Sirlin) or similar, is known to generate confusion with other sin2(theta).
   There is another important reason why it is better to make the comparison in terms of MW. To understand this point, look at Fig. 3.3 from the CDF upgrade proposal, which is included in this note. You will see that sin2(theta_bar) is weakly dependent on both MTOP and M(Higgs): Once you know MZ accurately (from LEP), and you calculate within the Minimal Standard Model, sin2(theta_bar) can be predicted within a narrow range, 0.228 < sin2(theta_bar) < 0.236 according to the figure,

if you restrict MTOP to be below 230 GeV/c**2. This range is substantially smaller than the range allowed by this measurement: sin2(theta_bar) = 0.228 +- 0.016. When comparing this measurement with MW measurements, it is therefore important to realize that:

a) To display the uncertainty from this measurement alone, one has to avoid using the implications of the LEP MZ measurement, directly or indirectly.
b) If MZ(LEP) is used, the uncertainty will be substantially reduced, but it no longer reflects the result of this measurement alone.

As shown above, one can cleanly compare to the CDF MW measurement. If one floats MZ, however, how exactly does one get from the CDF MW to 1 - (MW/MZ)**2 ?

SO WHAT IS WRONG WITH THE COMPARISON IN THE DRAFT PRL?

The draft contains a figure showing "the value of sin2(theta_Sirlin) (= 1-(MW/MZ)**2) derived from the asymmetry as a function of the top quark mass" plus corresponding text. Our MW measurement, in the guise of 1 - (MW/MZ)**2 with MW from CDF and MZ from LEP, is shown on the right of the figure to allow the comparison.

This way of comparing is WRONG because it does not obtain the correct result, MW = 80.5+-2.8 GeV/c**2, which was obtained above (in terms of 1 - (MW/MZ)**2, the correct result is 0.220+-0.055, where the central value uses MZ=91.177, while the size of the error bar is rather insensitive to the MZ value).

It is also highly MISLEADING: The construct "the value of MW derived from the asymmetry as a function of the top quark mass" says that you need MTOP in addition to sin2(theta_bar) to get MW, whereas we know that sin2(theta_bar) alone (together with alpha(MW), GF and MZ) is enough! There are only 4 independent parameters! (ignoring the weak Higgs mass dependence). Identical statements hold when MW is replaced by 1-(MW/MZ)**2.

Remark to the alert reader of the draft:
To obtain the functional MTOP dependence in the figure, one has to replace either MZ or GF by MTOP as one of the four independent input parameters. This means that a measured, fixed constant (MZ or GF as the case might be) is treated as a dependent, multivalued variable (i.e. the important constraint of fixed MZ is ignored). Using the approximation MZ = 38.45 GeV/SQRT(sin2(theta_bar)*cos2(theta_bar)), for instance, I calculate approximate values of MZ = 89.5 GeV, 91.6 GeV and 94.1 GeV for the three curves in the figure representing the CDF result (i.e. for sin2(theta_bar) of 0.244, 0.228 and 0.212, respectively).

Finally, I think, as stated above, that it would be much easier to express and understand if the comparison were done with MW rather than with 1 - (MW/MZ)**2.

CONCLUSION

I conclude that this figure plus the associated text must be removed from the paper. It is fortunate that this can be done easily: This comparison to MW has nothing do do with the sin2(theta_bar) measurement itself!

The correesponding plot must of course also be removed from the file of "blessed" CDF plots.

FINAL COMMENT

Let me conclude by advocating what I think is the nicest way for CDF to show comparisons of EWK variables: That is to use the plot of MW versus MTOP, both of which are quantities that we aim to measure. This plot can be used to show:

1) The calculated dependence MW = MW(alpha(MW),GF,MZ,MTOP)

2) Our (and other) MW measurements as horizontal bands, see for instance Fig. 3-1 in the CDF upgrade proposal.

3) The LEP measurement of sin2(theta_bar), converted to a predicted MW as shown above, as a horizontal band

4) Any and all other relevant measurements, such as nu-e scattering, again converted to a horizontal band via the relation

   MW = MW(alpha(MW),GF,MZ,new measurement)

Note: To deal with precision experiments, one will have to include the Higgs mass dependence.

Figure 3-3 from the Upgrade Proposal

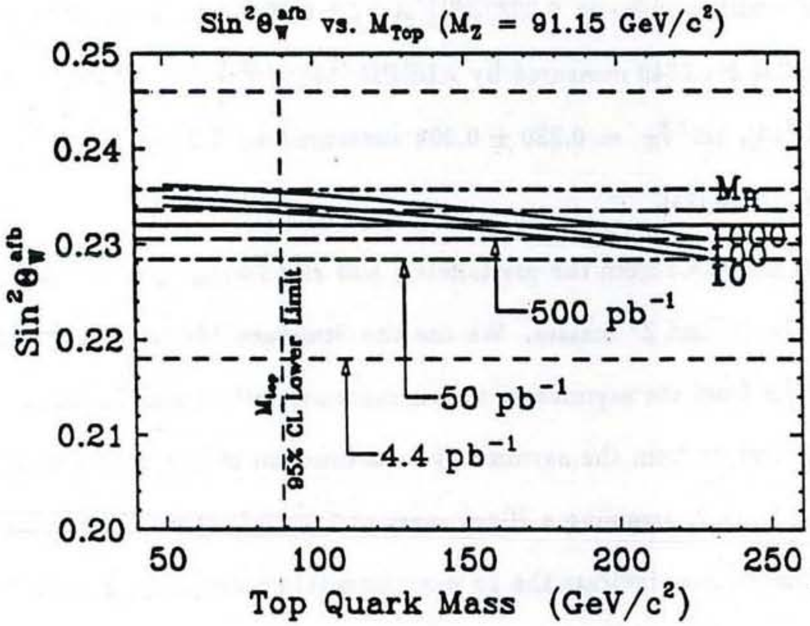

Figure 3-3: Predicted M_{top} dependence of $\sin^2\theta_W^{a/b}$ from the combined results of forward-backward asymmetry measurements in $Z \rightarrow e^+e^-$ and $Z \rightarrow \mu^+\mu^-$. Uncertainties are given for several values of integrated luminosity. Example given is for current CDF measurement.

Our final result, $\sin^2\bar{\theta}_W = 0.228^{+0.017}_{-0.015}(\text{stat}) \pm 0.002(\text{sys})$, is in good agreement with $\sin^2\bar{\theta}_W = 0.2291 \pm 0.0040$ measured by ALEPH [14], $\sin^2\bar{\theta}_W = 0.2309 \pm 0.0048$ measured by DELPHI [15], $\sin^2\bar{\theta}_W = 0.230 \pm 0.004$ measured by L3 [16], $\sin^2\bar{\theta}_W = 0.233^{+0.007}_{-0.006}$ measured by OPAL [17].

$\sin^2\bar{\theta}_W$ is measured from the asymmetry, and $\sin^2\theta_W|_{Sirlin} = 1 - M_W^2/M_Z^2$ [18] from the ratio of the W and Z^0 masses. We use the Standard Model to convert our measurement of $\sin^2\bar{\theta}_W$ from the asymmetry into a measurement of $\sin^2\theta_W|_{Sirlin}$ [19]. A plot of $\sin^2\theta_W|_{Sirlin}$ derived from the asymmetry as a function of the top mass is shown by the solid line in Figure 2, assuming a Higgs mass of 250 GeV (the Higgs mass dependence is small); the dashed lines indicate the 1σ experimental uncertainty. Figure 2 also shows the 1σ confidence region derived from recent Z^0 mass measurements [20] as well as a direct measurement of $1 - M_W^2/M_Z^2 = 0.232 \pm 0.008$[21] determined from the CDF W and LEP Z^0 masses. A_{FB} is not strongly dependent on the top mass; the top mass dependence in Figure 2 comes from higher order corrections in the Standard Model incurred in the conversion from $\sin^2\bar{\theta}_W$ to $\sin^2\theta_W|_{Sirlin}$.

In summary, we have measured $A_{FB} = (5.3 \pm 5.9(\text{stat}) \pm 0.4(\text{sys}))\%$ after background and QCD corrections, and $\sin^2\bar{\theta}_W = 0.228^{+0.017}_{-0.015}(\text{stat}) \pm 0.002(\text{sys})$ after background and radiative corrections. The systematic uncertainties are summarized in Table 1. Our measurement of $\sin^2\bar{\theta}_W$ is consistent, both without and with order α^3 radiative corrections,

with previous measurements of $\sin^2\bar{\theta}_W$ at the Z^0 mass. Our measurement of $\sin^2\theta_W|_{Sirlin}$ from the asymmetry is consistent with measurements of $\sin^2\theta_W|_{Sirlin}$ from the W and Z^0 mass ratio over a broad range of top quark masses.

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THE RELEVANT FIGURE FROM THE APRIL 11, 1991 PRL DRAFT

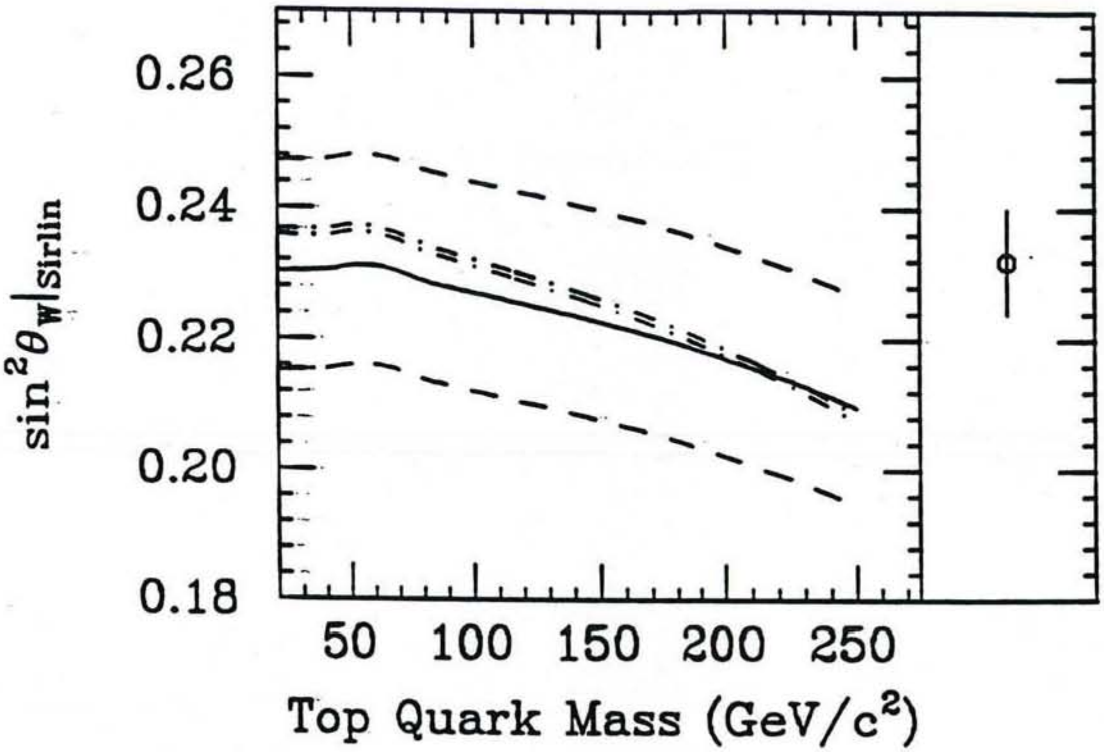

Figure 2: The solid line shows the central value of $\sin^2\theta_W|_{Sirlin}$ derived from the asymmetry as a function of the top quark mass; the dashed lines indicate the 1σ experimental uncertainty. The dot-dashed lines show the 1σ uncertainty on $\sin^2\theta_W|_{Sirlin}$ determined from Z^0 mass measurements. At right is the CDF value from $1 - M_W^2/M_Z^2$ [21] .